Homework 1 (Total out of 32 points + 1 bonus)

1) [16 points total] In the future, planetary astronomers discover an inhabited extrasolar planet, which they call "Houston2". This planet has a foul and noxious polluted atmosphere, which the media speculates is caused by incessant, unregulated industrial activity. However, all that is really known is that the atmosphere is rich in hydrocarbons, that the temperature at the 200-mbar level in the atmosphere is 273.16 K, and that the surface temperature is 546.32 K. The astronomers all agree that the atmosphere has an essentially adiabatic structure. But there are three camps regarding the atmospheric composition: (i) one group believes that the atmosphere is 50% by volume Xe and 50% methane. (ii) the other group believes that the atmosphere is pure lead tetraethyl $Pb(C_2H_5)_4$ --- the unwelcome substance found in leaded gasoline (iii) a final group argue for a composition of 50% He and 50% methane.

(a) What is the surface atmospheric pressure on "Houston2" in each model (i) through (iii)? [12 pts]

(b) Explain how atmospheres (i) and (iii) will differ in their vertical extent? You can assume that Houston2 has the same gravity as the Earth, and a useful thing to do would be to compare heights at the 0.2 bar level. [4 pts]

[In answering part (a), you will need heat capacity. Go and look at a thermal physics textbook. You'll find that each degree of freedom of a molecule or atom contributes $R/2$ to C_v, the molar heat capacity at constant volume. For a molecule of N atoms, there are 3N degrees of freedom. For example, for He there are 3N = 3 degrees of freedom corresponding to 3 translational modes in the x, y and z directions, so that $C_v = 3R/2$. For CO, for example, there are 3N = 6 degrees of freedom corresponding to 3 translational modes, 1 vibrational mode and 2 rotational modes. But whether certain modes are excited depends on temperature. H_2, for example, has C_v = 3R/2 at 40K, C_v ~ 5R/2 at 300K and CV = 6R/2 at 2000K and approaches 7R/2 in the limit (H_2 dissociates before it gets there). Each vibrational mode has a kinetic part associated with movement of atoms and a potential part like spring energy in a simple harmonic oscillator, which can constitute two dofs. So the maximum possible C_v = (3 trans. + 2 rots + 2 vibs)R/2 = 7R/2 for a diatomic molecule. At room temperature, vibrational modes tend to be inactive, so C_v = 5R/2 for *most* diatomic molecules. For the purpose of this homework, ignore these complications and assume that the 3N rule holds. Recall also that molar heat capacity at constant pressure is related to C_v via the equation $C_p = C_v + R$. (Data: Masses in grams/mole: Xe = 130, C = 12, H = 1, He = 4, Pb = 206)].

2) [4 points total] A spherical hot-air balloon is designed to float at the ~1 bar, 60 K level in the atmosphere of Neptune. Scientists hope to track the winds in the atmosphere of Neptune by following the motion of the balloon. The balloon is also instrumented with atmospheric sensors. The weight of the balloon and payload is 30 kg. What balloon volume and radius is required for a "hot air" temperature of 100 K?
(Data: mean molar mass of Neptune atmosphere = 2.6 g/mol). [4 pts]

3) [6 points total] In deriving the hydrostatic equation for an isothermal planetary atmosphere, we assumed that the scale height H was much smaller than the radius of the planet; that is, that g can be considered a constant throughout the atmosphere, as a reasonable approximation. Suppose early Titan had much hydrogen in its atmosphere. If so, our approximation is no longer valid because $g = (GM)/r^2$ at planetocentric distance r.

   (i) Re-derive the hydrostatic equation for this case, taking into account the variation of g with planetocentric distance. I suggest you start with $dP/P = -(\bar{m}g/R\bar{T})dr$. [3 pts]

   (ii) Assume that early Titan's atmosphere was made of pure hydrogen (H_2). What would be the scale height using the usual formula and assuming an isothermal temperature of 90 K? Look up Titan's radius. As a percentage, how much is the scale height a fraction of Titan's radius? [Titan g = 1.37 m s^{-2}] [2 pts]

   (iii) Why is a pure hydrogen atmosphere unlikely to be found around a planet like Titan? [1 pt]

4) [7 points] An Earth-based spectroscopic measurement of the strength of some features in the infrared spectrum of Mars reports a column abundance of CO_2 of 8000 cm-amagat.

   a. What is the corresponding surface pressure of carbon dioxide in millibars? [Data: molar mass of CO_2 is 44 g/mol; Martian gravity g = 3.72 m s^{-2}]. [4 pts]

   b. The paper reporting this result does not bother to tell us whether the abundance refers to the sub-Earth region of Mars (i.e. a vertical incidence region of Mars in the direct line-of-sight from Earth) or to an average over the entire disk of Mars. Would the surface pressure be less, the same, or greater than the value calculated in (a) if the reported abundance is actually a whole-disk average? Justify your answer in a qualitative fashion. (With a bit of thought and 'one step' arithmetic, it is possible to calculate disk-average pressure versus line of sight pressure. You're looking to deduce a weighting factor for a disk average rather than a direct path perpendicular to the surface at the equator. This gets a bonus point, but if you're not sure, don't sweat it because it's subtle). [2 pts + 1 bonus pt]