Two-Higgs model: problem 2.4 of BM

The two Higgs doublets are

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad \psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix} \tag{1}$$

Both are doublets under $\mathrm{SU}(2)_L$ and have hypercharge $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively.

The most general Higgs potential is a linear combination of

$$a, b, \mathrm{Re}\, c, \mathrm{Im}\, c, a^2, b^2, (\mathrm{Re}\, c)^2, (\mathrm{Im}\, c)^2, ab, a\,\mathrm{Re}\, c, a\,\mathrm{Im}\, c, b\,\mathrm{Re}\, c, b\,\mathrm{Im}\, c, \mathrm{Re}\, c\,\mathrm{Im}\, c \tag{2}$$

$d = \phi^\dagger \psi$ has hypercharge -1 and hence is not invariant under the Standard Model gauge group.

The covariant derivatives are

$$D_\mu \phi = \left(\partial_\mu - igA^a_\mu \frac{\tau^a}{2} - i\frac{g'}{2} B_\mu \right) \phi \tag{3}$$

$$D_\mu \psi = \left(\partial_\mu - igA^a_\mu \frac{\tau^a}{2} + i\frac{g'}{2} B_\mu \right) \psi \tag{4}$$

We expand around the vacuum state

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{v}{\sqrt{2}} + H \end{pmatrix}, \qquad \psi = \begin{pmatrix} \frac{u+iw}{\sqrt{2}} + \tilde{\chi} \\ \xi \end{pmatrix} \tag{5}$$

The scalar kinetic terms become:

$$|D_\mu \phi|^2 + |D\psi|^2 = \left| \partial_\mu \phi^+ - \frac{i}{2} gvW^+_\mu \right|^2 + \left| \partial_\mu H + \frac{i}{2\sqrt{2}} (gA^3_\mu - g'B_\mu) v \right|^2$$
$$+ \left| \partial_\mu \tilde{\chi} - \frac{i}{2\sqrt{2}} (gA^2_\mu - g'B_\mu)(u+iw) \right|^2 + \left| \partial_\mu \xi - \frac{i}{2} g(u+iw) W^-_\mu \right|^2 \tag{6}$$

We now impose the following gauge-fixing conditions:

$$v\phi^+ - (u+iw)\xi^* = 0 \tag{7}$$

$$-v(H - H^*) + (u-iw)\chi - (u+iw)\chi^* = 0 \tag{8}$$

We can impose these conditions because we have freedom to perform gauge transformations on the Higgs fields. The number of independent small gauge transformations is 4 (the number of generators of the SU(2)×U(1) group), which is equal to the number of gauge-fixing conditions in Eq. (7). After the conditions (7) are imposed, the gauge fields and the scalar fields are not mixed. The mass terms for the gauge fields is then

$$\frac{g^2}{4}(v^2 + u^2 + w^2) + \frac{1}{8}(v^2 + u^2 + w^2)(gA^3_\mu - g'B_\mu)^2 \tag{9}$$

The masses of the W and Z bosons therefore are given by the same formulas as in the Standard Model with one Higgs doublet, with the replacement of $v^2 \to v^2 + u^2 + w^2$. In particular, $M_W = M_Z \cos\theta$.

In general, one can have the following mass terms:

$$(\bar{e}_L\phi)e_R + \text{h.c.}, (\bar{u}_L\psi)u_R + \text{h.c.}, (\bar{d}_L\phi)d_R + \text{h.c.}, \tag{10}$$

$$(\bar{e}_L\tilde{\psi})e_R + \text{h.c.}, (\bar{u}_L\tilde{\phi})u_R + \text{h.c.}, (\bar{d}_L\tilde{\psi})d_R + \text{h.c.} \tag{11}$$

but if one requires an additional symmetry where e_R, u_R, d_R are rotated by $e^{i\theta}$ and ϕ, ψ are rotated by θ, then only terms on the first line are allowed. This symmetry also eliminates many terms in the Higgs potentials: c may appears only in the form of c^*c