C455A - Homework 5
Due Mon May 5 at **5PM**
Circle your answers. Submit your problems in order. ***Staple your work together*.**

Text Problems

Concept questions "Q" need only short (a few sentences max) answers

Q6.11 – concepts on Stern-Gerlach experiment
Q7.1 - concepts on SHO wave functions
Q7.2 – concepts on 2D rotation
Q7.13 – spherical harmonics and p- and d- wave functions

Problems
P7.2 – Compute commutators for angular momentum. Also answer, *what does this tell you about our ability to measure projections of angular momentum about two separate axes simultaneously?*

P7.24 – populations of vibrational and rotational states at room temperature.

Additional Problems
1) Given that $\hat{S}_y\alpha = i\hbar\beta/2$ and $\hat{S}_y\beta = -i\hbar\alpha/2$, construct and verify eigenfunctions (Y^+ and Y^-) for the Sy operator (measure spin component along y axis) operators.

2) Check that the spherical harmonics are solutions to the 'particle on a sphere' problem for (l=0, m=0), and (l=1,m=1) by substituting them into the time-independent Schrödinger equation for the particle on a sphere. Verify that the energies in each case are given by $E = \frac{\hbar^2}{2I} l(l+1)$

3) Show that the spherical harmonics are eigenfunctions of L^2 and L_z for both the (l=0, m=0) and (l=1,m=1) cases. Show that the eigenvalues are $\hbar^2 l(l+1)$, and $m\hbar$ in each case. (e.g. verify eqn 7.26 and 7.15 for the specific values of l and m given above)

4) When we worked the particle in the 3D box in lecture, we encountered degenerate energy levels for the first time. The particle on a sphere is a similar "high symmetry" problem, so we should again expect to encounter degeneracy.
a) Make an energy level diagram for the spherical harmonics as a function of m and l for l=0 to l=3. What is the degeneracy of each level?
b) The electromagnetic dipole selection rule for rotational transitions requires that $\Delta l = \pm 1$. For the case of transitions between the rotational states of the rigid rotor, does the $\Delta l = +1$ case correspond to absorption or emission of a photon?
c) Make a sketch (plotting absorbance vs. wavenumber) of what a pure rotational spectrum of a gas at room temperature might look like.

5) The l=0 to l=1 transition for carbon monoxide ($^{12}C^{16}O$) occurs at a microwave frequency of 1.153×10^5 MHz. Using the rigid rotor approximation and this information, calculate the bond length of carbon monoxide.