Atomic Interaction Effects in Precision Interferometry Using Bose-Einstein Condensates

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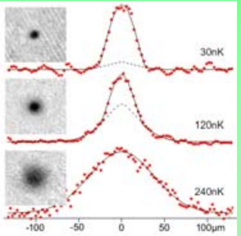
BEC Interferometry

Potential benefits of a Bose-Einstein Condensate(BEC) interferometer:

- Long coherence times
- Small initial momentum distribution
- Potentially sub-shot-noise accuracy



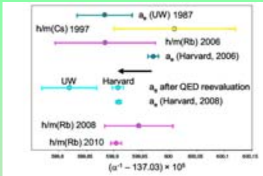
Finding α

The most precise determination of α at present comes from measurement of the electron's gyro-magnetic ratio (g) and complex QED calculations.

Recoil experiments compare the momentum and energy of an atom to determine h/m, from which α maybe calculated using (M_{Yb} currently being measured)[6]:

$$\alpha^2 = \left(\frac{e^2}{\hbar c}\right)^2 = \frac{2R_\infty}{c}\frac{M_{Yb}}{M_e}\frac{h}{m_{Yb}}$$

The plot below[4] compares recent measurements.

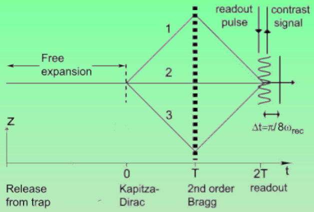


Contrast Interferometry for α

Measuring atomic recoil frequencies allows determination of the fine structure constant, α. A prototype experiment[1] (see accompanying figures) conducted in 2002 proved the robustness of the contrast interferometry technique. This symmetric three-arm interferometer has signal determined by

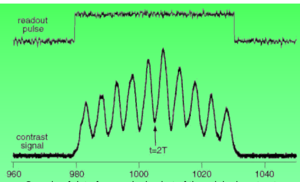
$$A(t)\sin^2\left(\frac{\phi_1(t)+\phi_3(t)}{2} - \phi_2(t)\right)$$

eliminating many systematic effects, and yet, the original experiment was plagued by a 200ppm systematic error. In simulations we find that the systematic shift in phase was due to mean-field effects, as originally hypothesized.



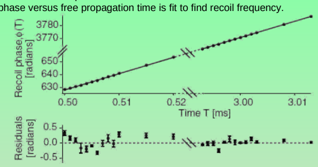
An improved experiment using Yb is in preliminary construction. In our lab we can cool Yb to a Bose-Einstein condensate. Yb has several important advantages

- Insensitive to magnetic fields
- Two available optical transitions (399nm and 556nm)
- Multiple stable bosonic isotopes
  - Allows comparisons of systematics between different isotopes
  - Variation of scattering length without introducing magnetic field
- Multiple stable fermionic isotopes
  - Interferometry with degenerate fermions a possible follow-up



Above: Sample of data from a single shot of the original contrast interferometry experiment

Below: Once the phase is extracted from traces such as the one above the phase versus free propagation time is fit to find recoil frequency.



Atomic Interaction Effects

Since we consider a regime with low density and temperature well below T_c, the Gross-Pitaevskii equation (GPE) with only two-body interactions gives a good description of the condensate dynamics.

$$i\hbar\frac{\partial}{\partial t}\phi(\vec{x},t) = -\frac{\hbar^2}{2m}\nabla^2\phi(\vec{x},t) + V(\vec{x})\phi(\vec{x},t) + \frac{4\pi\hbar^2(N-1)a}{m}|\phi(\vec{x},t)|^2\phi(\vec{x},t)$$

We developed a self-similar expansion model inspired by Castin and Dum's well-known result[2], postulating a wave function of form

$$\phi(\vec{x},t) = \frac{\phi_0\left(\frac{x_1}{\lambda_1(t)},\frac{x_2}{\lambda_2(t)},\frac{x_3}{\lambda_3(t)}\right)}{(\lambda_1(t)\lambda_2(t)\lambda_3(t))^{\frac{1}{2}}}e^{-i\theta(\vec{x},t)}$$

Treating distance from the BEC's center as a small parameter and defining $y_i = x_i/\lambda_i$ we find:

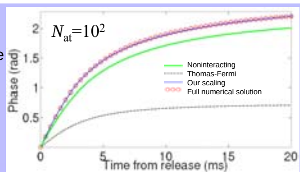
$$\theta(\vec{y},t) = f(t) - \frac{m}{2\hbar}\sum_{j=1}^{3}\dot{\lambda}_j\lambda_j y_j^2$$

$$\dot{f} = \frac{g}{\hbar}\frac{\alpha_0}{\lambda_1\lambda_2\lambda_3} - \frac{\hbar}{2m}\sum_{k=1}^{3}\frac{\alpha_k}{\lambda_k^2}$$

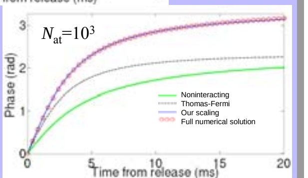
$$\alpha_0 \equiv (\phi_0(0))^2 \quad , \quad \alpha_k \equiv \frac{1}{\phi_0(0)}\left.\frac{\partial^2\phi_0}{\partial y_k^2}\right|_{\vec{y}=0}$$

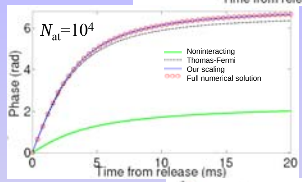
$$\ddot{\lambda}_j = \frac{g}{m}\frac{-\beta_{0j}}{\lambda_j\lambda_1\lambda_2\lambda_3} + \frac{\hbar^2}{2m^2}\frac{1}{\lambda_j}\sum_{k=1}^{3}\frac{\beta_{kj}}{\lambda_k^2}$$

$$\beta_{0j} \equiv \left.\frac{\partial^2\phi_0^2}{\partial y_j^2}\right|_{\vec{y}=0} \quad , \quad \beta_{kj} \equiv \left.\frac{\partial^2}{\partial y_j^2}\left[\frac{1}{\phi_0}\frac{\partial^2\phi_0}{\partial y_k^2}\right]\right|_{\vec{y}=0}$$

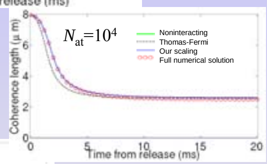


The top three plots show phase at the center of an expanding BEC as a function of time, comparing the noninteracting case and Thomas-Fermi (interaction dominated) case to full numerical solutions and our scaling solutions.





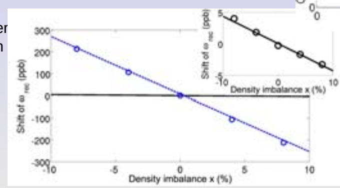
The plot to the right shows coherence length of an expanding BEC as a function of time, comparing the Thomas-Fermi (interaction dominated) case to full numerical solutions and our scaling solutions.



Slowly-Varying Envelope Approximation (SVEA)

The SVEA allows us to treat the various arms of the interferometer quasi-independently. Essentially, it amounts to viewing each arm in its rest-frame and then treating atom-laser and atom-atom interactions as ways to communicate between these frames. With ϕ_i the envelope for the i^{th} arm we obtain

$$i\hbar\frac{\partial\phi_j}{\partial t} = -\frac{\hbar^2}{2m}\nabla_j^2\phi_j + g\sum_{-l_1+l_2+l_3=j}\phi_{l_1}^*\phi_{l_2}\phi_{l_3}e^{-i(\omega_{l_2}+\omega_{l_3}-\omega_{l_1}-\omega_j)t}$$



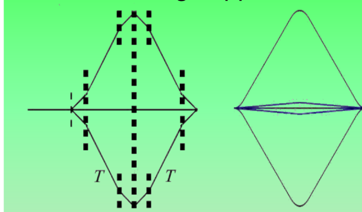
The plot to the left shows the shift of the recoil frequency that would be seen in a 22 ms long experiment with circles showing full numerical simulations and the lines showing scaling solutions plus an inter-arm perturbation correction. The blue data are for initial trapping frequency of 5 Hz while the black are for 200 Hz. Surprisingly, tighter traps can actually reduce interaction effects in the long run.

Projected Systematics

Systematics have been estimated for 30 mW Bragg beams of waist 8 mm red detuned from the 556 nm intercombination line. These parameters are achievable in our lab using known techniques.

Effect	$N=1$, $T=5$ms	$N=20$, $T=5$ms	Correctable?
Magnetic fields	0 ppb	0 ppb	easily testable
Electric fields	<0.1 ppb	<0.1 ppb	measurable
Gravitational curvature	<0.3 ppb	<0.3 ppb	scales with T^2
Differential AC Stark shift	<0.001 ppb	<0.001 ppb	
Wavefront curvature	0.2 ppb	0.2 ppb	maximize ϕ_{rec}
Beam alignment	0.6 ppb	0.6 ppb	maximize ϕ_{rec}
Index of refraction	20 ppb	1 ppb	scales w/ initial momentum or detuning
Readout back-action/ spontaneous scattering	---	---	degrades signal, no shift (currently simulating)
Atomic interactions	400 ppb	1 ppb	known scaling with atom number and scattering length

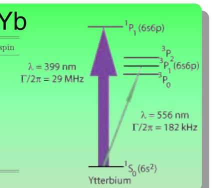
Getting to ppb



Achieving part-per-billion precision requires greater speeds, achieved with extra acceleration pulses. On the right, trajectories for N=1 (blue) and N=20 (black) contrast interferometry experiments of equal time are shown.

Yb

Isotope	Natural Abundance	Nuclear spin
^{168}Yb	0.0013	0
^{170}Yb	0.0305	0
^{171}Yb	0.143	1/2
^{172}Yb	0.219	0
^{173}Yb	0.161	5/2
^{174}Yb	0.318	0
^{176}Yb	0.127	0

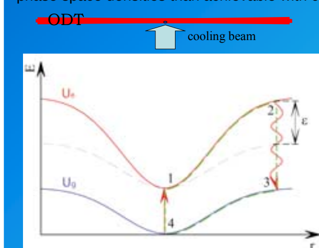


Sisyphus Cooling Driven by Spontaneous Decay in Optical Dipole Traps

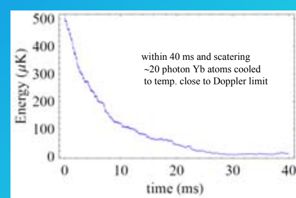
We propose a novel Sisyphus cooling method for atoms confined in an optical dipole trap. Utilizing their differential AC Stark shifts, two electronic levels of the atom are resonantly coupled by a laser only at the trap bottom. After resonance absorption, the atom loses energy by climbing the steeper potential, and then spontaneously decays away from the trap bottom.

Numerical simulations for the cases of ^{88}Sr and ^{174}Yb approach the expected limits of the recoil and Doppler temperatures, with much larger phase space densities than achievable with conventional laser cooling.

- T_{Dop} achievable at much higher densities than produced by conventional laser cooling
- Well-suited to optically trapped alkaline-earth-like atoms
- Very few scattering events needed to achieve T_{Dop}
  - More forgiving than conventional laser cooling
  - Potentially applicable to molecules
  - Up to 1/2 of atomic energy can be lost by scattering just one photon (in 1-D case)
  - Very short cooling time



Schematic of the proposed Sisyphus cooling method.



Numerical simulations of a single atom in an ODT formed by crossing of two Gaussian beams, each 2 W with waists of 3 and 3.5 μm. Three orthogonal cooling beams detuned at −γ.

	T_{rec}, μK	T_{Dop}, μK	T_{1D}, μK	T_{3D}, μK
Sr	0.46	0.18	0.8	1.4
Yb	0.35	4.3	6.0	8.5

Densities two orders of magnitude higher than achieved in laser cooling look plausible.

Requires: $|\alpha_P| > |\alpha_S|$

Narrow transition: $\hbar\gamma << U_P - U_S$ & $\gamma \le \omega_P$

$$\Delta\varepsilon = -k_B T\left(\frac{\alpha_S - \alpha_P}{\alpha_P}\right)\frac{2\omega_P^2\gamma}{\gamma^3 + 4\gamma\omega_P^2}$$

References

1. Gupta, S., et al. Phys. Rev. Lett. **89**, 140401 (2002)
2. Castin, Y. and Dum, R. Phys. Rev. Lett. **77**, 5315 (1996)
3. Chang, J.J., et al. Phys. Rev. Lett. **101**, 170404 (2008)
4. Bouchendira, et al., Phys. Rev. Lett. **106**, 080801 (2011)
5. Cadoret, M., et al. Phys. Rev. Lett. **101**, 230801 (2008)
6. Edmund Myers, private communication

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Group website: http://www.phys.washington.edu/users/deepg

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