ASTR 497 / ESS 490, **Planetary Sciences and Astrobiology** Prof. David Catling

HOMEWORK: PROBLEM SET 3

Qu. 1) In the core of the Sun, all atoms are ionized. The Sun shines by virtue of nuclear fusion, with the energy source being the fusion of hydrogen nuclei (protons) to helium nuclei.

a. Nuclear fusion does not happen in the center of Jupiter, otherwise Jupiter would be a star. For what physical reasons do you think that the higher density and temperature inside the Sun enable nuclear fusion? [2 pts]

b. Using the data at the end of this question, calculate how much energy is released from the fusion reaction $4p \rightarrow He + 2e^{+} + 2\nu$, where p is a proton, e^{+} is a positron, and ν (which has negligible mass) is a neutrino. [2 pts]

c. The Sun has an energy output of 4×10^{26} J s^{-1} = 4×10^{26} W. Assuming that all this energy comes from reactions like that described above, calculate how many reactions of the type described above occur every second. [1 pt]

d. The Sun's mass is 2×10^{30} kg. To get a rough estimate, in the following questions assume that the Sun is made up solely of protons.
   (i) How many protons are in the Sun? [1 pt]
   (ii) How many reactions could be undertaken in total to make He nuclei and what total cumulative energy would this generate if all protons were fused to helium nuclei? [2 pts]
   (iii) At the Sun's current output of energy, how long could the Sun shine? [1 pt]
   (iv) In fact, the Sun will only last about 11-12 billion years on the main sequence. Explain why your answer in (iii) is an overestimate? (If you're not sure, you may should read about the evolution of stars in an astronomy textbook or using reliable online sources). [3 pts]

[Data: He ions have a mass of 6.64476×10^{-27} kg; a proton mass = 1.672649×10^{-27} kg; a positron mass = 9.109534×10^{-31} kg; speed of light = 299792458 m/s].
[12 total]

Qu.2) This question is about greenhouse warming on other planets. Use the following information to calculate the effective temperature, T_e, of Mercury, Mars and Venus. The actual mean surface temperatures of Mercury, Mars and Venus are about 439-440 K, 218 K and 735 K. Deduce the magnitude of greenhouse warming on each of these planets. Albedos: 0.77 (Venus), 0.25 (Mars), Mercury (0.06). Distance from Sun in AU: 0.72 (Venus), 1.52 (Mars), 0.39 (Mercury). Take the solar constant at 1 AU as 1370 W m^{-2}. [6 total]

Qu. 3) During the 1970s, Carl Woese used molecular sequence data from ribosomal RNA to construct the first "tree of life" based on genetics. What are two reasons why genes for rRNA are a good choice for phylogenetic studies? [Advice: If you can't think of the answer try library resources/ internet.]
[2 total]

Qu. 4) Miranda, is the smallest and innermost of the 5 major moons (out of 27) of Uranus. It has a surface temperature of 86 K. Using an equation in the lectures notes, calculate the wavelength where the maximum blackbody radiation emission from Miranda occurs. What general part of the electromagnetic spectrum describes where this wavelength resides? Now consider Triton, Neptune 's largest moon, with a surface temperature of 38 K: What wavelength is its peak blackbody emission? [4 total]

Qu. 5) In class, we discussed how solar luminosity has increased roughly 30% since the Solar System formed. The Sun will continue to increase another 40% or so in brightness over the next 6 billion years and then will suddenly brighten by a factor of thousands. What do you think might happen to the atmosphere of the Earth as the Sun evolves over the next 6 billion years? How will the biosphere be affected? Which planet might Earth eventually come to resemble and why? (There is no absolutely correct answer to this question. I'm asking you to use logic and reason. Yes, that's right: you have to think). [5 points]

Qu. 6) Assume that the scale height of the Earth's atmosphere is 6.5 km. Imagine you are in an airplane. Using the hydrostatic equation derived in class, calculate the pressure outside the window of your seat in a plane. Assume the plane is at a height of 30,000 feet. How does the pressure compare to sea-level pressure as a percentage? (1 ft = 30.48 cm). [3 points]

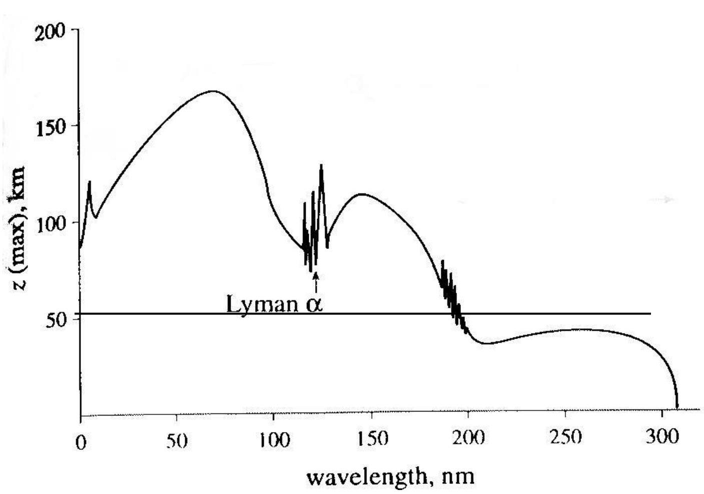
Qu. 7) As a complex, land-dwelling organism, you will appreciate that a nice part of living on Earth is that it is okay to go outside without getting hideously sunburned within tens of seconds. This is not so on Mars, for example, where UV-absorbing windows will be necessary if we send astronauts there. Earth's user-friendly environment arises from the absorption of harmful solar radiation by Earth's atmosphere. (Note: for this question, it was mentioned on the board in class how the energy of a photon is related to its frequency (and therefore its wavelength)).

(a) Calculate the wavelength in nm below at which photodissociation occurs for reactions listed in the table below, i.e., complete the table showing your working. Note that it does not necessarily follow that photodissociation will occur because the photon must be absorbed, but it is a reasonable approximation for us. [Planck's constant, h=6.626 $\times 10^{-34}$ J s; speed of light, c = 3×10^{8} m s^{-1}] [3 pts]

Photodissociation reaction	**Energy required (10^{-19} J)**	**Wavelength (nm)**	**Altitude (km)**
H_2 + hν = H + H	7.176		
N_2 + hν = N + N	15.64		
O_2 + hν = O + O	8.198		
O_3 + hν = O_2 + O	1.686		
CO_2 + hν = CO + O	8.740		

(b) Now examine the figure below. This figure shows the altitudes of maximum rates of photodissociation in the atmosphere at given wavelengths. You can think of the figure as effectively showing the altitude at which ultraviolet photons penetrate down into the Earth's atmosphere. For example, beyond about 310 nm, ultraviolet photons can reach the Earth's surface where they give most people a nice suntan and burn your fair-skinned instructor. From this figure make an estimate of the levels at which photodissociation occurs for each reaction in the table above. [2 pts]

[5 total]

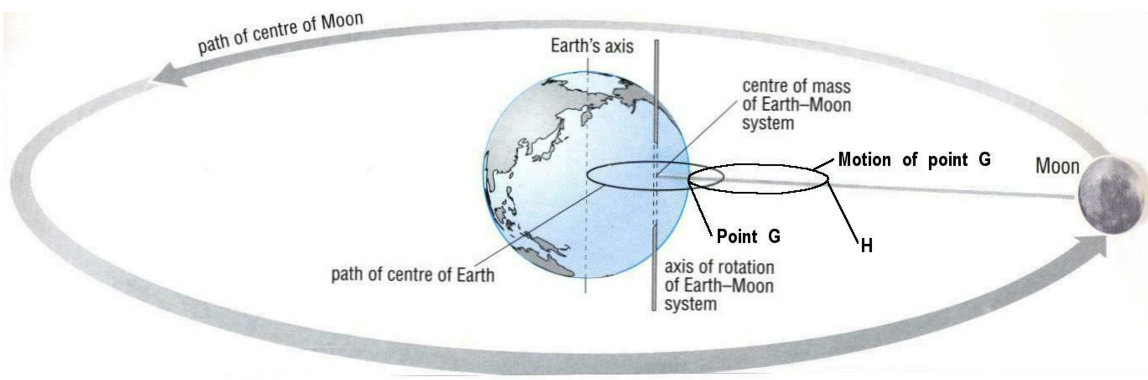


◀Figure showing the altitude, z, at which maximum dissociation rate occurs as a function of wavelength.

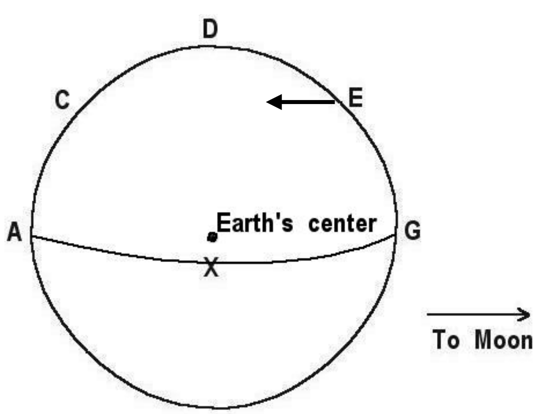
Qu. 8) In class, we used a rather simple approach to look at tides, essentially with the sole purpose of showing that the tidal forces on Earth depend on the cube of the distance of the Moon. In this question, I will walk you through some key concepts that that lie behind understanding tides in Earth's oceans [If you need help for this question as a whole, a great animation is http://www.pbs.org/wgbh/nova/earth/what-causes-the-tides.html]. In fact, the concepts have relevance to the detection of exoplanets and tides on other planets, which will become apparent later in the course. However, tides are tricky to understand, so don't bother attempting this question late at night or as a last minute effort; choose a better time.

(a) For the sake of argument, suppose that the Earth and the Moon were a gravitationally bound pair of planets equal in size, mass and density (i.e., a binary system). In this hypothetical case, about what point would both planets orbit? [1 pt]

In reality, the Moon has 81.3 times less mass than the Earth. But this still means that the Moon does not orbit around the center of the Earth. Instead both planets orbit around a common center of mass every 27.3 days. This center of mass lies about 5000 km from the center of the Earth, actually in Earth's mantle. The orbital motion of the Earth about the common center of mass in the Earth-Moon system is called the Earth's "**eccentric motion**". Because the Earth is going around in a small circle, there is centrifugal force associated with this eccentric motion. (We shall consider the Earth's eccentric motion to be circular, although in reality the motion is elliptical). DO NOT CONFUSE THE CENTRIFUGAL FORCE ASSOCIATED WITH THE ECCENTRIC MOTION WITH THE CENTRIFUGAL FORCE DUE TO EARTH'S SPIN. IN FACT, IGNORE THE LATTER FOR THE PURPOSE OF THIS QUESTION. Examine the picture below. You'll see that the center of the Earth goes around the common center of mass in the Earth-Moon system. Also all other points on Earth, such as point G, will execute a similar path. For example, when G has moved to the point marked H, the Moon will have moved to the left side of the diagram. (Of course, G executes motion on top of the eccentric motion due to the Earth spinning, but for now let's only consider the component of motion due to eccentric motion). The key point is that there is a centrifugal force resulting from the circular eccentric motion which is **equal** at all points on the Earth, directed parallel to the line joining the centers of the Earth and Moon, and always directed **away** from the Moon.



(b) Consider the diagram of the Earth below. The center of the Moon is assumed to lie above point G on the Earth's equator.



(i) Copy the diagram. Using color or different line styles, mark the centrifugal force due to Earth's **eccentric motion** on all the points (A, C, D, G, and X) as an arrowed vector. The centrifugal force is already shown for point E. Note that in this diagram, A, G and X are all meant to be on Earth's equator.

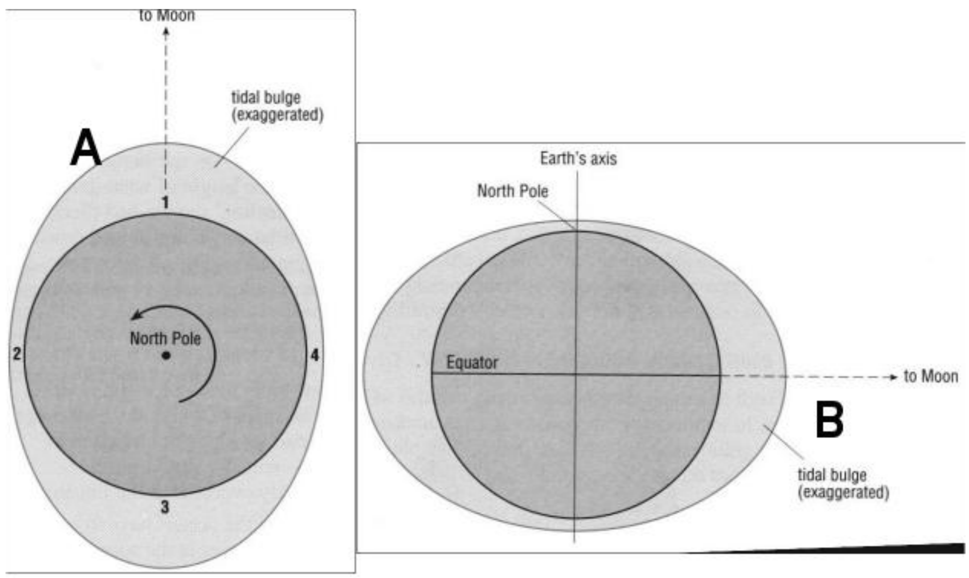
(ii) Using an arrowed vector mark the Moon's gravitational pull at each point (A, C, D, E, G, and X) with the length of each vector corresponding to the relative strength of the pull. (Hint #1: the Moon's gravitational pull is directed towards the center of the

Moon. Hint #2: At the center of the Earth, the gravitational pull of the Moon is equal to the centrifugal force of the eccentric motion (indeed, without this balance of forces we would collide with the Moon or lose it). But at any other point the gravitational pull of the Moon will be either smaller or greater than the centrifugal force at the center of the Earth.)

(iii) Except for point X, which is tricky to mark down, draw a resultant force at each point due to the vector combination of the Moon's pull and the eccentric motion centrifugal force. (The direction of the resultant force at X is into the Earth and so can't easily be drawn).
[4 pts]

(c) (i) With reference to your diagram produced in response to part (b) explain why ocean tidal bulges look like those shown below in diagram **B**. [4 pts]

(ii) With reference to the diagram below marked **A**, explain why beaches on the Earth experience two high tides and two low tides every day. [2 pts]

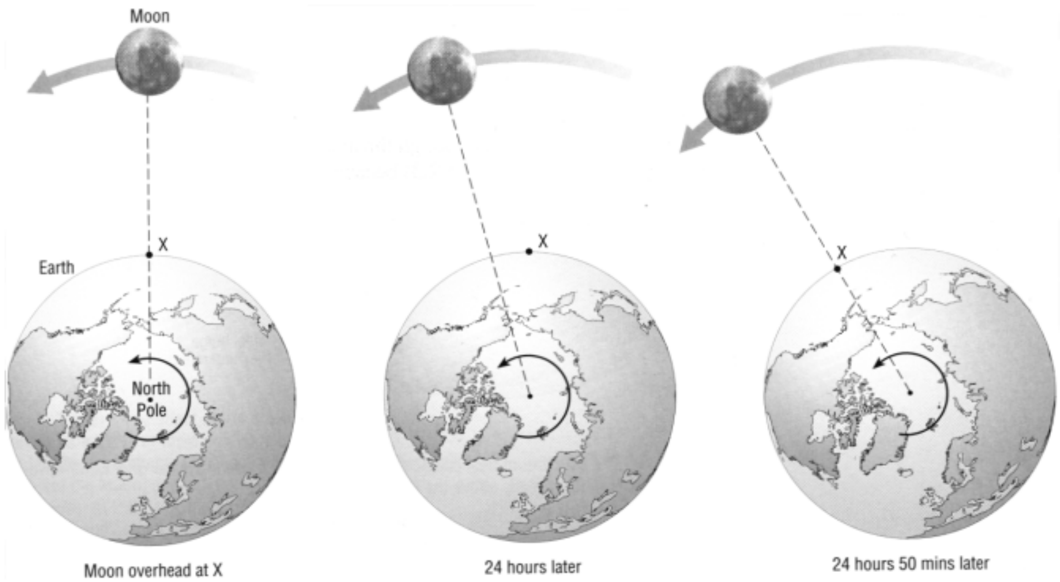


(d) The Moon continues to travel in its orbit as the Earth rotates. Examine the diagram below. Point X is on the Earth's equator when the Moon is directly overhead and comes back to its starting position 24 hours later. Meanwhile the Moon has moved on in its orbit so that point X has to rotate around (another 50 minutes worth) before it is once again directly under the Moon. In fact, the period of the Earth's rotation with respect to the Moon is called a "lunar day" and is defined as 24 hr 50 min.

(i) Using a value of 40,000 km for the Earth's circumference and a period of 24 hr 50 min, what speed would the tidal bulges have to move relative to the Earth's surface along the equator, in order to keep up with the Moon? [2 pts]

(ii) The theory for shallow water waves says that the speed of shallow waves is given by $v = (gd)^{½}$ where g is gravitational acceleration and d is depth. The average depth of oceans basins is about 4 km, while elsewhere, on continental shelves, the ocean depth becomes shallower and shallower. Calculate an approximate maximum speed of shallow water waves in the oceans. [g = 9.8 m s^{-2}]. [2 pts]

The speed you calculate in (ii) should be lower than that in (i). In fact, this means that tides cannot keep up with the Moon at the equator (although the speed required is less at high latitudes where the circumference is smaller, and the required speed is zero at the poles). If you ever go to a tropical island, you'll notice that tides arrive late, about 6 hours after the Moon has passed overhead. However, in Seattle – at higher latitude -- the lag time is very small.



[15 points total]

9) Mars has a 24.66 hr day. The distribution of the mass of the planet changes slightly with the seasons as carbon dioxide from its atmosphere condenses upon the winter pole and sublimes from the summer pole in an asymmetric fashion. In principle, would you expect this redistribution of mass to affect the rotation rate of Mars? Justify your answer (qualitatively). [4 points total]