Homework 4. Due Friday Oct 21 at 5pm in Prof. Ginger’s mailbox.
CIRCLE YOUR ANSWERS AND KEY INTERMEDIATE RESULTS
USE MAPLE WHENEVER POSSIBLE
STAPLE YOUR PAPERS TOGETHER
INCLUDE ALL COMPUTER PRINTOUTS (with commentary)

You will note that this problem set is shorter than usual because of the reduced time this week.

Levine Problems
4.16 most likely particle position
4.18 3D SHO
4.27 vibrations in LiH and ICl

Additional Problems
1) In lecture we introduced the lowering operator as 
\[ a^- = \frac{1}{\sqrt{2\hbar \omega}} (i\hbar \omega x + \mu \omega) \]
By arguing that energy can’t be negative, we reasoned that there must be some state \[ |\psi_0> \]
for which \[ a^- |\psi_0> = 0 \] Use this relationship to generate a first-order differential equation
for \[ \psi_0 \] and solve the differential equation (it should be a “simple” first-order equation) to
verify that \[ \psi_0 \] is the same as obtained with the power series solution.

2) Find \[ \psi_1, \] the first excited state of the SHO, by explicitly applying the raising operator,
\[ a^+ \] to \[ \psi_0 \]

3) The frequencies of the three normal modes of H2O are \( \omega_1=3833 \text{ cm}^{-1} \) \( \omega_2=1649 \text{ cm}^{-1} \) \( \) and \( \omega_3=3943 \text{ cm}^{-1} \). If we describe a vibrationally excited state by the notation \( (n_1n_2n_3) \) where \( n_i \) is the quantum number associated with the \( i^{th} \) normal mode, what is the energy
of the (121) state? What is the energy difference between the (112) and (010) state?

4) The spatial Parity operator \( P \) satisfies the eigenvalue equation: 
\[ P \psi = p \psi \]
where the eigenvalues of \( P \) are \( p=+1 \) (if \( \psi \) is even) and \( p=-1 \) (if \( \psi \) is odd) . Only even and odd
functions are eigenfunctions of \( P \). The symmetry of the Hamiltonian has important
consequences for the symmetry of the allowed wave functions which we examine below.

4a) Show that if the Hamiltonian is a symmetric (even) function, then \( [P,H]=0 \) (hint: what
is the parity of an even function times an even function, or an odd function times an odd
function, i.e. \( P(f_1f_2)=? ) \)

4b) Two operators will commute if and only if they have a simultaneous set of
eigenfunctions. Use this fact, and your result from a) to justify that statement that “for a
symmetric Hamiltonian, the only allowed energy eigenfunctions will be even and odd
functions.” The analysis of the spatial symmetry of a wavefunction is a very powerful
tool that allows you to predict properties (i.e. IR and Raman activity of vibrational
modes).