

Text Problems

Note: Concept questions “Q” need only short (a few sentences max) answers

Q6.11 – concepts on Stern-Gerlach experiment

Q7.1 - concepts on SHO wave functions

Q7.2 – concepts on 2D rotation

Q7.13 – spherical harmonics and p- and d- wave functions

P7.2 – Compute commutators for angular momentum. Also answer, *what does this tell you about our ability to measure projections of angular momentum about two separate axes simultaneously?*

P7.22 – S.H.O. energies compared to KE of gas

P7.24 – populations of vibrational and rotational states at room temperature.

Additional Problems

1) Construct and verify eigenfunctions (Y^+ and Y^-) for the S_y operator (measure spin along y axis) operators. Given that $\hat{S}_y\alpha = i\hbar\beta/2$ and $\hat{S}_y\beta = -i\hbar\alpha/2$

2) Check that the spherical harmonics are solutions to the ‘particle on a sphere’ problem for ($l=0, m=0$), and ($l=1, m=1$) by substituting them into the time-independent Schrödinger equation for the particle on a sphere. Verify that the energies in each case are given by $E = \frac{\hbar^2}{2I}l(l+1)$

3) Show that the spherical harmonics are eigenfunctions of L^2 and L_z ($l=0, m=0$), and ($l=1, m=1$) cases. Show that the eigenvalues are $\hbar^2l(l+1)$, and $m\hbar$ in each case. (e.g. verify eqn 7.26 and 7.15 for the specific values of l and m given above)

4) When we worked the particle in the 3D box in lecture, we encountered degenerate energy levels for the first time. The particle on a sphere is a similar “high symmetry” problem, so we should again expect to encounter degeneracy.

a) Make an energy level diagram for the spherical harmonics as a function of m and l for $l=0$ to $l=3$. What is the degeneracy of each level?

b) The electromagnetic dipole selection rule for rotational transitions requires that $\Delta l = \pm 1$. For the case of transitions between the rotational states of the rigid rotor, does the $\Delta l = +1$ case correspond to absorption or emission of a photon?

c) Make a sketch (plotting absorbance vs. wavenumber) of what a pure rotational spectrum of a gas at room temperature might look like.

5) The $l=0$ to $l=1$ transition for carbon monoxide ($^{12}\text{C}^{16}\text{O}$) occurs at a microwave frequency of 1.153×10^5 MHz. Using the rigid rotor approximation and this information, calculate the bond length of carbon monoxide.