

This is a self-test of the prerequisite math skills assumed for students entering the regular undergraduate physical chemistry sequence. I know sometimes you can get a little rusty over the summer so I've included it here.

This exercise consists of two "self-tests". Questions appear on one side of the sheet and the answers are on the back. Please try all of the problems in the first self-test before looking at the answers and recording your score below. If you get everything right on the first self-test, then there is no need to do the second one. Most students improve dramatically by the second time through.

Name: _____

Did you get all questions 1A-18A correct on the first try? (circle an answer) YES / NO

Mark an "X" through those questions you got wrong the first time you attempted them.

1A 2A 3A 4A 5A 6A 7A 8A 9A 10A 11A 12A 13A 14A 15A 16A 17A 18A

1B 2B 3B 4B 5B 6B 7B 8B 9B 10B 11B 12B 13B 14B 15B 16B 17B 18B

MATHEMATICS SELF-TUTORIAL, CHEM455

Evaluate and/or simplify the expressions without a calculator. While you are allowed to use a calculator on the homeworks and exams, most students who complain that the exams are “too long” also lose time on “remedial” topics.

Do not convert fractions to decimals. Complete this page entirely before turning to the next page.

QUESTION SET A

- 1A) use the quadratic equation to solve $2x^2 - x + 8 = 0$ for x
- 2A) if $y = (5+2i)$, what is $(y^*)(y)$
- 3A) express $(2+3i) / (4-2i)$ in the form $a+bi$
- 4A) $\int_0^{\pi} \sin(x) = ?$
- 5A) $(a^2bc^{-1})(ab^{-3}) = ?$
- 6A) $lil = ?$
- 7A) $|2-7il = ?$
- 8A) express $3+3i$ in $re^{i\theta}$ form
- 9A) express $2e^{(i\pi/3)}$ in $a+bi$ form
- 10A) Dimensional analysis: What are the units for angular momentum? Show that Planck's constant has units of angular momentum.
- 11A) For values of x which are close to 1, $\ln(x) \approx x - 1$. This problem uses a Taylor expansion to show that this is true. Use a Taylor Series to approximate $\ln(1.05) = ?$ to first order.
- 12A) $f(x) = (x^{4/3} - 2x^{-2})^5$ $df/dx = ?$ (no need to multiply out)
- 13A) $f(x) = (1 - x^4)/x$ $df/dx = ?$ (no need to multiply out)
- 14A) simplify $e^{2\ln 3}$
- 15A) find the maximum and minimum of the equation $f(x) = x^3 - 27x$ over the range $x=-5..5$
- 16A) $\int_{x=1}^3 2x^2 dx = ?$
- 17A) $\frac{\partial}{\partial x} \left(2x^4 y^6 + \frac{3}{x^2} \right) = ?$
- 18A) For objects with a radius of 2 meters, what are the a) circle's perimeter, b) circle's area, c) sphere's surface area, and d) sphere's volume? (express as multiples of π . Include units!)

Grade questions 1A-17A. Make sure that you understand the answers before turning to the next page.

<u>ANSWER SET A</u>	<u>REASONS</u>
1A) $\frac{1+3i\sqrt{7}}{4}, \frac{1-3i\sqrt{7}}{4}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2A) 29	$(a+bi)(a-bi) = a^2 + b^2$
3A) $\frac{1}{10} + \frac{4}{5}i$	$\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \left(\frac{c-di}{c-di}\right)$
4A) 2	$\int \sin(x) = -\cos(x)$
5A) $a^3b^{-2}c^{-1}$	$a^x a^y = a^{x+y}$
6A) 1	$ y = \sqrt{y^* y}$
7A) $\sqrt{53}$	$ y = \sqrt{y^* y}$
8A) $\sqrt{18}e^{i\pi/4}$	$a+bi = a+bi e^{i \operatorname{atan}(b/a)}$
9A) $1 + i\sqrt{3}$	$e^{i\theta} = \cos(\theta) + i \sin(\theta)$
10A) $\text{kg}\cdot\text{m}^2\cdot\text{s}^{-1}$	$\text{J}\cdot\text{s} = \text{N}\cdot\text{m}\cdot\text{s} = \text{kg}\cdot\text{m}\cdot\text{s}^{-2}\cdot\text{m}\cdot\text{s} = \text{kg}\cdot\text{m}^2\cdot\text{s}^{-1}$
11A) $\ln(x) = \ln(a) + (x-a)(1/a)$ $\ln(1.05) = 0 + (1.05 - 1)(1/1) = 0.05$	Taylor Series: $f(x) = f(a) + (x-a)f'(a) + (1/2!)(x-a)^2 f''(a) \dots$ To first order, for $x \approx 1$, set $a=1$ so that $\ln x \approx (x - 1)$
12A) $5(x^{4/3} - 2x^{-2})^4 (4/3x^{1/3} + 4x^{-3})$	for $f(x) = cx^n$, $df/dx = ncx^{n-1}$, where c is a constant
13A) $-4x^3(1/x) + (1-x^4)(-1/x^2)$	$d/dx(g(x)\cdot h(x)) = (d/dx g(x))(h(x)) + (g(x))(d/dx h(x))$ you will make fewer mistakes with the product rule
14A) $(e^{\ln 3})^2 = (3)^2 = 9$	$e^{\ln x} = x$
15A) $x = 3$ is min, $x = -3$ is max	min occurs at $df/dx = 0$ and $d^2f/dx^2 > 0$ max occurs at $df/dx = 0$ and $d^2f/dx^2 < 0$
16A) $\frac{2}{3}x^3 \Big _{x=1}^3 = \frac{2}{3}(27-1) = \frac{52}{3}$	$\int ax^n dx = \left(\frac{1}{n+1}\right) ax^{n+1}$ for $n \neq -1$
17A) $8x^3y^6 - 6/x^3$	Partial derivatives treat all other variables as constants.
18A) 4π meters, 4π (meters) ² 16π (meters) ² , $32/3 \pi$ (meters) ³	perimeter of circle = $2\pi r$, area of circle = πr^2 (hint: perimeter = $\frac{\partial}{\partial r}$ (area)) surface area of sphere = $4\pi r^2$, volume of sphere = $4/3 \pi r^3$ (hint: surface area = $\frac{\partial}{\partial r}$ (volume))

If you feel comfortable with questions 1A-18A and answered them all correctly on the first try, you may skip questions 1B-18B. If you want more practice or missed any of the previous questions, complete this page of questions. Complete this page entirely before turning to the next page, which contains the answers.

QUESTION SET B

- 1B) use the quadratic equation to solve $x^2 + x + 1 = 0$ for x
- 2B) if $y = (3-i)$, what is $(y^*)(y)$
- 3B) express $(5+i) / (1-9i)$ in the form $a+bi$
- 4B) $\int_0^{2\pi} \sin(x) = ?$
- 5B) $2^3 \cdot 4 = 2^?$
- 6B) $|1+il| = ?$
- 7B) $|3-il| = ?$
- 8B) express $3+4i$ in $re^{i\theta}$ form
- 9B) express $3e^{(i\pi/4)}$ in $a+bi$ form
- 10B) Dimensional analysis: Show that Planck's constant times the speed of light divided by wavelength has units of energy.
- 11B) For small values of θ , $\sin(\theta) \approx ?$
- 12B) $f(x) = (5x^3 + x^{2/3})^4$ $df/dx = ?$ (no need to multiply out)
- 13B) $f(x) = (1+x)/(x^2-2)$ $df/dx = ?$ (no need to multiply out)
- 14B) simplify $2e^{3\ln 2}$
- 15B) find the maximum and minimum of the equation $f(x) = x^3 - 48x$ over the range $x=-5..+5$
- 16B) $\int_{x=1}^2 2x^3 dx = ?$
- 17B) $\partial/\partial t (8st^{-2} - 3s^{-3}t^4)$
- 18B) For objects with a radius of 3 meters, what are the a) circle's perimeter, b) circle's area, c) sphere's surface area, and d) sphere's volume? (express as multiples of π . Include units!)

ANSWER SET B

If you need more practice with differentiation and integrals, check out the worked problems at:
<http://www.math.ucdavis.edu/~kouba/ProblemsList.html>

$$1B) \quad \frac{-1}{2} + \frac{i\sqrt{3}}{2}, \frac{-1}{2} - \frac{i\sqrt{3}}{2}$$

$$2B) \quad 10$$

$$3B) \quad \frac{-2 + 23i}{41}$$

$$4B) \quad 0$$

$$5B) \quad 5$$

$$6B) \quad \sqrt{2}$$

$$7B) \quad \sqrt{10}$$

$$8B) \quad 5e^{\operatorname{atan}(4/3)}$$

$$9B) \quad \frac{3\sqrt{2}}{2} + i \frac{3\sqrt{2}}{2}$$

$$10B) \quad \text{J} \cdot \text{s} \cdot \text{m} \cdot \text{s}^{-1} \cdot \text{m}^{-1} = \text{J}$$

$$11B) \quad \theta$$

$$12B) \quad 4(5x^3 + x^{2/3})^3 (15x^2 + 2/3x^{-1/3})$$

$$13B) \quad \frac{1}{x^2 - 2} - \frac{2x(1+x)}{(x^2 - 2)^2}$$

$$14B) \quad 2(e^{\ln 2})^3 = 2 \cdot 2^3 = 2^4 = 16$$

$$15B) \quad x = 4 \text{ is min, } x = -4 \text{ is max}$$

$$16B) \quad \frac{15}{2}$$

$$17B) \quad -16st^{-3} - 12s^{-3}t^3$$

$$18B) \quad 6\pi \text{ meters, } 9\pi \text{ (meters)}^2, 36\pi \text{ (meters)}^2, 36\pi \text{ (meters)}^3$$