Introduction

• a time series is a set of observations made sequentially in time (but techniques from time series analysis often applied to observations made sequentially in depth, along a transect, etc.)

• denote set by either \( \{x_t : t \in T\} \) or \( \{x(t) : t \in T\} \)

• \( x_t \) or \( x(t) \) is observation indexed by \( t \), where \( t \) is a value we can use to figure out the actual time (e.g., \( t = 50 \) might mean the 50th day of year 2016, i.e., Feb. 19th)

• \( T \) is the index set, which typically is either
  
  – discrete, e.g., \( T = \{1, 2, \ldots, n\} \) or \( T = \{\ldots, -1, 0, 1, \ldots\} \equiv \mathbb{Z} \) (set of all integers – here ‘\( \equiv \)’ is ‘equal by definition’); or
  
  – continuous, e.g., \( T = [a, b] \) (a closed interval such as the unit interval \([0, 1]\)) or \( T = (-\infty, \infty) \equiv \mathbb{R} \) (set of all real numbers)
Classification of Time Series: I

- by index set $T$ (discrete or continuous): ‘discrete parameter’ time series & ‘continuous parameter’ time series (‘parameter’ used here rather than ‘time’ to avoid overworking ‘time’!)
  - discrete parameter time series are typically – but not always – equally spaced (some are ‘gappy’ – stock market prices due to market closures on weekends, holidays etc.)
  - discrete parameter time series might be intrinsically discrete (number of planes departing Seatac every day) or might arise from underlying continuous parameter time series via
    * sampling (instantaneous temperature each day at noon)
    * aggregation (daily average temperature)
    * extrema (daily maximum temperature)
Classification of Time Series: II

- by number of values recorded
  - one value – univariate
  - more than one value – vector-valued or multivariate
    * special case: bivariate time series (two values)
Classification of Time Series: III

by type of value $x_t$ can assume

- real-valued, i.e., a value in $\mathbb{R}$ or subset thereof (temperatures)
- complex-valued (applications in electrical engineering)
- nonnegative integer (whale population)
- categorical (outcome of scheduled football match: win, lose, tie, forfeit, cancellation)
- binary-valued (coding for two categories)
- circular (wind direction)
Classification of Time Series: IV

- by predictability of $x_t$
  - deterministic (predictable perfectly or to a high degree)
    * horizontal displacement of a pendulum in a vacuum as a function of time
    * tidal levels as a function of time (but not individual waves)
  - stochastic (inherent degree of unpredictability)
    * stock market prices
    * physical phenomenon about which we do not have enough theory/information to fully predict (Yule’s pea shooter)

- emphasis here: discrete parameter time series $\{x_t : t \in T\}$, equally spaced, univariate (to start with), real-valued, stochastic in nature

- let’s look at some examples of ‘real-world’ time series
Sunspots (1700–2014)
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- index set is discrete parameter and equally spaced a year apart
- univariate time series
- $x_t$ can assume nonnegative rational values (average of daily counts)
- of interest to understand cycles for comparison with physical theories
- forecasting future values also of interest (in principle, if we knew enough about the sun, should be able to predict the number of spots)
Monthly Soil Temperature Versus Month Count

months after January 1976

$X_t$
Monthly Soil Temperature Versus Month Count

months after January 1976

$x_t$
Monthly Soil Temperatures in Mitchell, Nebraska

- index set is discrete parameter & equally spaced (ignoring fact that months have different lengths)
- univariate time series
- $x_t$ is average monthly temperature in centigrade (at depth of 20 cm) – values can’t be below $-273.15^\circ$ (absolute zero)
- strong seasonal component, but examining series for a long-term trend might be of interest
Galactic X-Ray Source GX 5–1
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- index set is discrete parameter and equally spaced (128 observations per second measured by Ginga satellite observatory)
- univariate time series
- $x_t$ can assume nonnegative integer values (counts within 1/128-sec intervals)
- understanding patterns of interest for comparison with physical theories
Australian Monthly Red Wine Sales

The graph represents the monthly red wine sales over the years 1980 to 1992. The x-axis indicates the years, while the y-axis shows the sales volume. The data shows a general upward trend with fluctuations each year.
Australian Monthly Red Wine Sales

- Index set is discrete parameter and equally spaced (again ignoring fact that months don’t have same number of days)
- Univariate time series
- $x_t$ can assume nonnegative real values (kiloliters)
- Might be of interest to compare seasonal patterns with what goes on in the Northern Hemisphere (does Christmas in summer rather than in winter matter?)
- Also of interest to forecast this series (perhaps looking at external factors such as average temperature/rainfall in growing areas, economic conditions, etc.)
All-Star Baseball Games (41 AL; 43 NL; 2 ties)
All-Star Baseball Games (First in Each Year)

<table>
<thead>
<tr>
<th></th>
<th>1940</th>
<th>1960</th>
<th>1980</th>
<th>2000</th>
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<tbody>
<tr>
<td>NL win</td>
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<td>tie</td>
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<td>AL win</td>
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BD–2–3 I–22
All-Star Baseball Games (Last in Each Year)

Outcome of game:
- NL win
- AL win
- Tie

Year:
- 1940
- 1960
- 1980
- 2000

Graph showing the outcome of the All-Star Baseball Games for each year from 1940 to 2000.
All-Star Baseball Games (1933–2015)

- index set is discrete parameter and equally spaced (almost!), but gappy (no game in 1945)
  - ‘almost’ because there were two games in each of 1959–1962
- univariate time series
- $x_t$ is categorical, but, if two ties eliminated, could regard as binary-valued
- 41 American League (AL) wins & 43 National League (NL) wins, but win/loss patterns seem to have streaks inconsistent with assumption of independence of outcomes (more later!)
- gamblers would be interested in forecasting this series!
Sea-Ice Profile

![Graph of the sea-ice profile with time (t) on the x-axis and the variable \( x_t \) on the y-axis. The graph shows fluctuations in the sea-ice profile over time.]
Sea-Ice Profile

- index set is discrete parameter and equally spaced, but gappy
- univariate time series, but here ‘time’ is distance along a transect
- $x_t$ is real-valued (meters)
- characterizing roughness of series is of interest for comparison with physical models
- filling in the missing observations is also of interest
Pressure Measurements from DART\textsuperscript{\textregistered} Buoy

![Graph showing pressure measurements over time from a DART buoy. The graph plots pressure values against hours from an earthquake event. The x-axis represents hours from the earthquake, ranging from -25 to 5, and the y-axis represents pressure values ranging from 5311.5 to 5312.1.]
Pressure Measurements from DART® Buoy

• index set of underlying measurements is discrete parameter and equally spaced (one value every 15 seconds), but available observations are gappy, and some are four-point (1 minute) averages

• univariate time series

• $x_t$ is real-valued (height of water column above instrument)

• of interest to remove variations due to tides (‘detiding’)

• after detiding, of interest to characterize nature of residuals from fits of detided data to numerical models
St Paul Temperatures

![Graph showing St Paul Temperatures over the years 1920 to 2000. The x-axis represents the years, ranging from 1920 to 2000, and the y-axis represents temperature, ranging from -10 to 5 degrees Celsius. The graph displays fluctuations in temperature over time.]
St Paul Temperatures and ENSO Index

- St Paul is island in Bering Sea (west of mainland Alaska and north of chain of Aleutian Islands; population 479 as of 2010)
- ENSO is abbreviation for ‘El Niño/Southern Oscillation’, which is a cyclic warming/cooling of eastern and central Pacific (‘El Niño’ is oceanic component; ‘Southern Oscillation’ is atmospheric component; ENSO is coupled system)
- index set is discrete parameter and equally spaced (one value every month), but temperature observations are gappy
- bivariate time series
- $x_t$ is real-valued
- of interest to deduce the relationship between the two variates
$x_t$ at Depths of 1, 5, 10, 15 & 20 Meters
Temperature Measurements at Wivenhoe Dam

- index set is discrete parameter and equally spaced (almost), but gappy
- multivariate time series, with depth relationship between variates
- $x_t$ can assume real values (degrees centigrade, but negative values highly unlikely!)
- daily, subannual and annual patterns are of interest for monitoring water quality
- of interest to use components of this series as covariates to predict, e.g., algae blooms
Uses/Objectives for Time Series Analysis

- forecasting/prediction (possible due to nonindependence of adjacent values in series – not living in ‘independent and identically distributed’ (IID) world!)
- compare/contrast two series (how are they related? – e.g., input/output to some system)
- data description (how does series vary?) and exploration (is there anything unusual?)
- process control (e.g., reservoir draining in response to rainfall)
- hypothesis testing
- will advocate data description/statistical approach, but other approaches are in use (see, e.g., article on ‘technical analysis’ in Wikipedia and critique by J.A. Paulos in his book *A Mathematician Plays the Stock Market*, 2003)
Reference

- G.U. Yule (1927), ‘On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer’s Sunspot Numbers,’ *Phil. Trans. (A),* 226, pp. 267–98