STATISTICS 220 **WINTER 2007** **MIDTERM EXAM**

Friday February 2, 2007; 8.30 a.m. - 9.20 a.m.

NAME _____ SECTION: (circle 1); AA AB AC AD AE AF

1.This is a closed-book, closed-notes exam, except that each student
may refer to one (two-sided) $8\frac{1}{2}$ by 11 page of notes (s)he has brought to the exam.
2.Use of a calculator is allowed.
However, in order to receive full credit, all computations must be shown.
3.Students must clearly explain each answer to receive full credit.
4.Students must follow a reasonable code of conduct.

Cheating or other dishonest practices will result in an examination grade of zero. Such practices include, but are not limited to (i) making use of books, papers, or memoranda other than those authorized, (ii) speaking or communicating with other students during the examination, (iii) purposely exposing written papers to the view of other students.

5.If you do not sign the Honor statement below, then your exam score will be zero.

A good strategy: Skim the entire exam.
Then work first the problems with which you feel most comfortable.
Answer as many of the questions as you have time for.
This is a long exam for 50 minutes; don't worry if you cannot complete it all.

Honor statement:
I have followed a reasonable code of conduct in conjunction with this exam, as outlined in point 4 above.

_____
Student signature

Please do not write in this space.

Question number	points	possible points
1		12
2		12
3		12
4		12
Total		48

1. A bypass is to be built around a small town. The State will pay for a surface option, but the Mayor wants a more expensive option in which the bypass is put in a deep cut. She agrees to put the issue to a popular vote. In the town there are 3100 voters of each of three neighborhoods (A,B, and C) and exactly half (4650) are low-income (L), and the others are classified as high-income (H).

(a) (4 points: **no explanations needed**)
As shown in the table, in neighborhoods A, B, and C, among the low-income voters, 60%, 50% and 20% will vote in favor. Among the high-income voters the percentages are 62%, 52% and 22%. Fill in the counts of votes in favor in the following table.

	low-income (L)			high-income (H)			
Neigh-borhood	total votes	% in favor	votes in favor	total votes	% in favor	votes in favor	Total votes
A	3000	60%	**1800**	100	62%	**62**	3100
B	1550	50%	**775**	1550	52%	**806**	3100
C	100	20%	**20**	3000	22%	**660**	3100
Total	4650	—	**2595**	4650	—	**1528**	9300

Deduct only one point total for minor arithmetic errors in above

(b) (4 points: **no explanations needed**)
(i) The percentage of low-income voters in favor is closest to
25% 33% 40% 50% **56%** 62% **(56% is correct)**

(ii) The percentage of high-income voters in favor is closest to
25% **33%** 40% 50% 56% 62% **(33% is correct)**

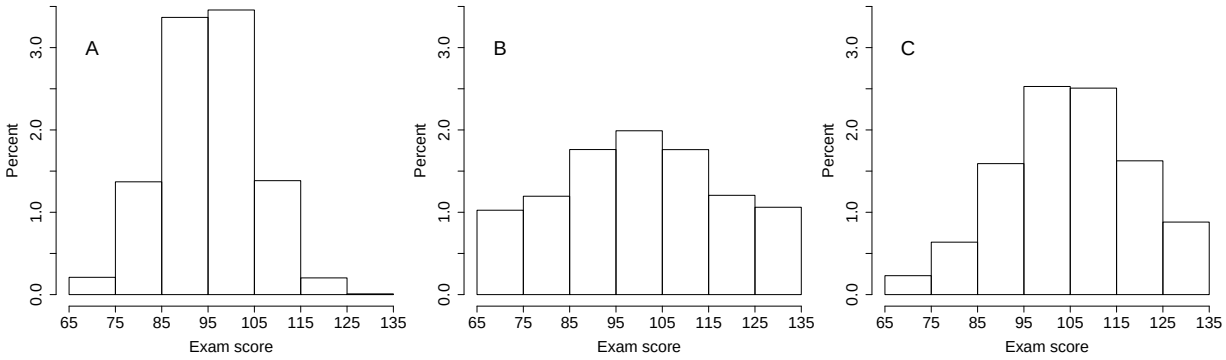
(iii) Does the initiative pass ? YES **NO** (NO is correct)

(c) (4 points) The Mayor says: "The majority of low-income voters are in favor. Also, in every neighborhood, high-income voters are going to be more willing than the low-income voters to pay for my deep-cut bypass option. So my initiative is certain to pass."
Is she right? **Explain why or why not.**

No, she is not right, as the numbers show.
EITHER: An example of Simpson's paradox. Within each neighborhood the high-income people are more in favor, but because of the association between neighborhood and income, overall the low-income are more in favor than the high-income.
OR: Neighborhood is a confounding factor. It is associated both with whether people are in favor and with whether they are high or low-income.

2. In a certain country, all school-children take a standardized nationwide exam at age 13 and at age 16. This question concerns the exam at age 13. Nationally, the exam scores are scaled so that the mean is 100, the SD is 15, and the scores are normally distributed. The pass mark is 85, so nationwide 16% of students fail the exam. Students scoring over 125 are classified as "gifted".

The following show the histograms of scores in three large school districts, Avondale (A), Burnaby (B) and Chester (C). The number of students in each district is about the same. The scale on the vertical axis is percent per unit exam score.



(a) (**no explanations needed**)
(i) (2 points) The median scores in the above three school districts are 105, 100 and 95. Which is which?

Avondale **95**	Burnaby **100**	Chester 105

(ii) (2 points) Which school district has the percentage of failing students equal to the nationwide average (16%).

Avondale (*Burnaby is higher, Chester is lower*)

(iii) (2 points) Which school district has the largest percentage of "gifted" students? About how large is this percentage?

Burnaby. Just about 10%

2 (b) (6 points) **Explain your answers**
Politicians argue about how to distribute funds to schools to bring about improvements.
Congressman Fred wants to distribute funds among schools with low median scores, while Senator Gina proposes a per-failing-student amount distributed to all school districts.
Secretary Hank wants to give funds to schools with high median scores, to develop their college prep classes, while Governor Juan proposes this should be a per-"gifted"-student amount distributed to all school districts.
Compare briefly how much funding Avondale, Burnaby and Chester will get, under the proposals of politicians Fred, Gina, Hank, and Juan.

Avondale has a low median, but only average numbers of failing students, and almost no "gifted" students. It will get funds under Fred's proposal, but not under the others.

Burnaby has high percentage of both failing and "gifted" students, but has a median right on the national average. It will do well with Gina's and Juan's proposals, but not Fred's and Hank's.

Chester has a high median and quite a few "gifted" students (not quite as many as Burnaby). It will do well under Hank's and Juan's proposals, but not Gina's or Fred's.

OR: it can be written the other way round ...
Fred's proposal is good for Avondale, because ...
Gina's is good for Burnaby, because ...
Hank's is good for Chester, because ..., and
Juan's is good for Burnaby and Chester.

3. (**Show your work**) To operate a certain machine in a factory, safety standards dictate that the machine operator be at least 65 inches tall, and no more than 73 inches tall. In the population of factory employees, the women have mean height 67 inches, and SD 2 inches, and the men have mean height 70 inches and SD 3 inches. In each population (men and women) height is approximately normally distributed.

(a) (2 points) What percentage of the women are too small to meet the safety standard to operate the machine?

For women: 65" is 1 SD below the mean. Middle area = 68% from table.
So half of 32%, that is 16% are too small.

(b) (2 points) What percentage of the men are too tall to meet the safety standard?

For men: 73" is 1 SD above the mean. Middle area = 68% from table.
So half of 32%, that is 16% are too tall.

(c) (3 points) Anna is at the 96 th. percentile of women's heights. Does she meet the safety standard?

We need 4% area at each end, and so 92% in middle.
From table, z-score = 1.75, so she is $67 + 2 \times 1.75 \ = \ 70.5$**" tall.**
So yes, she meets the safety standard.

(d) (2 points) Carl is at the 4 th. percentile of men's heights. Does he meet the safety standard?

Again, the middle area is 92%, so z-score = - 1.75
He is $70 - 1.75 \times 3 \ = \ 64.75$**" tall.**
So he just fails to meet the safety standard.

(e) (3 points: **Explain your answer.**) Do a higher percentage of the men or of the women in this population meet the safety standard? (Or is it the same percentage?)

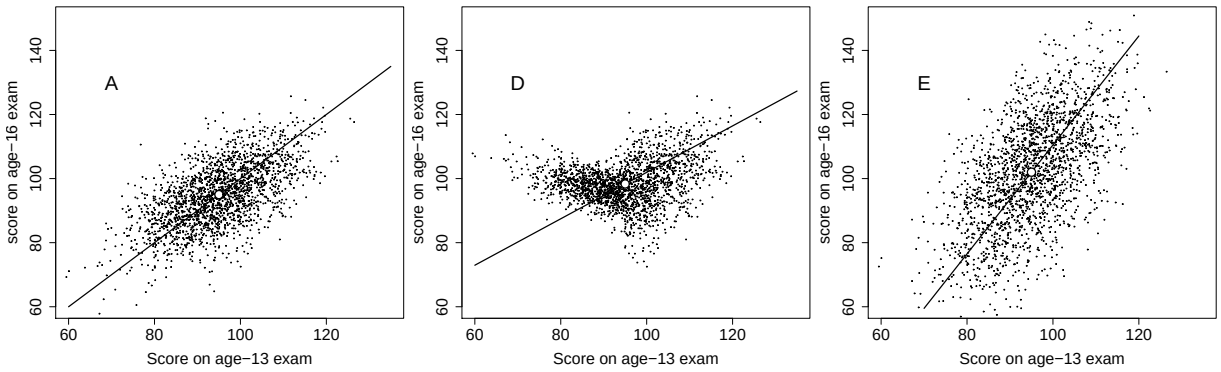
More men than women fail to meet the safety standard.
From (a) and (b) we see the same percent of women are too small as men too tall.
From (c) and (d) we see a higher percentage of men are too small than women too tall.

Saying only that men's SD is larger, so heights more spread gets 2 points – that is true, but depending on the means might give either a higher percentage of men or of women failing.

4. Two school districts, Dundee(D) and Ephrata(E), have age-13 exam scores very similar to Avondale (A) in question 2. Because the politicians want to study funding issues further, they implement a pilot initiative under which Avondale does not receive extra funding, but both Dundee and Ephrata do. Dundee focuses all its extra funding on classes only for students who scored below the district average on the age-13 exam Ephrata spreads its funding to improve classes for all students aged 13 to 16 in the district.

For the students who are 13 immediately before the new funding initiative starts, the three scatter plots below below show the scores of students on the age-13 exam (horizontal axis) and of the same students 3 years later on their age-16 exam (vertical axis). To make it easier to see, there are only 2000 points on each plot. The point of averages is shown as a blank spot in the center of each scatter plot, and the solid line is the SD-line.

Avondale's age-16 exam scores are very similar to the scores for all three school districts in the years before the new funding initiative.



(a) (3 points) **No explanation needed**
The three school districts have the same SD for their age-13 exam scores. For the age-16 exam scores, the three SD's are about 7, about 10, and about 15. Which is which?

Avondale **10**	Dundee **7**	Ephrata **15**

Parts (b) and (c) of this question are on the next page.

(b) (5 points) Dundee's School Superintendent says:
(i) "We have greatly decreased the percentage students failing the age-16 exam to way below the national average. However our mean value at age-16 is still below the national average of 100. How can this be?
(ii) Also, the correlation coefficient between the age-13 and age-16 scores has gone way down. This must mean that our good students are now performing worse than they were before the new funding initiative."

(i) Explain how his comment (i) can be so.

They have brought the "lower half" way up, but not above the national mean. Their mean is higher than Avondale's but not much. The SD is way down, so the percentage of students scoring below 85 is small
(*85 is in fact about 2 SD below the mean for the age-16 exam, but the distribution is not very normal anyhow.*)

(ii) It is true that the correlation coefficient has gone way down. Is the Superintendent correct in his conclusion? Why/Why not?

No the above-average students are performing exactly as before.
(In fact these are exact same points as Avoldale's to make this clear.)
The plot is a mixture of the two groups of students – below and above the age-13 exam mean. The relationship is not linear. The correlation coefficient is no longer a meaningful summary of the association.

(c) (4 points) Ephrata's School Superintendent says:
(i) "Our mean value is now above the national average. However, we have exactly the same percentage of students failing the age-16 exam as before. How can this be?
(ii) The statisticians tell me that the correlation coefficient between our between age-13 and age-16 exam scores is exactly as before. However, when I look at the scatter plot, the points appear less clustered about the SD line. How can this be?"

(i) Explain how his comment (i) can be so.

Ephrata has a football-shaped plot, but the age-16 SD has gone way up.
The spread of age-16 scores is large so even though the mean is above 100, the percent failing is the same.

In fact, the pass mark is about 1 SD below the mean, for both Avondale and Ephrata, at both age-13 and age-16.

(ii) It is true that the correlation coefficient is unchanged. How could the statisticians provide scatter plots that might convince the Superintendent that the correlation coefficients in plot A and in plot E are the same?

The points look less clustered on the Ephrata plot because the scores on the age-16 exam are more spread out. If the statistician plotted the standardized scores, standardized for each variable on each plot, then the Avondale and Ephrata plots would look just the same.