Precision Atom Interferometry with Bose-Einstein Condensates

Modeling Uncertainties for High-Precision Measurements

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BEC Interferometry

Potential benefits of a Bose-Einstein Condensate(BEC) interferometer:

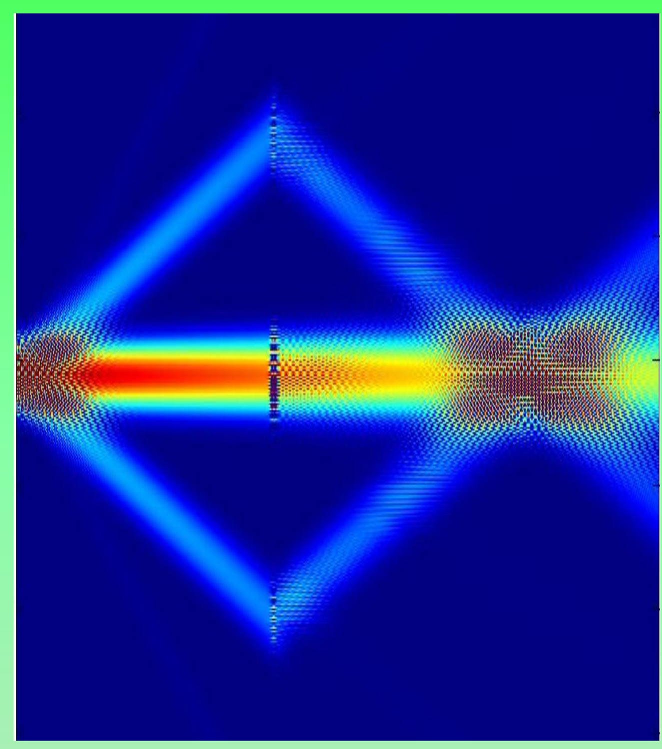
- Long coherence times
- Robust (~ million atom) signals
- Small initial momentum distribution
- Potentially sub-shot-noise accuracy using squeezed states

Principal draw-backs:

- Long (~minute) time to generate a single BEC
- Large systematic effects due to mean-field
- Production of unwanted collective effects---solitons and vortices
- Density dependent effects in atom-light interactions

The first of these has partially been solved with fast (few sec.) evaporative cooling in optical traps. Typical BEC production time scales are around ten seconds. We study, both numerically and analytically, the other problems. While we apply these techniques to a contrast interferometry experiment, they should have broad applicability. We have found ways to mitigate these draw-backs in future interferometry experiments using BEC sources.

Contrast Interferometry for α



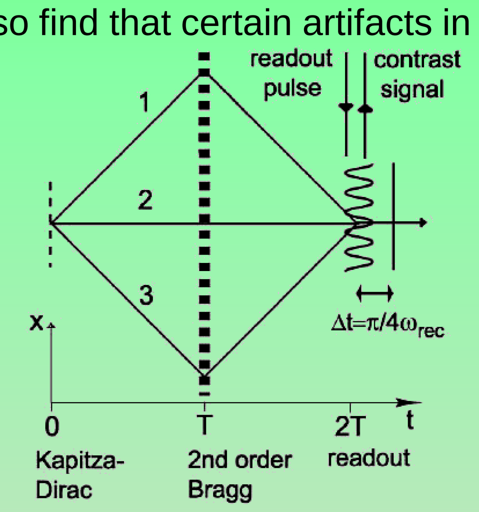
Above: Simulation of a contrast interferometry experiment showing magnitude of the wave function (x vertical, t horizontal)
Right: Schematic of the same experiment

Measuring atomic recoil frequencies allows determination of the fine structure constant, α. A prototype experiment[1] (see accompanying figures) conducted in 2002 proved the robustness of the contrast interferometry technique. This symmetric three-arm interferometer has signal determined by

$$A(t)\sin^2\left(\frac{\phi_1(t)+\phi_3(t)}{2}-\phi_2(t)\right)$$

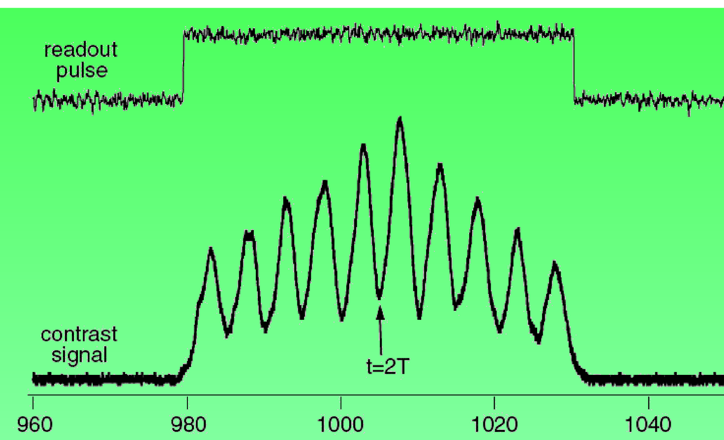
eliminating many systematic effects, and yet, the original experiment was plagued by a 2×10^{-4} systematic error.

In simulations we find that the systematic shift in phase was due to mean-field effects, as originally hypothesized. We also find that certain artifacts in the original data arise from asymmetric trap turn-off.

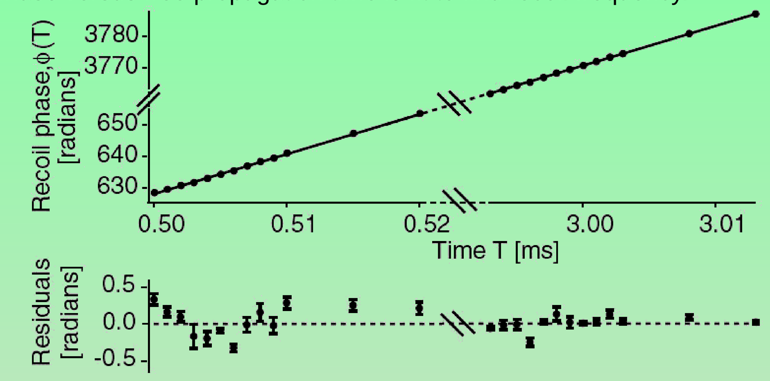


An improved experiment using Yb is in preliminary construction. In our lab we can efficiently laser cool Yb and subsequently load into a far-off-resonant optical trap. We have initiated the process of evaporative cooling toward degeneracy. Yb has several important advantages

- Insensitive to magnetic fields
- Two available optical transitions (399nm and 556nm)
- Multiple stable bosonic isotopes
  - Allows comparisons of systematics between different isotopes
  - Variation of scattering length without introducing magnetic field
- Multiple stable fermionic isotopes
  - Interferometry with degenerate fermions a possible follow-up



Above: Sample of data from a single shot of the original contrast interferometry experiment
Below: Once the phase is extracted from traces such as the one above the phase versus free propagation time is fit to find recoil frequency.



Atom-Atom Interaction Effects

Since we consider a regime with low density and temperature well below T_c, the Gross-Pitaevskii equation (GPE) with only two-body interactions gives a good description of the condensate dynamics.

$$i\frac{\partial\Phi}{\partial\tau}=-\frac{1}{2}\nabla^2\Phi+U(x)\Phi+|\Phi|^2\Phi$$

There are two important classes of atomic interaction effects modeled by the GPE: Collective nonlinear excitations and a systematic energy shift. We have studied each of these both through analytic modeling and numerical simulation. Our self-similar expansion model generalizes Castin and Dum's well-known result[2]. We postulate a wave function of form

$$\lambda^{-\frac{1}{2}}(t)\left(a-b\left(\frac{x}{\lambda(t)}\right)^2\right)^{\frac{1}{2}}\exp\left(if(t)-i\frac{1}{2}\frac{\dot\lambda(t)}{\lambda(t)}x^2\right)$$

and solve for λ and f. We find

$$\frac{df}{dt}=\frac{a}{\lambda}-\frac{b}{2a\lambda^2}\quad\text{and}\quad\frac{d^2\lambda}{dt^2}=-\frac{2b}{\lambda^2}-\frac{2b^2}{a^2\lambda^3}$$

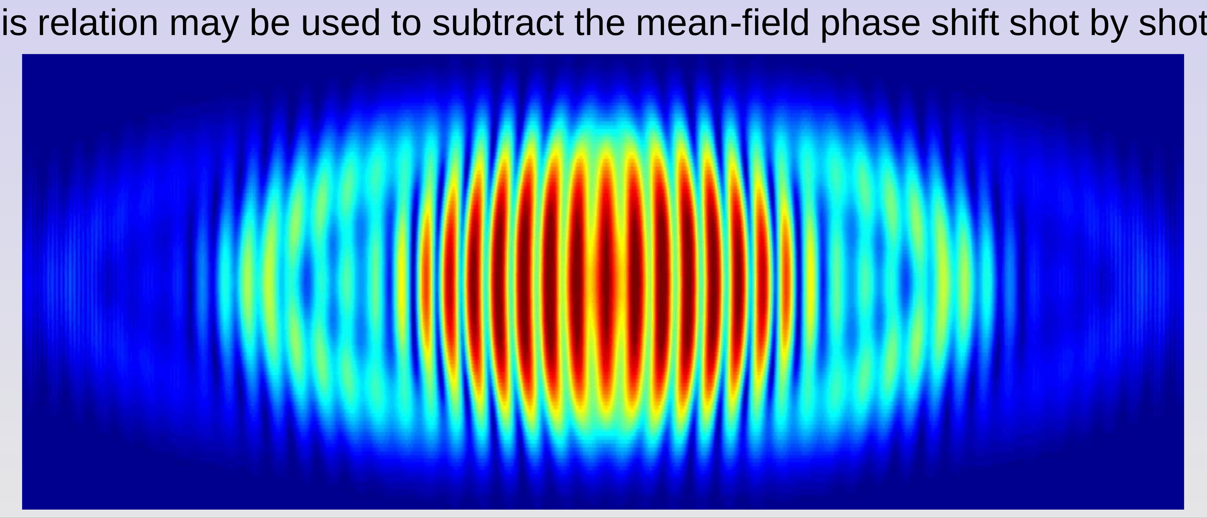
(The quantities *a* and *b* are fixed by initial conditions.) After the first laser pulse, a solution of this form is used for each of the recoil states.

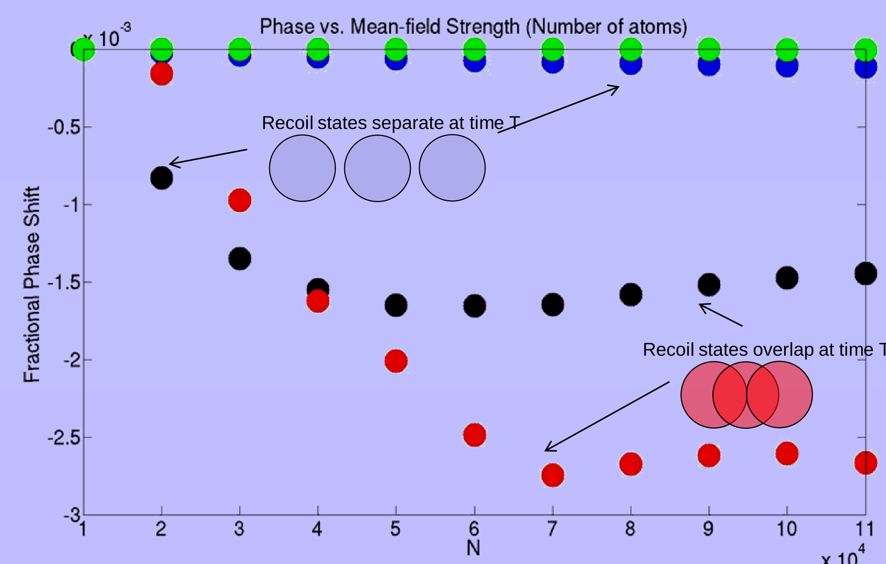
Collective Nonlinear Excitations

The GPE supports solutions containing solitons, vortices, and shockwaves. The literature contains rough estimates of conditions for producing such collective excitations. These are currently being verified in full numerical simulations of the GPE. An intuitive estimator is that the time for a sound wave to cross the condensate must be shorter than the time the condensate pieces are overlapped if vortices are to be formed. For our suggested experimental parameters this suggests we are far from the regime where vortices may be formed.

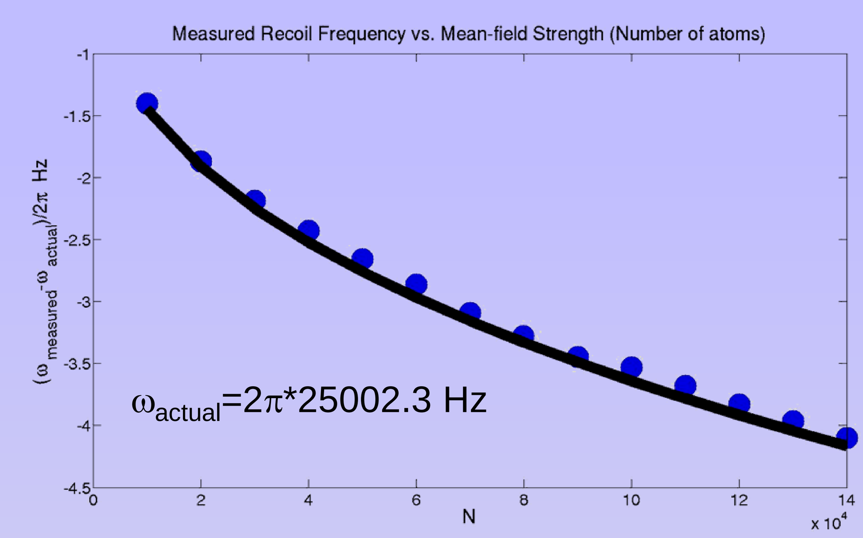
Energy Shift

We find good agreement between a self-similar expansion model and the numerically simulated mean-field shift in recoil frequency. Further, we see strong correlation between coherence time of the grating signal and mean-field shift. This relation may be used to subtract the mean-field phase shift shot by shot.

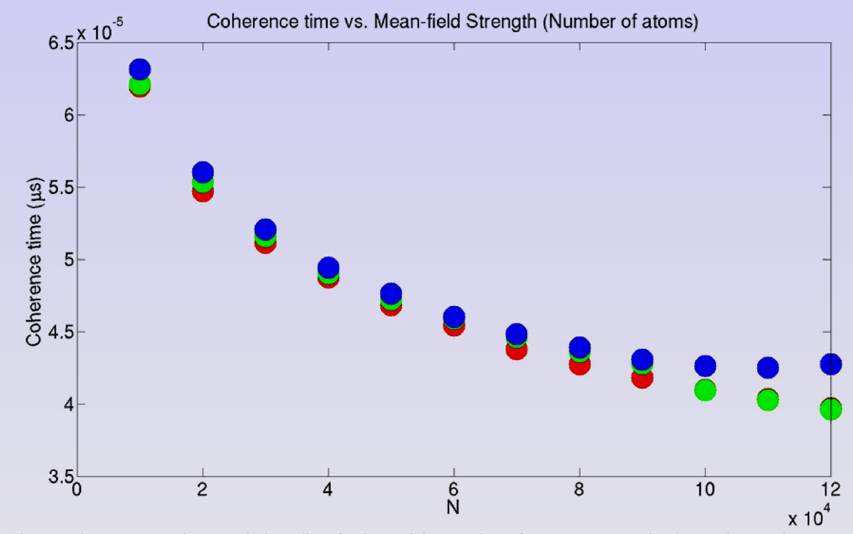




As atom number varies for small T, the recoil states will fail to completely separate for some values of N. For such Ts the phase shift is not an invertible function of atom number.



The black curve shows the self-similar expansion prediction. Here the recoil frequency is found using simulated data from T=3ms and T=4ms runs. Full separation of recoil states occurs, simplifying the model.



The coherence time of the final signal is a simple, monotonic function of mean-field strength when recoil states separate completely. For T=2ms (blue points) we begin to see the effects of partial overlap at high N.

Atom-Light Interactions

The refractive index of the cloud modifies the recoil momentum from its vacuum value. This effect is only relevant for splitting pulses, i.e. not for pure acceleration pulses.

- For N=1 this effect is of size $<2\times10^{-8}$. Since the relative error scales as N^{-2}, for N=20 the size is $<5\times10^{-11}$.

Beam alignment errors lead to reduced ($2\hbar k\cos{}^{\theta}/_2$ rather than $2\hbar k$) momentum transfer per pulse

- Alignment accuracy of 50μrad gives 3×10^{-10} error, and even better alignment is possible.

Several systematic uncertainties are due to the gaussian pulse shapes. Finite available laser power limits the waist size we can use and still have sufficient intensity to drive Bragg transitions. The spatial shape of the pulse is governed by

$$E=E_0\frac{w_0}{w(z)}\exp\left(-\frac{r^2}{w^2(z)}\right)\exp\left(-ikz-ik\frac{r^2}{2R(z)}+i\operatorname{atan}\left(\frac{z}{z_R}\right)\right)$$

$$w(z)=w_0\left(1+\left(\frac{z}{z_R}\right)^2\right)^{\frac{1}{2}}\qquad z_R=\frac{\pi w_0^2}{\lambda}\qquad R(z)=\frac{z_R^2}{z}\left(1+\left(\frac{z}{z_R}\right)\right)$$

Wave-front curvature reduces momentum transfer because the finite width of the beam gives a spread of transverse momenta.

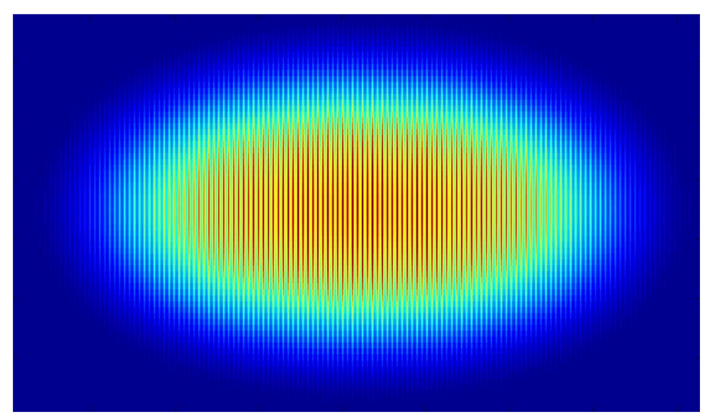
Guoy phase refers to the lag or advance of the phase along the beam axis relative to a pure plane wave.

Both effects are inevitable for any real beam, but can be reduced by enlarging the spot size at the focal point.

- Detailed calculations[5] give 2×10^{-10} errors for our planned 8mm beam waist

Additionally, the reduction of beam intensity both as the atoms move along the beam axis and as they fall out of the beam axis lead to differing AC Stark shifts for the various branches of the interferometer.

- For our planned parameters this effect is of relative size 2×10^{-15}



Magnitude of the wavefunction (cross-section) from a three dimensional simulation during the initial splitting pulse.

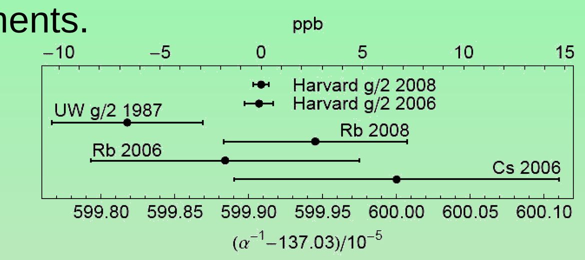
Finding α

The most precise determination of α at present comes from measurement of the electron's gyromagnetic ratio (g). This entails complex calculations in perturbative QED.

Atomic recoil experiments compare the energy and momentum of an atom to determine h/m. This is then used in the equation

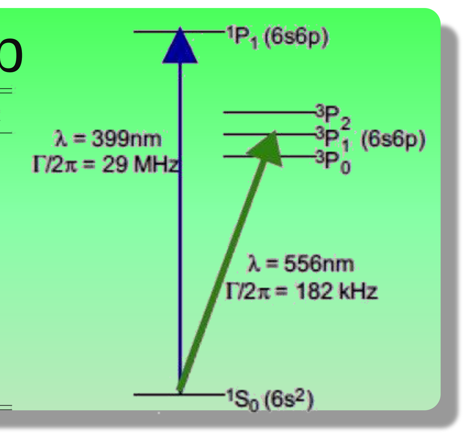
$$\alpha^2=\left(\frac{e^2}{\hbar c}\right)^2=\frac{2R_\infty}{c}\frac{M_{\rm Yb}}{M_e}\frac{h}{m_{\rm Yb}}$$

to find the value of α. All terms in this equation are easily accessible at better than ppb accuracy except for the h/m term. The plot below[4] shows current best measurements of α compared to that found from recent g/2 measurements. Points labeled Rb and Cs correspond to recoil experiments.



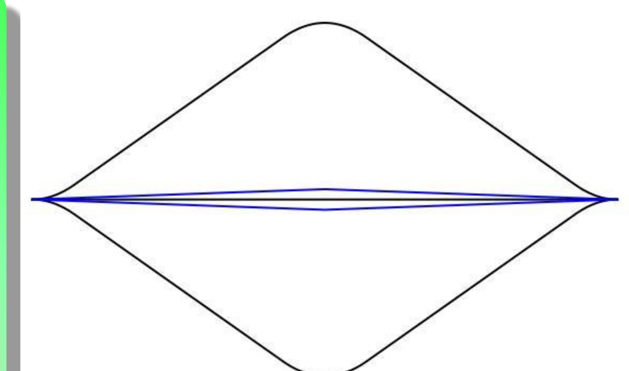
Yb

Isotope	Natural Abundance	Nuclear spin
^{168}Yb	0.0013	0
^{170}Yb	0.0305	0
^{171}Yb	0.143	1/2
^{172}Yb	0.219	0
^{173}Yb	0.161	5/2
^{174}Yb	0.318	0
^{176}Yb	0.127	0



Experimental Plan

We will measure the recoil frequency, ω_{rec}, of Yb atoms in a BEC using light red-detuned from the narrow 556nm transition. The wavelength of this light will be measured using an optical frequency comb. Since ω_{rec} scales quadratically with the recoil momentum, we will use multiple Bragg π-pulses to accelerate the atoms. Using N=20 acceleration pulses and a free propagation time of T=5ms in each direction we expect to reach ppb precision in roughly one day of running.



Trajectories for N=1 (blue) and N=20 (black) contrast interferometry experiments of equal time.

References

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Group website: http://www.phys.washington.edu/users/deepg

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