Feedforward Control: Theory and Applications

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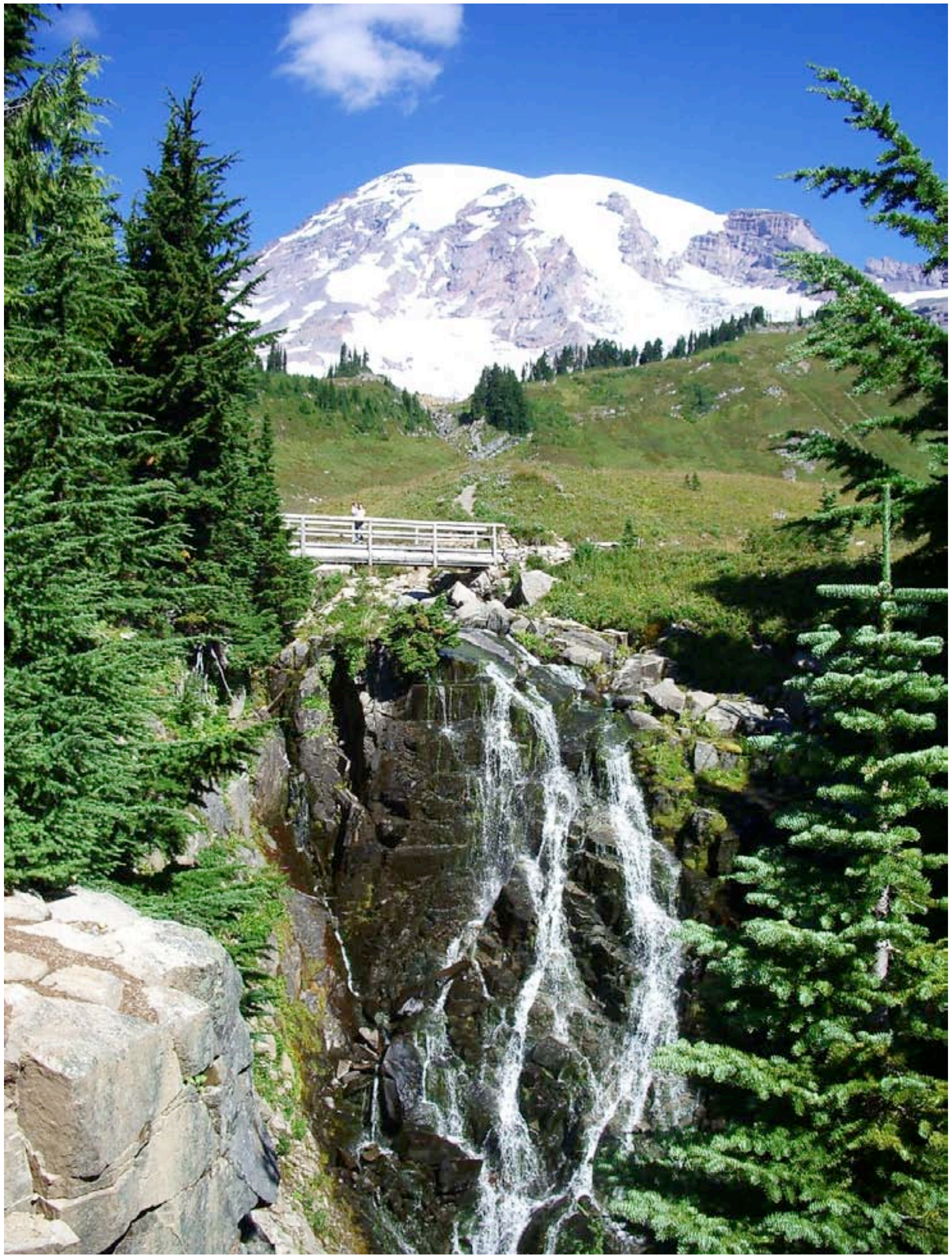
Seattle, WA

Outline of talk

1. **Brief intro to U. of Washington**
2. Motivation --- nanopositioning
3. The good and the bad
4. Approach: Inversion-based feedforward
5. Connections to ZPET, Robust, Optimal
6. Experimental Results
7. The ugly --- unresolved challenges
8. Conclusions

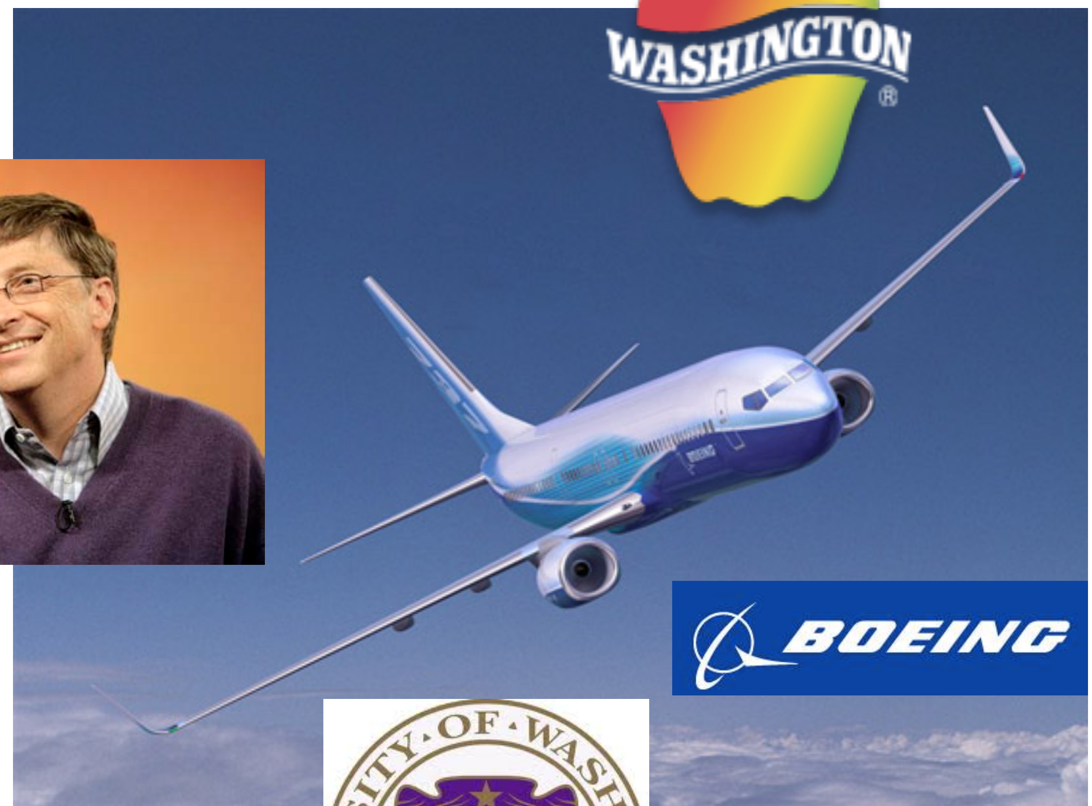
Where is UW (Seattle)?

Seattle is very Scenic



Picture by Szu-Chi Tien

Local Industries



Boeing Commercial Aircraft Division (www.boeing.com)
Microsoft (www.microsoft.com)
Amazon.com (www.amazon.com)
Starbucks (www.starbucks.com)
COSTCO, APPLES, UPS (1907) and UW

University of Washington at a Glance

- Founded in 1861
- 49,000 students (fall of 2010)
- Faculty of nearly 4,000 includes:
  - Six Nobel Prize winners
- Research budget (2010)
  more than US $ 1 billion
- Ranked in top 20 of world universities
  (16th) http://www.arwu.org

- Overall --- a nice to work

- **…and a nice place to visit ☺**

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My Current Research Areas

1. **Air Traffic Control**
   (PhD Student: Jeff Yoo)

Picture from IEEE Control Systems Magazine

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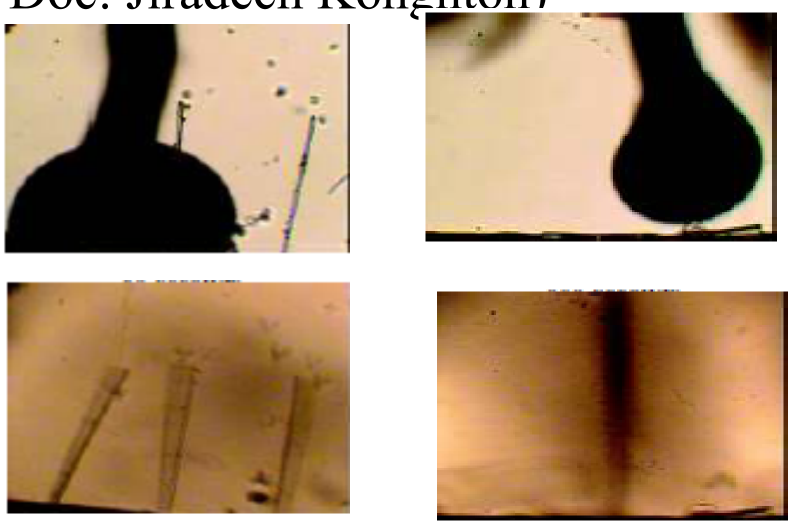
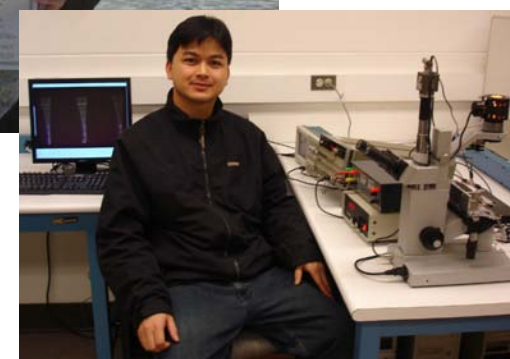
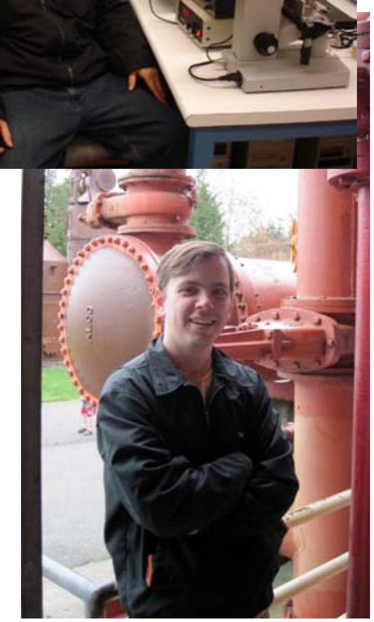


Ink drop and
90 seconds later
With Cilia

Ink drop and
900 seconds later
Without Cilia





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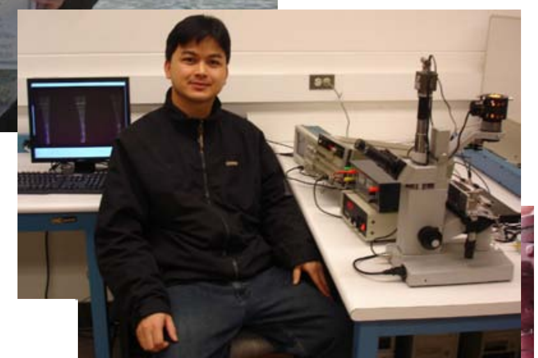

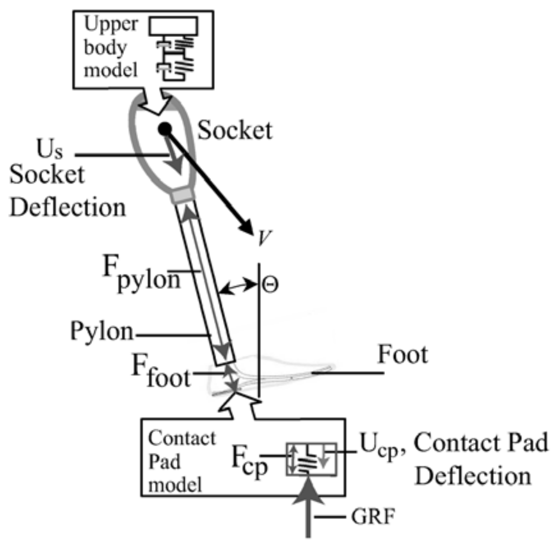

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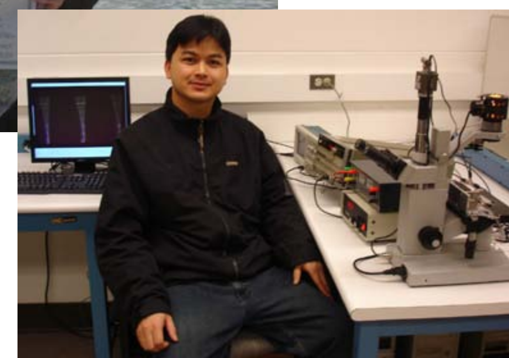

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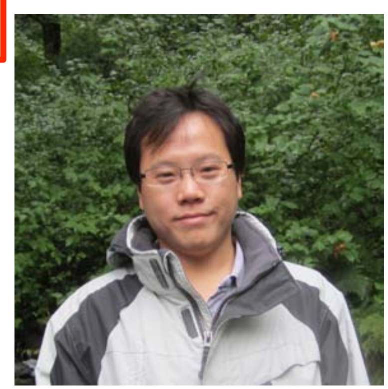
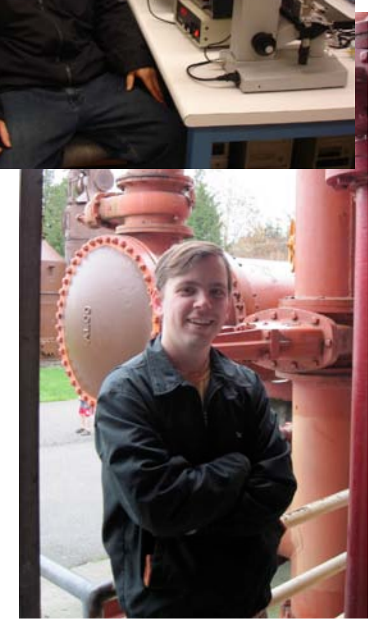
4. **High-Speed AFM for imaging human cells**
   (PhD Student: Arom Boekfah)
5. **Large-Range Nanopositioners**
   (PhD Student: Scott Wilcox)





Talk based on review article in ASME

A Review of Feedforward Control Approaches in Nanopositioning for High Speed SPM

***ASME J. of Dyn. Sys., Meas. and Control*,**

131 (6), Article number 061001, pp. 1-19, Nov. 2009

PDF of talk: http://faculty.washington.edu/devasia/

Acknowledgment: Covers work with a number of collaborators and their slides ☺

Don Croft
Raytheon Systems Arizona
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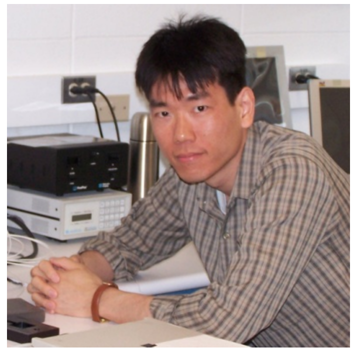
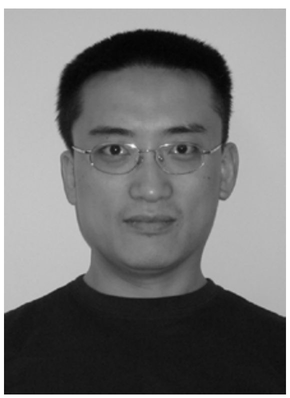


Szu-Chi Tien
Asst Prof, NCKU Taiwan

Hector Perez
Research Prof, U. Pontificia Bolivariana, Columbia



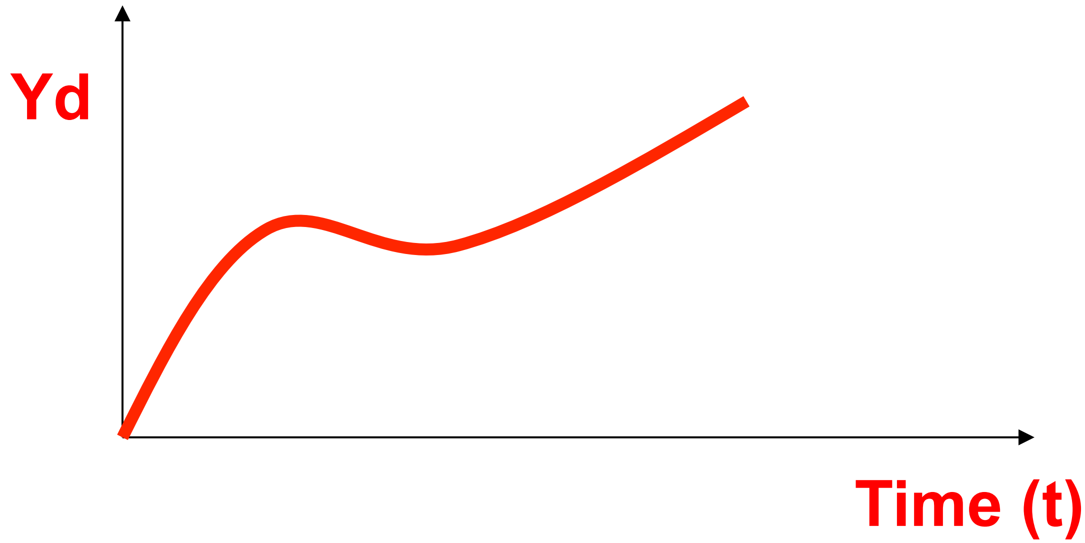
Qingze Zou
Associate Prof, Rutgers U.

Kam Leang
Associate Prof, U. Of Nevada, Reno

Garrett Clayton
Asst Prof, Villanova

The Research Problem

Find the input u that achieves a desired output time-trajectory



Why precision output trajectory tracking?

1) Medical robotics --- e.g., robotics based surgery, where positioning is needed to achieve a cut along a desired path

INTUITIVE SURGICAL®

Why precision output trajectory tracking?

1) Medical robotics --- e.g., robotics based surgery, where positioning is needed to achieve a cut along a desired path

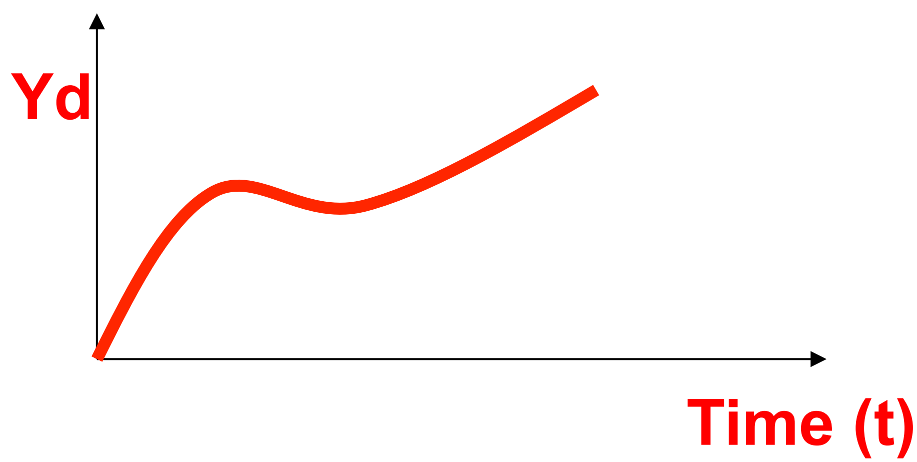
2) Manufacturing robotics --- Similarly, in robotics-based welding of complex parts.

Why precision output trajectory tracking?

1) Medical robotics --- e.g., robotics based surgery, where positioning is needed to achieve a cut along a desired path

2) Manufacturing robotics --- Similarly, in robotics-based welding of complex parts.

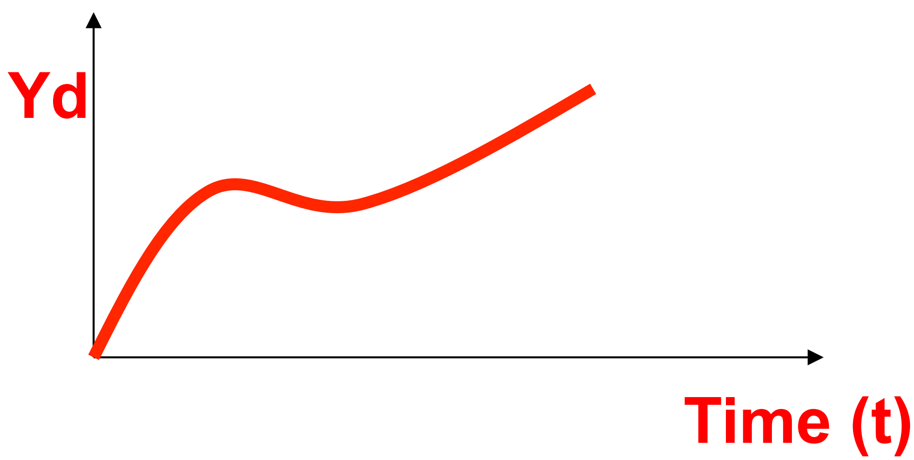
3) Spatial and temporal aspects are important
   e.g., rate of weld is imp
   for quality



Maneuver Regulation --- time not important

If time is not important, but spatial form is important,

then we have more flexibility & maneuver regulation (John Hauser) would be more appropriate



Nano-Position-Transition Problems

1) Positioning of the end point of a flexible structure such as the read-write head in a disk drive

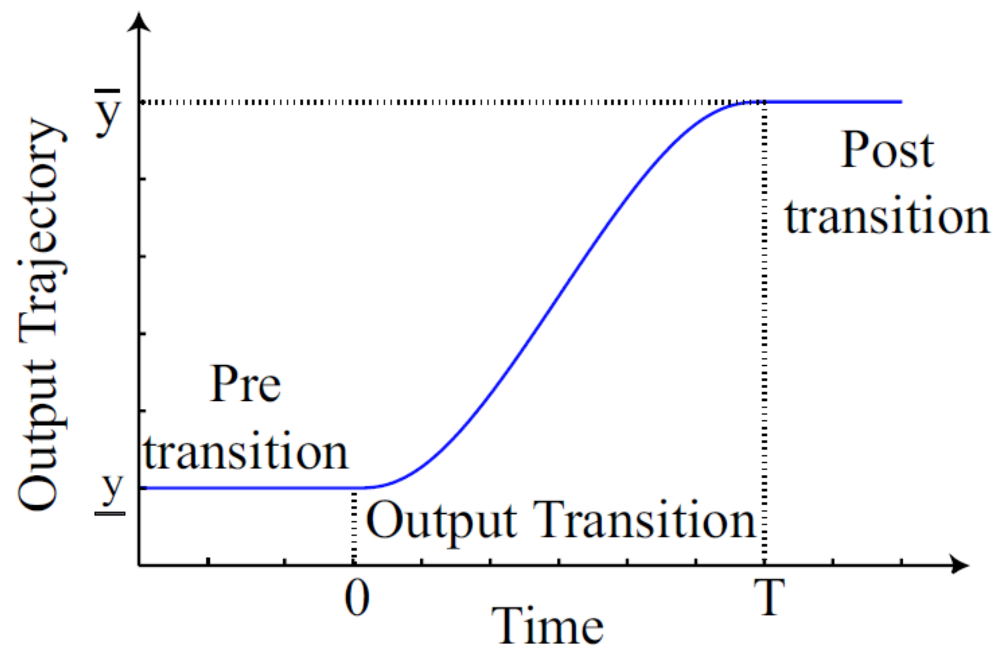
--- becomes more important as size of memory becomes smaller for higher-density storage

--- competition from flash memory (still about 4 time costlier)

semiaccurate.com

The Transition Problem

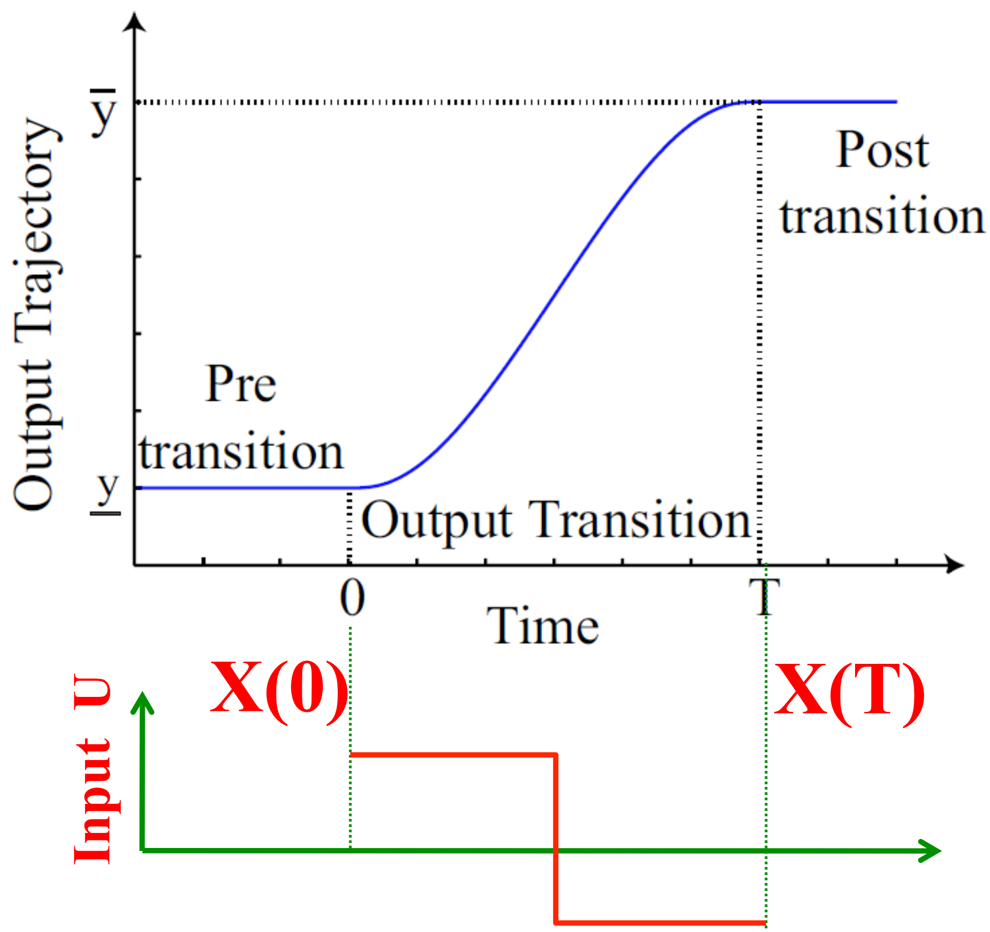
- **Goal:** Output transition Y(0) → Y(T)
- **Applications:**
  1) Disk drives,
  2) Nano-fabrication
  Change operating point between desired locations
- **Requirement:** Maintain constant output outside [0, T]
- **Key Issue:** Minimize Transition Time T



Minimize transition time T

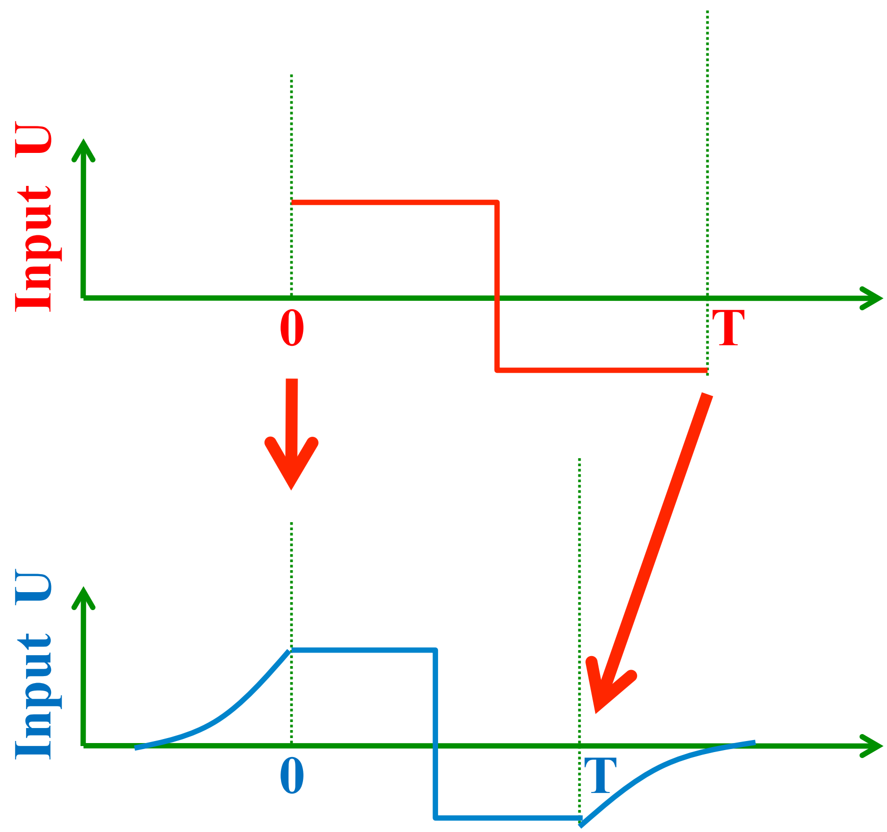
Standard State Transition SST

- **Approach:** Find equilibrium states X(0) and X(T) corresponding to outputs Y(0) and Y(T)
- **Problem:** Minimum time state transition X(0) → X(T)
- **Standard Solution:** Bang-Bang inputs
- **No Pre- and Post-actuation:** Input applied during transition time interval [0,T]



What is new?

- **Approach:**
  OOT: Y(0) → Y(T)
  instead of
  SST: X(0) → X(T)
- **What is new?**
  OOT uses pre- and post-actuation
- **Advantage:** More time for input --- outside [0,T].
- **Reduce transition time** "T" for OOT (compared to SST)

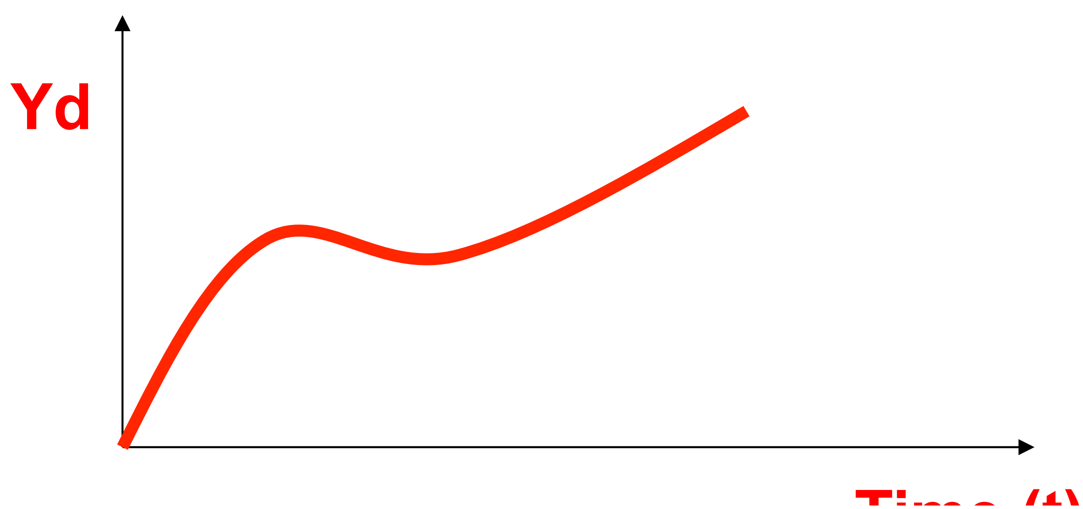


D. Iamratanakul and S. Devasia
"Minimum-Time/Energy, Output Transitions for Dual-Stage Systems," *ASME JDSMC*, 2009

Today's talk is on tracking at the nanoscale

Positioning in Scanning Probe Microscopes (AFM, STM, etc…)
--- e.g., high-speed **nano-scale imaging of soft samples**

Find the input u that achieves a desired output time-trajectory



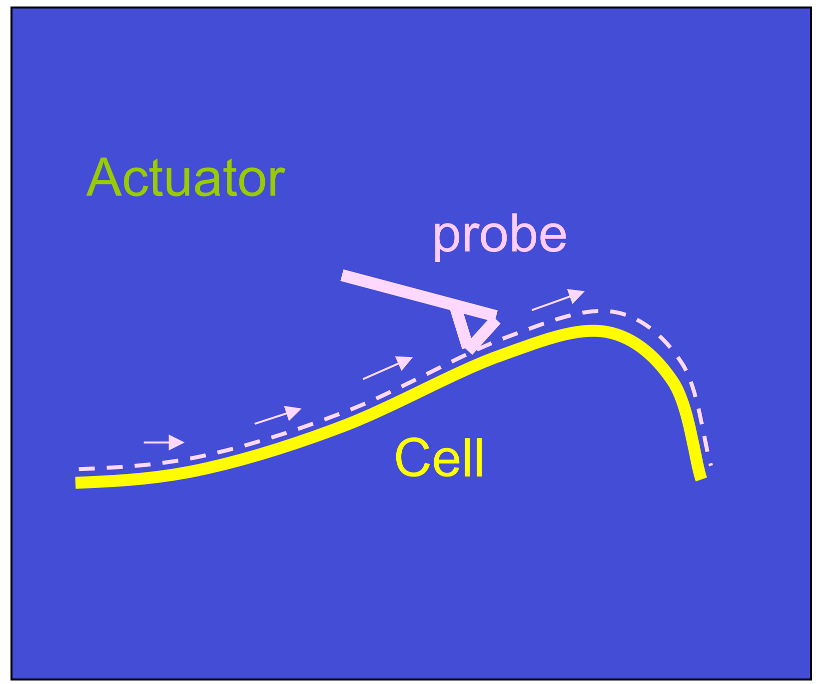
Example: Cell Imaging with AFM

Investigate, reasons for abnormal cell behavior, e.g., due to aging or cancer, and how to correct it

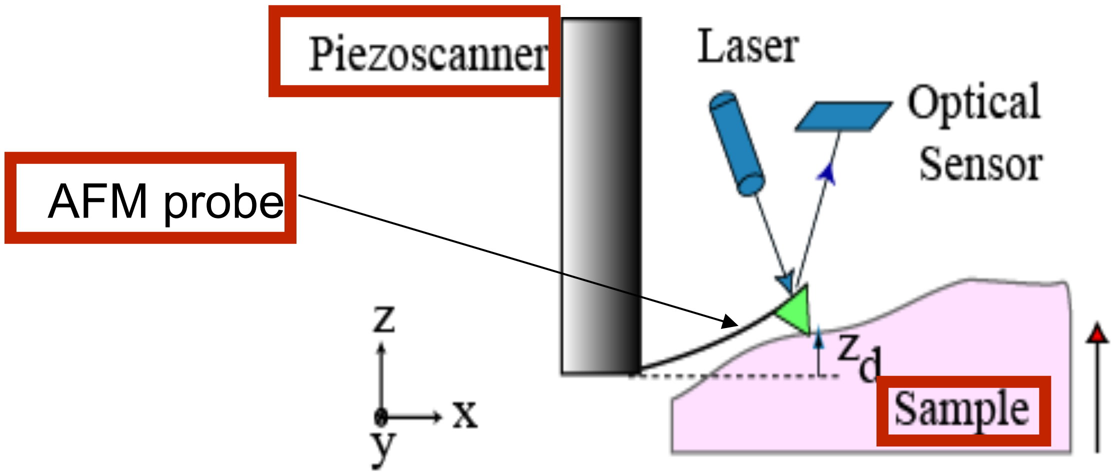
Similar to Doctor tapping on stomach to diagnose reason for abdominal pain

AFM probe is used to tap on a human cell

But with very small forces (pN) 10^{-12}N

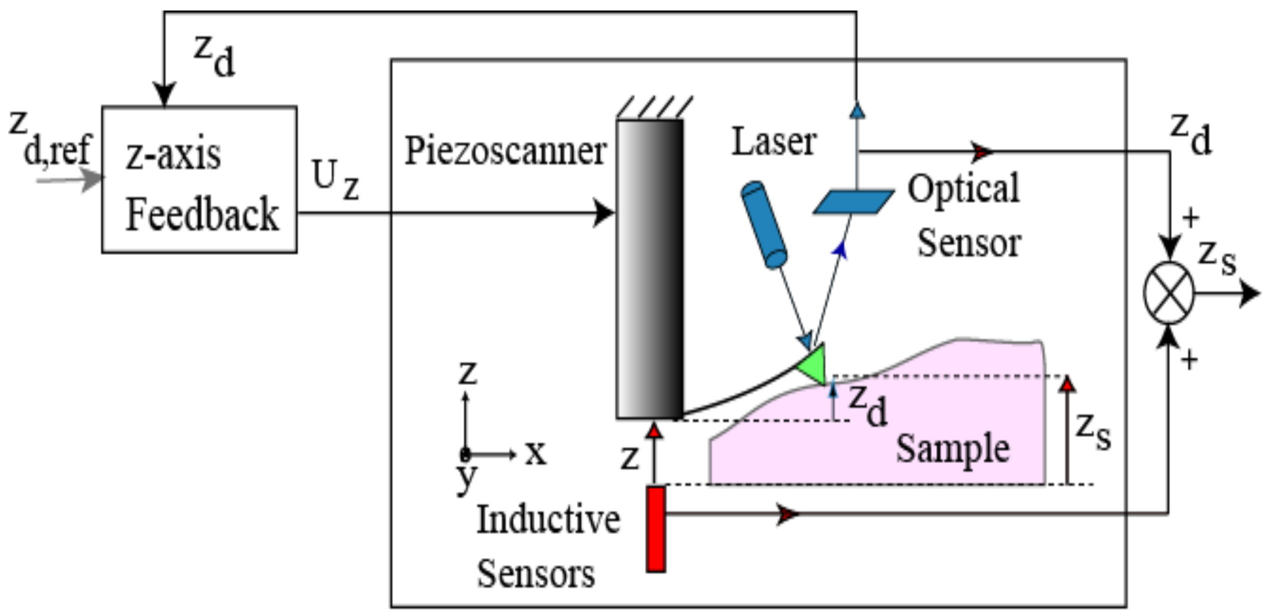


Vertical Control of SPM

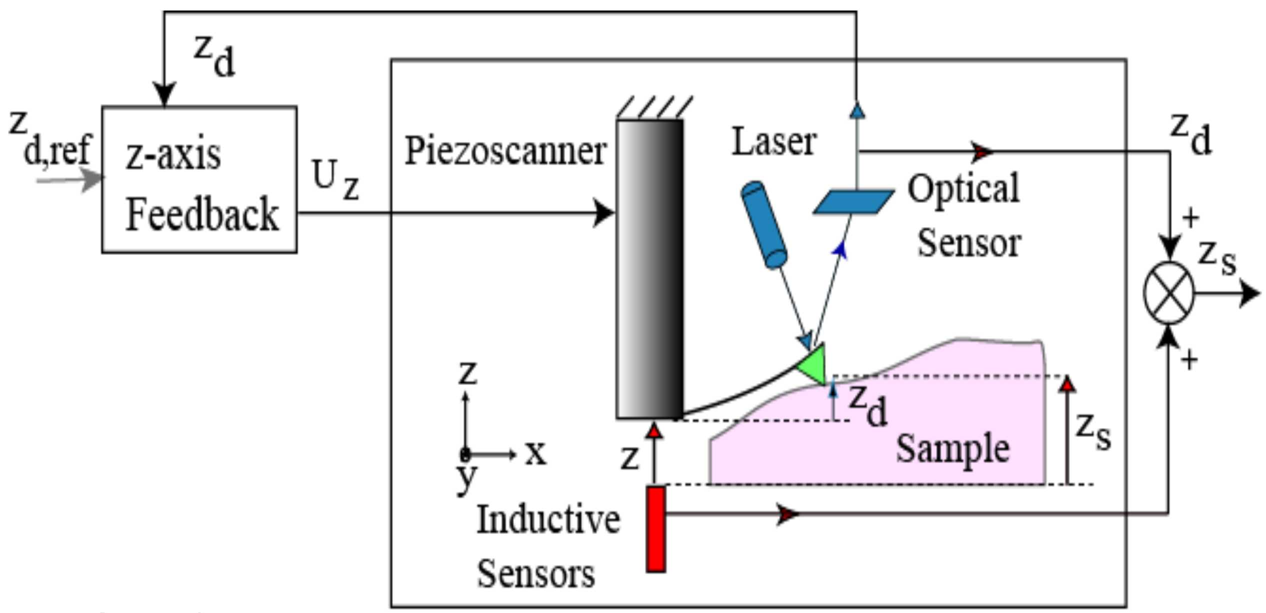


Vertical positioning is critical to maintain small forces and reduce sample damage

Feedback is used to control position



Position control critical to force control

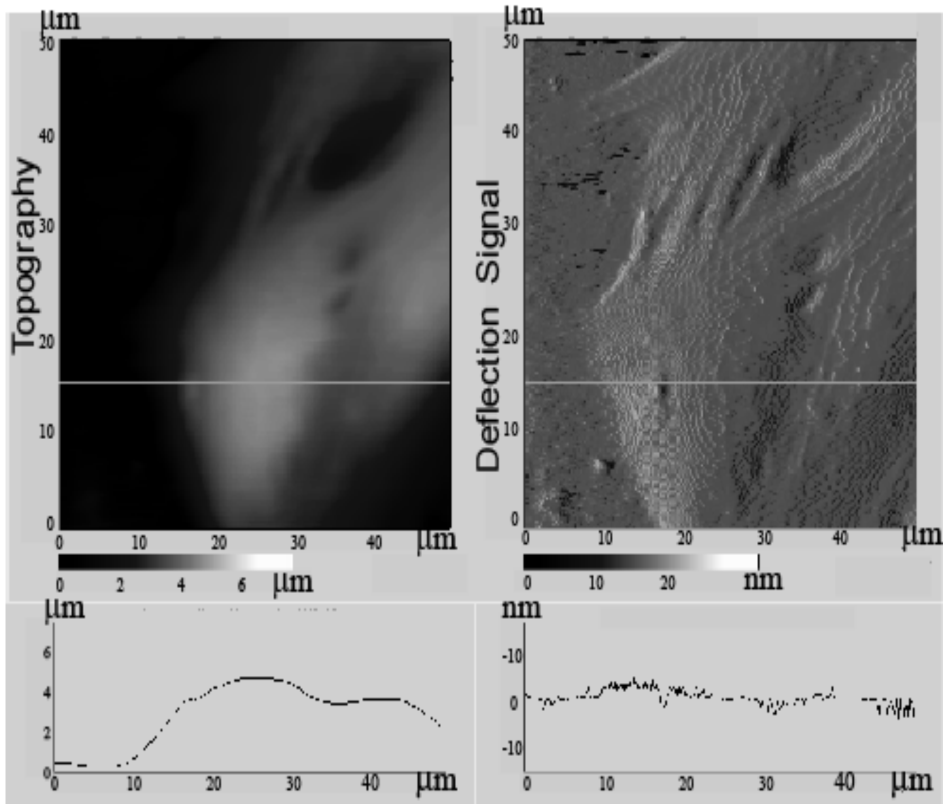


Force = stiffness * deflection
= (0.01 N/m) * deflection

Force variations less than 0.1nN → deflection error less than
0.1nN/(0.01N/m) = 10nm.

Critical during AFM operation over soft biological samples and polymers

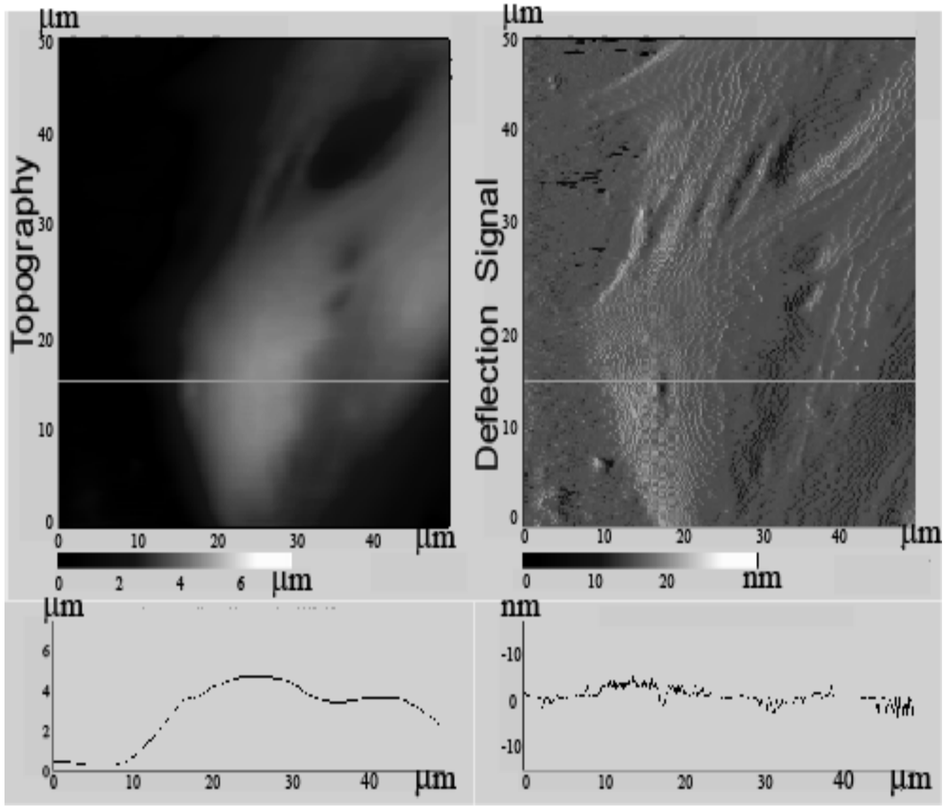
AFM Imaging of soft cells is slow!



If you are slow --- a good integral controller (PID) can track with very high precision --- due to robustness of “I”

But slow -- About 20 minutes … cells can change during this time

AFM Imaging of soft cells is slow!

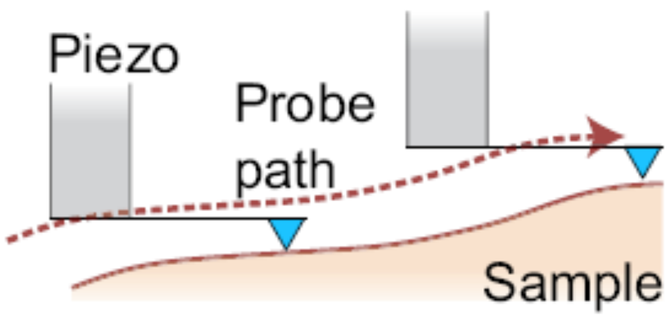


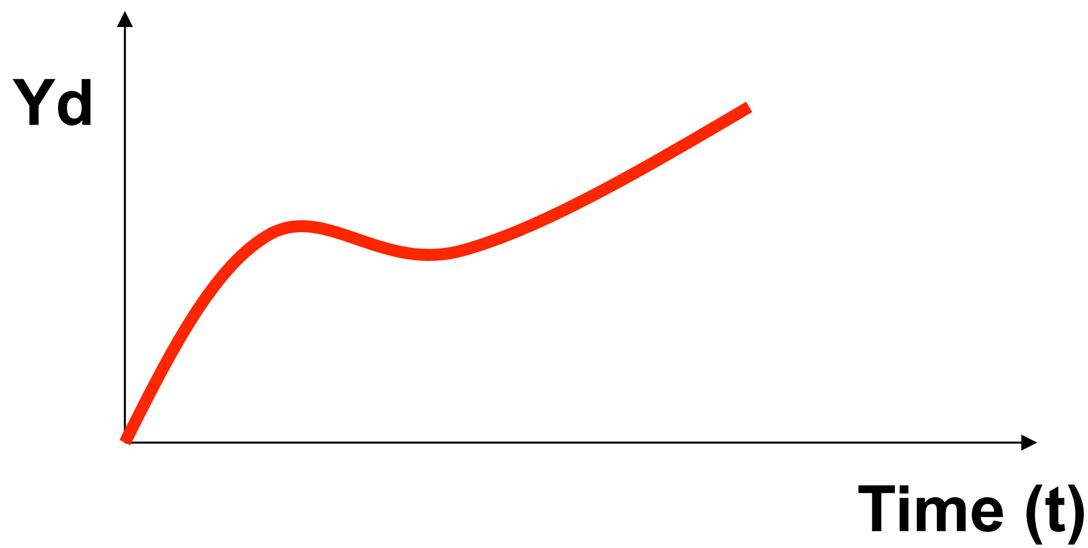
About 20 minutes … cells can change during this time

Can image faster; will still get an image (cell can withstand some abuse) – but unclear if it is a good image, i.e., if the sample is damaged/modified…

Typical goals in positioning control

Find the input u that achieves the desired output (position) time-trajectory





Goals: High-speed, high-precision, large-range

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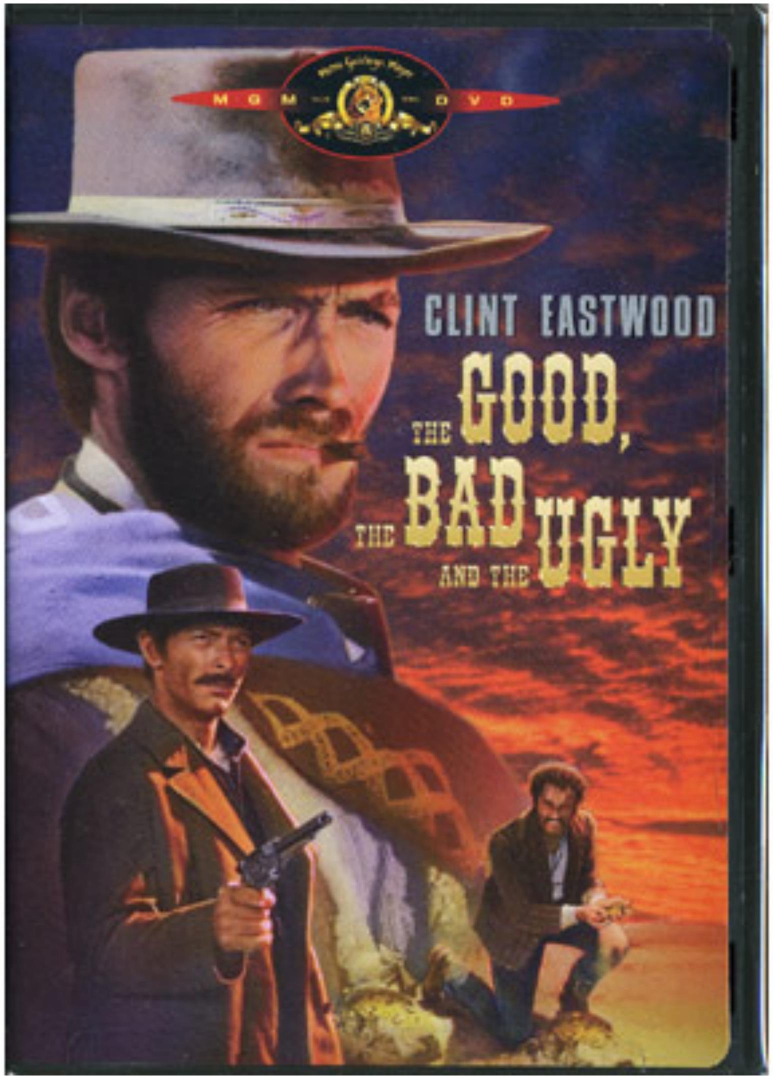
The good, the bad, and the ugly in Nanopositioning

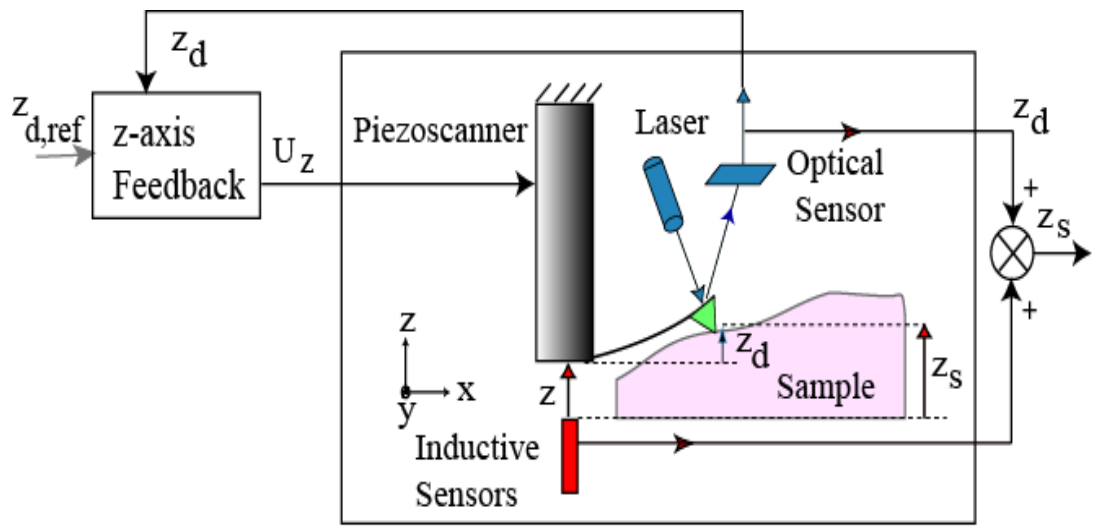


The good: Piezos as actuators

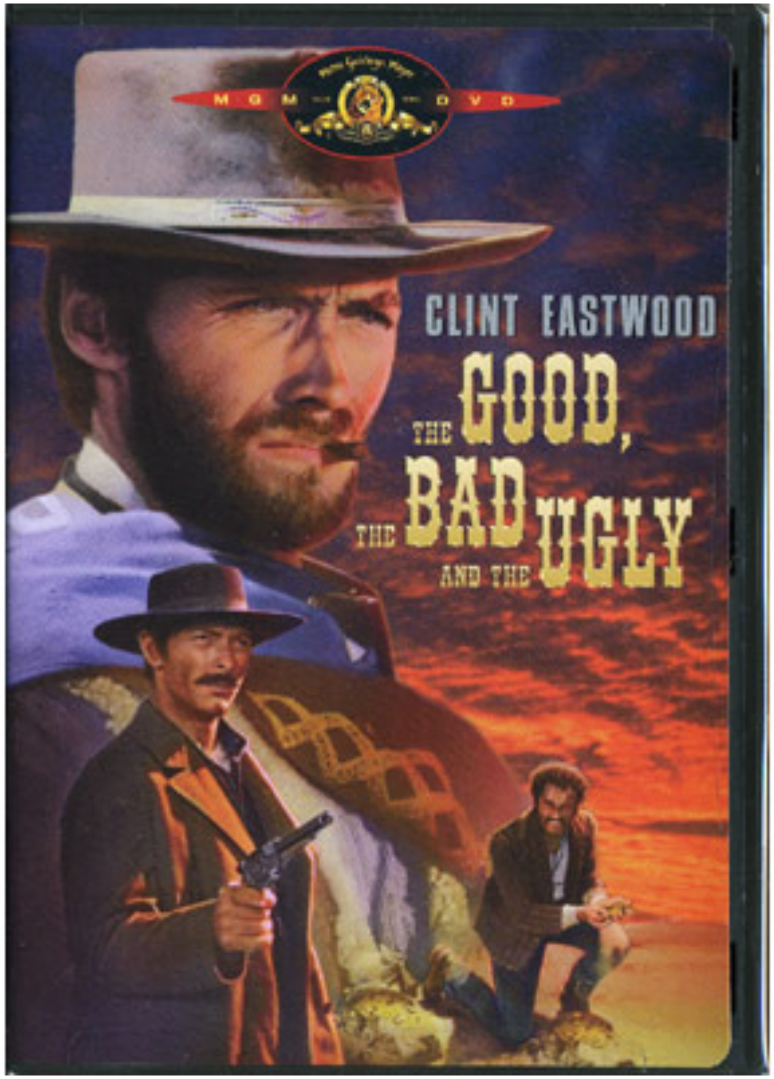
No sliding friction (stiction effects)

Can achieve very-high (sub-nano) resolution

With simple integral controllers



The good, the bad, and the ugly in Nanopositioning

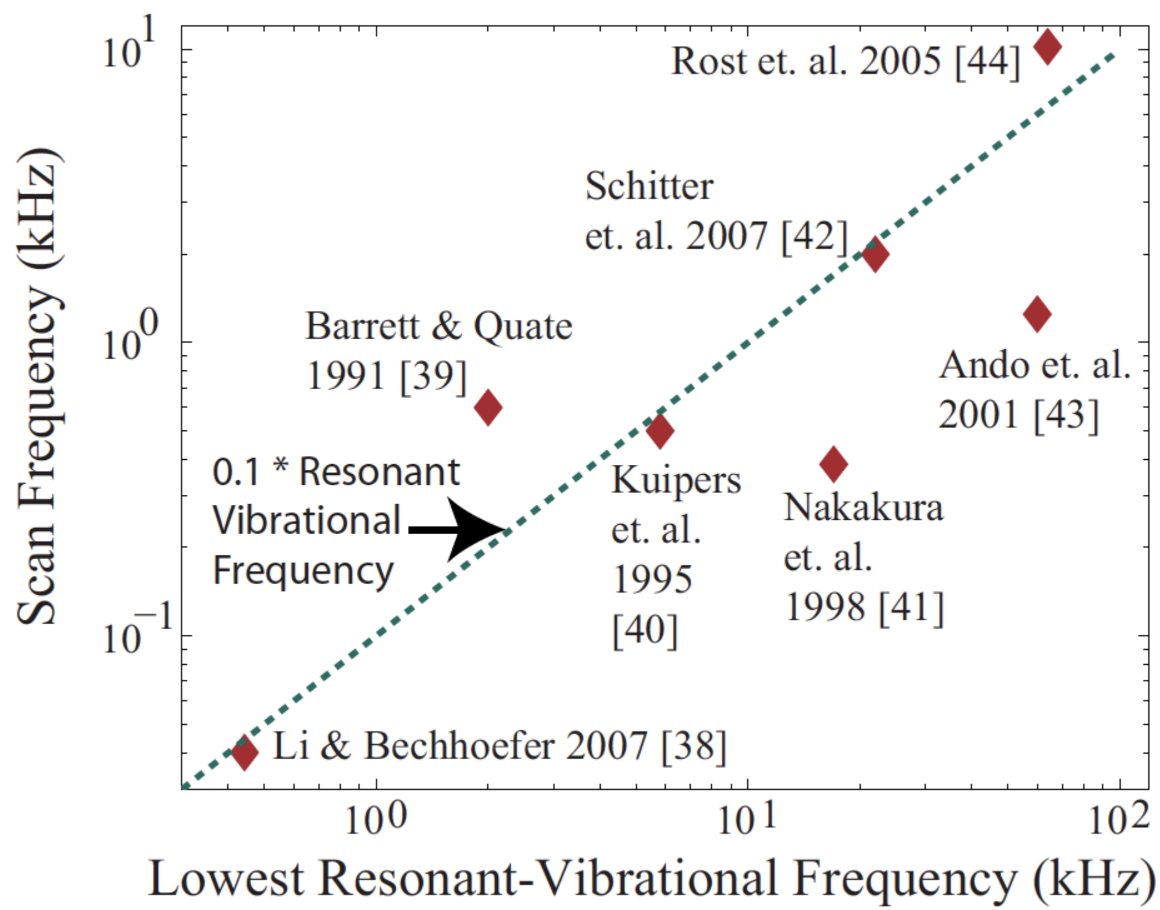


The bad: low positioning bandwidth

How fast (at what frequency) can you scan across a surface?

Depends of precision needed as well as surface topography

Scan frequencies are much less than 1/10th to 1/100th of the lowest resonance frequency



Dynamics limits bandwidth

- **Controller needs to overcome three problems**

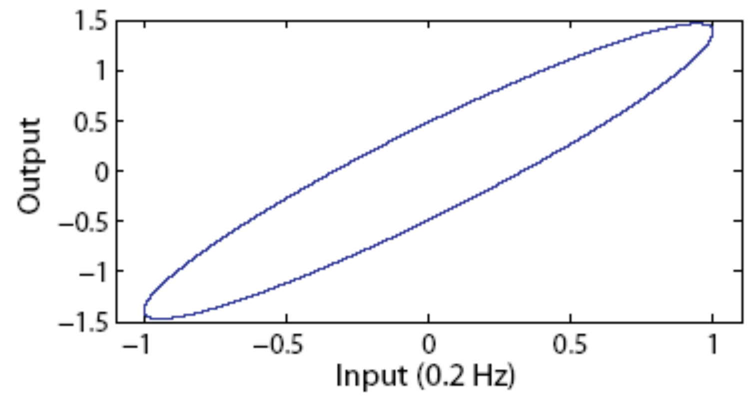
- 1) Creep
- 2) Hysteresis
- 3) Vibrations

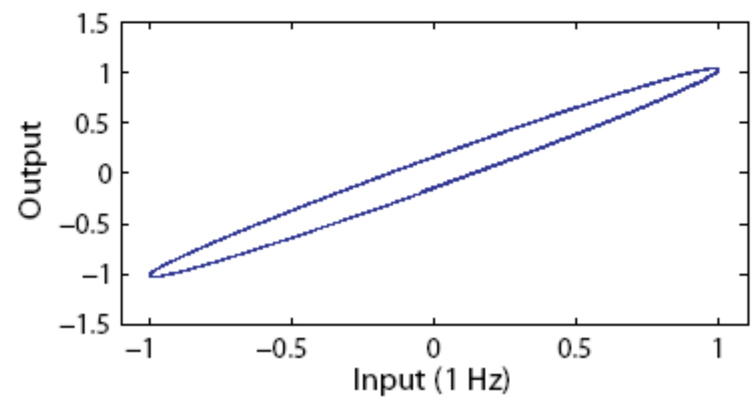
Creep

A low-frequency effect

Can be modeled using springs and dampers

$$\mathcal{G}_c(s) = \frac{1}{k_0} + \sum_{i=1}^{N} \frac{1}{c_i s + k_i},$$





It is frequency dependent -- See figure on right

(1Hz result different from 0.2Hz)

$$\mathcal{G}_c(s) = 1 + \frac{1}{s+1}$$

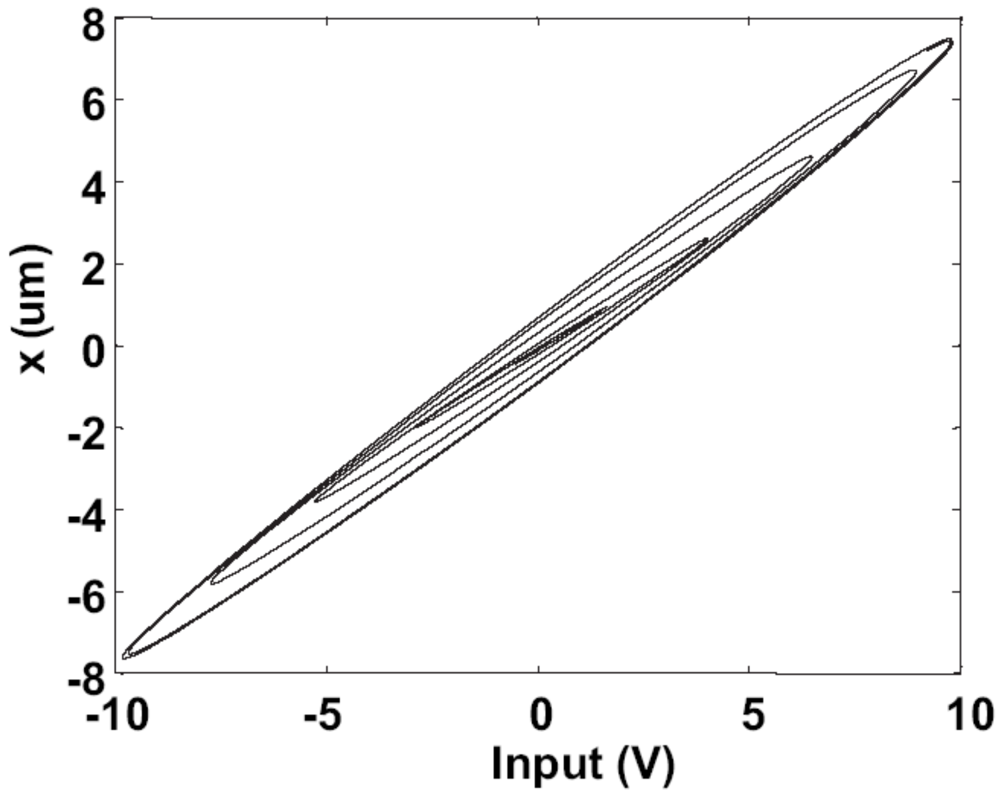
Hysteresis

A memory effect
(see figure)

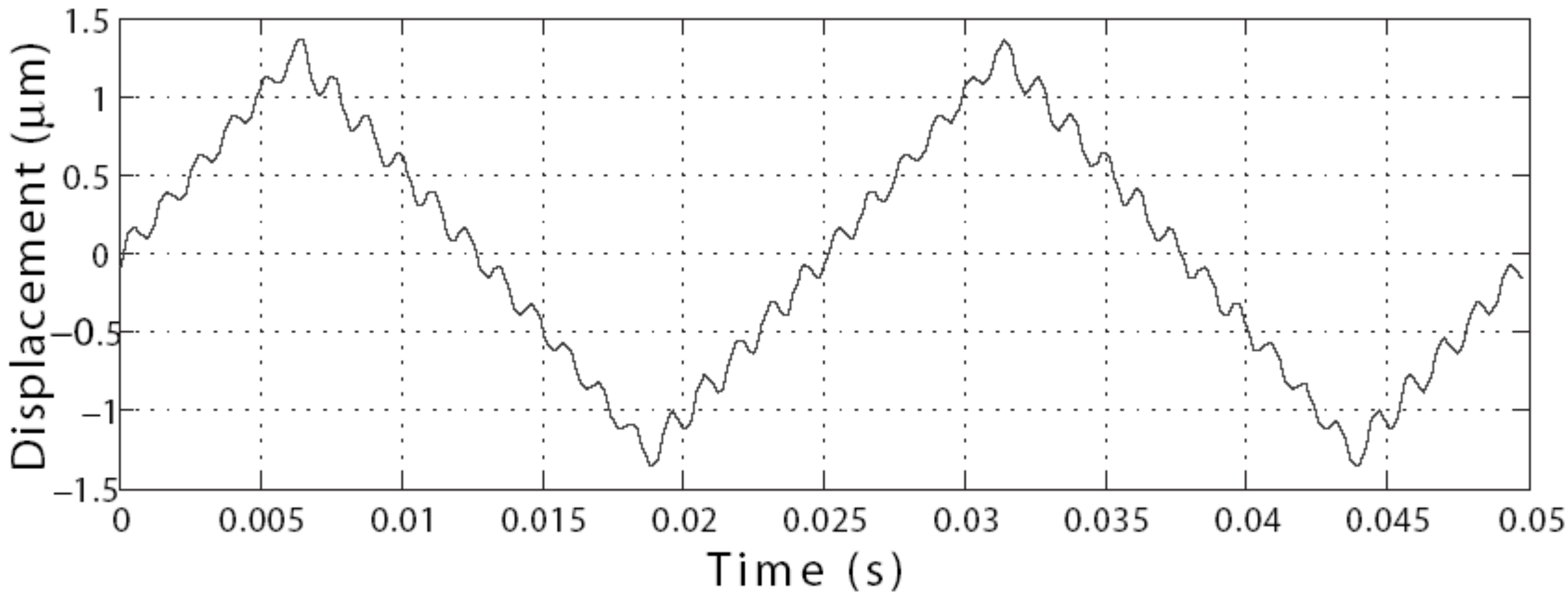
Inner-loops are a challenge to model

Substantial efforts in modeling hysteresis:

We used
Preisach Models



Vibrations



- A high-speed positioning phenomena
- Example -- 40 Hz triangle wave, resonance at 850 Hz.
- Vibrations Limit bandwidth
- Modeling errors --- unmodeled high frequency resonances, and coupling between vibrations in different axes (X,Y,Z)

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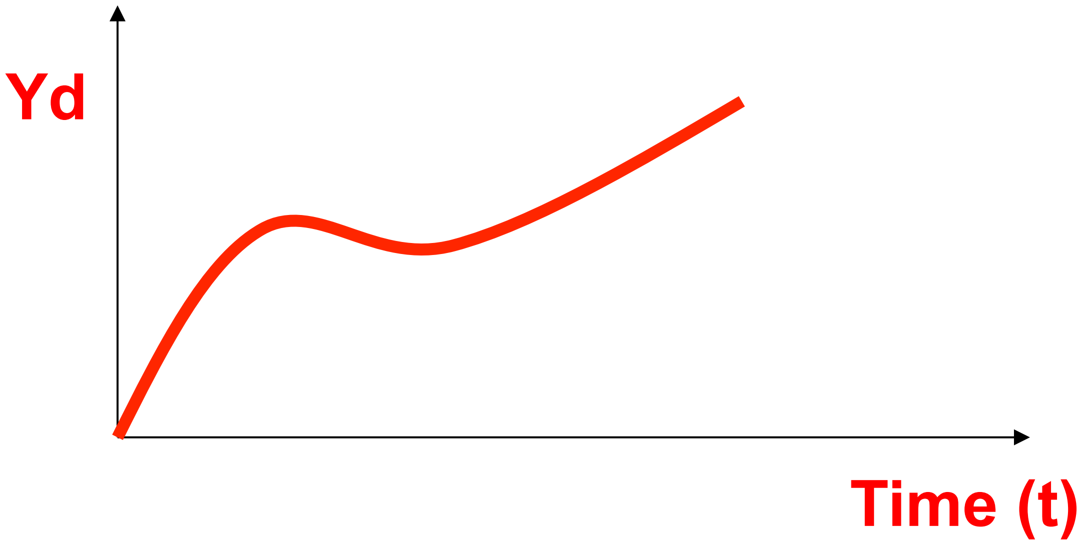
The Research Problem in high-speed positioning

- Find the input u that achieves a desired output y_d --- we use inversion approach



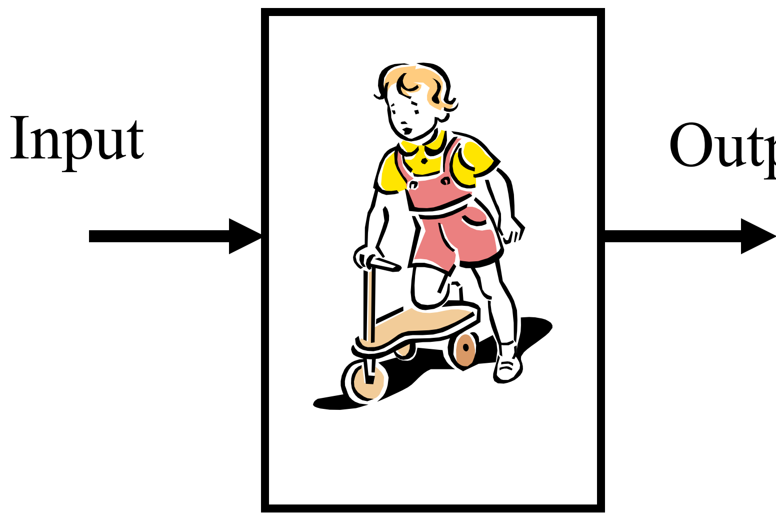
What is Inversion-Based Control?

Two parts

Part 1: the concept

Part 2: theoretical challenge

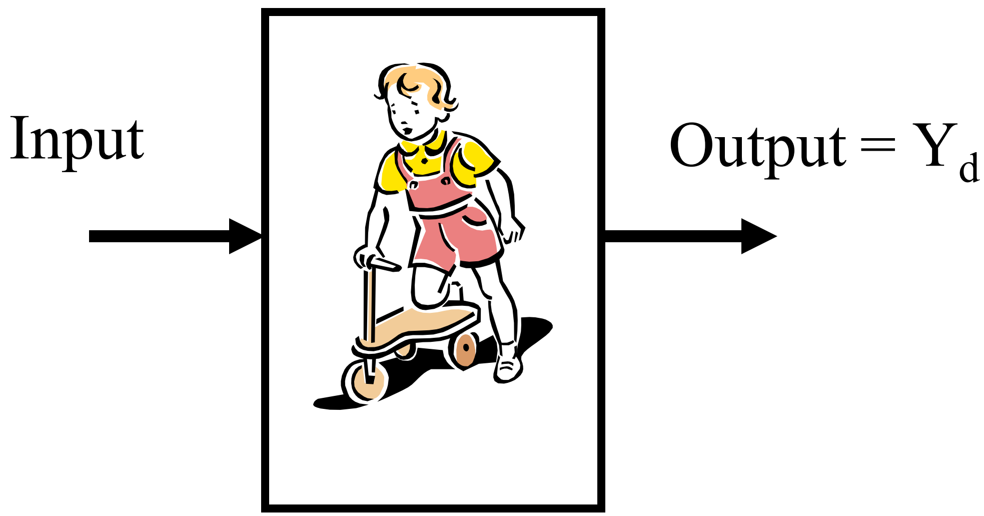
What is Inversion-Based Control?



Consider a System --- My Nephew

Let the **desired output be, say, eat dinner!**

What is Inversion-Based Control?

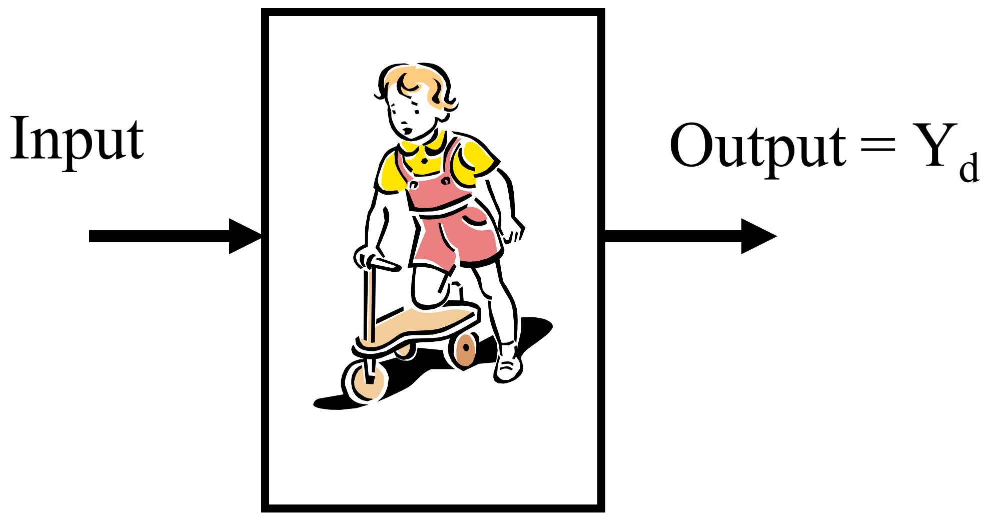


Let the desired output be, say, eat dinner!

Question: What input should you apply?

(**negotiate, encourage, ??**?)

What is Inversion-Based Control?

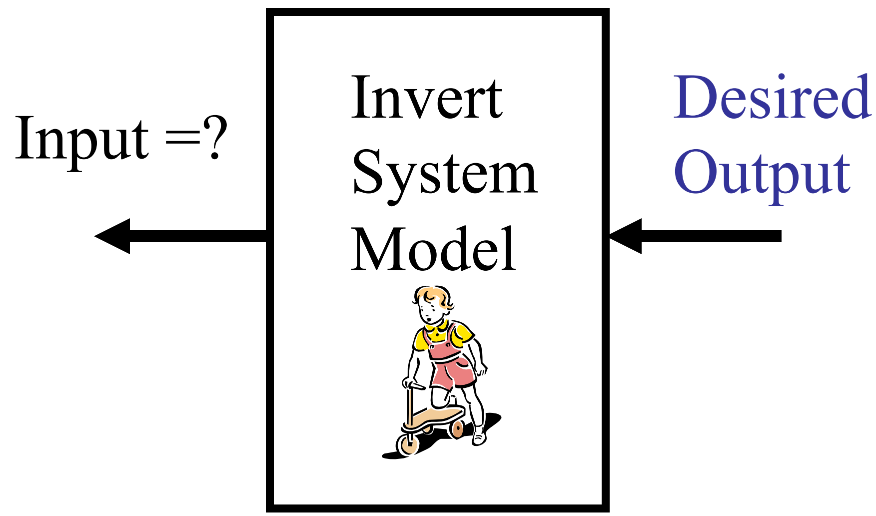


Let the desired output be, say, eat dinner!

Question: What input should you apply?

(**negotiate, encourage, bribe <u>always works for me</u>**!)

The Inversion-Problem

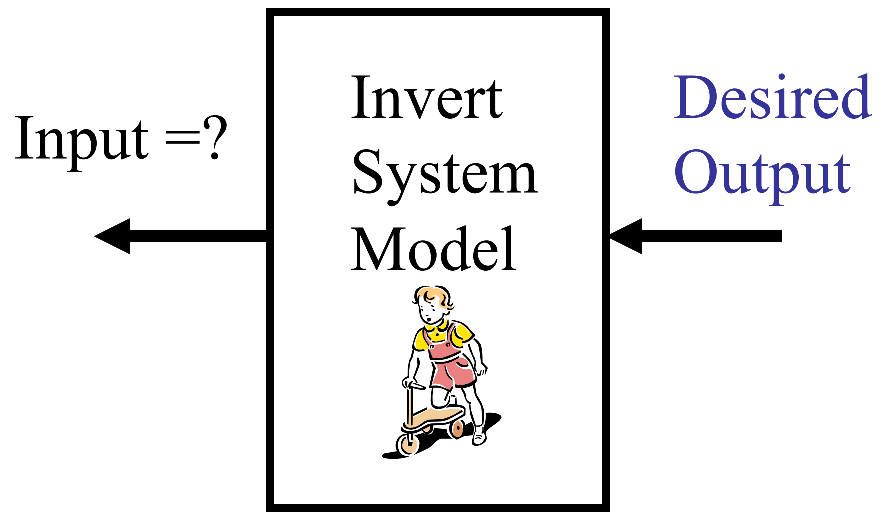


Prior Knowledge

Invert the known system model (**G_0**) to find input.

Input = **G_0^{-1}** [Desired Output]

The Inversion-Problem



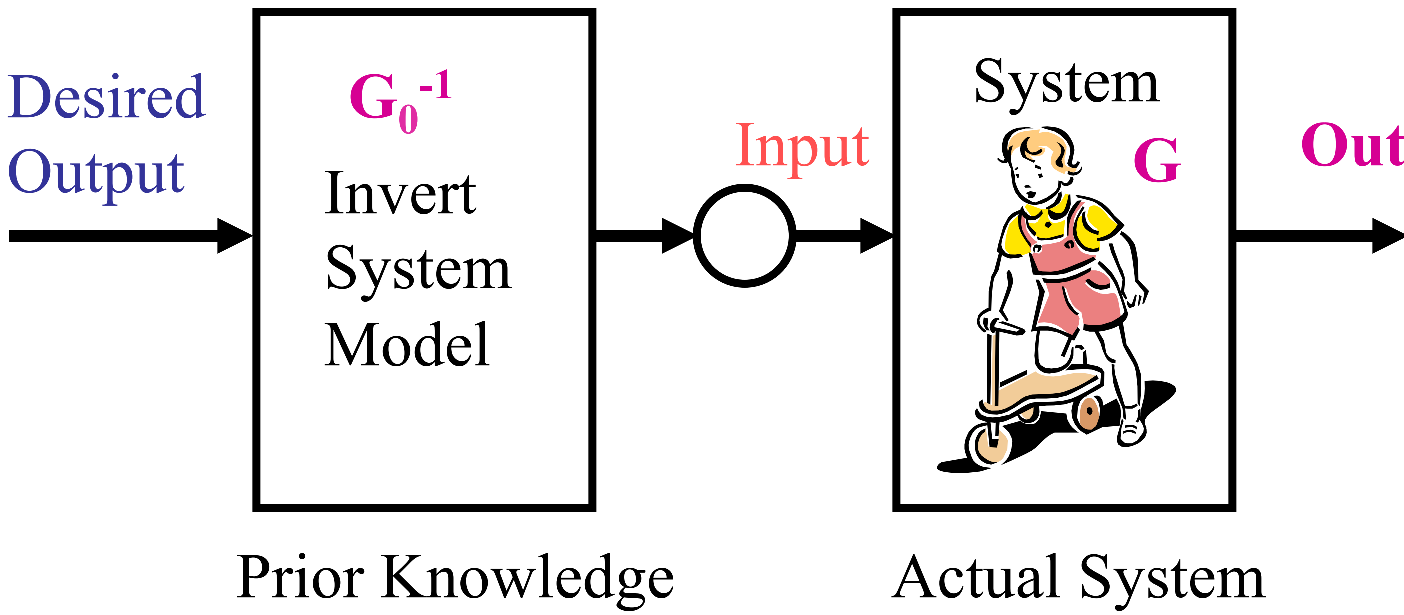
Prior Knowledge

Invert the known system model (**G_0**) to find input.

Input = **G_0^{-1}** [Desired Output]

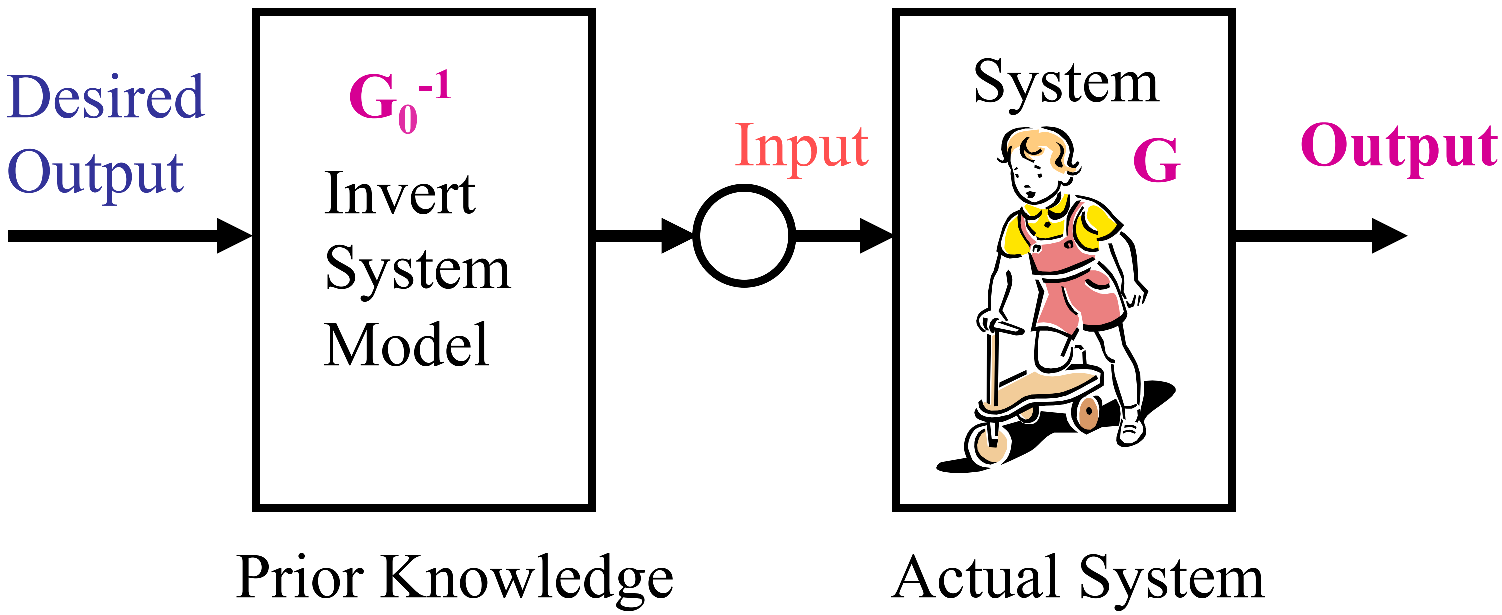
(His Mom know' s how --- she has a reasonable model)

The Control method using Inversion



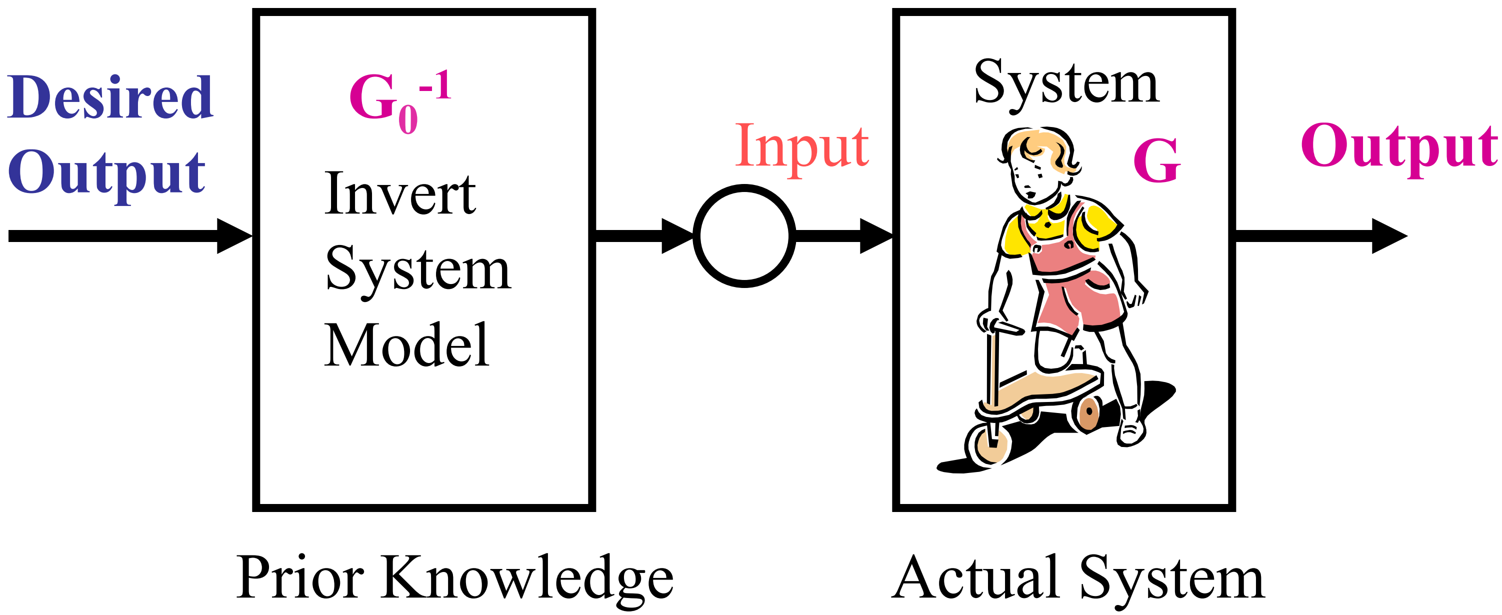
Use Inverse input as the feedforward input to system

Feedforward is Common in Human Systems



Examples:
Walking, Playing Baseball, Driving a Car

Problem --- model uncertainty

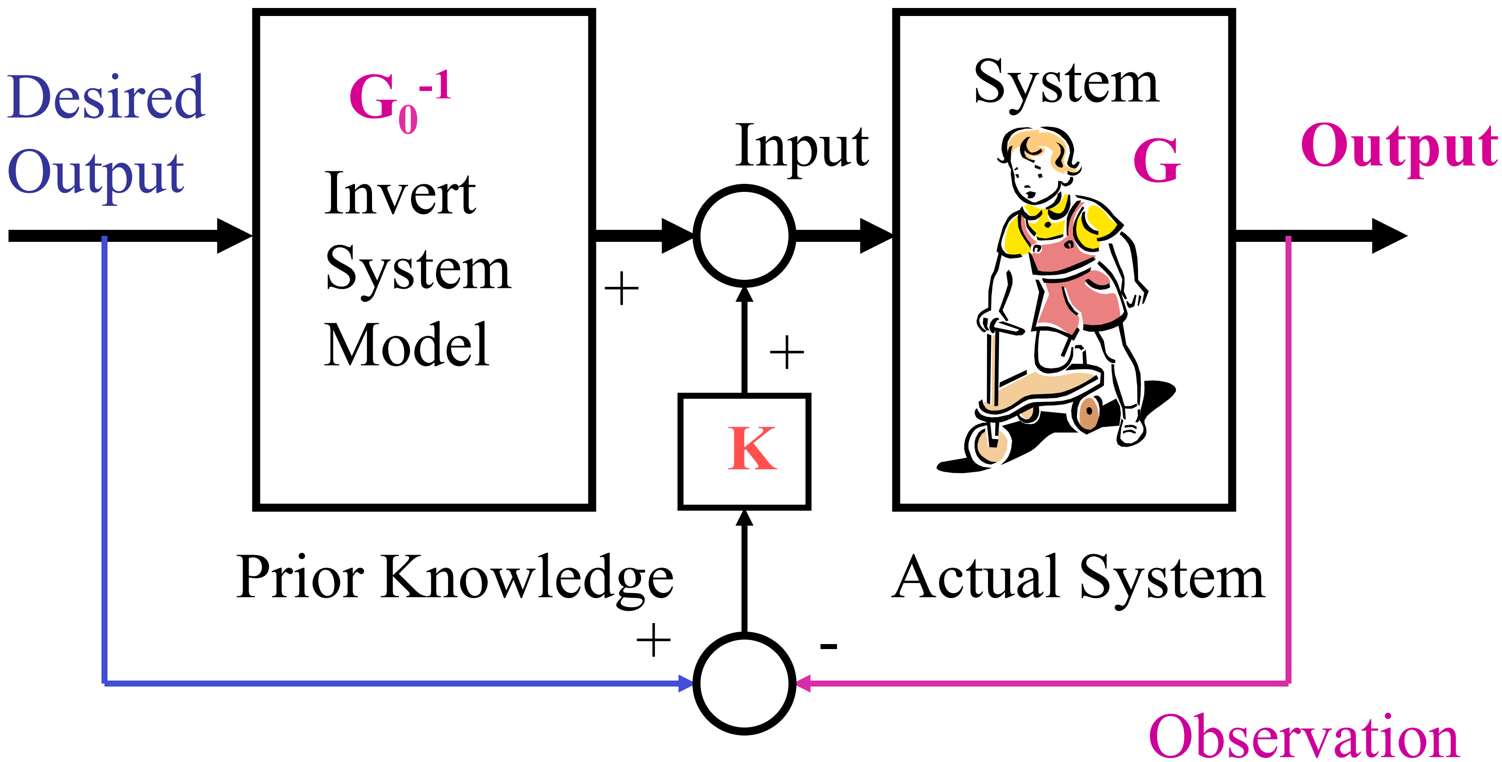


Is Desired output = Output?

Yes if we know the model perfectly!

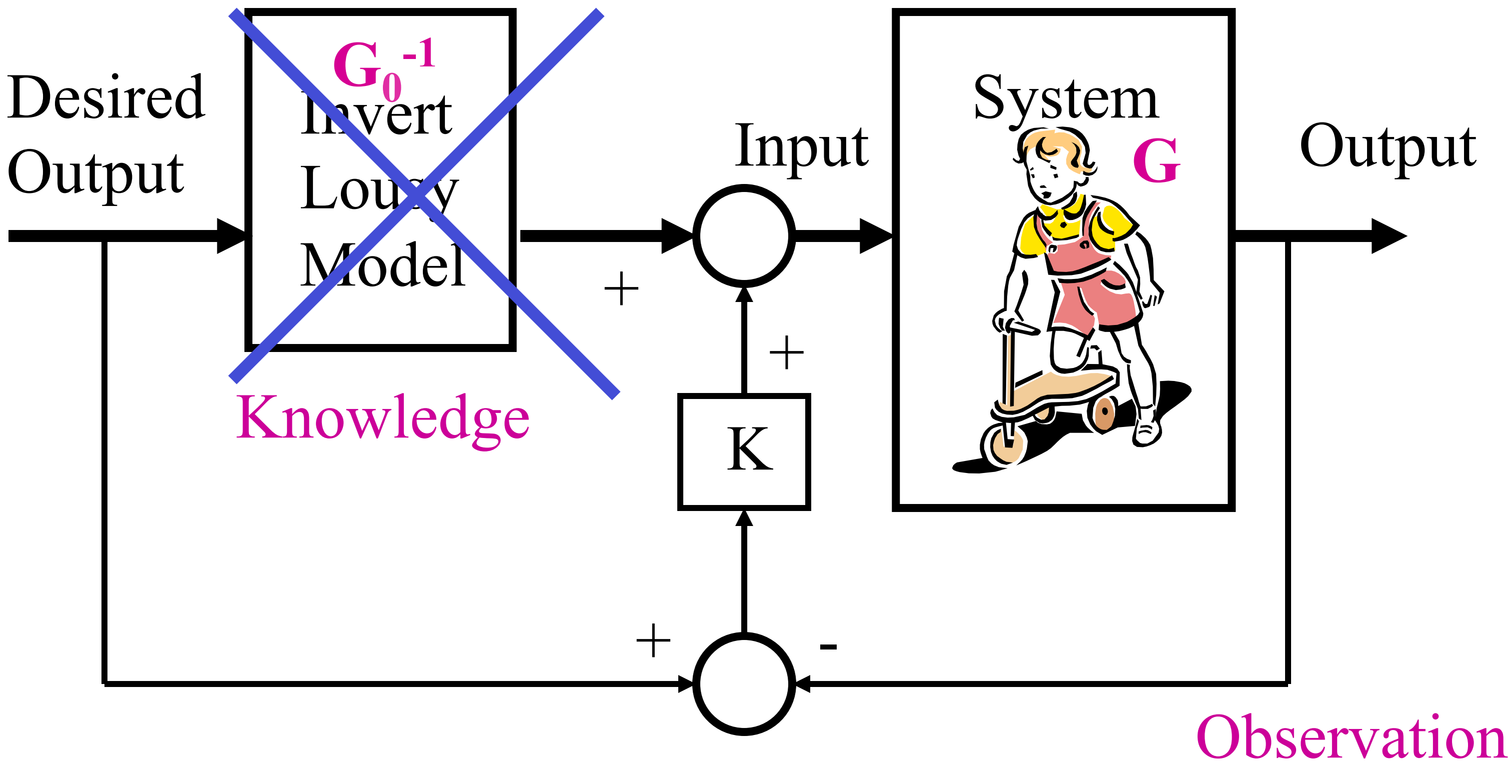
But, we rarely know a system perfectly ($G_0 \neq G$, $G_0^{-1} \neq G^{-1}$)

Resolution: Addition of Feedback



Exploit knowledge of the system through feedforward input
Account for errors (uncertainties, perturbations) using feedback

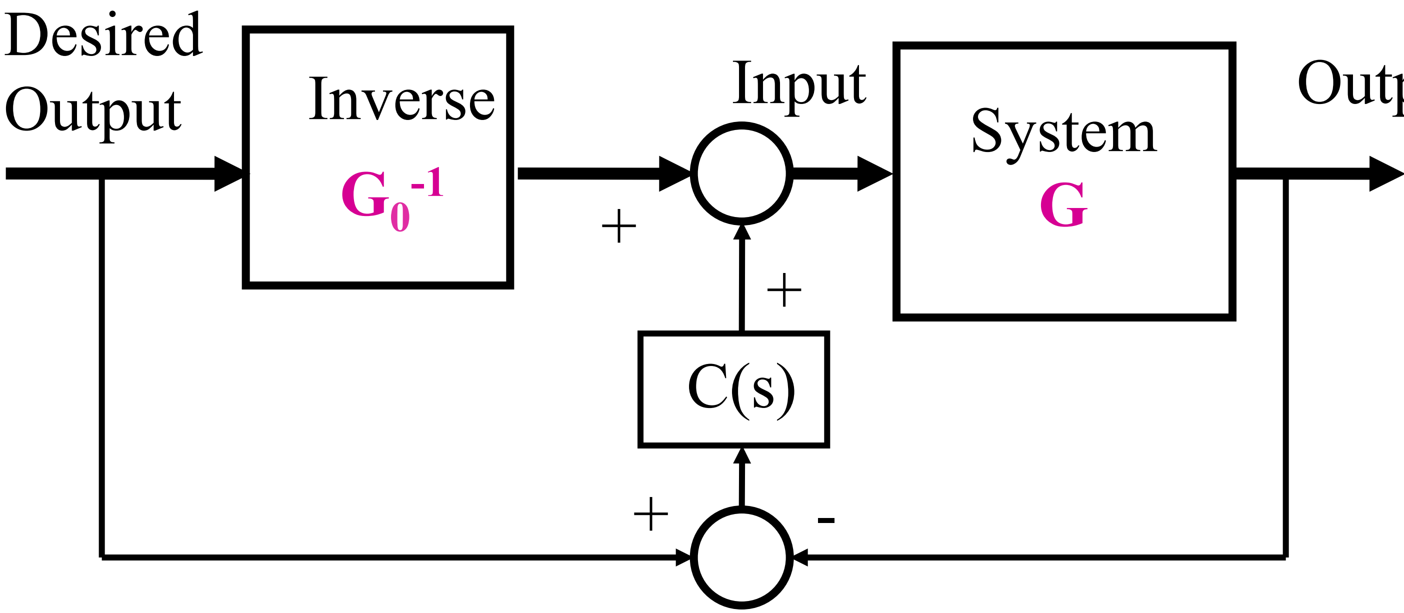
Feedforward under Uncertainty?



As the kid grows up the model gets lousy! $\Delta(\omega) = G_0(\omega) - G(\omega)$

Maybe it is better to use pure feedback without feedforward?

Feedforward under Uncertainty?



Let the Error in model be $\Delta(\omega) = G_0(\omega) - G(\omega)$

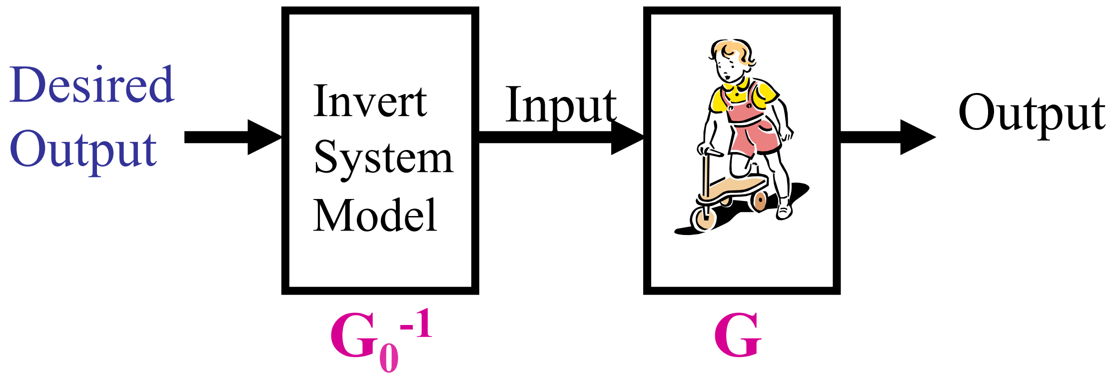
For SISO Case, Feedforward always improves output tracking for any feedback if

$$|\Delta(\omega)| < |G_0(\omega)|$$

More generous conditions than for robust-feedback

Re-Cap

- **Key Idea: Feedforward Input is found using System Inversion**



(1) Feedforward input uses system knowledge to control the output
(2) Feedforward should be integrated with feedback
(3) Performance better than the use of feedback alone if uncertainty is not too large $|\Delta(\omega)| < |G_0(\omega)|$

What is Inversion-Based Control?

Two parts

Part 1: the concept

Part 2: theoretical challenge

Difficulty of inverting nonminimum phase systems

Given

$$\frac{y(s)}{u(s)} = \frac{(s - 1)}{(s+2)(s+3)}$$

Find the inverse of a desired output y_d

$$u_{inv}(s) = \frac{(s+2)(s+3)}{(s - 1)} y_d(s)$$

This inverse is unbounded!

Inversion is difficult for nonminimum phase systems with zeros on the right hand side of the imaginary axis

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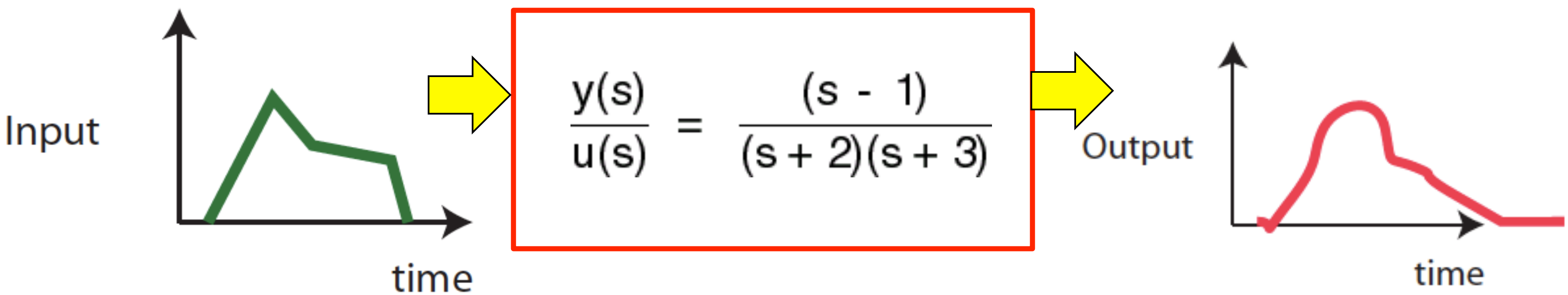
Inversion is difficult for nonminimum phase systems with zeros on the right hand side of the imaginary axis.

Question: Does this imply that the inverse does not exist?

Does nonminimum phase imply inverse does not exist?

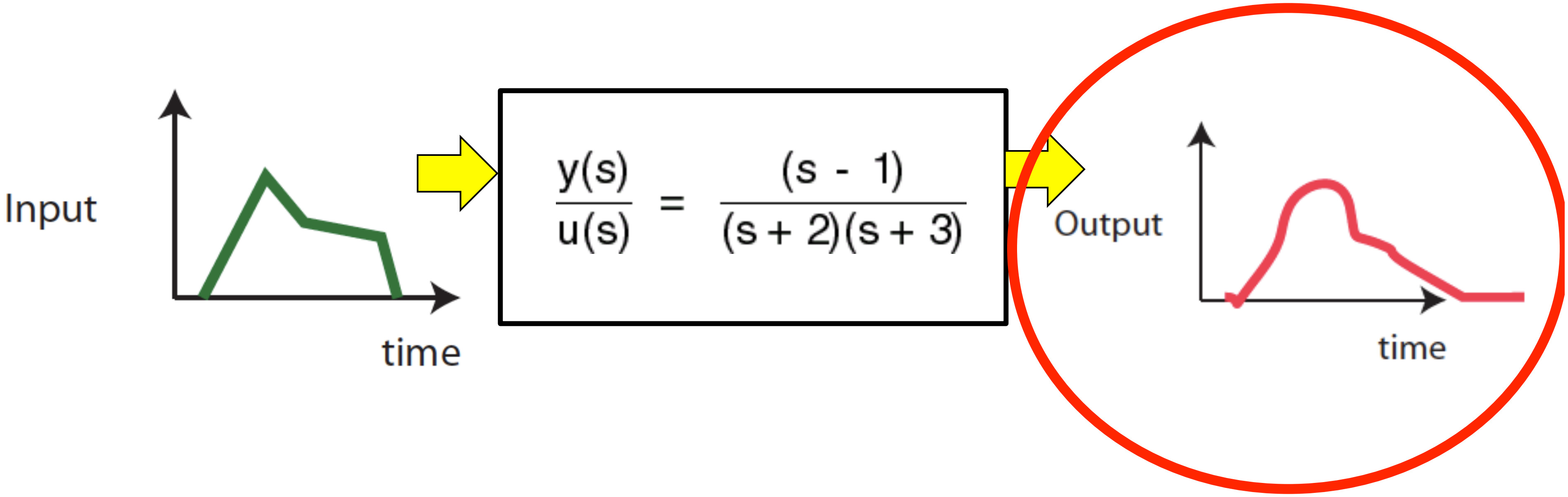
Apply an input U_d to the system --- find the resulting output



Does nonminimum phase imply inverse does not exist?

Apply an input U_d to the system --- find the resulting output

Choose this output as the desired output Yd



$$\frac{y(s)}{u(s)} = \frac{(s - 1)}{(s+2)(s+3)}$$

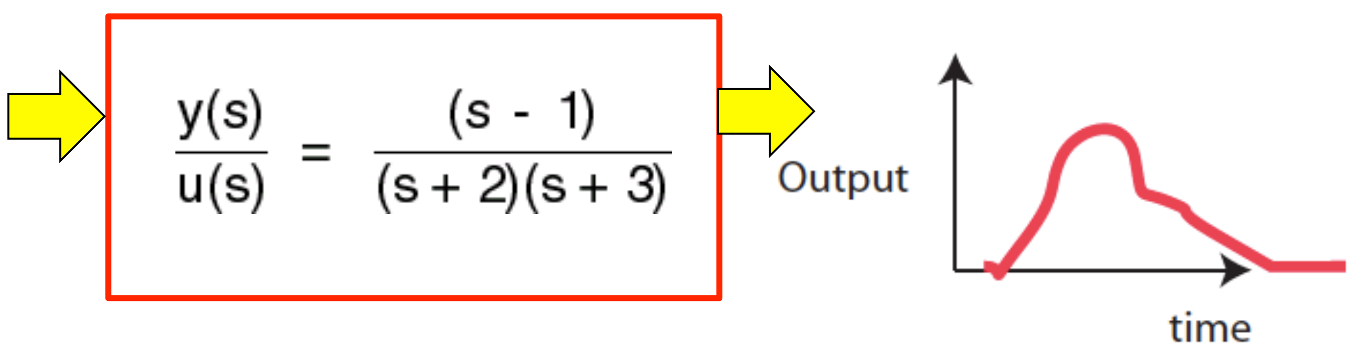
Does nonminimum phase imply inverse does not exist?

Apply an input U_d to the system --- find the resulting output
Choose this output as the desired output Yd

$$Y_d(s) = \frac{(s-1)}{(s+2)(s+3)} U_d(s)$$

Does the inverse of this output Yd exist?

?



Does the inverse exist for this yd?

Given

$$\mathbf{Y_d(s)} = \frac{(s - 1)}{(s+2)(s+3)} \mathbf{U_d(s)}$$

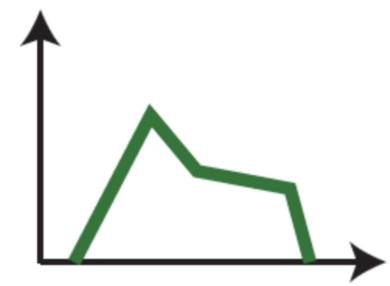
$$u_{inv}(s) = \frac{(s+2)(s+3)}{(s - 1)} y_d(s)$$

This inverse U_{inv} is still unbounded!

But we know there is an inverse!

Given

$$Y_d(s) = \frac{(s-1)}{(s+2)(s+3)} U_d(s)$$



$$u_{inv}(s) = \frac{(s+2)(s+3)}{(s-1)} y_d(s)$$

This inverse U_{inv} is still unbounded!

But we know there is a bounded inverse (U_d)!
Issue: how to find this bounded inverse?

Other approaches to output-tracking of nonminimum-phase system

1. Regulator approach: (Asymptotic tracking for certain trajectories)
    1) Francis, 1977—Linear multivariable regulator problem.
    2) Isidori and Byrnes, 1990—Extension to the nonlinear case (solving a partial differential equation is required).
    3) Huang and Rugh, 1992—Approximate method to nonlinear servomechanism problem.
    4) Di Benedetto and Lucibello, 1993—Existence of initial conditions that can lead to exact inverse for nonminimum phase systems.
2. Approximation method (Nonminimum-phase by Minimum-Phase)
    1) Gurumoorthy and Sanders, 1993, Gopalswamy and Hedrick, 1993—Approximation technique. Modification of the desired trajectory to make the system minimum phase.
    2) Tomizuka (1987), Hauser, Sastry and Meyer (1992)—Approximate by a minimum-phase system.

Some Approximation methods

$$\frac{y(s)}{u(s)} = \frac{(s - 1)}{(s+2)(s+3)}$$

Neglect zero (same gain)

$$\frac{y(s)}{u(s)} = \frac{-1}{(s+2)(s+3)}$$

Replace nonminimum phase zero with minimum phase zero

$$\frac{y(s)}{u(s)} = \frac{-(s+1)}{(s+2)(s+3)}$$

Zero phase error (replace zero by stable pole)

$$\frac{y(s)}{u(s)} = \frac{-1}{(s+1)(s+2)(s+3)}$$

Fourier Approach (by Bayo)

Given

$$\frac{y(s)}{u(s)} = \frac{(s - 1)}{(s+2)(s+3)}$$

Find the inverse of a desired output y_d

$$u_{inv}(s) = \frac{(s+2)(s+3)}{(s - 1)} y_d(s)$$

$$u_{inv}(j\omega) = \frac{(j\omega + 2)(j\omega + 3)}{(j\omega - 1)} y_d(j\omega)$$

This inverse is bounded but non-causal (Bayo)

Extension to Nonlinear Systems?

Time-Domain Inversion: The Linear Case

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t), \\
y(t) &= Cx(t)
\end{aligned}
$$

Find $u_{ff}(t)$ by differentiating $y(t)$:

$$
\begin{aligned}
\dot{y}(t) &= C\dot{x}(t) = CAx(t) + CBu(t) = CAx(t) \\
&\vdots \\
y_d^{(r)}(t) &= M_x x(t) + M_u u(t)
\end{aligned}
$$

$$\Downarrow$$

$$u_{ff}(t) = M_u^{-1}(y_d^{(r)}(t) - M_x x(t))$$

Inverse Control Law

Find the inverse control law

$$u_{ff}(t) = M_u^{-1}(y_d^{(r)}(t) - M_x x(t))$$

Inverse Control Law

By using $\begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} = Tx(t)$

$$\xi = \begin{bmatrix} y \\ \dot{y} \\ \cdots \\ y^{(r-1)} \end{bmatrix}$$

to rewrite inverse control law as

$$u_{ff}(t) = M_\eta \eta(t) + M_Y Y_d(t)$$

and system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) \end{aligned}$$

$$Y_d = \begin{bmatrix} \xi_d & y_d^{(r)} \end{bmatrix}^T$$

as

$$\dot{\eta}(t) = A_\eta \eta(t) + B_\eta Y_d(t)$$

Internal Dynamics

Key: Solve the internal Dynamics

$$u_{ff}(t) = M_u^{-1}(y_d^{(r)}(t) - M_x x(t))$$

Inverse Control Law

By using $\begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} = Tx(t)$

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$$Y_d = \begin{bmatrix} \xi_d & y_d^{(r)} \end{bmatrix}^T$$

as

$$\dot{\eta}(t) = A_\eta \eta(t) + B_\eta Y_d(t)$$

Internal Dynamics

Solving the (unstable) internal dynamics

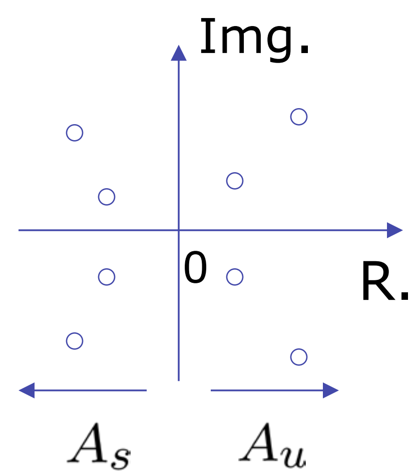
To solve the internal dynamics

$$\dot{\eta}(t) = A_\eta \eta(t) + B_\eta Y_d(t), \qquad (1)$$

$$A_\eta = T \begin{bmatrix} A_s & 0 \\ 0 & A_u \end{bmatrix} T^{-1} \qquad \text{(a1)}$$

rewrite Eq. (1) by state-transformation (a1) as:

$$\begin{aligned} \dot{\eta}_s(t) &= A_s \eta_s(t) + B_s Y_d(t), && (2) \\ \dot{\eta}_u(t) &= A_u \eta_u(t) + B_u Y_d(t) && (3) \end{aligned}$$



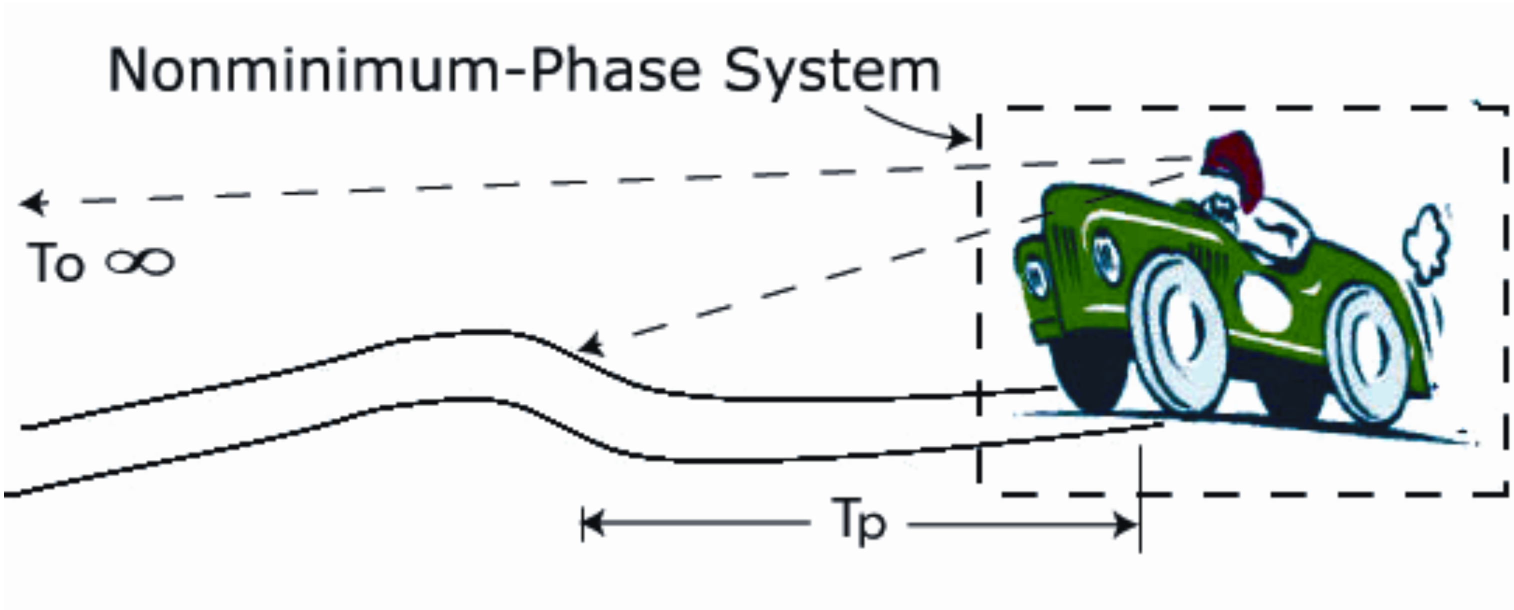
Solve Eqs. (2) and (3) by:

$$\begin{aligned} \eta_s(t) &= \int_{-\infty}^{t} e^{A_s(t-\tau)} B_s Y_d(\tau) d\tau && (4) \\ \eta_u(t) &= -\int_{t}^{\infty} e^{-A_u(\tau-t)} B_u Y_d(\tau) d\tau && (5) \end{aligned}$$

Noncausal!

▪ ***Question**: How much preview time do we need to compute the inverse input within desired precision?*

Physical intuition: Car Driving Example



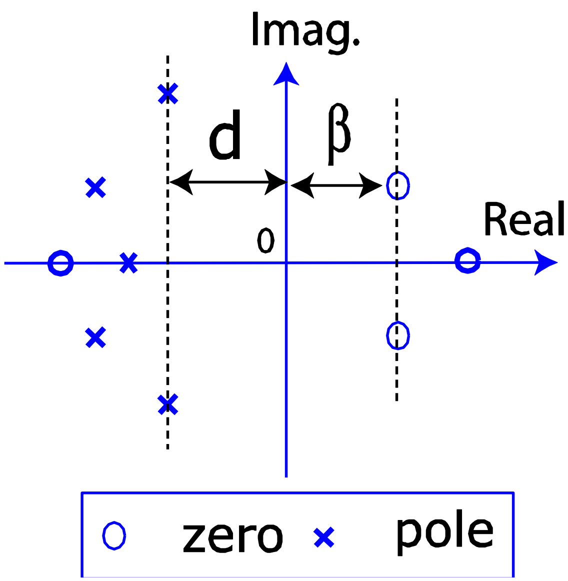
$$\eta_s(t) = \int_{-\infty}^{t} e^{A_s(t-\tau)} B_s Y_d(\tau) d\tau \quad (4)$$

$$\eta_u(t) = -\int_{t}^{\infty} e^{-A_u(\tau-t)} B_u Y_d(\tau) d\tau \quad (5)$$

$$\bar{\eta}_u(t) = -\int_{t}^{t+T_p} e^{-A_u(\tau-t)} B_u Y_d(\tau) d\tau$$

▪ ***Question:*** *How much preview time do we need to compute the inverse input within desired precision?*

$$\bar{\eta}_u(t) = -\int_t^{t+T_p} e^{-A_u(\tau - t)} B_u Y_d(\tau) d\tau$$



Preview time: $T_p^* \approx \frac{4 \sim 6}{\beta}$

Settling time: $t_s \approx \frac{4 \sim 6}{d}$

Finding the inverse control law

$$u_{ff}(t) = M_u^{-1}(y_d^{(r)}(t) - M_x x(t))$$

Inverse Control Law

By using
$$\begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} = Tx(t)$$

to rewrite inverse control law as

$$\xi = \begin{bmatrix} y \\ \dot{y} \\ \dots \\ y^{(r-1)} \end{bmatrix}$$

$$u_{ff}(t) = M_\eta \eta(t) + M_Y Y_d(t)$$

and system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) \end{aligned}$$

$$Y_d = \begin{bmatrix} \xi_d & y_d^{(r)} \end{bmatrix}^T$$

as

$$\dot{\eta}(t) = A_\eta \eta(t) + B_\eta Y_d(t)$$

Internal Dynamics

Nonlinear Stable-Inversion

Linear Case:

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t)$$

Find $u_{ff}(t)$ by differentiating $y(t)$:

$$y_d^{(r)}(t) = M_x x(t) + M_u u(t)$$

$$\Downarrow$$

$$u_{ff}(t) = M_u^{-1}(y_d^{(r)}(t) - M_x x(t))$$

Nonlinear Case:

$$\dot{x} = f(x) + \sum_{i=1}^{p} g_i(x) u_i,$$
$$y = h(x)$$

Find $u_{ff}(t)$ by differentiating $y(t)$:

$$y_d^{(r)}(t) = \psi_1(x) + \psi_2(x)u(t)$$

$$\Downarrow$$

$$u_{ff}(t) = \psi_2(x)^{-1}(y_d^{(r)}(t) - \psi_1(x)x(t))$$

Nonlinear Stable-Inversion

<u>Linear Case:</u>

$$\dot{x}(t) = Ax(t) + Bu(t), \quad \text{(a1)}$$
$$y(t) = Cx(t) \quad \text{(a2)}$$

$$u_{ff}(t) = M_u^{-1}(y_d^{(r)}(t) - M_x x(t)) \quad \text{(a3)}$$

$$\xi = \begin{bmatrix} y \\ \dot{y} \\ \dots \\ y^{(r-1)} \end{bmatrix} \qquad \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} = Tx(t)$$

$$u_{ff}(t) = M_Y Y_d(t) - M_\eta \eta(t)$$
$$\dot{\eta}(t) = A_\eta \eta(t) + B_\eta Y_d(t)$$

$$Y_d = \begin{bmatrix} \xi_d & y_d^{(r)} \end{bmatrix}^T$$

<u>Nonlinear Case:</u>

$$\dot{x} = f(x) + \sum_{i=1}^{p} g_i(x) u_i, \quad \text{(b1)}$$
$$y = h(x) \quad \text{(b2)}$$

$$u_{ff}(t) = \psi_2(x)^{-1}(y_d^{(r)}(t) - \psi_1(x)x(t)) \quad \text{(b3)}$$

$$\xi = \begin{bmatrix} y \\ \dot{y} \\ \dots \\ y^{(r-1)} \end{bmatrix} \qquad \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} = T(x(t))$$

$$u_{ff}(t) = \mathcal{P}(\eta(t), Y_d(t))$$
$$\dot{\eta}(t) = s(\eta(t), Y_d(t)) \quad \text{(b4)}$$

Solving the nonlinear internal dynamics

$$\begin{aligned}\dot{\eta}(t) &= s[Y_d(t), \eta(t)] \\ &= A_\eta \eta + [s[Y_d, \eta] - A_\eta \eta] \\ &\triangleq A_\eta \eta + \phi(\eta, Y_d) \qquad (1)\end{aligned}$$

$$A_\eta = \left.\frac{\partial s[Y_d, \eta]}{\partial \eta}\right|_{Y_d = 0,\ \eta = 0}$$

$$A_\eta = \begin{bmatrix} A_s & 0 \\ 0 & A_u \end{bmatrix}$$

$$\phi(\eta, Y_d) \triangleq \begin{bmatrix} \phi_s \\ \phi_u \end{bmatrix}$$

$$\begin{bmatrix} \dot{\eta}_s(t) \\ \dot{\eta}_u(t) \end{bmatrix} = \begin{bmatrix} A_s & 0 \\ 0 & A_u \end{bmatrix} \begin{bmatrix} \eta_s \\ \eta_u \end{bmatrix} + \begin{bmatrix} \phi_s(\eta_s, \eta_u, Y_d) \\ \phi_u(\eta_s, \eta_u, Y_d) \end{bmatrix}$$

Picard-Like Iteration Process (2)

$$\eta_0(t) = 0, \quad \text{for } t \in \Re$$

$$\begin{aligned}\dot{\eta}_{s,m}(t) &= A_s \eta_{s,m}(t) + \phi_s(\eta_{s,m-1}(t), \eta_{u,m-1}(t), Y_d) \\ \dot{\eta}_{u,m}(t) &= A_u \eta_{u,m}(t) + \phi_u(\eta_{s,m-1}(t), \eta_{u,m-1}(t), Y_d)\end{aligned}$$

- **Challenge is to prove Convergence: Establish conditions for an argument based on the *contraction mapping theorem.***

Outline of talk

1. Brief intro to U. of Washington
2. Motivation --- nanopositioning
3. The good and the bad
4. Approach: Inversion-based feedforward
5. **Connections to ZPET, Robust, Optimal**
6. Experimental Results
7. The ugly --- unresolved challenges
8. Conclusions

Connections with other methods

- 1) Robust Feedforward
- 2) ZPET (zero phase-error tracking)

Optimal Inverse

Position = Transfer Function * Input Voltage

P = G * V

Error = desired position – achieved position

E = (Pd - P)

$$J(V) = \int_{-\infty}^{\infty} \{V^*(j\omega)R(j\omega)V(j\omega) + E_P^*(j\omega)Q(j\omega)E_P(j\omega)\}d\omega$$

Input cost   Tracking error cost

Optimal Inverse

$$J(V) = \int_{-\infty}^{\infty} \{V^*(j\omega)R(j\omega)V(j\omega) + E_P^*(j\omega)Q(j\omega)E_P(j\omega)\}d\omega$$

Input cost　　　　Tracking error cost

Such cost-function is used for **finding robust feedforward Gff, where P = G V**

$$J_{H_\infty}(G_{\mathrm{ff}}) = \left\| \begin{matrix} r(\cdot)G_{\mathrm{ff}}(\cdot) \\ q(\cdot)[1 - G_{\mathrm{ff}}(\cdot)G(\cdot)] \end{matrix} \right\|_\infty$$

but typically restricted to causal feedforward

Optimal Inverse

$$J(V) = \int_{-\infty}^{\infty} \{V^*(j\omega)R(j\omega)V(j\omega) + E_P^*(j\omega)Q(j\omega)E_P(j\omega)\}d\omega$$

Our approach: Solve over all feedforward --- causal as well as non-causal

Optimal Inverse

$$J(V) = \int_{-\infty}^{\infty} \{V^*(j\omega)R(j\omega)V(j\omega) + E_P^*(j\omega)Q(j\omega)E_P(j\omega)\}d\omega$$

Our approach: Solve over all feedforward --- causal as well as non-causal

Yields an easy to compute solution

$$V_{\text{opt}}(j\omega) = \left[\frac{G^*(j\omega)Q(j\omega)}{R(j\omega) + G^*(j\omega)Q(j\omega)G(j\omega)}\right]P_d(j\omega)$$

$$= G_{\text{opt}}^{-1}(j\omega)P_d(j\omega)$$

This is the best (& robust) feedforward …

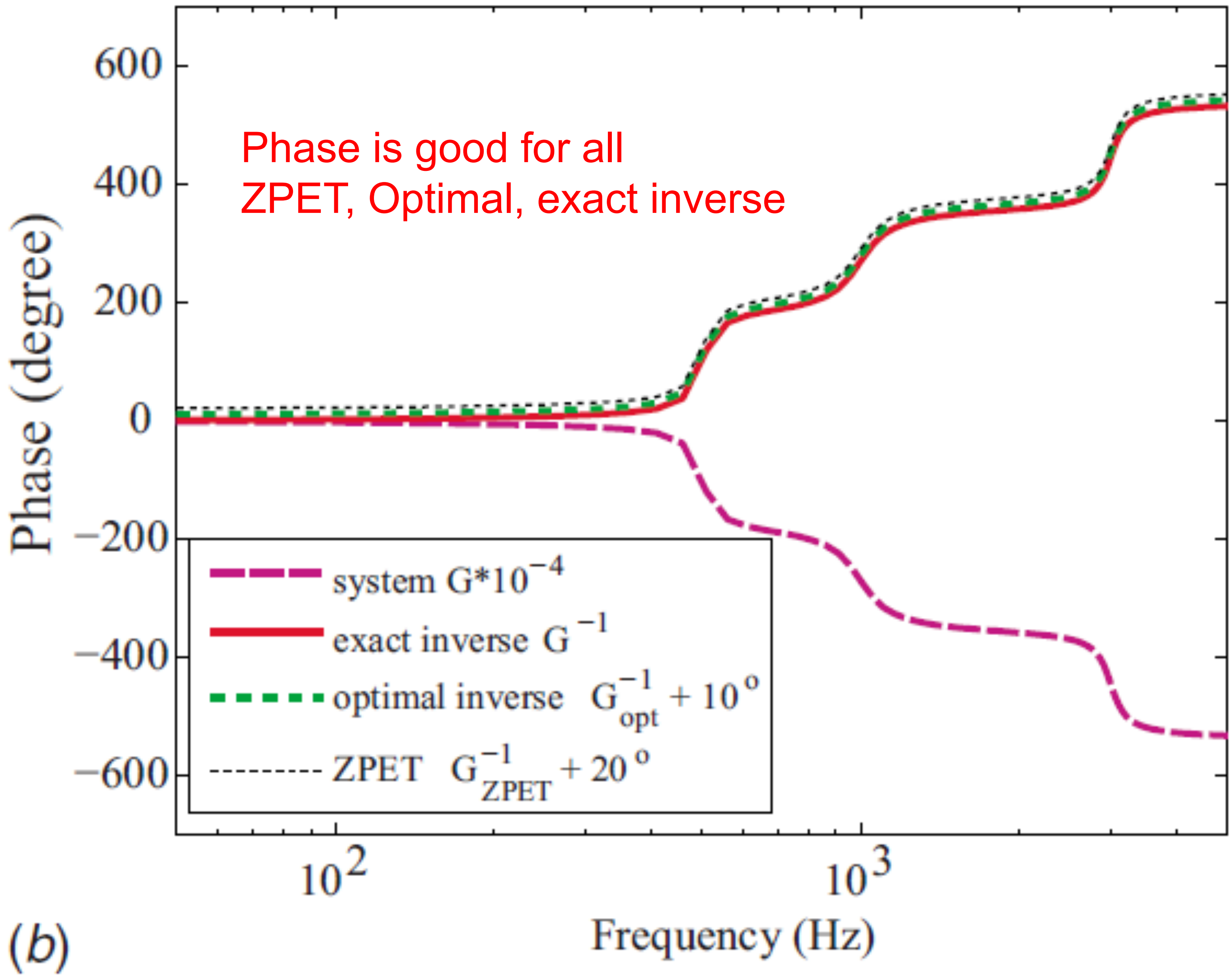
2) Comparison with ZPET

$$\frac{y(s)}{u(s)} = \frac{(s - 1)}{(s+2)(s+3)}$$

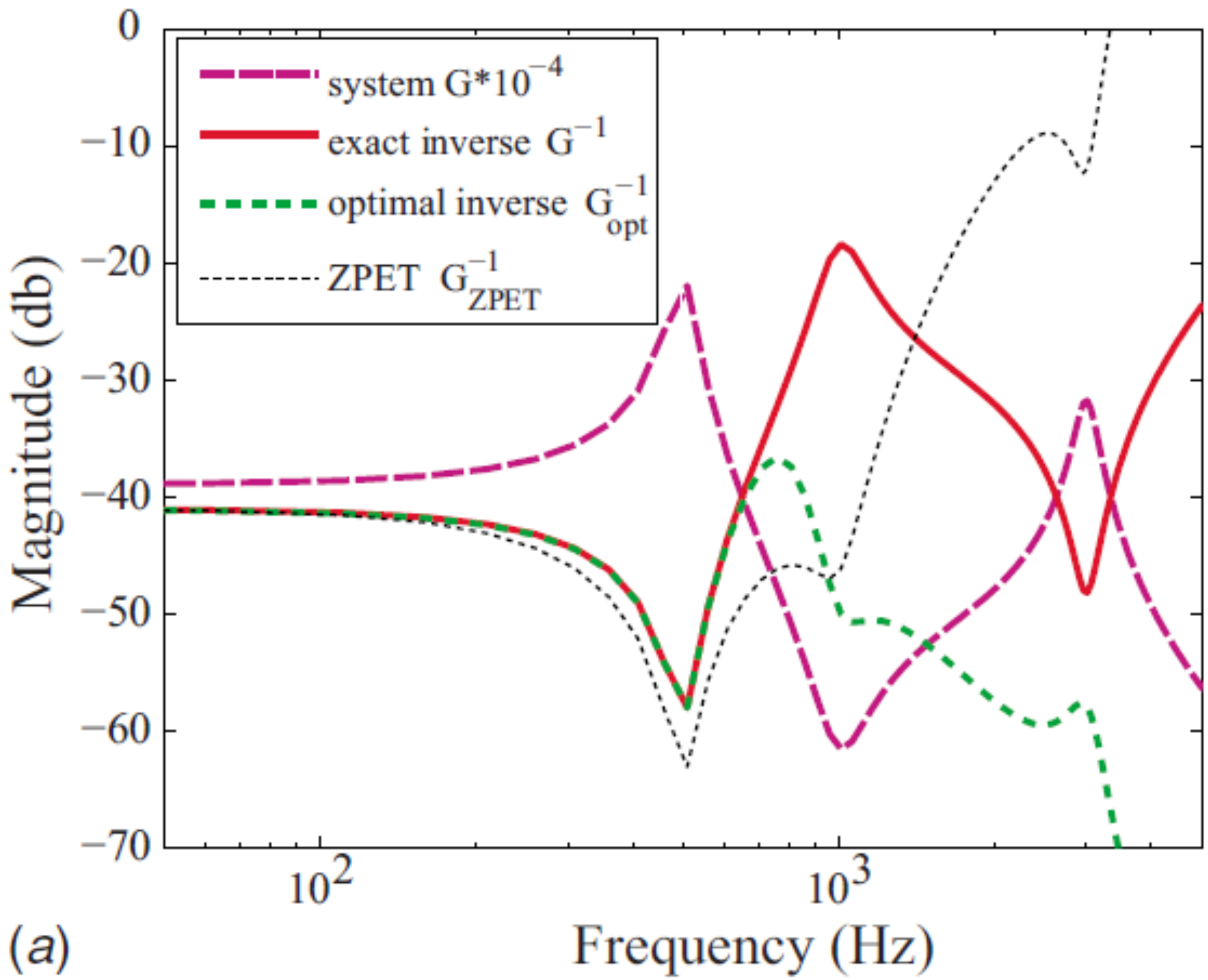
Zero phase error (replace zero by stable pole)

$$\frac{y(s)}{u(s)} = \frac{-1}{(s+1)(s+2)(s+3)}$$

Comparison with ZPET



Comparison with ZPET



Tracking bandwidth: ZPET < Optimal Inverse < Exact Inverse

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Nanoscale Positioning in AFM

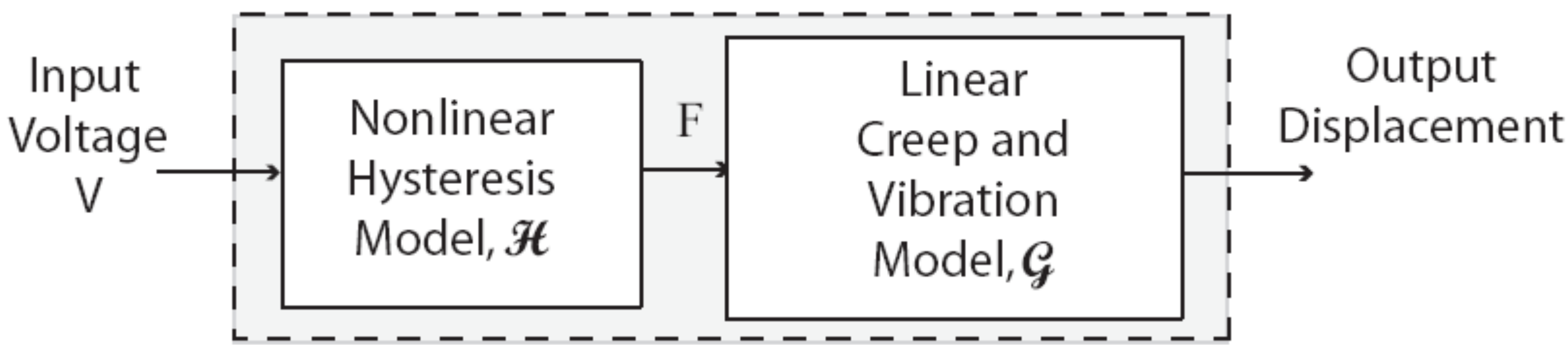
- **Three problems**

- 1) Creep
- 2) Hysteresis
- 3) Vibrations

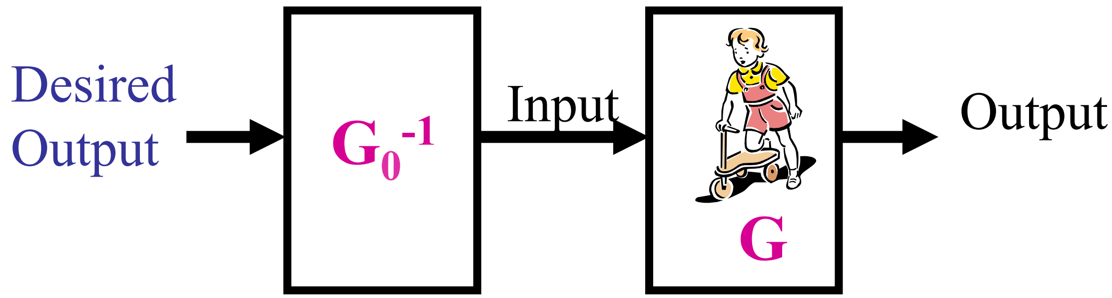
Key Issues in Modeling

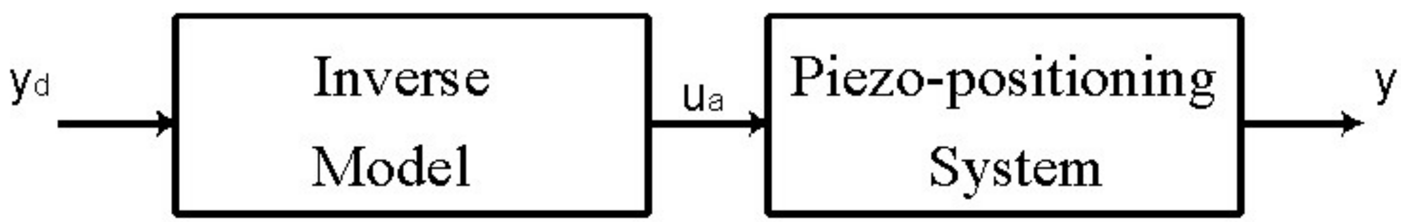
Need to capture all three effects: nonlinear Hysteresis, linear creep and vibrations

Modeling should account for the coupling between these effects --- For example, some of the time dependence of hysteresis might be modeled as linear creep!



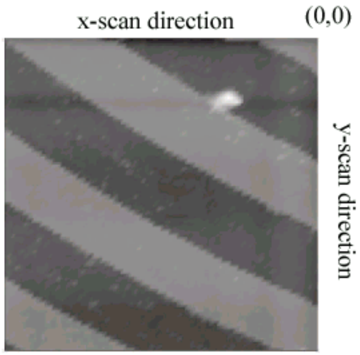
Use in Piezo Nanopositioners



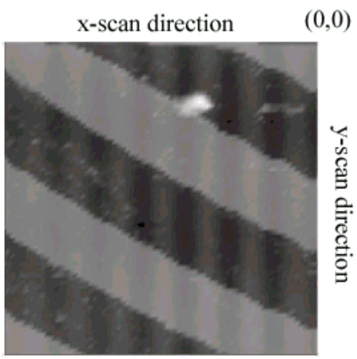


- System inverse is used to find input voltages, u_a , which compensate for positioner dynamics and achieve the desired output, i.e. $y = y_d$

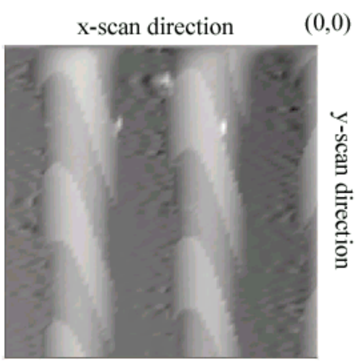
Application to Atomic Force Microscope



a) No Compensation (5 Hz)



c) No Compensation (30 Hz)



e) No Compensation (100 Hz)

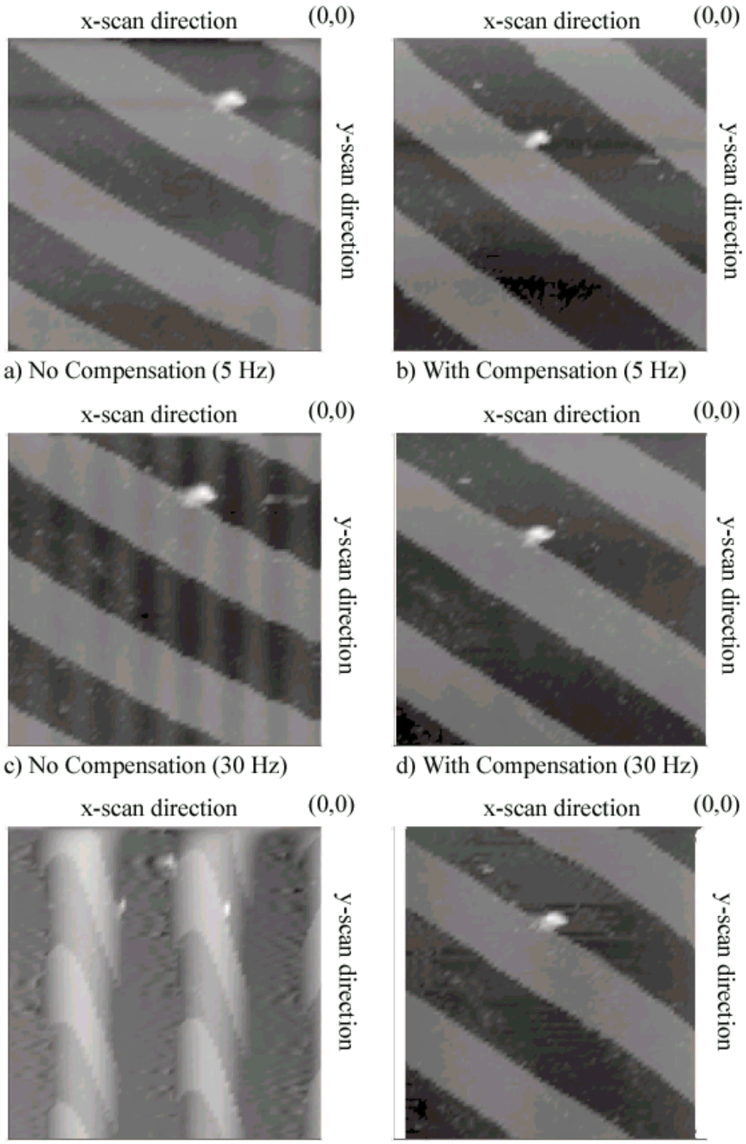
Large-range Image (50 microns) compared to sub-nano for STM….

Distortions in images due to positioning errors

(a) Creep and Hysteresis at low speeds

(b) Vibrations as speed is increased

Application to Atomic Force Microscope



a) No Compensation (5 Hz)

b) With Compensation (5 Hz)

c) No Compensation (30 Hz)

d) With Compensation (30 Hz)

e) No Compensation (100 Hz)

f) With Compensation (100 Hz)

We increased the scan speeds from 1-2 Hz to about 100Hz

Key point --- All three effects -- creep, hysteresis, and vibration --- can be corrected with feedforward

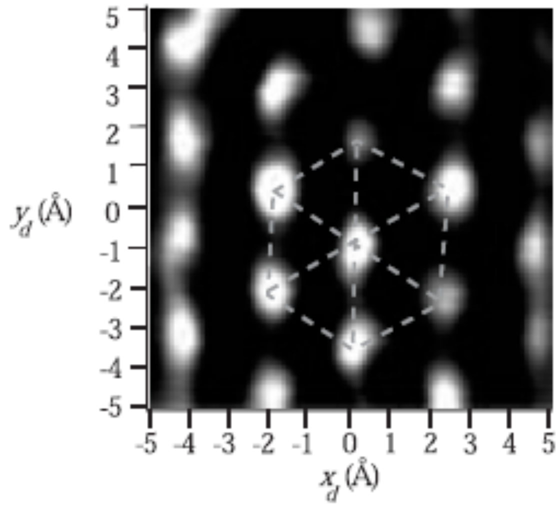
feedback can improve results further

Image-based Sub-nano Control

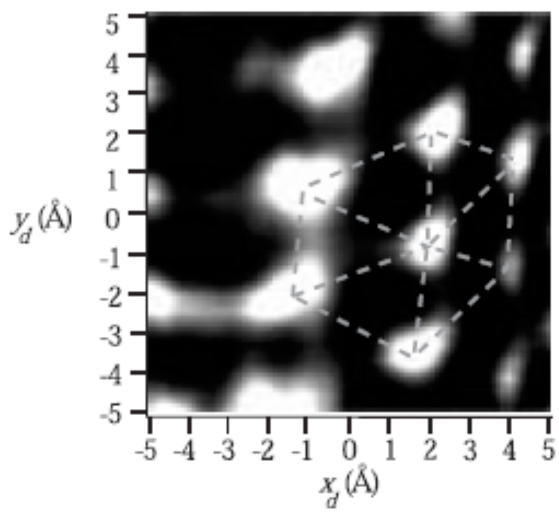
Goal: **High-speed Sub-angstrom positioning --- Image size is about 1nm (carbon atoms in graphite)**

Sensors do not have high-resolution and high bandwidth (noise issues)

Sensors cannot measure lateral position of atomic tip of SPM probe directly --- esp if you are using large arrays of probes



(a)

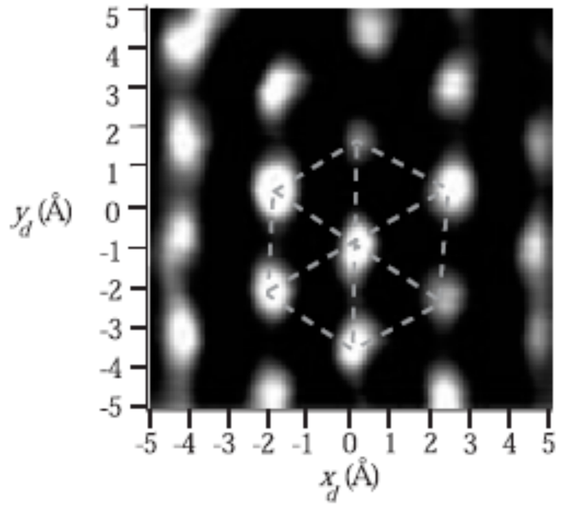


(b)

Key Idea

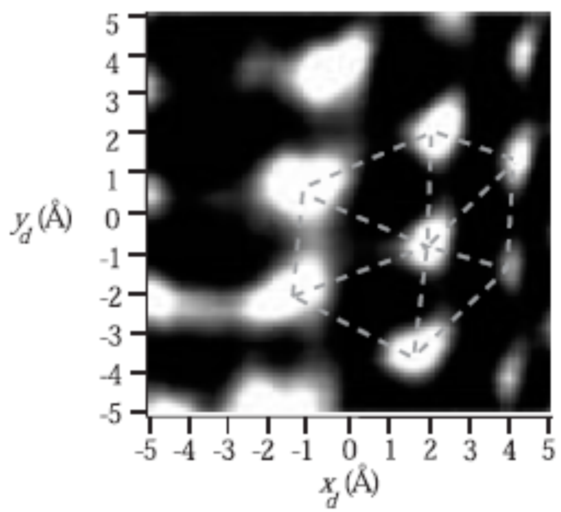
Distortion of the image

has information about positioning errors



(a)

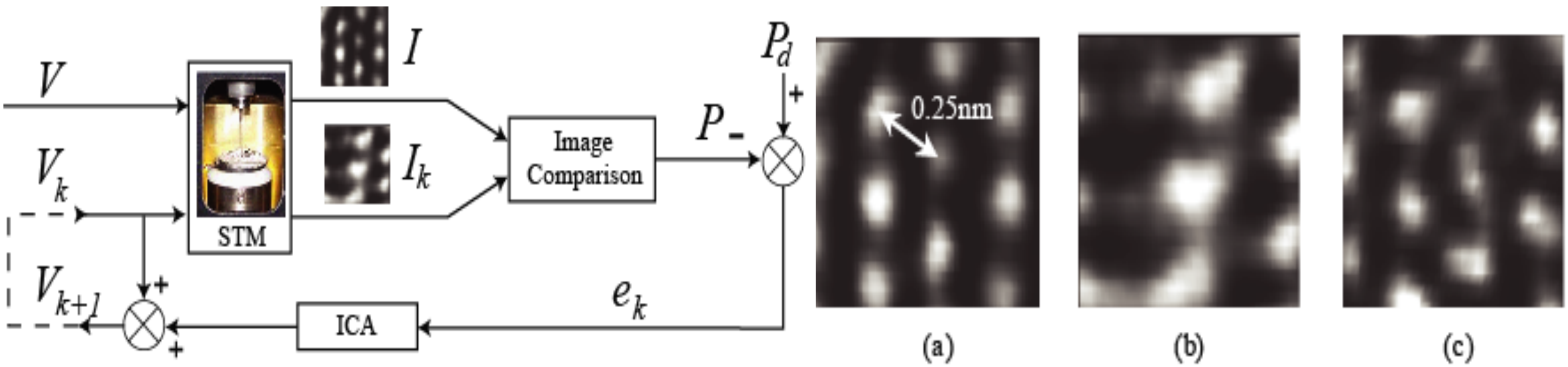
USE DISTORTION TO CORRECT DISTORTION



(b)

compare low and high frequency images to obtain positioning error and then find inputs to correct the distortions

Image-based Iterative Control



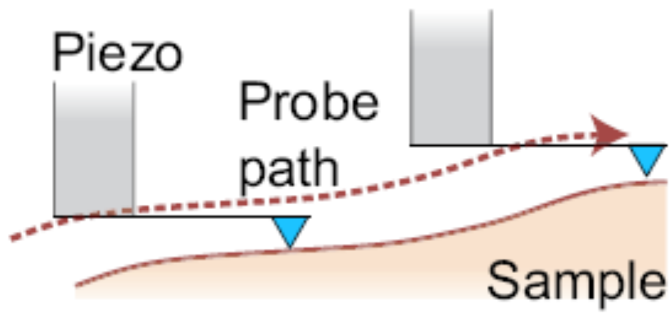
Iteration Scheme: Compare images; find error; correct.
Results: Able to recover periodic lattice in image
Advantage:
Increase throughput
Does not need external sensors
Can be used with **large arrays of sensors and actuators**

Current Efforts

- Imaging Soft Samples: in particular micro-vascular endothelial cells

Inversion-based approach

$$V_{inv}(j\omega) \ = G^{-1}(j\omega)P_d(j\omega).$$

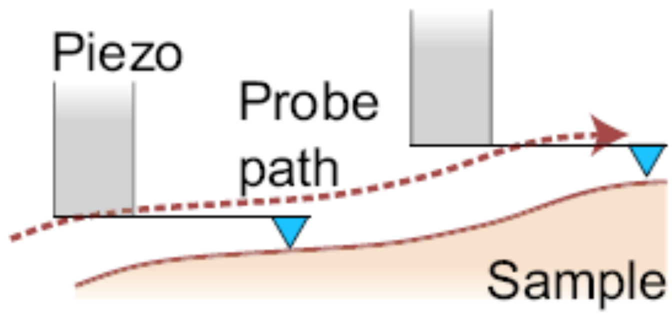


Pd is the desired position over the cell and G is the model of the positioning dynamics

Problem with inversion

Inversion Approach for precision positioning

$$V_{inv}(j\omega) \ = G^{-1}(j\omega)P_d(j\omega).$$



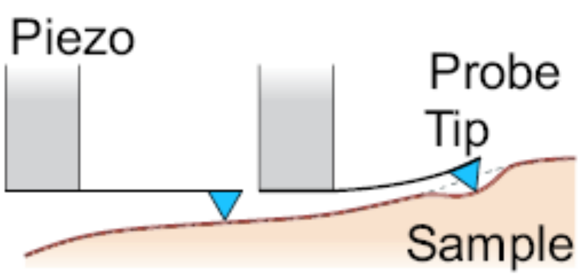
Problem: Don't know the cell profile Pd before imaging
→ so we cannot find the inverse input!

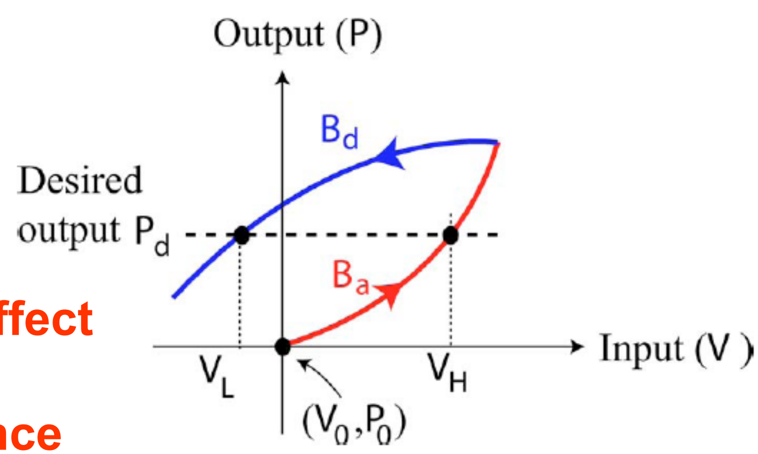
Approach: Iterative control

Apply some input; find error and then correct iteratively

$$V_{ff,k+1}(j\omega) = V_{ff,k}(j\omega) + \rho(j\omega)\left[G^{-1}(j\omega)\right]\left[E_k(j\omega)\right]$$

Only need the measured error (excess deflection)





Need to worry about convergence!

(a) Frequency domain convergence + noise effect

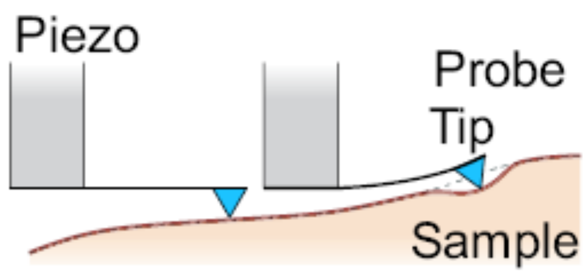
(b) Nonlinear Hysteresis effects on convergence

Approach: Iterative control

Apply some input; find error and then correct iteratively

$$V_{ff,k+1}(j\omega) = V_{ff,k}(j\omega) + \rho(j\omega)\left[G^{-1}(j\omega)\right]\left[E_k(j\omega)\right]$$

Only need the measured error (excess deflection)



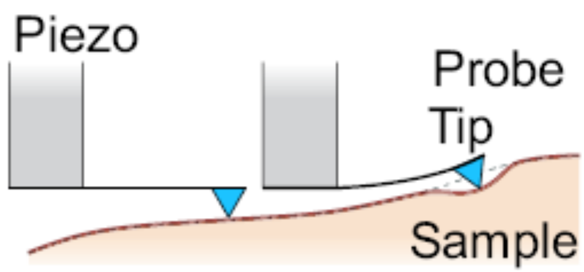
Problem --- initial error (deflection) can be too large!

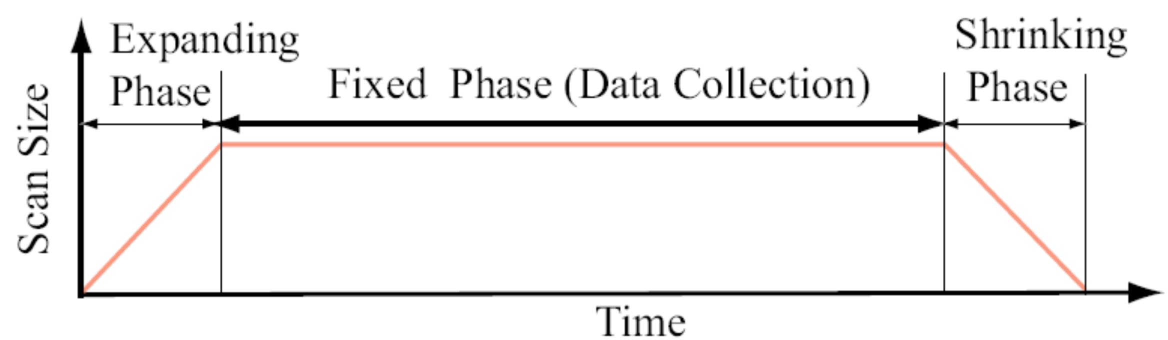
Once damaged, no point imaging further.

Zoom-out/Zoom-in Approach

Still use iterative control

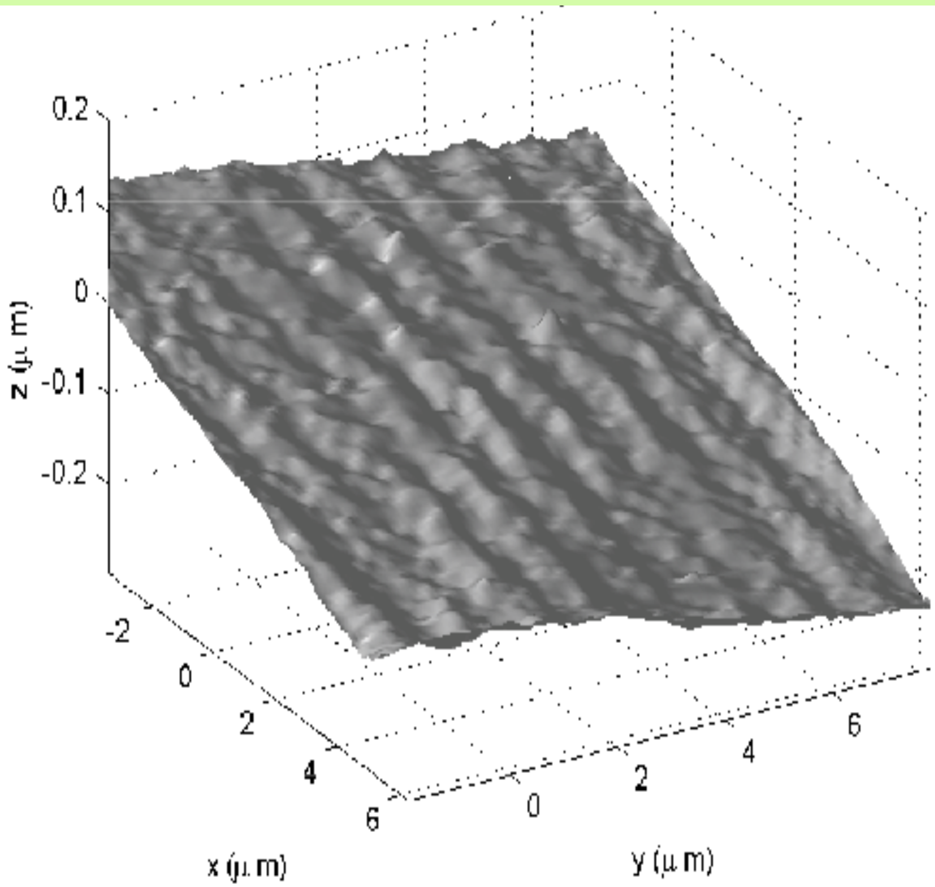
$$V_{ff,k+1}(j\omega) = V_{ff,k}(j\omega) + \rho(j\omega)\left[G^{-1}(j\omega)\right]\left[E_k(j\omega)\right]$$





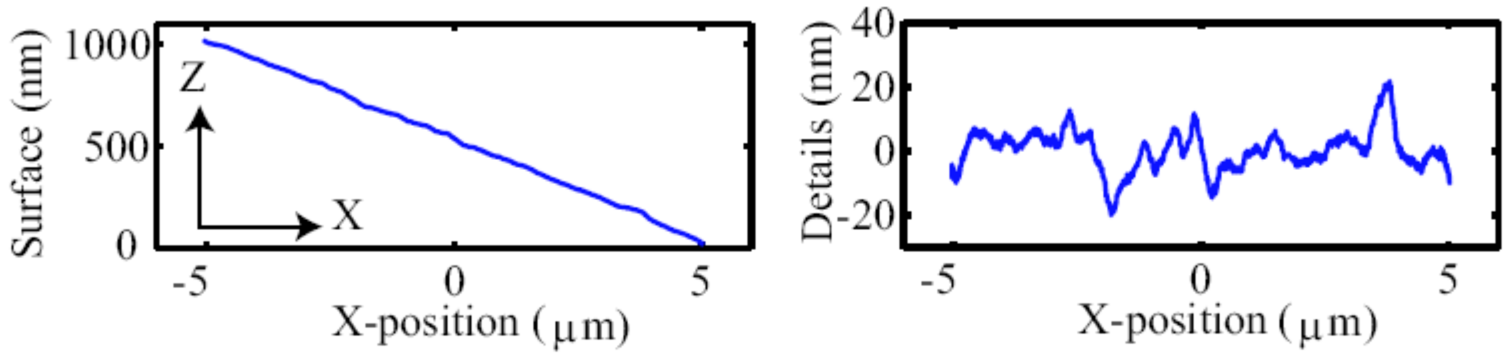
Increase scan area slowly → initial height changes are small → initial deflection (forces) are small

Results



Soft hydrogel (contact lens) samples in liquid

Details Sample



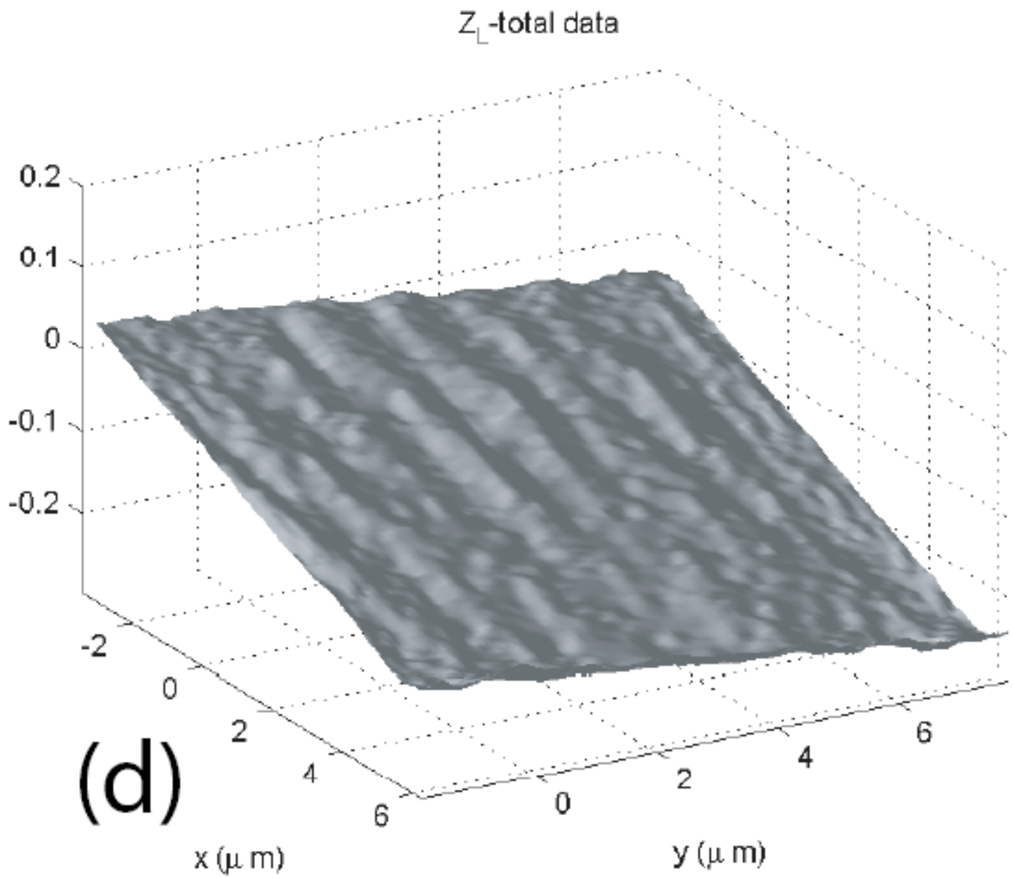
Soft Hydrogel sample (Contact lens) in saline solution

Large scan (10 micron)

Height variation = 1 micron

Sample is not changing --- so easy to compare low speed scans with high-speed scans (critical for evaluating performance)

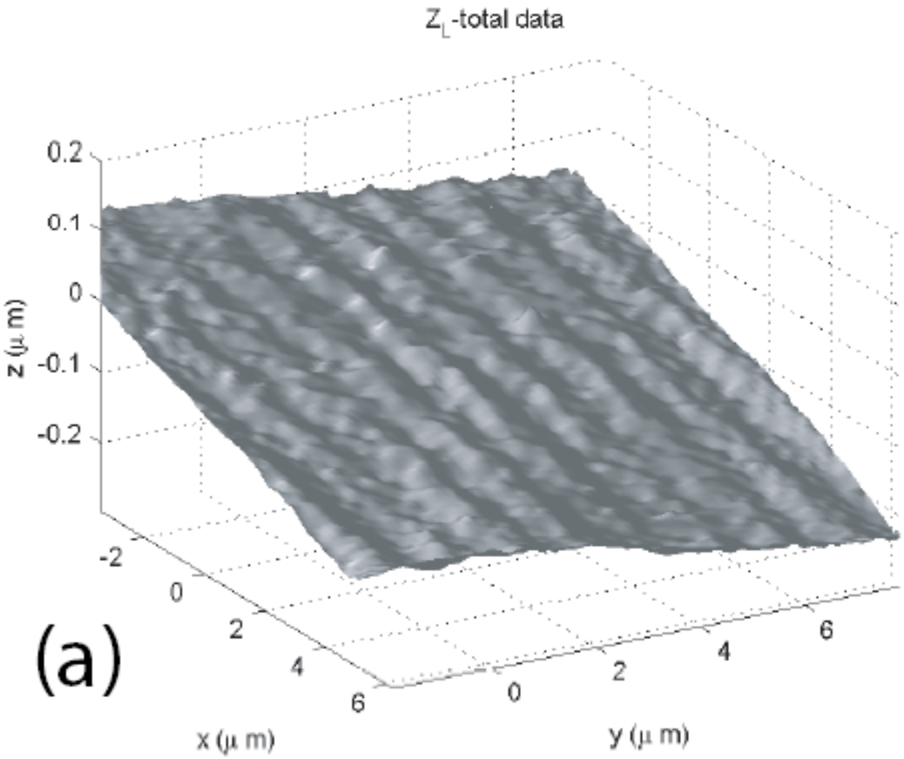
Able to image at 30 Hz



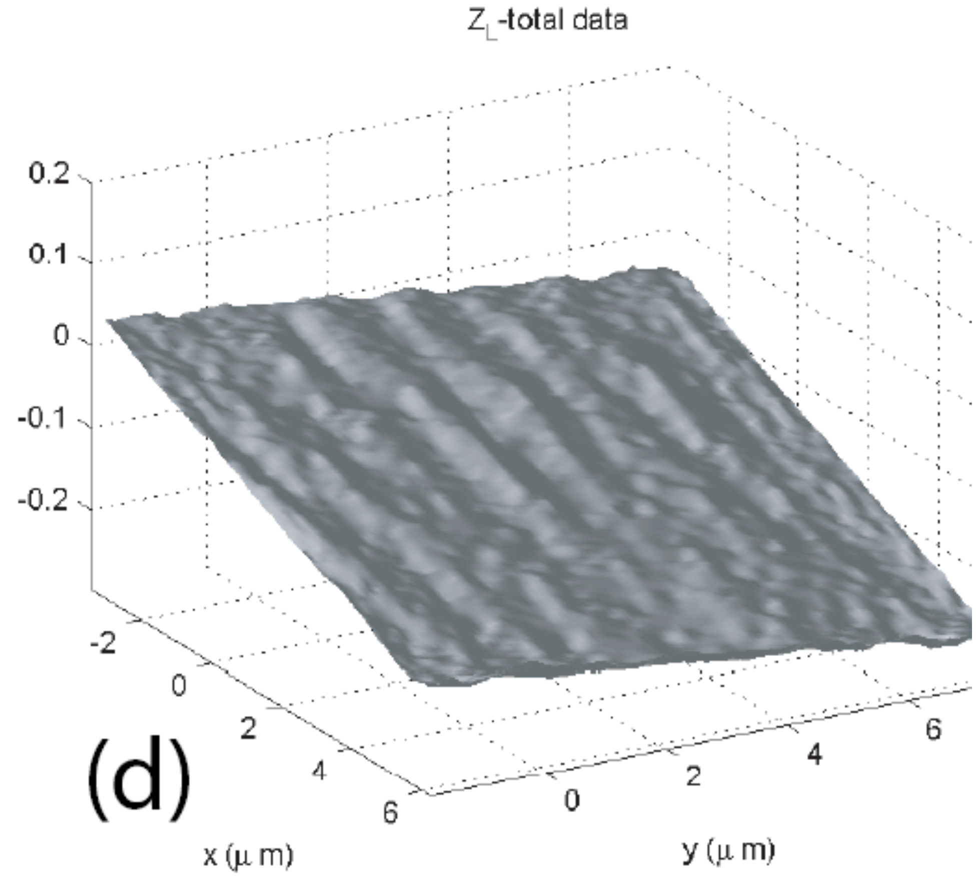
30 Hz

Forces are less than 500 pN

Image comparable to low speed



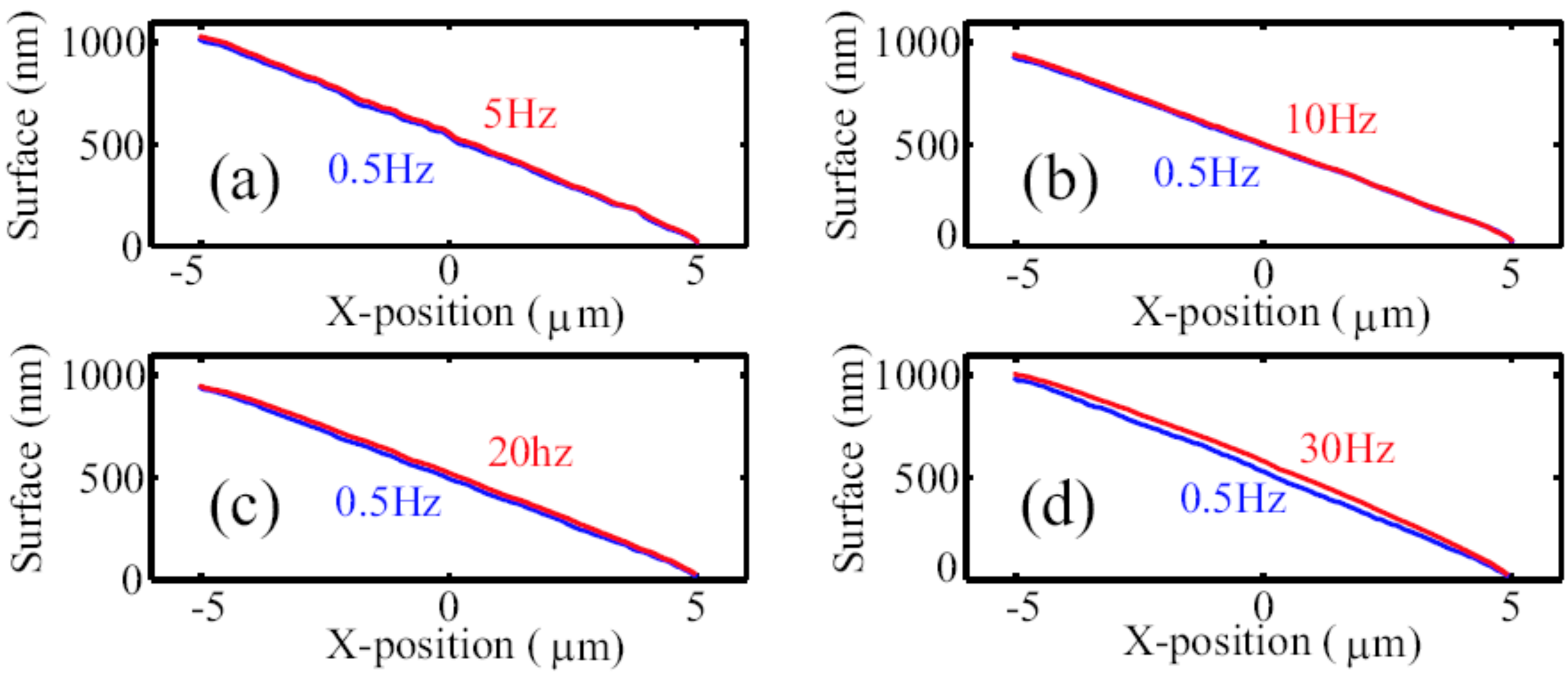
1 Hz



30 Hz

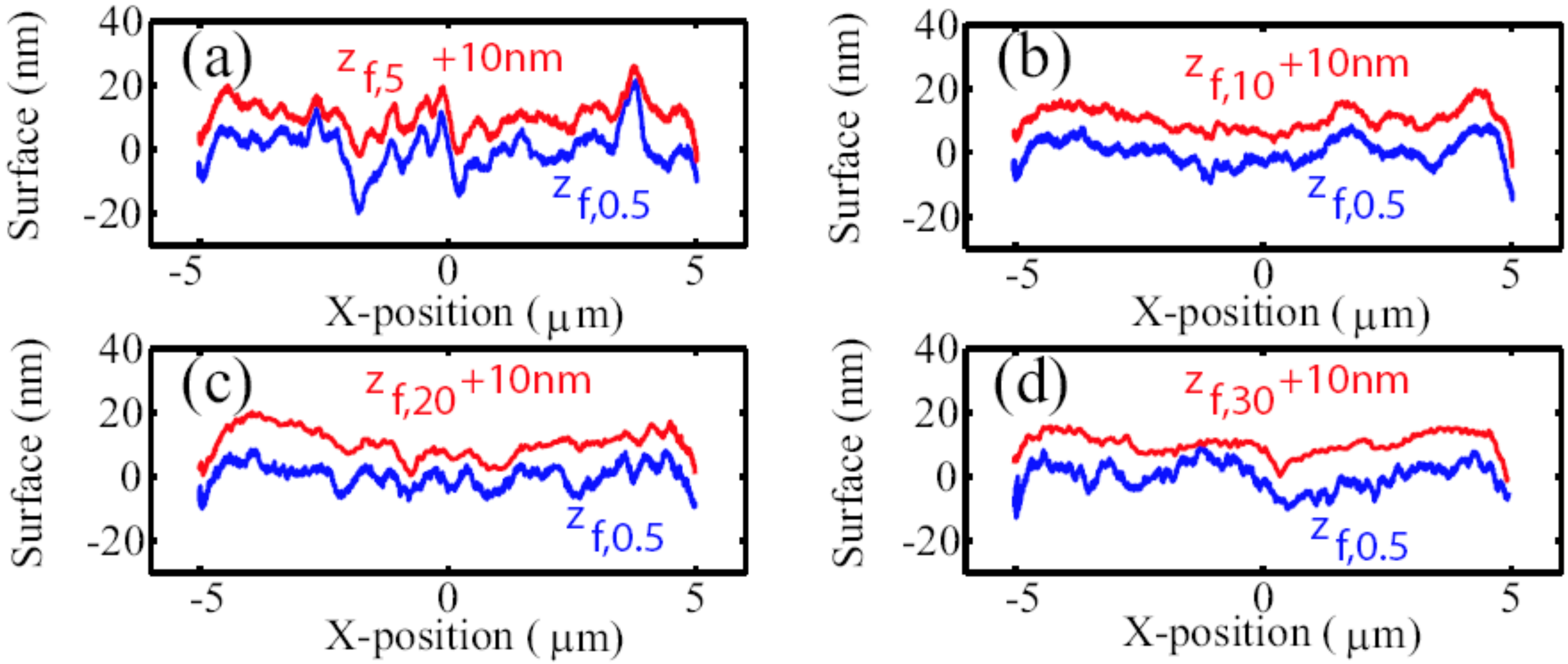
Features are similar;
Comparison is challenging; drift

Comparisons of Estimated Surface (Large scan)



Large details are reasonably easy to capture

Comparison of Estimated Surface (Details)



At 20 Hz you can still see details quite well
At 30 some of the details are being lost
Scan rate increase: 1-2 Hz to about 20 Hz (soft samples)

Note: an active research area

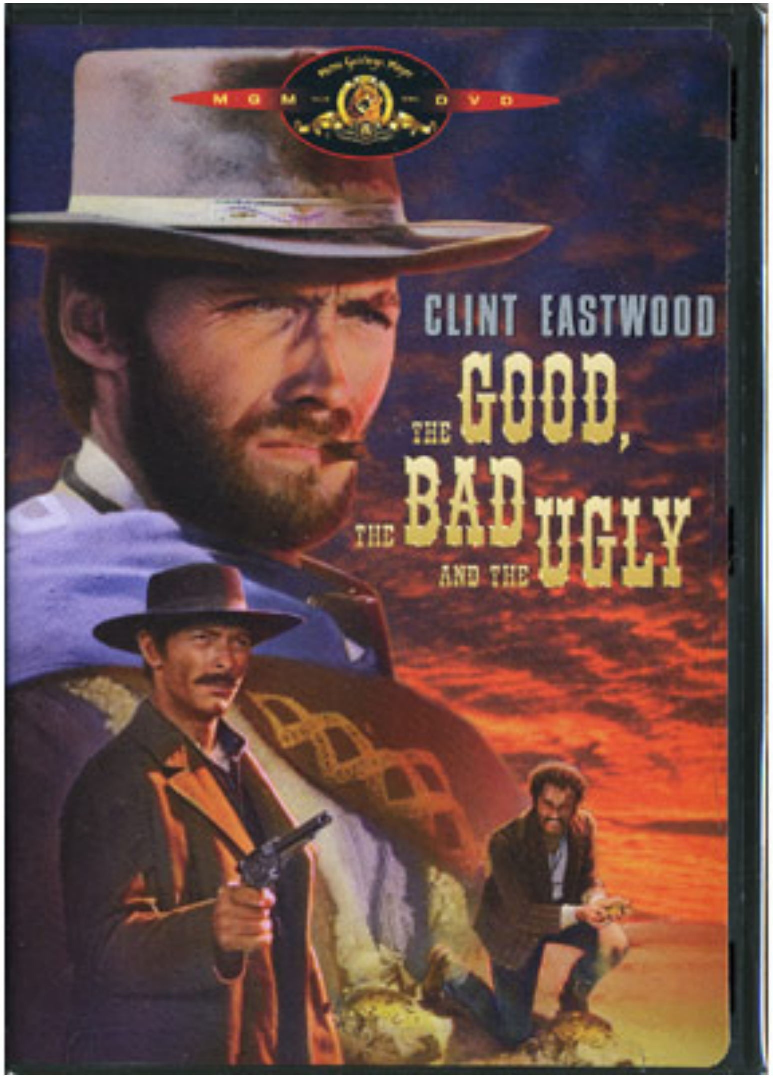
1) Qingze Zou (Rutgers) & John Bechhoefer (Simon Fraser U.) --- model-less iteration approaches
2) Kam Leang (U of Nevada) --- repetitive control methods for AFM imaging
3) Reza Moheimani (Newcastle, Australia) --- spiral scan methods to increase speeds
4) Sean Anderson (Boston U.) --- non-raster scans for tracking multiple particles
5) M Salapaka (Minnesota) --- error-estimates for measured topographies
6) ... and others (mechanics, hysteresis etc…)
7) --- Still remains difficult for soft cells at high speeds

Outline of talk

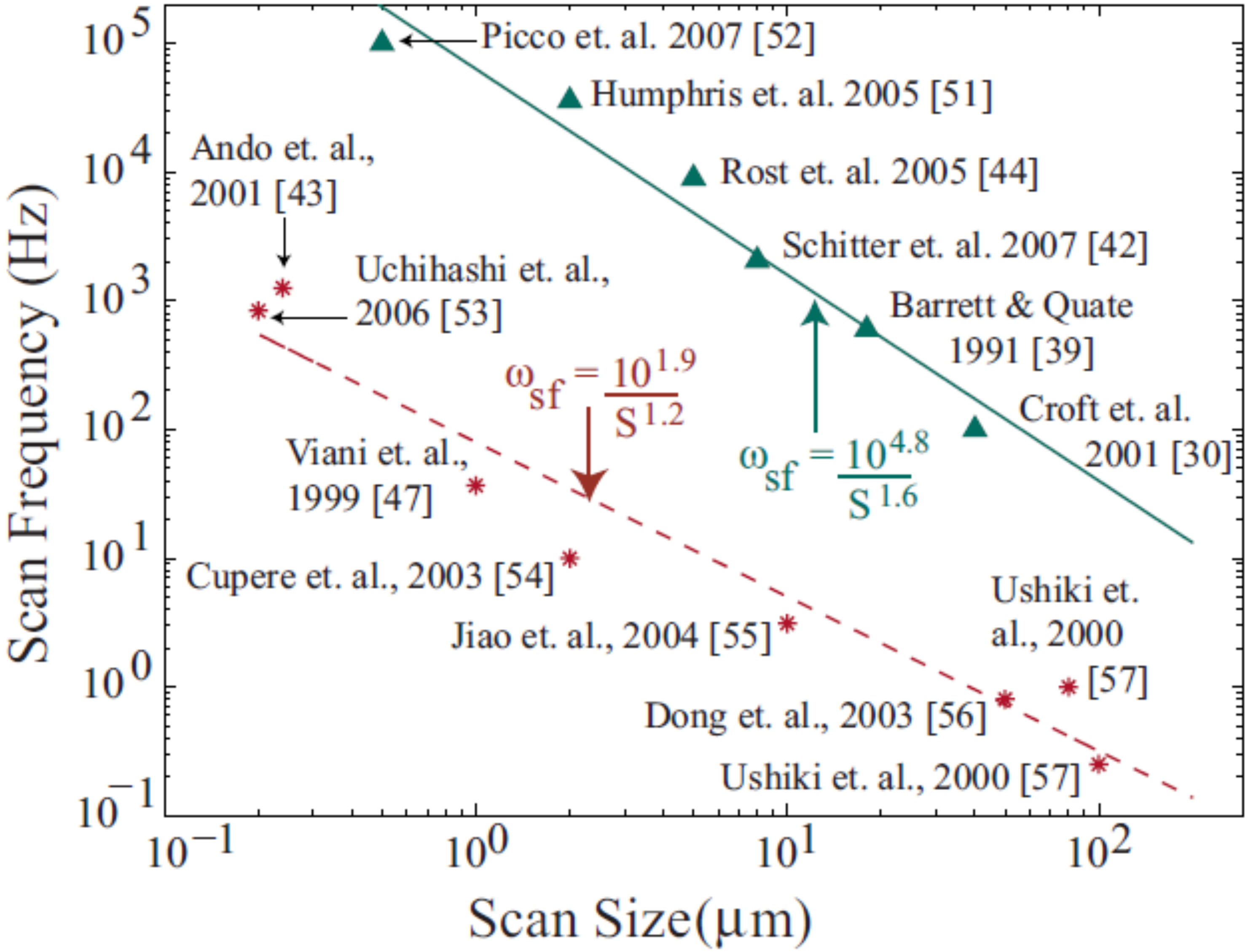
1. Brief intro to U. of Washington
2. Motivation --- nanopositioning
3. The good and the bad
4. Approach: Inversion-based feedforward
5. Connections to ZPET, Robust, Optimal
6. Experimental Results
7. **The ugly --- small range of piezos**
8. Conclusions

The good, the bad, and the ugly

Piezos have small range

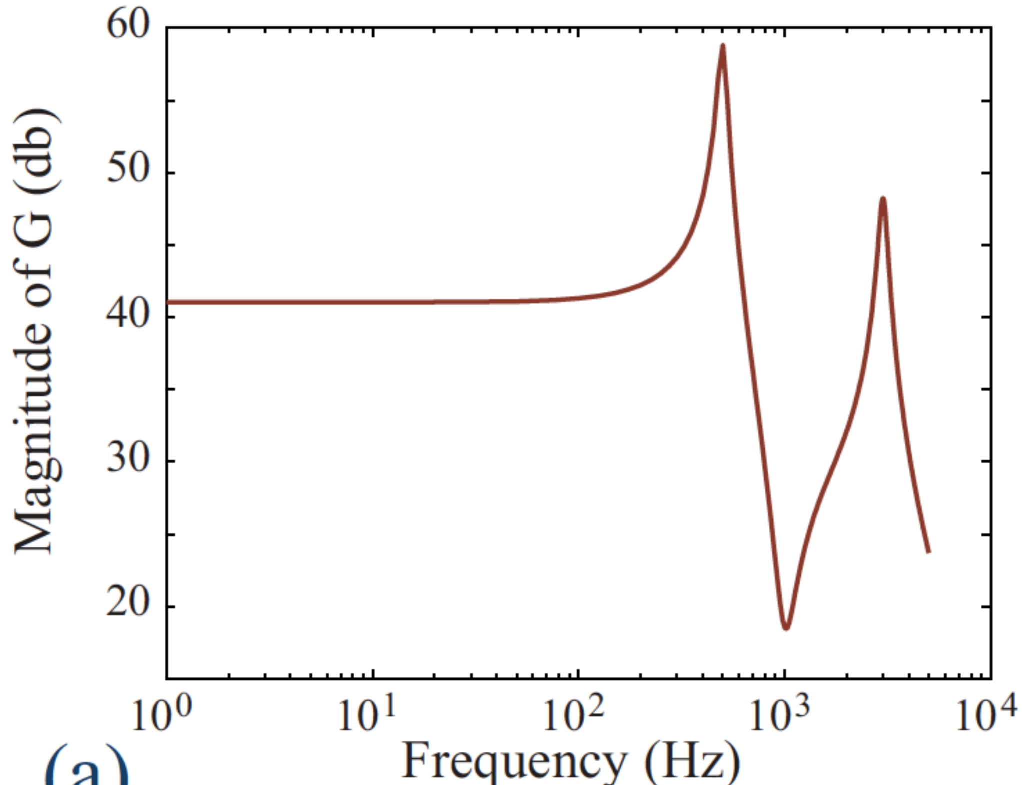


Piezos have small range --- larger piezos have smaller bandwidth



Ref: Review article in ASME J Dy. Systems, Meas. and Control, 2009

Zeros limit positioning bandwidth

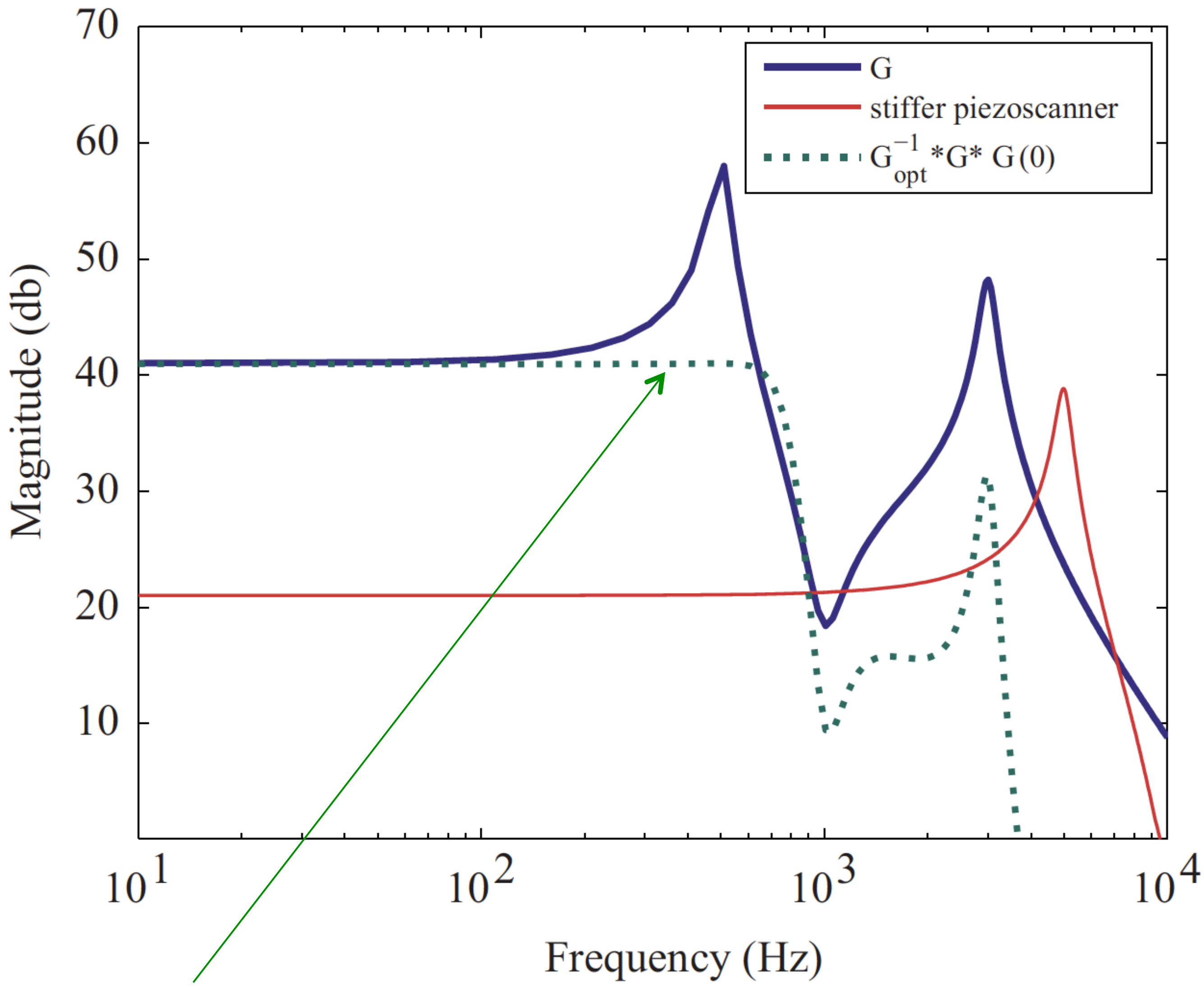


(a)

Resonances (vibrations) cause distortions in positioning --- difficult to track beyond the first resonance frequency
(approximately, the bandwidth --- frequencies up-to which we can track well)

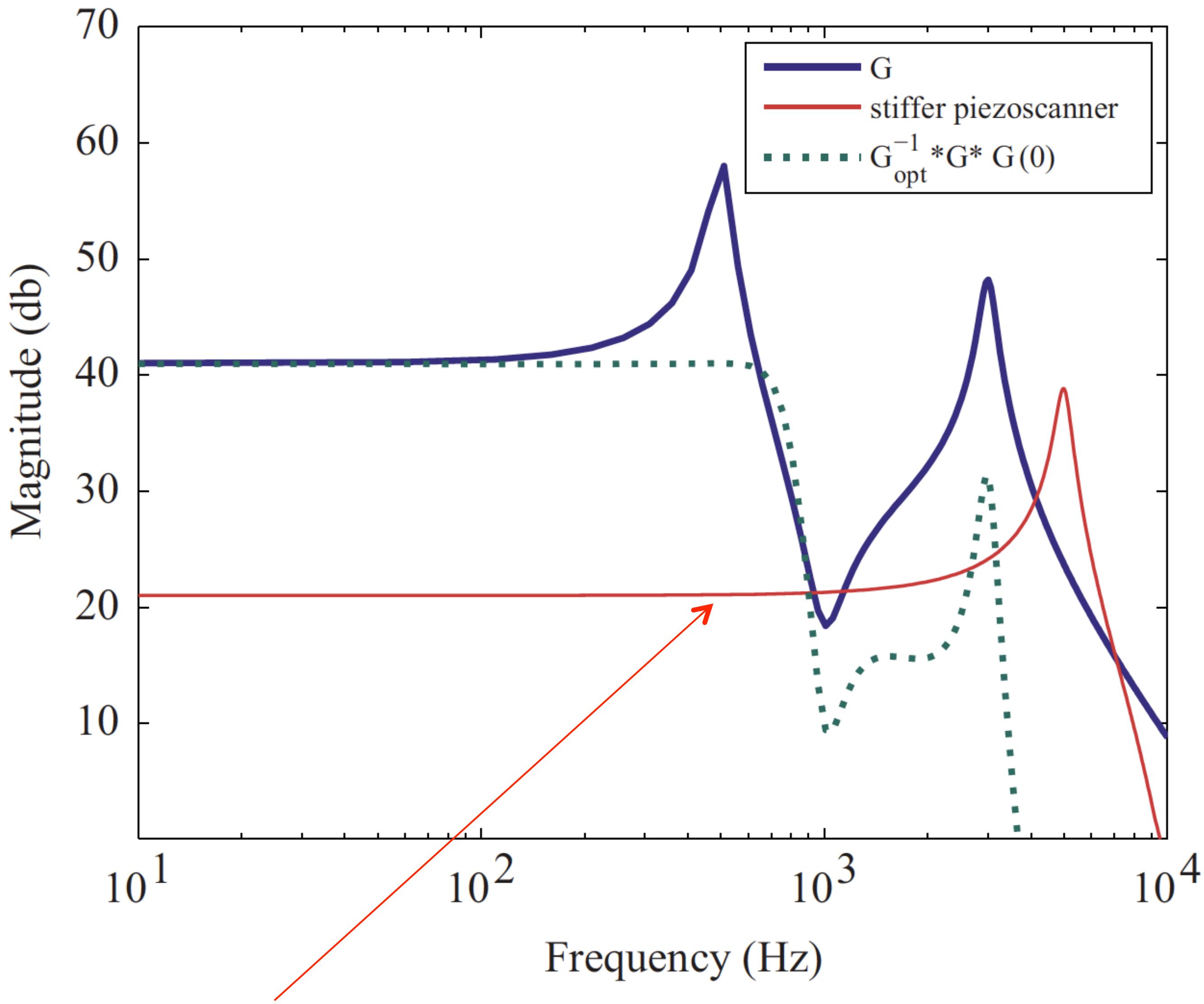
Q: how can we increase the bandwidth?

Increasing bandwidth --- with controls



Flatten the response (with controls); less vibrations but bandwidth still limited by zeros …

Increasing bandwidth --- with design



Approach 2: Use shorter piezos --- increases bandwidth since Resonance is higher --- but shorter range

Why? Resonance Frequency is inversely proportional to Size (L^2)

- First resonance freq (possible bandwidth) increases as dimensions

$$\omega_1 = \frac{1.875^2}{L^2}\sqrt{\frac{EI}{\rho A}} = \frac{1.875^2}{4L^2}\sqrt{\frac{E[D^2 + (D - 2h)^2]}{\rho}}$$

- **Piezo-tube** L= length, D = Diameter, h= thickness ρ=Density, E=Youngs Modulus

However: range is proportional to Size (L^2)

$$\omega_1 = \frac{1.875^2}{L^2}\sqrt{\frac{EI}{\rho A}} = \frac{1.875^2}{4L^2}\sqrt{\frac{E[D^2 + (D-2h)^2]}{\rho}}$$

$$R = \frac{2\sqrt{2}d_{31}L^2}{\pi D}\frac{v_{\max}}{h}$$

- **Piezo-tube** : Vmax = max voltage, d31 = piezo constant

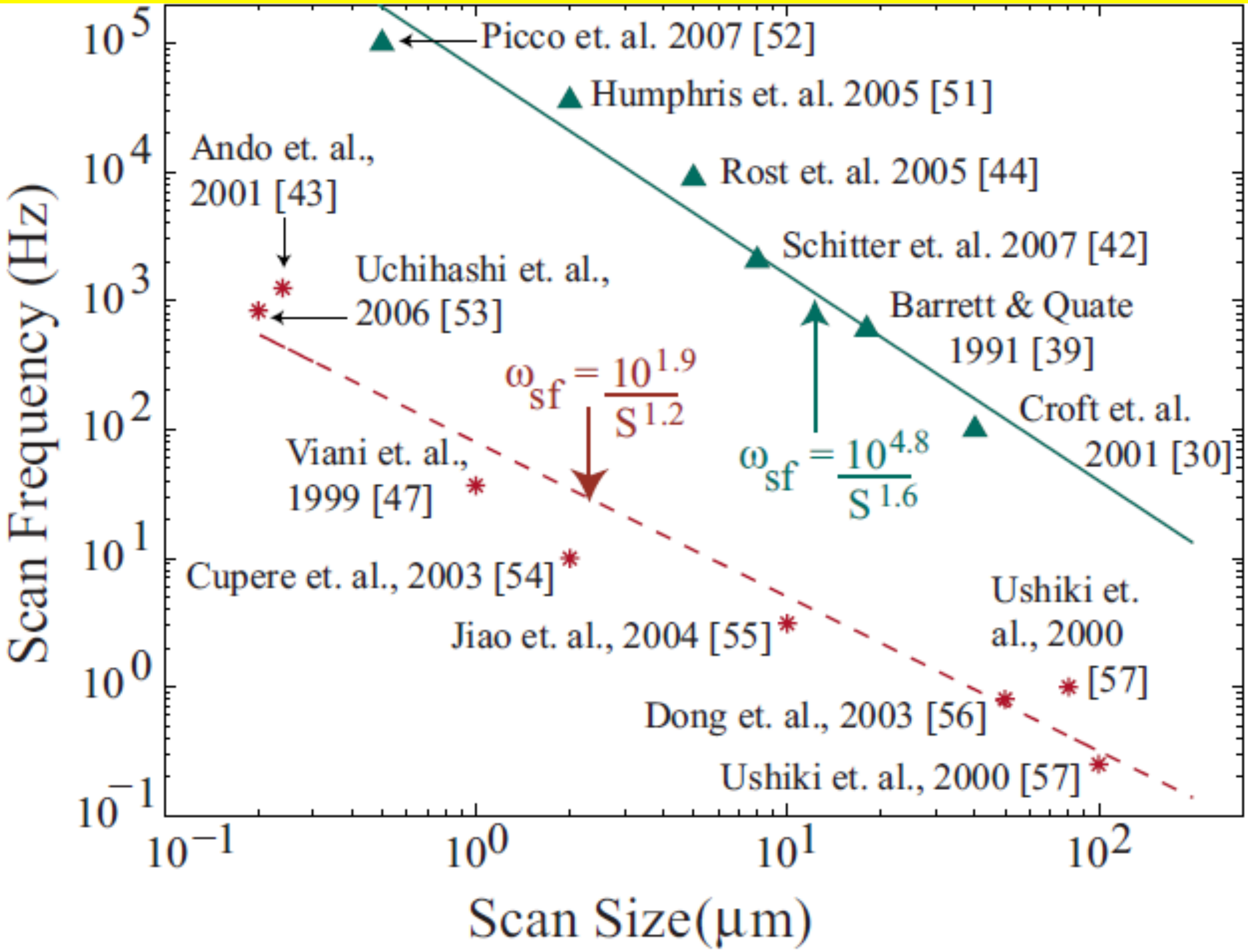
Main Problem: Smaller piezos increase bandwidth but reduce range

$$\omega_1 = \frac{1.875^2}{L^2}\sqrt{\frac{EI}{\rho A}} = \frac{1.875^2}{4L^2}\sqrt{\frac{E[D^2 + (D-2h)^2]}{\rho}}$$

$$R = \frac{2\sqrt{2}d_{31}L^2}{\pi D}\frac{v_{\max}}{h}$$

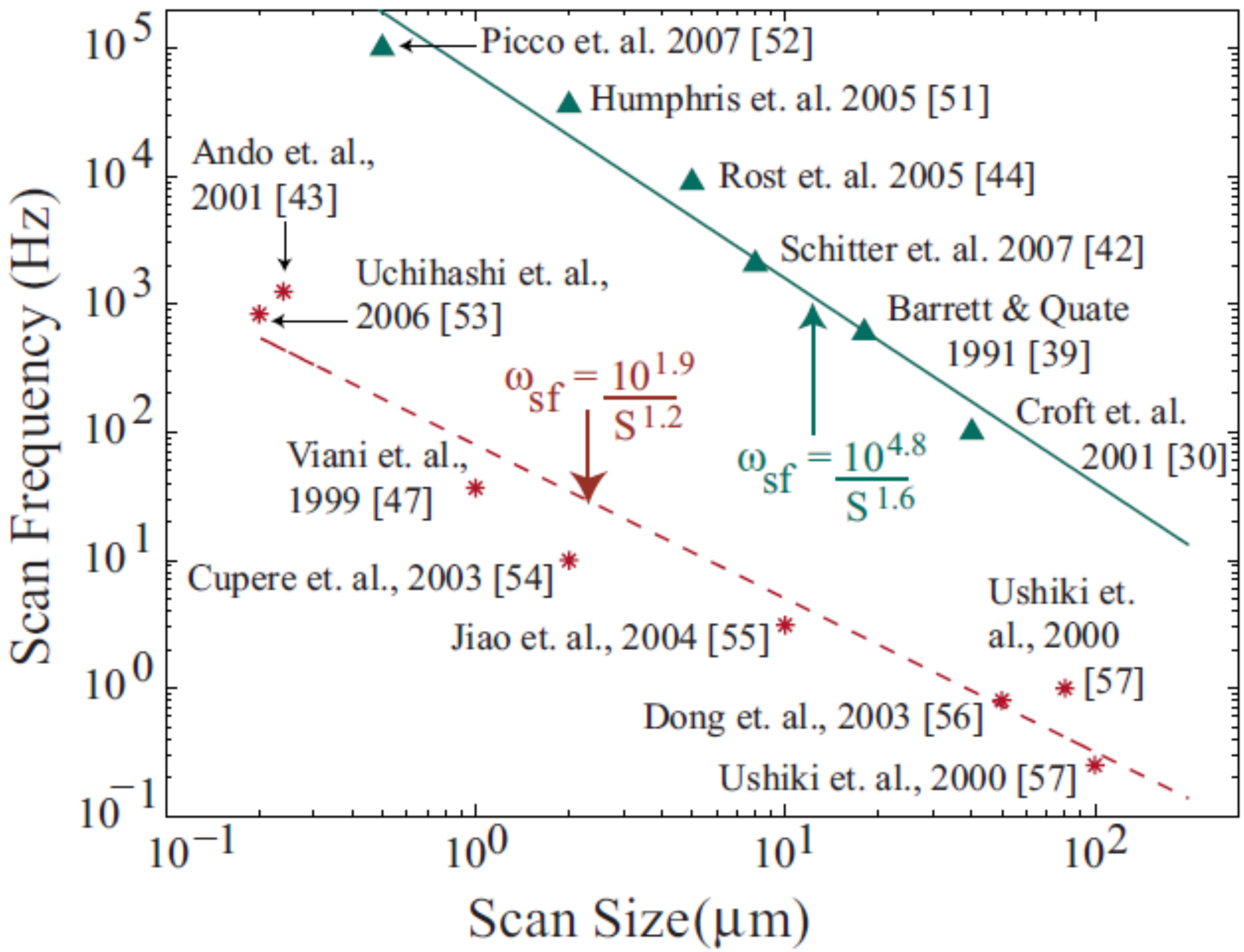
$$\omega_1 = 0.8 d_{31}\frac{v_{\max}}{h}\frac{1}{R}\sqrt{\frac{E}{\rho}[1 + (1 - 2h/D)^2]} \propto \frac{1}{R}$$

The Scan Frequency decrease with Scan Size is seen in range of SPM control methods



Ref: Review article in ASME J Dy. Systems, Meas. and Control, 2009

Scanning is even more slower for soft samples!



Slower by about 100 times on soft samples in liquid --- potential for control improvements

An unresolved issue in nanopositioning

Want high precision (piezo type positioner)

but

We also want high bandwidth & large range

Main Concept --- stepping

Piezos are small → small step (high bandwidth)

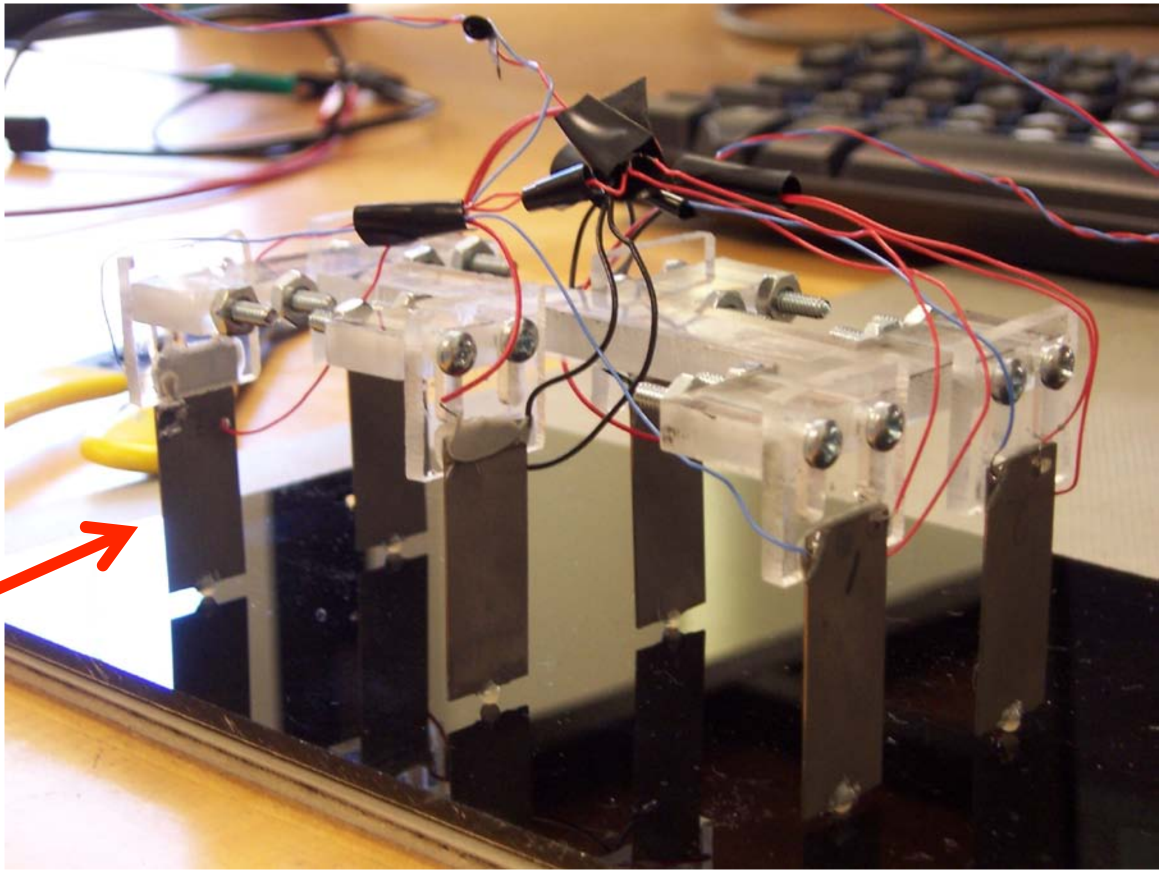
multiple steps → large overall range

Small Steps -- Large Motion

Common in nature inchworms, humans

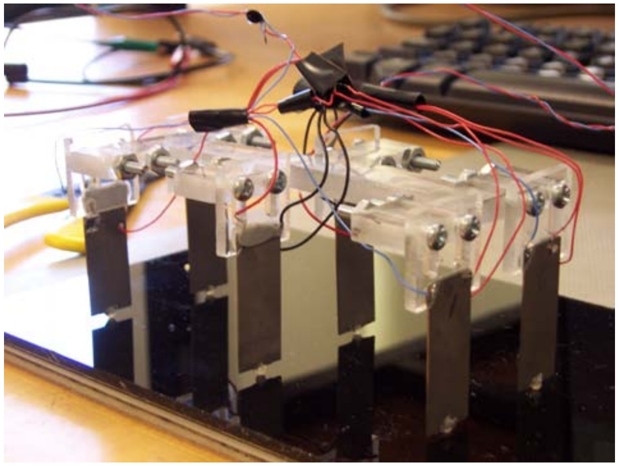
www.random-charm.com

Experimental Nanostepper System

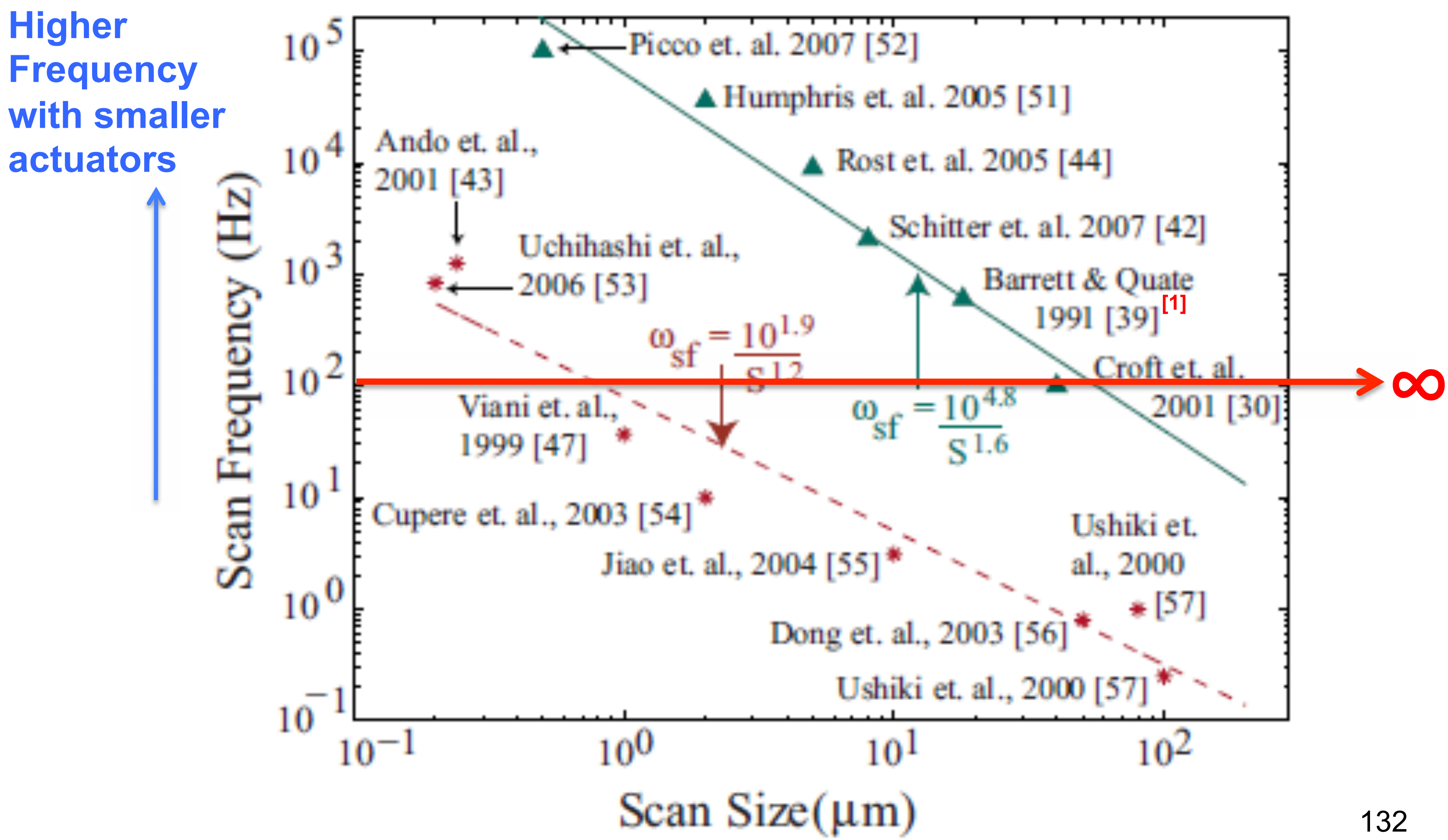


Piezo Actuators
(small range)

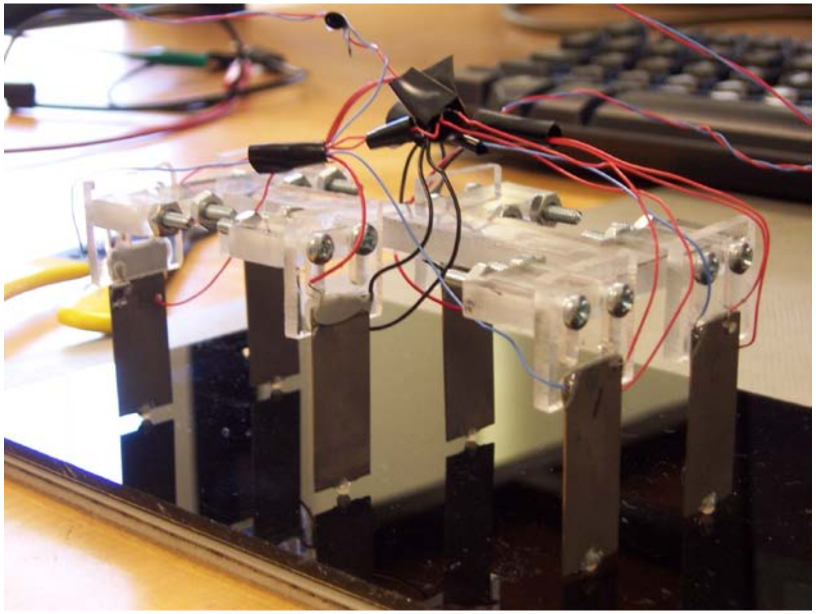
Videos



Nanostepper Advantages



Current Challenges



--- Motion of each leg: vibrations during each step needs to be reduced
--- Number and pattern of excitation of legs
--- reduce the size (footprint)

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8. **Conclusions**

Conclusions 1/3

(a) Growing demand for Biological Imaging (SPM plays a niche role)

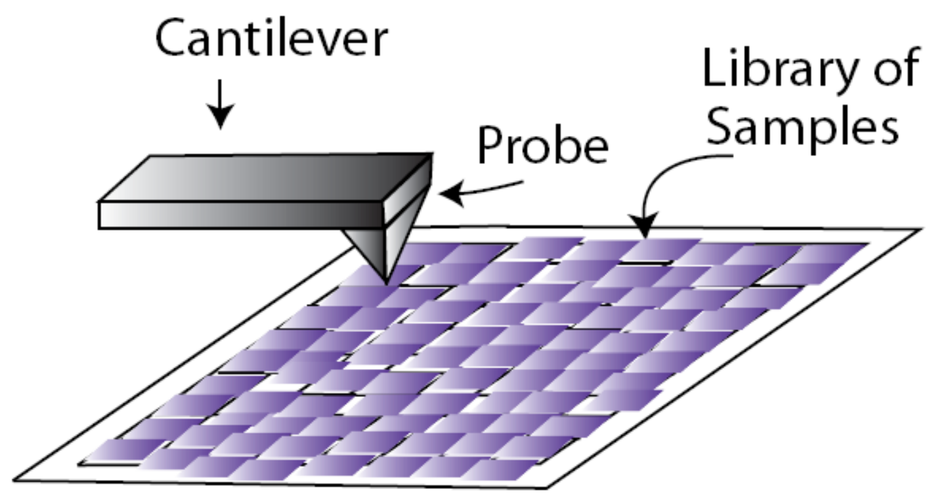
Conclusions 1/3

(a) Growing demand for Biological Imaging (SPM plays a niche role)

(b) Evaluating large arrays of samples **(combinatorial chemistry)**

Combinatorial AFM

Image from Qingze Zou
Rutgers



Conclusions 1/3

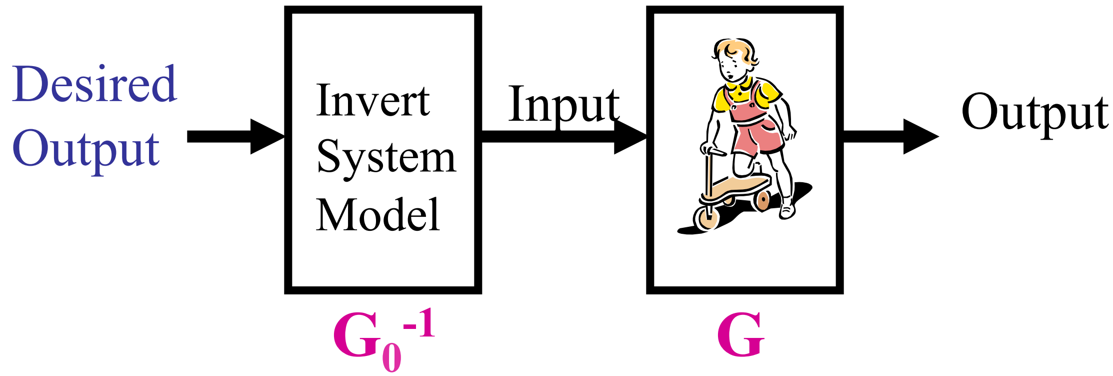
(a) Growing demand for Biological Imaging (SPM plays a niche role)

(b) Evaluating large arrays of samples

Main Themes

- Increase Precision: large errors lead to large forces (imaging soft samples), wrong features (distortions in nanofabrication)
- Increase Range: Nanofeatures imaged/fabricated over tens of micron
- Increase Bandwidth: Increase throughput of imaging/fabrication → **parallelism**

Conclusions 2/3
What is the Role of Feedforward?



- **Feedforward --- inversion, uses known system model**
- **Iterative approaches --- tracking error reduced to noise range**
- **Uncertainty --- Feedforward + feedback → guaranteed improvement**

- **Application to SPM --- increases the operating speed of SPM**
- **Recent works --- soft samples**
- **Emerging areas --- highly-parallel systems**
  & large-range positioner design (+ feedforward)

Conclusions 3/3
Positioning is an intellectually rich area

Broad applications
1) Nanotechnologies (SPM)
2) Disk Drive Industries (Dual-stage)
3) Aircraft Control (VTOL hover control)
4) Robotics

Neat Theory Problems
1) Is it possible to achieve a given position trajectory?
2) If so, how do we find the input to achieve it?
3) If not, how do you re-design the trajectory (optimally)?

Some advantages of working in positioning
1) Can choose from a large set of areas for research (broad applications)
2) Fundamental theoretical issues
3) Nice interaction between theory and application

Thank You