HOMEWORK PROBLEMS SET 2

SOLAR SYSTEM ORIGIN AND PLANET FORMATION

Qu. 1) Many chondrules contain iron sulfide (FeS) mixed in with silicate materials. It we assume that the FeS has not grown within the chondrules after they formed, what constraint does this put on the temperature at which the chondrules formed? (Hint: Consider the notes on condensation sequence]. [2 pts]

Qu. 2) The mass of a planetary embryo increases with the semi-major axis of its orbit around the Sun. The rationale for this is that the runaway growth of an embryo should stop when the embryo has accreted and exhausted most of the mass available in an annulus of width comparable to its Hill's radius, which is its gravitational "feeding zone". The Hill's radius of a body of mass M depends on the heliocentric distance a (i.e. distance from the Sun) according to the following formula,

$$R_H = a\left(\frac{M}{3M_{\odot}}\right)^{1/3} \qquad (1)$$

where $M_{\odot}$ is the mass of the Sun.

(a) Suppose the solar nebula has a column density (kg m^{-2}) of solid material denoted σ. Write down an expression for the mass, M, available in an annulus of width R_H at radius a from the Sun. (Hint: consider the circumference at radius a and assume that the R_H is a small delta on top of a).
[**2 pts**]

(b) Substitute R_H from equation (1) into your expression from (b) and rearrange to get an expression for M, the mass of an embryo at the end of its runaway growth. The column density, σ, is usually assumed to vary with distance a from the Sun proportional to a factor between $1/a$ and $1/(a^{3/2})$. By combining this fact with your solution for M, answer the following question: how does the mass of a planetary embryo vary with distance from the Sun?
[**3 pts**]

(c) Why is this model for planetary embryos consistent with the core-instability hypothesis for how Jupiter formed?
[**2 pts**]

Qu. 3) Planets grow slowly when planetary embryos and/or planetesimals collide. A simple model for the growth rate of a terrestrial planet results in the following formula:

$$\frac{dR}{dt} = \frac{F_g \sigma n}{4\rho}\sqrt{\frac{3}{\pi}}$$

where σ is the column density of solid material in the solar nebula, ρ is the density of the planetary embryo, R is the radius of the growing planet, F_g is a constant called the "gravitational enhancement factor" that takes account of gravitational effects when planetesimals closely approach each other, n is a constant, and t is time.

(a) Assume that the Earth grew at a constant rate. Use the above equation to work out how long it took (in years) for the Earth to grow. You can assume the following parameters: σ = 10 g cm^{-2} (the minimum solar nebula value), $n = 2\times10^{-7}$ s^{-1}, ρ = 4.5 g cm^{-3}, the radius of the Earth = 6371 km, and $F_g = 7$. [2 pts]

(b) The value you calculated in (a) is about 10 times less than the growth time for the Earth discussed in class. Suggest a reason why the estimate in (a) for growth time is too short. [2 pts]

(c) Now calculate the timescale for Neptune. Take $\sigma = 0.2$ g cm^{-2} at the distance of Neptune. Neptune has radius 24300 km. Otherwise, assume that the other parameters are the same. Why is this timescale far too long to be reasonable? Clearly, the simple theory breaks down. As described in lectures, one current theory (the "Nice model") favored by some planetary scientists suggests outward migration of the ice giants and that Neptune formed closer to the Sun. [2 pts]

Qu. 4) This question is about planetary orbits and introduces some basic physics about them.

(a) Newton's first law says that a body continues at constant velocity in a straight line unless it is acted upon by a force that causes an acceleration (i.e., a change in velocity). Most planetary orbits are nearly circular. A body in circular motion is obviously not moving in a straight line and so must have a force acting upon it. In circular motion, basic physics tells us that the centripetal acceleration at orbital distance r is given by v^2/r, where v is the tangential speed. Using Newton's second law, force = mass × acceleration, the centripetal force of circular motion is given by $F = mv^2/r$, where m is the mass of the planet. Newton's universal law of gravitation tells us that the gravitational force, F, from the Sun of mass M at orbital radius r is $F = GMm/r^2$, where G is the universal gravitational constant ($G = 6.672\times10^{-11}$ N m^2 kg^{-1}). The centripetal force is provided by the gravitational force. Equate these two forces and show that the tangential velocity is $v = (GM/r)^{1/2}$. Considering the circumference of a circular orbit and the time to go round an orbit, derive an expression that proves Kepler's third law. [3 pts]

(b) Another velocity of interest is that required to escape from the Solar System, which we denote v_{escape}. The kinetic energy of a body is given by the well-known expression $(\frac{1}{2})mv^2$. The kinetic energy of a body falling from infinite distance to an orbital distance around the Sun is equal to the gravitational work done on that body during its fall. (Recall that in physics, work is energy in units of Joules given by work = force × distance, by definition). So the kinetic energy gained by a body falling towards the Sun will be equal to the integral of the gravitational force times the distance from infinity to the orbital radius about the Sun r. The argument can be reversed. The kinetic energy required to just escape from an orbit around the Sun, $\frac{1}{2}mv_{escape}^2$, is equal to that integral of gravitational force times distance. Using this argument and the expression for Newton's gravitational force in (a), show that the escape velocity, v_{escape}, is related to the circular orbital velocity by $v_{escape} = v\sqrt{2}$. What speed directed away from the Sun would be required for the Earth to escape from its orbit? (The orbital distance of the Earth from the Sun is 1.5×10^{11} m and the mass of the Sun is 1.99×10^{30} kg).
[3 pts]