QUANTUM MICRO-MECHANICS WITH ULTRACOLD ATOMS

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In many experiments, isolated atoms and ions have been inserted into high-finesse optical resonators for fundamental studies of quantum optics and quantum information. Here, we introduce another application of such a system, as the realization of cavity optomechanics where the collective motion of an atomic ensemble serves the role of a moveable optical element in an optical resonator. Compared with other optomechanical systems, such as those incorporating nanofabricated cantilevers or the large cavity mirrors of gravitational observatories, our cold-atom realization offers direct access to the quantum regime. We describe experimental investigations of optomechanical effects, such as the bistability of collective atomic motion and the first quantification of measurement backaction for a macroscopic object, and discuss future directions for this nascent field.

Keywords: Quantum micro-mechanics; ultracold atoms; optomechanical systems

1. Introduction

Cavity opto-mechanics describes a paradigmatic system for quantum metrology: a massive object with mechanical degrees of freedom is coupled to and measured by a bosonic field. Interest in this generic system is motivated by several considerations. For one, the system allows one to explore and address basic questions about quantum limits to measurement. In this context, quantum limits to quadrature specific and non-specific measurements, both for those performed directly on the mechanical object and also those performed through the mediation of an amplifier have been derived.[1]

Second, as a detectors of weak forces, cavity opto-mechanical systems in the quantum regime may yield improvements in applications ranging from the nanoscale (e.g. for atomic or magnetic force microscopies) to the macroscale (e.g. in ground- or space-based gravity wave observatories). Finally, such systems, constructed with ever-larger mechanical objects, may allow one to test the validity of quantum mechanics for massive macroscopic objects. Striking developments in this field were presented at ICAP 2008 by Harris and Kippenberg.

Our contribution to this developing field is the realization that a cavity opto-mechanical system can be constructed using a large gas of ultracold atoms as the mechanical object. Having developed an apparatus that allows quantum gases to be trapped within the optical mode of a high-finesse Fabry-Perot optical resonator, we are now able to investigate basic properties of opto-mechanical systems. Several of these investigations are described below. The atoms-based mechanical oscillator may be considered small by some, with a mass ($\simeq 10^{-17}$ g) lying geometrically halfway between the single-atom limit explored at the quantum regime in ion and atom traps (10^{-22} g) [2,3, for example], and the small ($\simeq 10^{-12}$ g) nanofabricated systems now approaching quantum limits.[4,5] Nevertheless, our system offers the advantages of immediate access to the quantum mechanical regime, of the *ab initio* theoretical basis derived directly from quantum optics and atomic physics, and of the tunability and amenability to broad new probing methods that are standard in ultracold atomic physics. Our motivation for probing cavity opto-mechanics with our setup is not just to poach the outstanding milestones of this field (e.g. reaching the motional ground state or observing measurement backaction and quantum fluctuations of radiation pressure with a macroscopic object[6]). Rather, we hope to contribute to the development of macroscopic quantum devices by clarifying experimental requirements and the role of and limits to technical noise, developing optimal approaches to signal analysis and system control, exploring the operation and uses of multi-mode quantum devices, and defining different physical regimes for such systems. Also, our opto-mechanical system may have direct application as part of an atom-based precision (perhaps interferometric) sensor.

2. Collective modes of an intracavity atomic ensemble

The theoretical reasoning for considering a trapped atomic gas within a high-finesse optical resonator as a macroscopic cavity opto-mechanical system is laid out in recent work.[6] Recapping that discussion, we consider the dispersive coupling of an ensemble of N identical two-level atoms to a single standing-wave mode of a Fabry-Perot cavity, obtaining the spectrum of "bright" eigenstates of the atoms-cavity system according to the following Hamiltonian:

$$\mathcal{H} = \hbar\omega_c\hat{n} + \sum_i \frac{\hbar g^2(z_i)}{\Delta_{ca}} + \mathcal{H}_a + \mathcal{H}_{in/out}. \tag{1}$$

Here $\hat{n}$ is the cavity photon number operator, $\Delta_{ca} = \omega_c - \omega_a$ is the difference between the empty-cavity and atomic resonance frequencies, and $g(z_i) = g_0 \sin(k_p z)$ is the spatially dependent atom-cavity coupling frequency with z_i being the position of atom i and k_p being the wavevector at the cavity resonance. The term $\mathcal{H}_a$ describes the energetics of atomic motion while $\mathcal{H}_{in/out}$ describes the electromagnetic modes outside the cavity. Note that this expression already treats the atom-cavity coupling to second order in g. Repeating this analysis starting from the first-order term does not change our conclusions substantially.

Now, let us assume that all the atoms are trapped in harmonic potentials with "mechanical" trap frequency ω_z and neglect motion along directions other than the cavity axis. Further, we treat the atomic motion only to first order in atomic displacements, δz_i, from their equilibrium positions, $\bar{z}_i$; i.e. we assume atoms to be confined in the Lamb-Dicke regime with $k_p \delta z_i \ll 1$. We now obtain the canonical cavity opto-mechanical Hamitonian[7] as

$$\mathcal{H} = \hbar\omega_c'\hat{n} + \hbar\omega_z\hat{a}^\dagger\hat{a} - F\hat{Z}\hat{n} + \mathcal{H}_a' + \mathcal{H}_{in/out}. \tag{2}$$

We make several steps to arrive at this expression. First, we allow the cavity resonance frequency to be modified as $\omega_c' = \omega_c + \sum_i g^2(\bar{z}_i)/\Delta_{ca}$, accounting for the cavity resonance shift due to the atoms at their equilibrium positions. Second, we introduce the collective position variable $\hat{Z} = N_{\text{eff}}^{-1} \sum_i \sin(2k_p\bar{z}_i)\delta z_i$ that, along with a weighted sum $\hat{P} = \sum_i \sin(2k_p\bar{z}_i)p_i$ of the atomic momenta p_i, describes the one collective motion within the atomic ensemble that is coupled to the cavity-optical field. The operators $\hat{a}$ and $\hat{a}^\dagger$ are defined conventionally for this mode. In our treatment, without the presence of light within the optical cavity, this mode is harmonic, oscillating at the mechanical frequency ω_z, and endowed with a mass M equal to that of $N_{\text{eff}} = \sum_i \sin^2(2k_p\bar{z}_i)$ atoms. Third, we summarize the opto-mechanical coupling by the per-photon force $F = N_{\text{eff}}\hbar k g_0^2/\Delta_{ca}$ that acts on the collective mechanical mode. Finally, we lump all the remaining atomic degrees of freedom, and also the neglected higher order atom-cavity couplings, into the term $\mathcal{H}_a'$.

With this expression in hand, we may turn immediately to the literature on cavity opto-mechanical systems to identify the phenomenology expected for our atoms-cavity system. Several such phenomena are best described by referring to the opto-mechanical force on the collective atomic mode, given as

$$\hat{\mathcal{F}}_{\mathrm{opto}} = -M\omega_z^2 \hat{Z} + F\hat{n}. \tag{3}$$

We consider the following effects:

- If we allow the state of the cavity to follow the atomic motion adiabatically ($\omega_z \ll \kappa$), neglect quantum-optical fluctuations of the cavity field, and assume the collective atomic displacement remains small, the linear variation of $\langle \hat{n} \rangle$ with $\hat{Z}$ modifies the vibration frequency of the collective atomic motion. Here, κ is the cavity half-linewidth. This modification, known as the "optical spring," has been observed in various opto-mechanical systems and has been used to trap macroscopic objects optically.[8–10] We have made preliminary observations of the optical spring effect in our system as well.
- For larger atomic displacements, the opto-mechanical force may become notably anharmonic, and even, under suitable conditions, bistable.[11] Our observations of the resulting opto-mechanical bistability[12] are discussed in Sec. 4.
- When the cavity field no longer follows the atomic motion adiabatically, the opto-mechanical potential is no longer conservative. The dramatic effects of such non-adiabaticity are the cavity-induced damping or coherent amplification of the mechanical motion.[13] Such effects of dynamical backaction have been detected in several micro-mechanical systems[14–16] and also for single[17] or multiple atoms[18] trapped within a cavity.
- Finally, we consider also the effects of quantum-optical fluctuations of the intracavity photon number and, thereby, of the optical forces on the atomic ensemble. It can be shown that these force fluctuations represent the backaction of quantum measurements of the collective atomic position,[6] as described in Sec. 5.

3. Collective atomic modes in various regimes

The theoretical treatment described above is suitable in the Lamb-Dicke regime of atomic confinement and under the condition that the linear opto-mechanical coupling term ($F\hat{Z}\hat{n}$) is dominant (i.e. that the intracavity atomic gas is not tuned to positions of exclusively quadratic sensitivity). These conditions are met in our experiments at Berkeley, where an ultracold gas of about 10^5 atoms of ^{87}Rb is transported into the mode volume of a high-finesse Fabry-Perot optical resonator. The resonator length is tuned so that the resonator supports one TEM_{00} mode with wavelength $\lambda_T = 850$ nm (trapping light) and another within a given detuning Δ_{ca} (in the range of 100's of GHz) of the D2 atomic resonance line (probe light). Laser light with wavelength λ_T is sent through the cavity to generate a 1D optical lattice potential in which the cold atomic gas is trapped (Fig. 1). The gas is strewn across over > 100 contiguous sites in this 1D optical lattice. Within each well, atoms are brought by evaporative cooling to a temperature $T \sim 700$ nK. At this temperature, the atoms lie predominantly in the ground state of motion along the cavity axis, with $\hbar\omega_z/k_B \simeq 2\,\mu\mathrm{K} \gg T$, and the Lamb-Dicke condition is satisfied with respect to the wavevector of probe light ($k_p \simeq 2\pi/(780\,\mathrm{nm})$) used to interrogate the atomic motion.

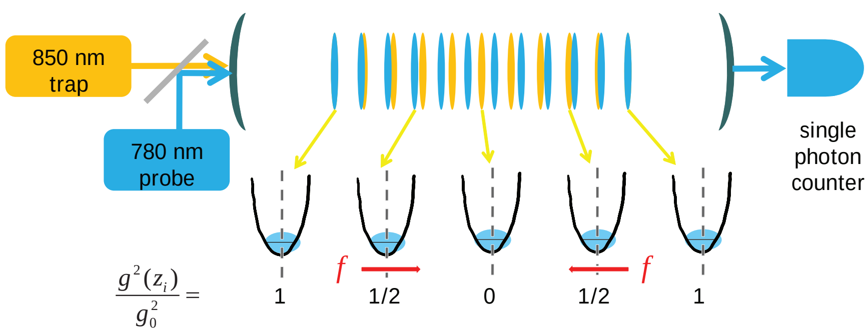

Fig. 1. Scheme for opto-mechanics with ultracold atoms in the Lamb-Dicke confinement regime. A high finesse cavity supports two longitudinal modes – one with wavelength of about 780 nm that is near the D2 resonance of ^{87}Rb atoms trapped within the resonator, and another with wavelength of about 850 nm. Light at the 850 nm resonance produces a one-dimensional optical lattice, with trap minima indicated in orange, in which atoms are confined within the lowest vibrational band. These atoms induce frequency shifts on the 780 nm cavity resonance. The strength of this shift, and of its dependence on the atomic position, varies between the different sites of the trapping optical lattice, as shown. Nevertheless, in the Lamb-Dicke confinement regime, the complex atoms-cavity interactions reduce to a simple opto-mechanical Hamiltonian wherein a single collective mode of harmonic motion, characterized by position and momentum operators $\hat{Z}$ and $\hat{P}$, respectively, is measured, actuated, and subjected to backaction by the cavity probe.

The opto-mechanics picture of atomic motion in cavity QED has also been considered recently by the Esslinger group in Zürich.[19] There, a continuous Bose-Einstein condensate of ^{87}Rb is trapped in a large-volume optical trap within the cavity volume. Yet, in spite of the stark differences in the external confinement and the motional response of the condensed gas, a similar opto-mechanical Hamiltonian emerges. In our prior description of the Lamb-Dicke regime, optical forces due to cavity probe light are found to excite and, conversely, to make the cavity sensitive to a specific collective motion in the gas. In the case of a continuous condensate, the cavity optical forces excite atoms into a specific superposition of the $\pm 2\hbar k_p$ momentum modes. Interference between these momentum-excited atoms and the underlying condensate creates a spatially (according to k_p) and temporally (according to the excitation energy) periodic density grating that is sensed via the cavity resonance frequency. Thus, by identifying operators $\hat{a}$ and $\hat{a}^\dagger$ with this momentum-space excitation and the operator $\hat{Z}$ with the density modulation, we arrive again at the Hamiltonian of Eq. 2.

We can attempt to bridge these two opto-mechanical treatments by tracking the response of an extended atomic gas to spatially periodic optical forces (due to probe light at wavelength 780 nm) as we gradually turn up the additional optical lattice potential (due to trapping light at wavelength 850 nm). In the absence of the lattice potential, a zero-temperature Bose gas forms a uniform Bose-Einstein condensate. The excitations of this system are characterized by their momentum and possess an energy determined by the Bogoliubov excitation spectrum; in Fig. 2(a), we present this spectrum as a free-particle dispersion relation, neglecting the effects of weak interatomic interactions. The spatially periodic optical force of the cavity probe excites a superposition of momentum excitations as described above.

Adding the lattice potential changes both the state of the Bose-Einstein condensate, which now occupies the lowest Bloch state, and also the state of excitations, which are now characterized by their quasi-momentum and by the band index. There are now many excitations of the fluid that may be excited at the quasi-momentum selected by the spatially periodic cavity probe. In the case that the lattice is very shallow, shown in Fig. 2(a), the cavity probe will still populate only one excited state nearly exclusively. Given the relation between the wavelengths of the trapping (850 nm) and cavity-probe light (780 nm), this excited state lies in the second excited band. As the lattice is deepened, however, matrix elements connecting to quasi-momentum states on other bands will grow (shown in Fig. 2(c)). Now our simple opto-mechanical picture is made substantially more complex, with multiple mechanical modes oscillating with differing mechanical frequencies all influencing the optical properties of the cavity.

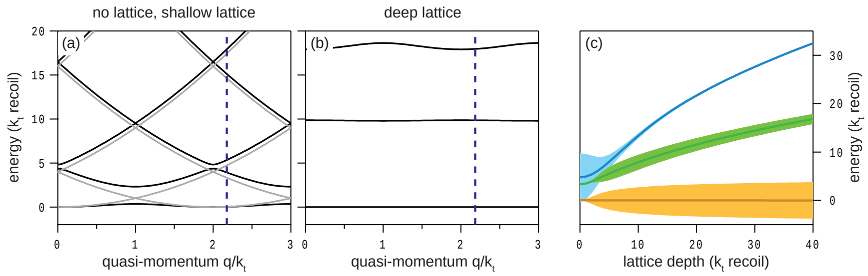

Fig. 2. Influence of band structure on the opto-mechanical response of an ultracold atomic gas confined within a high-finesse Fabry-Perot optical resonator. We consider the relevant macroscopic excitation produced by cavity probe light at wavevector $k_p = 2\pi/(780\,\text{nm})$ within a Bose gas confined within a one-dimensional optical lattice formed by light at wavevector $k_t = 2\pi/(850\,\text{nm})$ and with variable depth. The gas is cooled to zero temperature, non-interacting, and extended evenly across many lattice sites. Energies are scaled by the recoil energy $E_r = \hbar^2 k_t^2/2m$ and wavevectors by k_t. (a) With a weak lattice applied ($2E_r$), the band structure for atomic excitations (black lines) is slightly perturbed from the free-particle excitations in the absence of a lattice (gray). The cavity probe excites atoms primarily to states with quasi-momenta $\pm 2k_p$ within the second excited band, corresponding closely to momentum eigenstates in the lattice-free regime. (b) In a deep lattice ($15E_r$), energy bands show little dispersion and are spaced by energies scaling as the square root of the lattice depth. (c) Lines show the energies of the three lowest energy states at quasi-momentum $2k_p$ as a function of the lattice depth. The relative probability for excitation by cavity probe light to each of these states, taken as the square of the appropriate matrix element, is shown by the width of the shaded regions around each line. At zero lattice depth, cavity probe light excites the second excited band exclusively. At large lattice depth, the excitation probability to the first excited band grows while excitation to higher bands is suppressed. At intermediate lattice depths, several excited states are populated, indicating the onset of complex multi-mode behaviour.

Continuing to deepen the optical lattice, this complexity will be alleviated when we reach the Lamb-Dicke regime, i.e. as the Lamb-Dicke parameter $k_p\delta z$ becomes ever smaller, the probabilities of excitation from the ground state via the cavity probe zero in on the first excited band. We calculate such probabilities as $p_i \propto |< 2k_p; i|\cos(2k_p z)|g >|^2$ where the bra is the $2k_p$ quasi-momentum Bloch state in the ith band, and the ket is the ground state in the lattice considered. Here, we interpret excitations to higher bands as being controlled by terms of higher order in the Lamb-Dicke parameter, e.g. excitations to the second excited band result from couplings that are quadratic in the atomic positions.

Thus, we confirm that a simple opto-mechanics picture emerges for the collective atomic motion within a cavity both in the shallow- and deep-lattice limits. We note, however, that these limits differ in two important ways. First, we see that the mechanical oscillation frequency for the collective atomic motion is constrained to lie near the bulk Bragg excitation frequency in the shallow-lattice limit, whereas it may be tuned to arbitrarily high frequencies (scaling as the square root of the lattice depth) in the deep-lattice limit. The ready tunability of the mechanical frequency in the latter limit may allow for explorations of quantum opto-mechanical systems in various regimes, e.g. in the the resolved side-band regime where ground-state cavity cooling and also quantum-limited motional amplification are possible.[20,21] Second, we see that the mechanical excitation frequency has a significant quasi-momentum (Doppler) dependence in the shallow-lattice limit. This dependence makes it advantageous to use low-temperature Bose-Einstein condensates for experiments of opto-mechanics, as done in the Zürich experiments, so as to minimize the Doppler width of the Bragg excitation frequency. In contrast, the excitation bandwidth is dramatically reduced (exponentially with the lattice depth) in the deep-lattice limit. Thus, one can conduct opto-mechanics experiments with long-lived mechanical resonances in the deep-lattice limit without bothering to condense the atomic gas. Nevertheless, we note that variations in the mechanical frequency due to the presence of significant radial motion (not considered in this one-dimensional treatment) do indeed limit the mechanical quality factor in the Berkeley experiments.

4. Effects of the conservative optomechanical potential: optomechanical bistability

The observation of cavity nonlinearity and bistability arising from collective atomic motion is described in recent work.[12] Briefly, we find that the optical force due to cavity probe light will displace the equilibrium collective atomic position $\langle \hat{Z} \rangle$, leading to a probe-intensity-dependent shift of the cavity resonance frequency. By recording the cavity transmission as the cavity probe light was swept across the cavity resonance, we observed asymmetric and shifted cavity resonance lines, and also hallmarks of optical bistability.

Refractive optical bistability is well studied in a variety of experimental systems.[22] One unique aspect of our experiment is the observation of both branches of optical bistability at average cavity photon numbers as low as 0.02. The root of such strong optical nonlinearities is the presence within the cavity of a medium that responds significantly to the presence of infrequent cavity photons (owing to strong collective effects) and recalls the presence of such photons for long coherence times. Here, the coherence is stored within the long-lived collective motion of the gas. It is interesting to consider utilizing such long-lived motional coherence, rather than the shorter-lived internal state coherence typically considered, for the various applications of cavity QED and nonlinear optics in quantum information science, e.g. photon storage and generation, single-photon detection, quantum logic gates, etc.

Such motion-induced cavity bistability can also be understood in the context of the opto-mechanical forces described by Eq. (3). Neglecting the non-adiabatic following of the cavity field to the collective motion (essentially taking $\kappa/\omega_z \to \infty$ so that dynamical backaction effects are neglected) and also the quantum fluctuations of the cavity field, we may regard atomic motion in an optically driven cavity to be governed by an opto-mechanical potential of the form

$$U(Z) = \frac{1}{2}M\omega_z^2 Z^2 + n_{\max}\hbar\kappa \arctan\left(\frac{\Delta_{pc} - FZ/\hbar}{\kappa}\right). \tag{4}$$

Here Δ_{pc} is the detuning of the constant frequency probe from the modified cavity resonance frequency ω_c', and $n_{\max}$ is the average number of cavity photons when the cavity is driven on resonance.

The form of this potential is sketched in Fig. 3 for different operating conditions of the atoms-cavity system. Cavity bistability[12] is now understood as reflecting an effective potential for the collective atomic variable Z that has two potential minima. Remarkably, these potential minima may be separated by just nanometer-scale displacements in Z. Even though the inherent quantum position uncertainty of each individual atom (10's of nm) is much larger than this separation, the reduced uncertainty in the collective variable Z allows for these small displacements to yield robust and distinct experimental signatures in the cavity transmission.

5. Quantum fluctuations of the optomechanical potential: measurement backaction

Aside from the conservative forces described above, the intracavity atomic medium is subject also to dipole force fluctuations arising from the quantum nature of the intracavity optical field. Indeed, should these force fluctuations be especially large, the picture of cavity optical non-linearity and bistability described in the previous section, in which we implicitly assume that the collective atomic motion may follow adiabatically into a local minimum of an opto-mechanical potential, must be dramatically modified. To assess the strength of such force fluctuations, let us consider the impact on

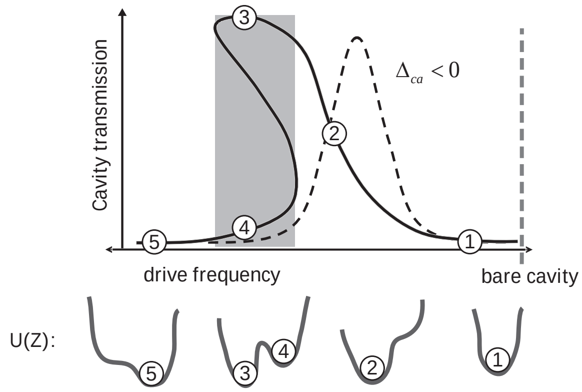

Fig. 3. For different stable regimes of cavity operation, the cavity-relevant collective mode of the intracavity atomic ensemble is trapped in a particular minimum of the effective potential $U(Z)$ (bottom). In the regime of bistability, the two stable cavity states reflect the presence of two potential minima.

the atomic ensemble of a single photon traversing the optical cavity. During its residence time of $\sim 1/2\kappa$, such a photon would cause a dipole force that imparts an impulse of $\Delta P = f/(2\kappa)$ on the atomic medium, following which the collective mode is displaced by a distance $\Delta Z = \Delta P/(M\omega_z)$; in turn, this displacement will shift the cavity resonance frequency by $F\Delta Z/\hbar$. Comparing this single-photon-induced, transient frequency shift with the cavity half-linewidth leads us to define a dimensionless "granularity parameter" as

$$\epsilon = \sqrt{\frac{F\Delta Z}{\hbar\kappa}} = \frac{FZ_{\rm ho}}{\hbar\kappa}, \tag{5}$$

where $Z_{\rm ho} = \sqrt{\hbar/2M\omega_z}$ is the harmonic oscillator length for the atomic collective mode. The condition $\epsilon > 1$ marks the granular (or strong) opto-mechanical coupling regime in which the disturbance of the collective atomic mode by single photons is discernible both in direct quantum-limited measurements of the collective atomic motion and also in subsequent single-photon measurements of the cavity resonance frequency. In our experiments, the granularity parameter is readily tuned by adjusting frequency difference between the cavity and atomic resonance Δ_{ca}. Under conditions of our recent work, the granular regime is reached at $|\Delta_{ca}|/(2\pi) < 27$ GHz.

In recent work, we have focused on effects of fluctuations of the dipole force in the non-granular regime, attained at atom-cavity detunings in the 100 GHz range. As described in our work,[6] and also derived in earlier

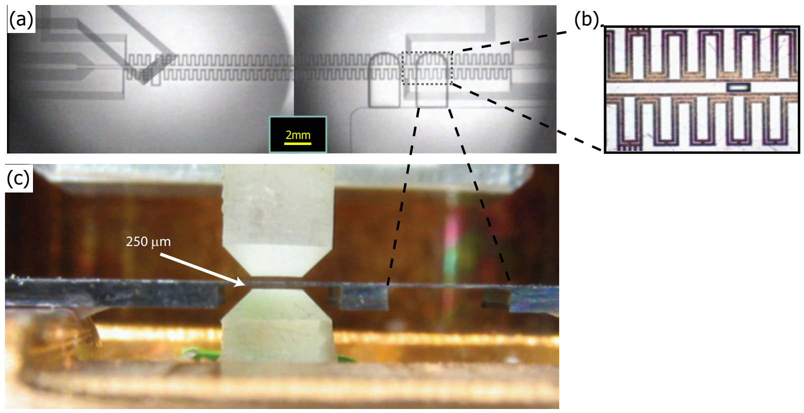

Fig. 4. An integrated cavity QED/atom chip. (a) A top view of the microfabricated silicon chip shows etched trenches, later electroplated with copper and used to tailor the magnetic field above the chip surface. The left portion of the image shows wire patterns used for producing the spherical-quadrupole field of a magneto-optical trap and also the Ioffe-Pritchard fields for producing stable magnetic traps. Serpentine wires spanning the entire chip form a magnetic conveyor system to translate atoms to the optical cavities that are located in the right half of the image. (b) A detailed view shows the serpentine wires and also a two-wire waveguide surrounding a central, rectangular hole that pierces the atom chip. (c) Fabry-Perot cavities are formed by mirrors straddling the atom chip. Between the mirrors, the chip is thinned to below 100 μm – outlines of the thinned areas are seen also in (a). The cavity mode light passes unhindered through the chip via the microfabricated holes shown in (b).

treatments,[20,23–25] these fluctuations will cause the motional energy of the collective atomic mode to vary according to the following relation:

$$\frac{d}{dt}\langle a^\dagger a\rangle = \kappa^2\epsilon^2\left[S_{nn}^{(-)} + \left(S_{nn}^{(-)} - S_{nn}^{(+)}\right)\langle a^\dagger a\rangle\right] \tag{6}$$

Here, the relevant dipole force fluctuations are derived from the spectral density of intracavity photon number fluctuations at the mechanical frequency ω_z, calculated for a coherent-state-driven cavity as $S_{nn}^{(\pm)} = 2\langle n\rangle\kappa/\left(\kappa^2 + (\Delta_{pc} \pm \omega_z)^2\right)$ Eq. 6, which can be derived readily from a rate-equation approach,[20] reveals two manners in which the mechanical oscillator responds to a cavity optical probe: momentum diffusion, which raises the mechanical oscillator energy at a constant rate, and the dynamic backaction effects of cavity-based cooling or amplification of the mechanical motion, described by an exponential damping or gain.

The mechanical momentum diffusion in an opto-mechanical system plays the essential metrological role of providing the backaction necessary in a quantum measurement, as discussed, for example, by Caves in the context of optical interferometry.[26] For $\omega_z \ll \kappa$, we see that, at constant circulating power in the cavity, this diffusion is strongest for probe light at the cavity resonance and weaker away from resonance. This dependence on the intensity and detuning of the cavity probe light precisely matches the rate of information carried by a cavity optical probe on the state of the mechanical oscillator. To elucidate this point, we recall that, under constant drive by a monochromatic input field, the intracavity electric field oscillates at the input field frequency with complex amplitude $E_{cav} = \eta/(\kappa - i\Delta_{pc})$. A displacement by ΔZ of the mechanical oscillator varies the probe-cavity detuning by $F\Delta Z/\hbar$. In response, the electric field in the cavity varies as

$$E_{cav} \simeq E_0 \left(1 + \frac{i}{\kappa - i\Delta_{pc}} \frac{F\Delta Z}{\hbar}\right) = E_0 + E_{sig}, \tag{7}$$

where E_0 is the cavity field with the cantilever at its equilibrium position and we expand to first order in ΔZ. The sensitivity of the cavity field to the cantilever displacement, at constant intracavity intensity (constant E_0), is determined by the magnitude of $|E_{sig}/E_0|^2 \propto 1/(1 + \Delta_{pc}^2/\kappa^2)$; this functional dependence matches that of the momentum diffusion term, supporting its representing measurement backaction.

To measure this backaction heating, we take advantage of several features of our experiment. First, by dint of the low temperature of our atomic ensemble, we ensure that the effects of dynamical backaction (cooling and amplification) are negligible. Second, the low quality-factor of our mechanical oscillator ensures that the momentum diffusion of the collective atomic motion leads to an overall heating of the atomic ensemble, allowing us to measure this diffusion bolometrically. Third, the large single-atom cooperativity in our cavity QED system implies that this backaction heating of the entire atomic ensemble dominates the single-atom heating due to atomic spontaneous emission. And, fourth, owing to the finite, measured depth of our intracavity optical trap, backaction heating can be measured via the light-induced loss rate of atoms from the trap. The measured light-induced heating rate was found to be in good agreement with our predictions, providing the first quantification of measurement backaction on a macroscopic object at a level consistent with quantum metrology limits.

6. Future developments: cavity QED/atom chips

While continuing explorations of quantum opto-mechanics in our existing apparatus, we are also developing an experimental platform that integrates the capabilities of single- and many-atom cavity QED onto microfabricated atom chips. Similar platforms have been developed recently by other groups.[27,28] Aside from enabling myriad applications in quantum atom optics and atom interferometry, we anticipate the cavity QED/atom chip to provide new capabilities in cold-atoms-based opto-mechanics. For instance, the tight confinement provided by microfabricated magnetic traps will allow atomic ensembles to be confined into single sites of the intracavity optical lattice potential, providing a means of tuning the opto-mechanical coupling between terms linear or quadratic in $\hat{Z}$. As emphasized by Harris and colleagues,[29] a purely quadratic coupling may allow for quantum non-demolition measurements of the energy of the macroscopic mechanical oscillator.

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