HOMEWORK 3: Some radiation plus atmospheric escape on Earth, Venus, Titan.

Qu. 1) [9 total] As mentioned in class, hydrogen escape has been important in the evolution of the Earth's atmosphere and is ultimately linked to the origin of our oxygen-rich atmosphere and the existence of animals like ourselves. In the 1920s, James Jeans explained the theory for the thermal escape flux of molecules from the top of the atmosphere. Consequently, the process of escape from the exosphere due to thermal motion is called "Jean escape". Jeans gave the escape flux as

$$F = \frac{nu}{2\pi^{1/2}} e^{-v^2/u^2} \left(\frac{v^2}{u^2} + 1 \right)$$

where n is the number density of molecules of a certain species at the exobase (the escape level), v is the escape velocity for the Earth, and u is the most probable molecular velocity (not to be confused with mean velocity). The latter is given by kinetic theory as

$$u = \sqrt{\frac{2kT}{m}}$$

where m is the molecular mass and k is Boltzmann's constant (1.38066×10^{-23} J K^{-1})

Unfortunately, Jeans wrongly believed that Earth's upper atmosphere was extremely cold and deduced that essentially no hydrogen escaped from the Earth. In fact, we now know that the exosphere has a high kinetic temperature ~1000 K.

(a) The lifetime of a molecular or atomic species at the exobase is proportional to n/F. From the above equations and an expression for escape velocity, deduce the ratio of the lifetime of atomic oxygen to atomic hydrogen, assuming a temperature of 1000 K. (Be careful with the algebra. Hint: an elegant solution would work through algebra for the ratio $(n_O/F_O)/(n_H/F_H)$ and then substitute numerical values for velocities only after simplifying the algebra. An inelegant, messy solution would substitute numerical values at the beginning). [7 points].

(b) A hydrogen atom, once it reaches the exobase, escapes within hours. Using your answer to (a), what does this tell you about how much oxygen has escaped from the Earth over its history? [2 points]

[Data: An H atom mass is about 1 atomic mass unit, 1.66×10^{-27} kg; Earth's radius is 6371 km and the exobase height is around 500 km; O has a mass of 16 atomic mass units].

Qu. 2) [9 total] Venus probably started out with an inventory of volatile substances (carbon, sulfur, water, etc.) similar to the Earth's. Presumably, Venus had a primordial ocean. If Venus lost its primordial ocean because water vapor was photolyzed and hydrogen escaped to space, a question arises about where all the oxygen went. The surface of Venus is red-colored suggesting the presence of fully oxidized iron (Fe_2O_3 = hematite) and perhaps partially oxidized iron (Fe_3O_4= magnetite). Originally Venus would have had a gray surface characterized by iron in the Fe^{2+} state (FeO), like the Moon. At some point the crust oxidized.

Suppose the endpoint of FeO oxidation in the crust was predominantly Fe_3O_4 (magnetite). For a rough calculation, let us assume that the crust was oxidized to 30 km depth (a reasonable depth for "tectonic mixing") and started out with 10% (by mass) FeO that all got oxidized to magnetite via:

$$3FeO + 0.5O_2 = Fe_3O_4$$

(a) Calculate how much oxygen (in kg of O per square meter) the crust would absorb in this case, noting data at the end of this question. Consider a column of crust. [4 points]

(b) Given that the mean radius of Venus is 6052 km, what is the total mass (in kg) of oxygen absorbed by the crust in this scenario? [1 point]

(c) The oxygen in your answer to (b) would have been derived from splitting liquid water. Given you answer in (b), calculate how much water gave rise to the oxygen taken up by the crust. [2 points]

(d) How deep would this amount of water be if spread all around the surface of primordial Venus? [1 point]

(e) The mass of Earth's oceans is 1.4×10^{21} kg. How much, as a fraction of the Earth's oceans, is your answer in (c). [1 point]

A similar calculation, assuming hematite as the end-point of FeO oxidation rather than magnetite, suggests that the crust would absorb 1/3 of an Earth ocean. Perhaps at some point in Venusian history the crust was saturated with oxygen so that O_2 began to accumulate in the atmosphere to significant concentrations and oxygen was lost to space by non-thermal processes. A similar fate may await the Earth in several billion years time when it loses its oceans as the Sun warms up. Some people speculate that a few hundred bar pressure of O_2 may then accumulate in Earth's atmosphere after the accessible crust becomes saturated with oxygen. But I wouldn't lose sleep over it.

[Data: assume the density of Venus crust is ~2500 kg m^{-3}; density of water = 1000 kg m^{-3}; take atomic masses: Fe = 56g/mole, O = 16g/mole; molar mass of water = 18g/mole]

Qu. 3) [13 total] The estimated photolysis rate for the mass of methane in a column of Titan's atmosphere is 4×10^{-13} g cm^{-2} s^{-1}. Titan's total atmospheric surface pressure is 1.5 bar. Take the CH_4 mixing ratio as 5%. (Titan surface g = 1.354 m s^{-2}; mean molar mass of Titan air = 28.6 grams; molar mass of CH_4 = 16 grams).

(a) Calculate how long (in years) it takes for all the methane in Titan's atmosphere to be destroyed by photolysis. [5 points]

(b) Your answer in (a) is a short timescale compared to the age of the Solar System. What are two different explanations for why there is presently so much methane in Titan's atmosphere despite methane's constant destruction [2 points]

(c) Assume that photolysis of methane results solely in the formation of ethane, which accumulates on the surface as liquid ethane (overall: $2CH_4 + h\nu \rightarrow C_2H_6 + 2H$, where H escapes to space). Calculate how much ethane would accumulate on the surface (in units of g cm^{-2}) over the age of the Solar System (4.5 billion years), assuming a constant photolysis rate. (Molar mass of ethane = 30 grams) [3 points]

(d) The density of ethane on the surface of Titan is 645 kg m^{-3}. Calculate the depth (in meters) of liquid ethane corresponding to your answer in (c). [3 points] (Of course, there is no evidence for a liquid ocean covering all of Titan.)