Clouds in planetary atmospheres: A useful application of the Clausius–Clapeyron equation

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The Clausius–Clapeyron equation is used to do a comparative study of the properties of the clouds that form in planetary atmospheres. Simple static atmospheric models for various planets, the satellite Titan, and the extrasolar planet HD209458b are used together with the saturation vapor pressure curves of the different kinds of molecules to determine the pressure, density, and scale height of the clouds in each body. This application of the Clausius–Clapeyron equation extends our knowledge of terrestrial water clouds to different exotic clouds present in other planets. © 2004 American Association of Physics Teachers.

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I. INTRODUCTION

The study of the solar system with ground-based and space telescopes as well as with visiting spacecrafts has shown that most of the planets and some massive satellites have atmospheres with clouds and suspended aerosol particles.1,2 Clouds are expected to be present in the new family of recently discovered giant extrasolar planets.3 In most cases the clouds determine the visual appearance of these bodies and serve as tracers of meteorological phenomena (see Fig. 1). Clouds also play an important role in determining the heat balance and global atmospheric dynamics.

Cloud formation is very complicated and involves many facets of atmospheric science such as cloud particle physics (microphysics)4 and thermodynamics. Suspended particles in the atmosphere can form from a variety of natural processes including condensation, chemical and photochemical reactions, outgassing or lifting from the surface (dust, evaporation and sublimation, volcanic activity), and particle bombardment of atoms and molecules in the upper atmosphere. Except for the Earth, most of the fundamental aspects of cloud constitution and formation mechanisms on the planets are poorly known. However, the most massive clouds in planetary atmospheres form from the condensation of minor atmospheric constituents. Thus, a simple approach to cloud formation and to their basic properties in planets and satellites can be derived from fundamental physical assumptions. In this paper we compare the nature, location, and expected properties of the condensate clouds of planetary atmospheres. We also give a practical example of the use of the Clausius–Clapeyron equation which can be incorporated into courses on thermodynamics and meteorology.

We study those planets that possess massive atmospheres (Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune), a satellite of Saturn (Titan), and the best characterized extrasolar planet (HD 209458b). The comparison of planetary clouds will help students gain a broad perspective on how different types of condensate clouds can form under very different environments than that found on Earth.

To understand the physics of planetary clouds in terms of a condensation process, we define the reference thermal state and chemical composition of the atmosphere in Sec. II. We introduce the Clausius–Clapeyron equation in Sec. III and express it in a useful form for atmospheric physics. At this stage we have all the necessary tools to explore what type of clouds and at what altitude will form in any planet (Sec. IV A). The data needed to perform these calculations for each planet are given in tabular form. A comparison between modeled and real clouds is presented for the Earth and Venus. The Clausius–Clapeyron equation also lets us obtain the expected cloud density, vertical extent of the clouds, and how the vertical temperature profile is modified by cloud formation (Secs. IV B and IV C). In Sec. V we discuss some properties of clouds that cannot be treated in this study but that can be useful for teachers to know.

II. VERTICAL TEMPERATURE PROFILE IN ATMOSPHERES

For simplicity, we assume the atmosphere to be at rest (with no heat transport by motion). Because it is in hydrostatic equilibrium, the temperature will depend only on altitude; this dependence is called the vertical temperature profile. To calculate it simply we need to make additional assumptions. We assume that the atmosphere obeys the ideal gas law

$$P = \rho R^* T,$$

where $P$ is the pressure, $T$ is the temperature, $\rho$ is the density of the gas, and $R^*$ is the specific gas constant (the universal gas constant divided by the mean atmospheric molecular weight $\mu$). Because the atmosphere is assumed to be in hydrostatic balance, we have

$$\frac{dP}{dz} = -\rho g,$$

where $z$ is the vertical coordinate and $g$ is the acceleration due to gravity. Upon integration we obtain the altitude–pressure relation, which requires knowledge of the vertical temperature profile $T(z)$,

$$P(z) = P_0 \exp \left[ -g \int_{z_0}^{z} \frac{dz'}{R^* T(z')} \right],$$

where $P_0 = P(z_0)$. The vertical temperature profile $T(z)$ depends on radiative and dynamic (advective or convective) energy transfer. Here we concentrate on the troposphere (the weather layer), which is the atmospheric layer where the temperature decreases with altitude ($dT/dz < 0$) and most of
the atmospheric mass resides and clouds form. We consider two situations:

(a) An isothermal atmosphere, $T(z) = T_0$, so that Eq. (3) becomes

$$P(z) = P_0 \exp\left(-\frac{z}{H}\right),$$

where we have introduced the scale height $H = R_a T_0 / g$. $H$ represents a useful length scale to characterize vertical motions in the atmosphere.

(b) For an adiabatic (an atmosphere for which there is no heat exchange) and a dry atmosphere, we can use the first law of thermodynamics, Eqs. (1) and (2), and the fact that $c_p = c_v + R_a$ to obtain $\Gamma_a = -T_0 dT / dz = g c_p$, where $\Gamma_a$ is the adiabatic gradient and $c_p(c_v)$ is the specific heat at constant pressure (volume) (per unit mass). If we assume $\Gamma_a = \text{const}$ ($g$ is constant and $c_p$ does not depend on the temperature), we have

$$T(z) = T_0 - \Gamma_a (z - z_0).$$

If we substitute Eq. (5) into Eq. (3), we obtain

$$P(z) = P_0 \left[1 - \frac{\Gamma_a(z - z_0)}{T_0}\right]^{g \Gamma_a R_a}.$$  

Note that in real atmospheres the vertical temperature gradient is lower than $\Gamma_a$, as long as condensation releases heat. This point will be discussed in Sec. IV C. Here we assume a dry atmosphere where condensation occurs. Equations (5) and (6) then give the dry adiabatic profile

$$P(T) = P_0 \left(\frac{T}{T_0}\right)^{g \Gamma_a R_a}.$$  

In Table I we present the data for planetary atmospheres needed to calculate the average $T(P)$ or $T(z)$ profiles. Some comments are necessary to clarify the values presented in Table I. First, the atmospheres of the terrestrial planets (Venus, Mars, and Earth) were formed by the outgassing of volatiles during their hot formation phase and have largely evolved in time. On the contrary, the giant planet atmospheres (Jupiter, Saturn, Uranus, and Neptune) are practically of primordial origin, that is, their composition has remained practically unchanged since the origin of our solar system. Second, for the terrestrial planets and the satellite Titan, we take for $T_0$ and $P_0$ the surface values $T_S$ and $P_S$. However, the atmospheres of the giant planets are outward extensions of the fluid planetary bodies, extending deeply into the interior and up to pressures of $\sim 1-2$ Mbar, where molecular hydrogen is converted gradually to a metallic (liquid) phase. Because no ground is present in these planets, we take as reference values $P_0 = 1$ bar and $T_0 = T(P_0)$. In general, the value of $T_0$ is close to the equilibrium temperature of the planet, that is, to the temperature that a planet would have in thermal balance with the solar radiation. However, the greenhouse effect modifies the surface temperature of the terrestrial planets (the extreme case is Venus), and the internal heat source modifies $T_0$ in Jupiter, Saturn, Neptune, and probably in the giant extrasolar planets. Curiously, Uranus does not have an internal energy source so $T_0$ is the equilibrium temperature due to solar radiation.

III. THE CLAUSIUS–CLAPEYRON EQUATION

The gases composing planetary tropospheres are volatile because of the low temperatures and pressures. The main intervening elements in their composition are C, H, O, N, and S. They form molecules such as $\text{N}_2$, $\text{O}_2$, $\text{H}_2$, $\text{H}_2\text{O}$, $\text{CO}_2$, $\text{CH}_4$, $\text{NH}_3$, and $\text{SH}_2$. Those substances, or their reaction products, whose bond strength is weakest, tend to form clouds at the lowest temperature by condensing from the gas phase to the liquid or solid phases. The abundance of the condensable gases is measured in terms of the mass mixing ratio $m_c = \rho_v / \rho$ (or the ratio of the vapor mass to the total mass). Because the partial pressure of the condensable

![Fig. 1. A panorama at visual wavelengths of the eight solar system bodies (not to scale) whose clouds' properties are discussed in the text. From left to right first row: Venus, Earth, Mars, Titan (a satellite of Saturn), and in the second row: Jupiter, Saturn, Uranus and Neptune. Image composed from NASA images.](image-url)
gases satisfies $P_p = P$ (total pressure), we can use Dalton’s and ideal gas equations to obtain $m_c = \varepsilon P_v / P$, where $\varepsilon = \mu_{\text{vap}} / \mu$ is the ratio between the vapor molecular weight and the mean atmospheric molecular weight (in this context $m_c$ is also called the specific humidity). If saturation occurs, then the saturated mass mixing ratio is given by

$$m_3(T, P) = \varepsilon \frac{P_v(T)}{P}, \quad (8)$$

where $P_v(T)$ is the saturated vapor pressure. The relative humidity is defined as $f(T, P) = 100P_v / P_v(T) = 100(m_c / m_v)$. Another way to measure the abundance of a gas mixture is by means of the volume mixing ratio (also called the molar mixing ratio or mole fraction)

$$X_c = \frac{V_c}{V} = \frac{P_p}{P} = \frac{m_c}{\varepsilon}. \quad (9)$$

Note that in general for a mixture of $i$ gases with volume mixing ratios $X_i$, we have

$$\mu = \sum_i \mu_i X_i. \quad (10)$$

The slope of the vapor pressure curve $P_v(T)$ in Eq. (8) marks the phase transition where two phases are in equilibrium and is known as the Clausius–Clapeyron equation:

$$\frac{dP_v}{dT} = \frac{L}{T(V_2 - V_1)}, \quad (11)$$

where $L$ is the latent heat of the phase transition (in J g$^{-1}$) and $V_i = 1/\rho_i$ is the specific volume (phase 1 = vapor phase, 2 = liquid or ice phase). In Fig. 2 we show the pressure-temperature phase diagram for water, which is a common liquid in planetary atmospheres.

A useful approximation to Eq. (11) can be obtained under the following assumptions: the temperature variations in the latent heat can be neglected, the vapor is an ideal gas, and the specific volume of the liquid or solid phases is neglected compared to that of the vapor, that is, $V_2 - V_1 \sim V_{\text{vapor}} \sim 1/\rho_v = R_v T / P$, with $R_v$ the specific gas constant for the vapor. Then Eq. (11) can be rewritten as

$$\frac{dP_v}{dT} = \frac{LP_v}{R_v T^2}, \quad (12)$$

which is the approximate form of the Clausius–Clapeyron equation employed in introductory meteorological textbooks. If we integrate Eq. (12) with $L = \text{const}$, we obtain

$$P_v(T) = P_0 \exp \left[ L \left( \frac{1}{R_v T_0} - \frac{1}{R_v T} \right) \right], \quad (13)$$

where $P_0$ is the saturation vapor pressure at temperature $T_0$. A more accurate expression for the latent heat can be obtained by integrating the relation

$$\left| \frac{\partial L}{\partial T} \right| = \Delta C_p, \quad (14)$$

where $\Delta C_p$ is the change of the specific heat between the two phases. In the following $\Delta$ denotes an increment. If we expand the specific heat for each phase as $C_p(T) = \alpha + \beta T + \cdots$, we obtain

$$L = L_0 + \Delta \alpha T + \frac{\Delta \beta}{2} T^2 + O(T^3), \quad (15)$$

where $L_0$ is an integration constant, $\alpha$ and $\beta$ are empirically determined constants for each phase, and $\Delta \alpha$ and $\Delta \beta$ indicate the change of the constants $\alpha$ and $\beta$ between the two phases. We combine Eqs. (12) and (15) to find the general form for the saturation vapor pressure curve:

$$\ln(P_v) = \ln(C) + \frac{1}{R_v} \left[ -\frac{L_0}{T} + \Delta \alpha \ln T + \frac{\Delta \beta}{2} T + O(T^2) \right], \quad (16)$$
In this paper we will use the saturation vapor pressure curves for each molecule as determined empirically for the vapor-liquid or vapor-solid equilibrium transitions in the range of temperatures found for each planet. In Table II we give the appropriate values for the coefficients in Eq. (16).

### IV. CLOUDS IN PLANETARY ATMOSPHERES

#### A. Condensation level and cloud formation

We shall assume that condensation and cloud formation takes place whenever the partial pressure \( P_p \) of the vapor exceeds the saturation vapor pressure \( P_v(T) \), that is, when supersaturation occurs (a process known as homogeneous condensation),

\[
P_p = X_C P(T) > P_v(T) \quad \text{(17)}
\]

Equation (17) is equivalent to stating that the relative humidity, \( f \), of the condensing layer is greater than 100%. Thus, for a given atmosphere, a comparison of its temperature vertical profile \( T(P) \) as given in Sec. II, with the saturation vapor pressure curve \( P_v(T)/X_C \), tells us which gases condense to form clouds and at what pressure (altitude) the clouds form. The cloud base is located where \( P_{c}(T_{cl}) = P_v(T_{cl})/X_C \). In Fig. 3 we show the vertical temperature profiles for each planetary atmosphere as well as the saturation vapor pressure curves for the different condensates. The crossing point between the thermal profile and the saturation vapor pressure curve marks the cloud base. In some cases different condensation lines have been plotted for the same molecule because they depend on values of the concentration of the gas that are not well known or vary at different locations.

In Table III we list the values of the measured molar fractions of each planet condensate and the derived mean properties. Of course, the condensed phase (liquid or solid) depends on the atmospheric temperature-pressure relation and on the condensate phase diagram. As an example, we add to the phase diagram in Fig. 2 the pressure-temperature ranges for the different planets where water forms. In the first column in Table III we give the expected phase of the corresponding condensate.

An examination of the Earth’s atmosphere gives some insight. The average temperature profile in the lower troposphere has a vertical temperature gradient \( \Gamma \sim 6.5 \text{ K/km}^2 \). We use this value of \( \Gamma \) in Eq. (7) and the surface temperature and pressure values \( T_0, P_0 \) in Table I to find the empirical relation:

\[
P(T) = 1.013 \left( \frac{T(K)}{288} \right)^{5.2} \quad \text{(18)}
\]

The Earth’s molar mixing ratio for water (wet atmosphere) is \( X_C = 0.015 \). The saturation vapor pressure curve in Eq. (16) for water can be obtained from Table II and Eq. (17):

\[
P_v(T) = \frac{1}{0.015} \exp \left[ 25.096 - \frac{6823.15}{T(K)} - 0.019 T(K) \right] \quad \text{(19)}
\]

This system of equations (18) and (19) is best resolved graphically as can be seen in Fig. 3, which gives \( T = 285 \text{ K} \).
and $P = 0.96$ bar. If we use the hydrostatic approximation Eq. (4), we can convert the pressure into height from the surface leading to clouds at $z=500$ m. For a dry atmosphere the mixing ratio is $X_C = 2.5 \times 10^{-4}$ and clouds are formed at $z \sim 10$ km. As another example, the lower troposphere temperature gradient for Venus is $\Gamma \sim 8$ K/km, and we find clouds at $P \sim 1$ bar or $P \sim 11.3$ bar depending on the local abundance of sulphuric acid. Their heights from the surface are $z \sim 45$ km and $z \sim 23$ km, which reproduce the levels of Venus' main clouds.²

### B. Cloud density and vertical extent

The vertical extent of the clouds can be derived using Eq. (12) using $T_0 = T_{cl}$ for the temperature at the cloud base,

$$P(T) = P_{V,cl} \exp \left[ -\frac{L(T_{cl} - T)}{R VT_{cl}} \right].$$  

From Eq. (5) we have $T_{cl} - T = (g/c_p)(z - z_{cl})$ and because $(T-T_{cl})/T \ll 1$ in general, we can take $T \sim T_{cl}$, and write

$$P(T) = P_{V,cl} \exp \left[ -\frac{L(g/c_p)(z - z_{cl})}{R_{T cl}^2} \right].$$  

We define a vertical scale height for the cloud as

$$H_c = \frac{R_{T cl}^2 c_p}{gL}.$$  

A useful parameter for comparing the cloud vertical extent among the different planets is the ratio of the cloud and the atmospheric scale heights,

$$\frac{H_c}{\bar{H}} \sim \frac{c_p T_{cl} R_V}{L \bar{R}}.$$
If we compare the data for the different planets given in Table III, we see that in general $H_{c}/H \sim 0.05-0.2$, and thus cloudy layers are expected to be thin relative to the atmospheric scale height.

The cloud density is the ratio of the condensate mass (mass mixing ratio times the atmospheric mass) to the total volume:

$$\rho_{cl} = m_{cl} \frac{P_{cl}}{g H_{c}}. \quad (24)$$

Note that $\rho_{cl}$ is the maximum cloud density because we have assumed that all the vapor in the atmosphere condenses with no precipitation. In general, Eq. (24) must be multiplied by a factor ranging from 0.01 to 1 to treat a more realistic cloud density distribution.

C. Change in the temperature profile by cloud condensation

The latent heat released during cloud formation heats the atmosphere locally, changing the vertical temperature gradi-
ent to a wet adiabatic gradient, $\Gamma_S$. This gradient can be obtained by another application of the Clausius–Clapeyron equation. We start with the first law of thermodynamics and the hydrostatic equation. We assume that a single component condenses and that the amount of heat removed from an ascending parcel by the condensed phase is small compared with that remaining in the parcel, a process referred as pseudo-adiabatic.\(^6\) At saturation, the amount of heat deposited in the condensing layer is $\delta Q = -L dm_S$, and the first law of thermodynamics can be written as

$$c_p dT = -g dz - L dm_S.$$  \hfill (25)

Here $dm_S$ is the change in the saturation mixing ratio of the condensable gas, that is, the mass of the vapor that condenses out per gram of dry gas (formed by the noncondensable species). We take logarithms in Eq. (8), differentiate, and obtain

$$\frac{dm_S}{m_S} = \frac{dP_V}{P_V} = \frac{dP}{P}.$$  \hfill (26)

If we use Eq. (12) for $dP_V/P_V$ and the hydrostatic relationship for $dP$ and introduce both in Eqs. (25) and (26), we obtain the pseudo-adiabatic gradient

$$\Gamma_S = -\frac{dT}{dz} = \frac{g}{c_p} \left( 1 + \frac{L m_S}{R V} \right),$$  \hfill (27)

Inside the cloud, the vertical temperature gradient becomes $\Gamma_S$. It is always found that $\Gamma_S \approx \Gamma_a$, due to the latent heat release at condensation. In Table III we give the value of the pseudoadiabatic gradient for the condensing clouds for each planet. The ratio $\Gamma_S / \Gamma_a$ can be used to estimate the influence that condensation and cloud formation have on the thermal structure of the planetary atmospheres.

V. ADDITIONAL COMMENTS ON PLANETARY CLOUDS

A look at Table III shows that a large variety of cloud compositions and properties are present in planetary atmospheres. There are two cases where the clouds come from chemical reactions.\(^3\) One is the Venussian clouds composed of sulfuric acid ($H_2SO_4$) as the result of a reaction cycle between $SO_2$ and $H_2O$ as the main ingredients. The other case is that of ammonium hydrosulfide ($NH_4SH$) clouds, which are present in the giant and icy planets that result from a reaction between $NH_3$ and $SH_2$.

Some clouds are really exotic, such as when particles condense in the atmospheres of the giant extrasolar planets that are very close to their stars.\(^3\) These planets are nicknamed “hot Jupiters” because they are gas giants at high temperature (close to 1500 K) because of the intense stellar radiation. In such hot environments, only clouds of refractory materials like iron and enstatite ($MgSiO_3$) are able to condense.\(^9\)

Mars’ main condensate is a special case. The main atmospheric constituent ($CO_2$) can condense when the temperatures become low enough. Thus, in the polar areas, great deposits of carbonic snow form during the winter season, with the mean atmospheric pressure dropping when the atmospheric mass is removed (the Martian atmospheric pressure fluctuates following the seasonal cycle).\(^10\) In addition, the atmosphere usually has important quantities of dust lifted from the ground by the blowing winds. A rare but important event in this context is the large-scale dust storms that develop when the winds are intense enough to lift and spread the dust along the entire planet. Then surface markings become invisible, but the reason is not the condensing clouds but the suspended dust.

The clouds predicted by thermochemical studies are essentially colorless (white). However, the visible clouds of Venus and those of the giant planets show a faint color.\(^1\) The agent that makes the Venussian clouds yellowish and the Jovian clouds brown or red is unknown. Most surely, these contaminant molecules must be the result of nonequilibrium processes in the atmosphere. In addition, the formation of high altitude hazes by photochemical processes in the giant and icy planets, together with the absorption and Rayleigh scattering of the solar radiation by the gases, cause the visual appearance of cloudy planetary atmospheres. The same also occurs on the satellite Titan which has a dense and vertically extended reddish haze layer made of hydrocarbons.

A significant difference between the Earth and the other planets is that although the terrestrial clouds are composed of a single component, the giant and icy planets have a multilayer system of clouds with the composition changing with altitude. Most clouds in planets are probably of the stratiform type, as predicted by models\(^5,10,11\) and found by in situ measurements made by probes on Venus, Earth, Mars, and Jupiter. The predicted vertical extent of these cloudy layers is always between 5% and 20% of the atmospheric scale height ($H_C/H \sim 0.05–0.2$). However, vertical mixing between the layers resulting from dynamics is expected. It occurs on Earth most intensely during convective storms (cumulus thunderstorms) which extend vertically a height ($H_C/H \sim 10$ km), reaching the tropopause. There is evidence of convective water clouds on the giant planets Jupiter\(^12,13\) and Saturn.\(^14\) However, there is an important difference with the Earth. The condensate on Earth has a smaller molecular weight (18 for water) than the bulk atmosphere (29 for air), but, in contrast, the condensates on the giant planets have higher molecular weight than their light hydrogen bulk atmosphere. This difference has important consequences for convective storm development.\(^14\)

Finally, we note that although Earth and Mars have partial cloud coverage, Venus, the giant planets and Titan, have fully cloud covered atmospheres.

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\(^6\) W. Sears, An introduction to Thermodynamics, the Kinetic Theory of Gases and Statistical Mechanics (Addison-Wesley, Reading, MA, 1953).
THE MEASUREMENT PROBLEM IN BASEBALL

The old National League umpire Bill Klem told a story about a batter who persisted in arguing with him over the way he was calling balls and strikes. "Any idiot could see that last pitch was a ball!" the batter screamed. "Listen," Klem responded calmly. "Until I called it, it wasn’t anything; it became a strike when I called it a strike." I doubt if Klem was a student of Bohr’s philosophy, but that remark summed it up beautifully.


Submitted by Alan DeWeerd.

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