## Homework \#2: Radiation [total = 54 points]

1) [6 points total] You are asked to be part of a NASA team designing an ultraviolet spectrometer to operate on the surface of Mars in the future. The spectrometer will look vertically upward and measure the flux over the wavelength range 180 nm to 400 nm in spectral intervals $\sim 1 \mathrm{~nm}$ wide.
(a) During an early team meeting, a question is raised about whether the spectrometer could be made sensitive enough to measure the column abundance of ozone, even in the low latitudes where the $\mathrm{O}_{3}$ abundance is generally low. You know that the column abundance of $\mathrm{O}_{3}$ has been measured remotely and is estimated as $\sim 5 \times 10^{15}$ molecules $\mathrm{cm}^{-2}$ in the Martian low-latitudes. Meanwhile, a handbook gives the $\mathrm{O}_{3}$ absorption cross-section as $10^{-21} \mathrm{~m}^{2}$ at the deepest point in the Hartley bands, which occurs around a wavelength of 255 nm . You also know that as a rule-ofthumb, in order to discern a signal you need at least ten times better resolution than the signal itself. Estimate a minimum signal-to-noise ratio required by the UV spectrometer to measure column $\mathrm{O}_{3}$ abundance in the low latitudes of Mars with an overhead Sun. [3 pts]
[Hint: Part (a) is not concerned with spectral resolution because $\mathrm{O}_{3}$ absorption is very broad. It's asking 'how precisely do you need to measure the flux at 255 nm to determine the absorption by overhead ozone?']
(b) While thinking about the problem in (a), you realize from the expected UV spectrum on Mars could also be used in another way: the UV spectrometer could effectively act as a barometer, measuring the column abundance of $\mathrm{CO}_{2}$. Discuss what characteristic of the UV spectrum on Mars could be used to do this? [3 pts]
[This question is an account of a real-life situation encountered by your instructor some years ago.]
2) [4 points total] The eccentricity of Mars' orbit is 0.0934 , while the eccentricity of Earth's orbit is 0.0167 . How much greater, as a percentage, is the equilibrium temperature of Mars and Earth at perihelion compared to aphelion for each planet?
3) [5 points total] At the surface of a planet of Earth mass (...in a galaxy far, far away) about $10 \%$ of the violet ( 450 nm ) radiation from its sun is scattered during vertical incidence transit through its atmosphere.
(a) If the atmosphere has a pressure of 2 bar at the surface and the mean molar mass is 38 grams, estimate the mean Rayleigh scattering crosssection (i.e., cross-section (in $\mathrm{cm}^{2}$ ) per molecule) for the atmospheric gas mixture. You can assume that the attenuation is due solely to Rayleigh scattering rather than any other form of extinction. [3 pts]
(b) Estimate the fraction of incident red ( 900 nm ) light that is scattered by passing through the same atmosphere when the sun is $30^{\circ}$ above the horizon. [2 pts]
4) [16 points total] Astronomers discover a large asteroid on a collision course with the Earth. It is expected to hit somewhere in the Middle East. Sadly, it is too late to stop the asteroid. But it can be deflected so that it lands in an unpopulated area. Government A argues that the asteroid should be deflected into the desert of Iraq. However, Government B argues that that the asteroid should be deflected into the Indian Ocean.

The question is: which is worse for the inhabitants of the Earth: a land impact or an ocean splash?

For climate risk assessment, you have the following information:

- In the event of a desert impact, dust lifted into the atmosphere will decrease the planetary albedo for reflected sunlight from the top of the atmosphere from 0.3 to 0.25 , while the absorption of solar radiation in the atmosphere will increase from $20 \%$ to $50 \%$ as a percentage of the incoming solar flux.
- If the asteroid lands in water, the amount of water vapor introduced into the atmosphere will make the lower atmosphere entirely opaque to thermal infrared radiation for several weeks.
(a) The media dubs one scenario analogous to a "nuclear winter" and the other a "hothouse Earth". Before calculating anything, which of the scenarios do you think will be analogous to a "nuclear winter" and which would be the "hothouse". [2 pts]
(b) Consider a simplified model for the impact on land where the atmosphere consists of a single, slab layer above the surface at temperature $T_{1}$. Assume the ground is at temperature $T_{\mathrm{g}}$. Before the impact, the time- and space-averaged solar radiation on the planet is reflected according to albedo, and the atmosphere absorbs $20 \%$ of the solar radiation (total, not minus the reflected amount) before it reaches the ground. Assume the surface emits like a blackbody and assume the atmospheric layer emits like a blackbody. Take the solar constant $S=1360 \mathrm{~W} \mathrm{~m}^{-2}$. The StefanBoltzmann constant $=\sigma=5.67 \times 10^{-8} \mathrm{~W} \mathrm{~m}^{-2} \mathrm{~K}^{-4}$.
(i) Evaluate the temperature of the ground before the impact. (Hint: Draw a diagram and consider a balance of fluxes in and out of an atmosphere layer at temperature $T_{1}$, the ground at temperature $T_{\mathrm{g}}$, and the influx or outflux of solar, reflected solar, and infrared fluxes to space from the atmosphere layer; draw arrows for the fluxes; remember that the atmospheric layer emits downward and upward. Represent the solar absorption factor by coefficient, a = 0.2. Apply the Stefan-Boltzmann law, in particular). [4 pts]
(ii) Evaluate the temperature of the ground after impact. [4 pts]
(iii) Explain qualitatively why, in a more complex calculation, the ground temperature is likely to depend on the vertical position of the dust in the atmosphere (e.g., troposphere versus stratosphere). [4 pts]
(c) For the ocean impact, consider a modified model. Add a second layer to the atmosphere at temperature $T_{2}$ beneath the layer at temperature $T_{1}$. The lower layer is opaque to thermal infrared radiation from the ground. Assume that the absorption of total solar radiation of $20 \%$ is equally divided between the two atmospheric layers. What is temperature of the ground after the impact into the Indian Ocean, using this simple model? [4 pts]

5) [4 points total] Jupiter's atmosphere receives energy from the Sun and from Jupiter's interior. In this question, you may find the following information useful: Jupiter's Bond albedo $=0.343$, the solar constant at Earth is $1360 \mathrm{~W} \mathrm{~m}^{-2}$. The mean Earth-Sun distance is $1.496 \times 10^{11} \mathrm{~m}$; the mean Jupiter-Sun distance is $7.78 \times 10^{11} \mathrm{~m}$; Jupiter's radius $=71,492 \mathrm{~km}$. Stefan-Boltzmann constant $=5.67$ $\times 10^{-8} \mathrm{~W} \mathrm{~m}^{-2} \mathrm{~K}^{-4}$.
(i) What is the equilibrium temperature of Jupiter based on the input of solar energy only? [2 pts]
(ii) Spacecraft have measured the effective temperature of Jupiter as 124.4 K. The difference between the measured and the calculated value of the temperature is thought to be due to energy from the interior of the planet. How large a flux $\left(\mathrm{W} \mathrm{m}^{-2}\right)$ is required to account for the difference? How much energy (W) is emanating from the interior of the Jupiter? (Comparisons: 15 TW energy use by human civilization or 45 TW emanating from the Earth's interior). [Hint: In the lecture notes online, you will find an equation that includes a planet's interior flux].
6) [12 points total]

The figures below are coincident measurements of IR emission of the terrestrial cloud-free atmosphere (a) 20 km looking downward over a polar ice sheet (b) at the surface looking upward.


Answer the following. In each case explain how you deduce the answer with reference to the part of the spectrum or feature that you're looking at:
(a) What is the approximate temperature of the surface of the ice sheet? [3 $\mathrm{pts}]$
(b) What is the approximate temperature of the near-surface air? [3 pts]
(c) What is the approximate temperature of the air at the aircraft's altitude of 20 km ? [3 pts]
(d) Which band is responsible for the feature between 9 and 10 microns? [1pt]
(e) Is there any evidence for an inversion in the near-surface temperature profile or not? [2 pts]
7) [4 points total]. A scientist wants to measure the top-of-atmosphere solar intensity. She uses a ground-based radiometer operating at $\lambda=0.45 \mu \mathrm{~m}$ to measure the solar intensity at the ground, $\mathrm{I}_{\lambda}(0)$. For a solar zenith angle $\theta=30^{\circ}$, $\mathrm{I}_{\lambda}(0)=1.74 \times 10^{7} \mathrm{~W} \mathrm{~m}^{-2} \mu \mathrm{~m}^{-1} \mathrm{sr}^{-1}$. For $\theta=60^{\circ}, \mathrm{I}_{\lambda}(0)=1.14 \times 10^{7} \mathrm{~W} \mathrm{~m}^{-2} \mu \mathrm{~m}^{-1} \mathrm{sr}^{-1}$. From this information, determine the top-of-the-atmosphere solar intensity $\mathrm{S}_{\lambda}$ (in $\mathrm{W} \mathrm{m}{ }^{-2} \mu \mathrm{~m}^{-1} \mathrm{sr}^{-1}$ ) and the atmospheric optical thickness, $\tau_{\lambda}$.
8) [3 points total]. In another galaxy far away, an Earth-sized exoplanet in an Earth-sized orbit has an albedo of 0.3, which is the same as the Earth's. But this exoplanet exists around a hotter star with a maximum emission per unit wavelength of $0.4 \mu \mathrm{~m}$ instead of $0.502 \mu \mathrm{~m}$ for our Sun. What is the difference in effective temperature for this exoplanet compared to the Earth's 255 K? (For the sake of this question, we will ignore an astronomical nuance that a hotter star will tend to have greater radius that the Sun and so more luminosity just due to size. In this question, we assume that the star has the same size.).

