Introduction

- a *time series* is a set of observations made sequentially in time (but techniques from time series analysis often applied to observations made sequentially in depth, along a transect, etc.)
- denote set by either $\{x_t : t \in T\}$ or $\{x(t) : t \in T\}$
- x_t or $x(t)$ is observation indexed by t, where t is a value we can use to figure out the actual time (e.g., $t = 50$ might mean the 50th day of year 2020, i.e., Feb. 19th)
- T is the index set, which typically is either
  - discrete: $T = \{1, 2, ..., n\}$ or $T = \{\ldots, -1, 0, 1, \ldots\} \stackrel{\text{def}}{=} \mathbb{Z}$ (set of all integers – here '$\stackrel{\text{def}}{=}$' is 'equal by definition'); or
  - continuous: $T = [a, b]$ (a closed interval such as the unit interval $[0, 1]$) or $T = (-\infty, \infty) \stackrel{\text{def}}{=} \mathbb{R}$ (set of all real numbers)

Classification of Time Series: I

- by index set T (discrete or continuous): 'discrete parameter' time series & 'continuous parameter' time series ('parameter' used here rather than 'time' to avoid overworking 'time'!)
  - discrete parameter time series are typically – but not always – equally spaced (some are 'gappy' – stock market prices due to market closures on weekends, holidays etc.)
  - discrete parameter time series might be intrinsically discrete (number of planes departing Seatac every day) or might arise from underlying continuous parameter time series via
    - sampling (instantaneous temperature each day at noon)
    - aggregation (daily average temperature)
    - extrema (daily maximum temperature)

Classification of Time Series: II

- by number of values recorded
  - one value: univariate
  - more than one value: vector-valued or multivariate
    - special case: bivariate time series (two values)

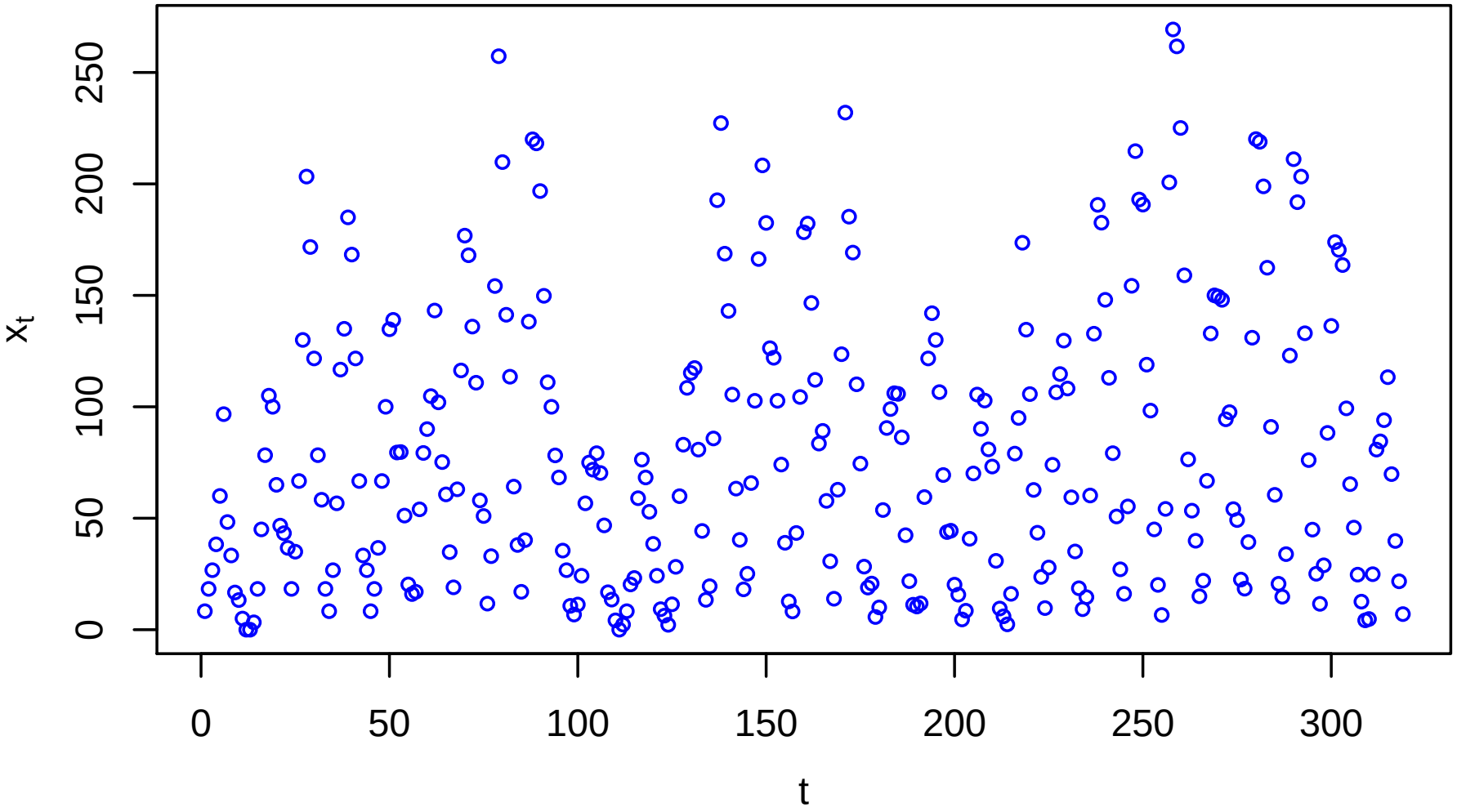
Classification of Time Series: III

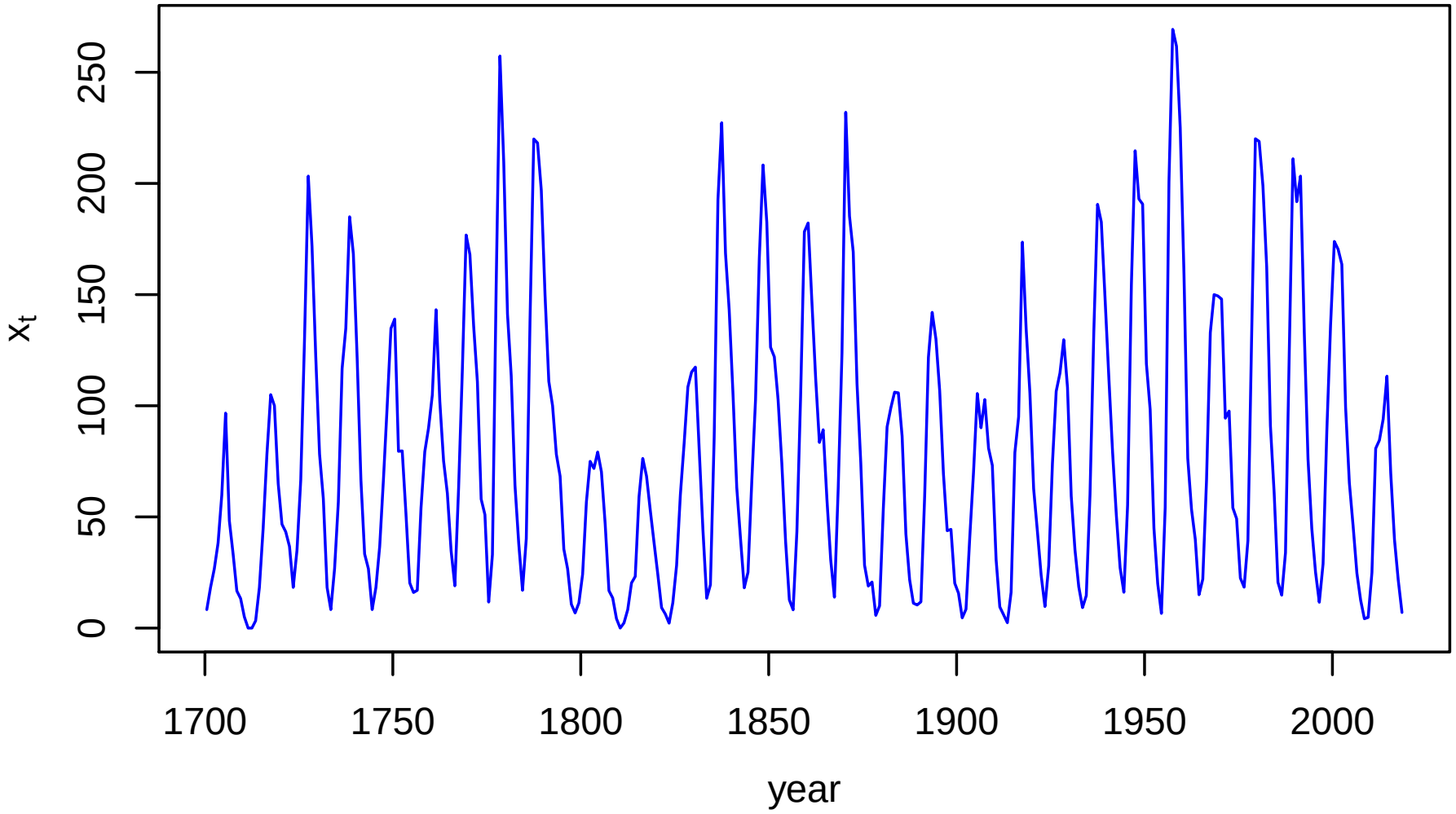
- by type of value x_t can assume
  - real-valued: a value in $\mathbb{R}$ or subset thereof (temperatures)
  - complex-valued (applications in electrical engineering)
  - nonnegative integer (whale population)
  - categorical (outcome of scheduled football match: win, lose, tie, forfeit, cancellation)
  - binary-valued (coding for two categories)
  - circular (wind direction)

Classification of Time Series: IV

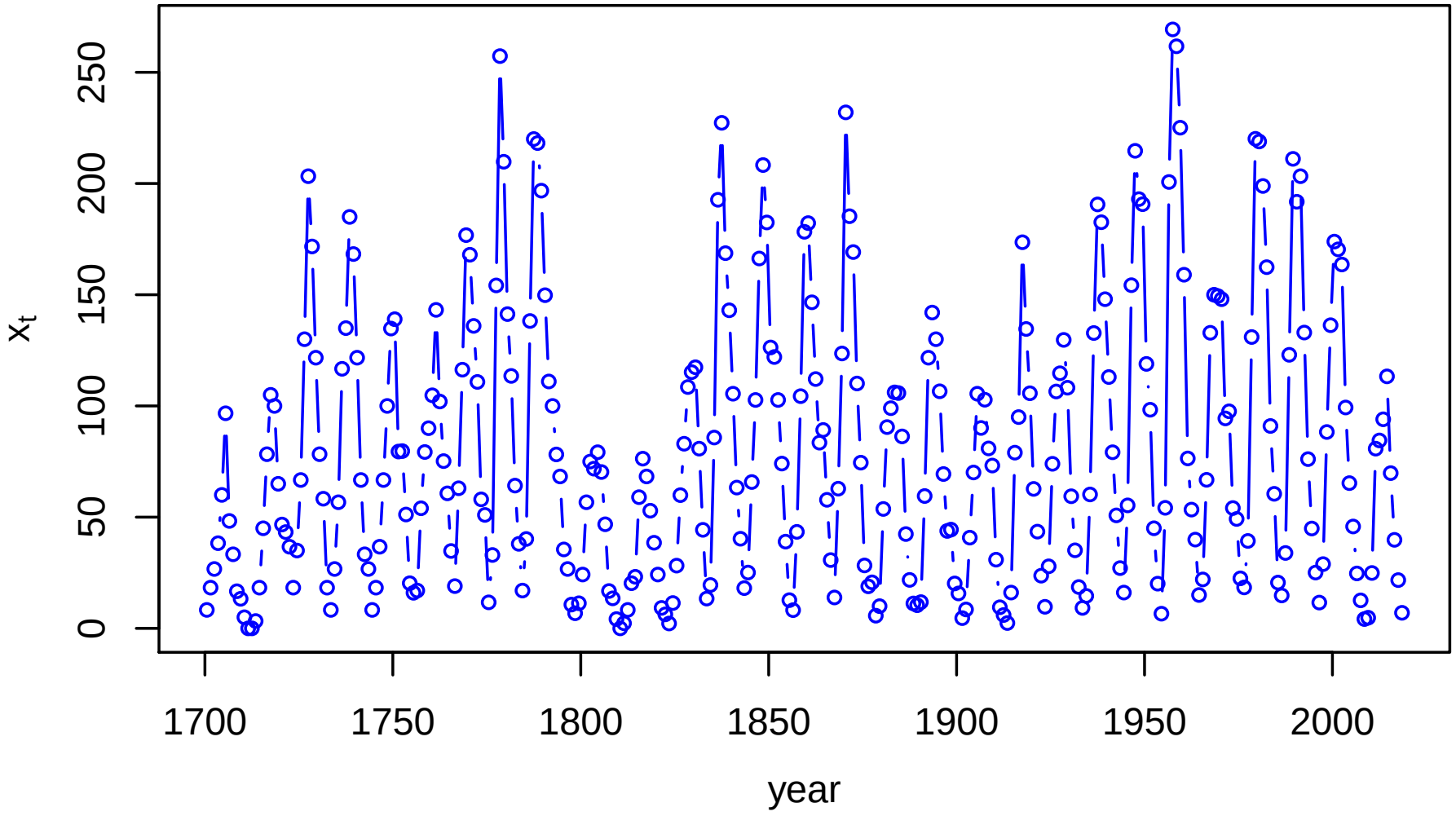
- by predictability of x_t
  - deterministic (predictable perfectly or to a high degree)
    - horizontal displacement of a pendulum in a vacuum as a function of time
    - tidal levels as a function of time (but not individual waves)
  - stochastic (inherent degree of unpredictability)
    - stock market prices
    - physical phenomenon about which we do not have enough theory/information to fully predict (Yule's pea shooter)
- emphasis here: discrete parameter time series $\{x_t : t \in T\}$, equally spaced, univariate (to start with), real-valued, stochastic in nature
- let's look at some examples of 'real-world' time series

Sunspots (1700–2018)

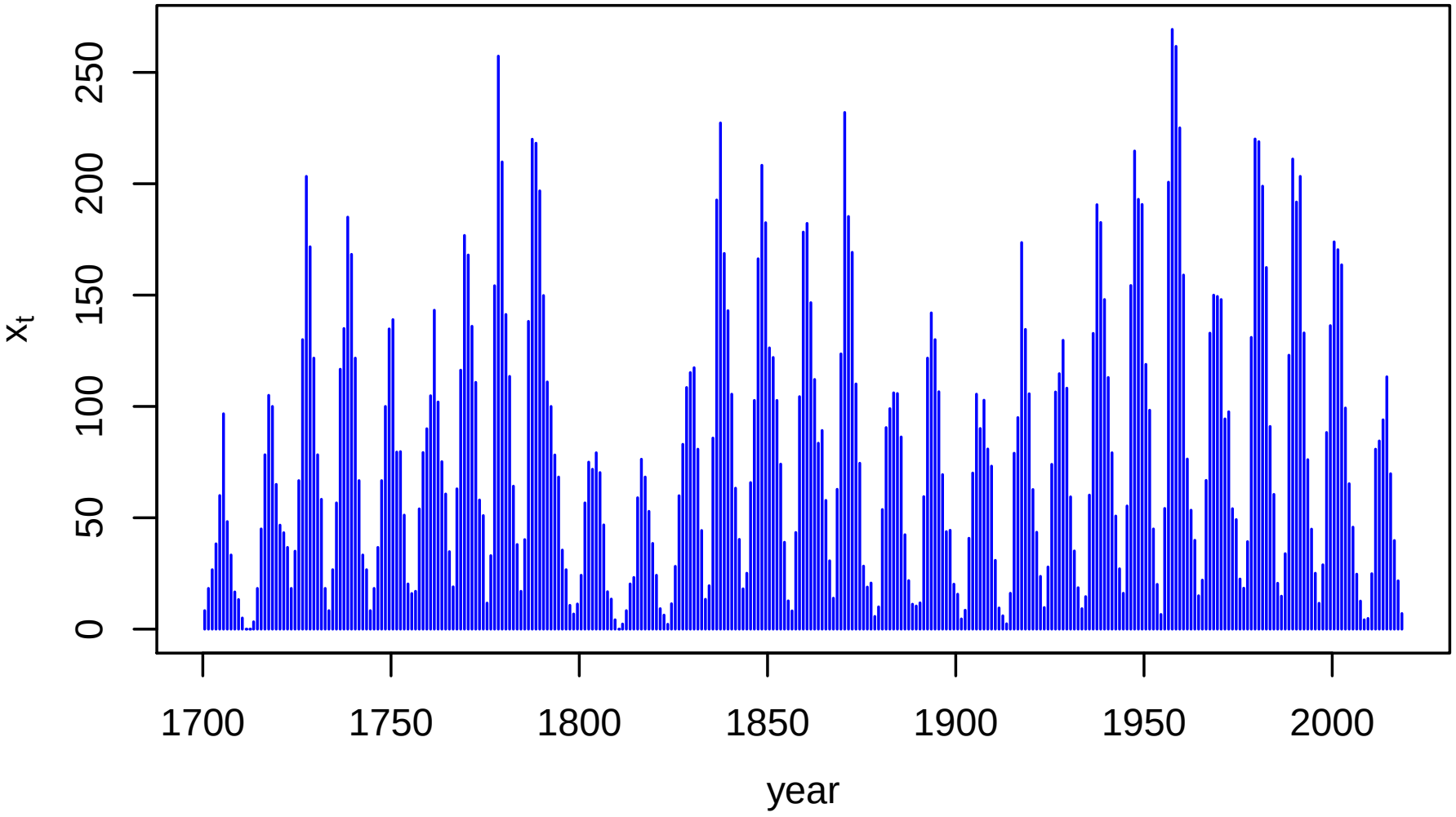



Sunspots (1700–2018)
x_t
0
50
100
150
200
250
1700
1750
1800
1850
1900
1950
2000
year

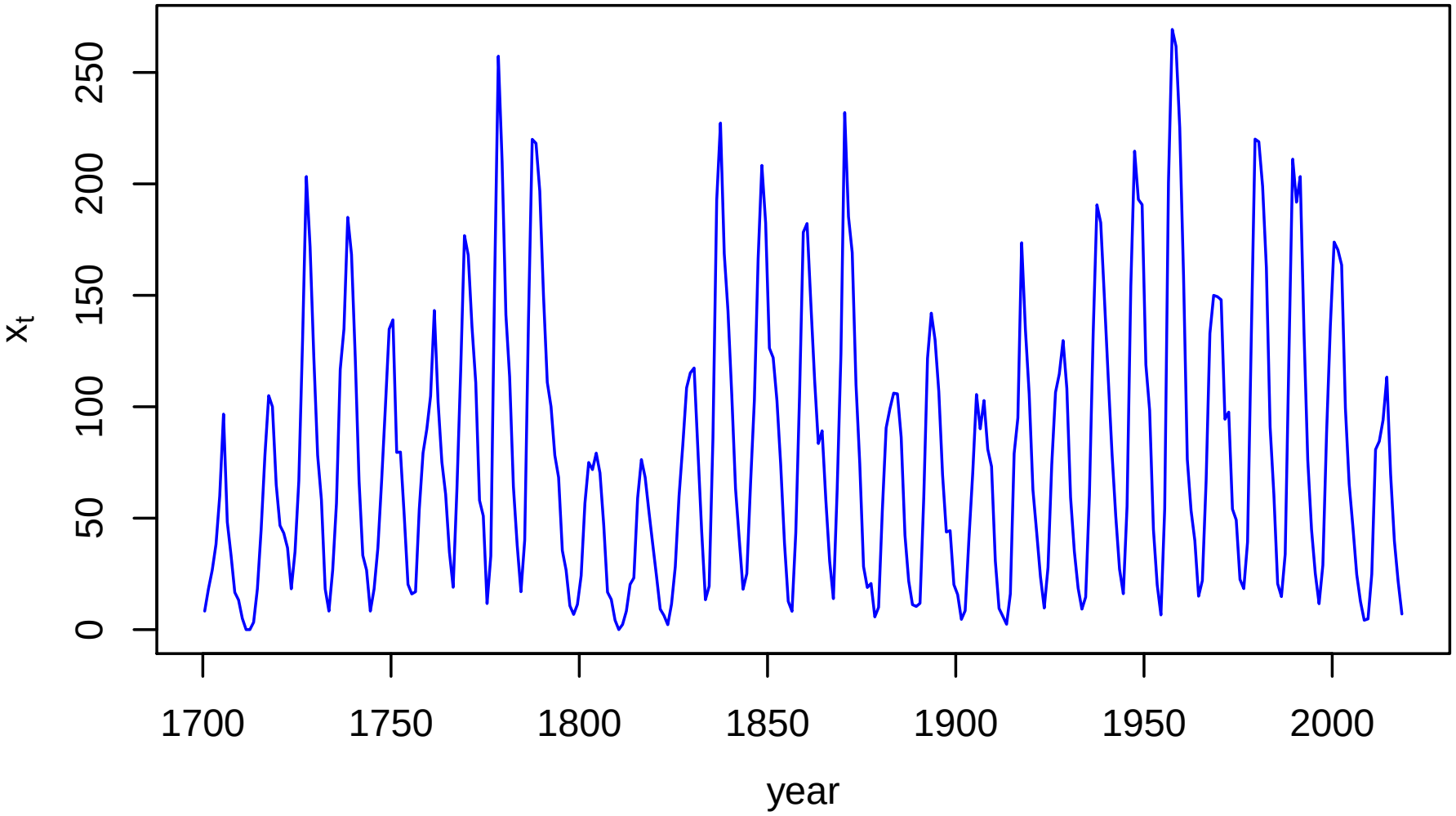
Sunspots (1700–2018)



Sunspots (1700–2018)



Sunspots (1700–2018)



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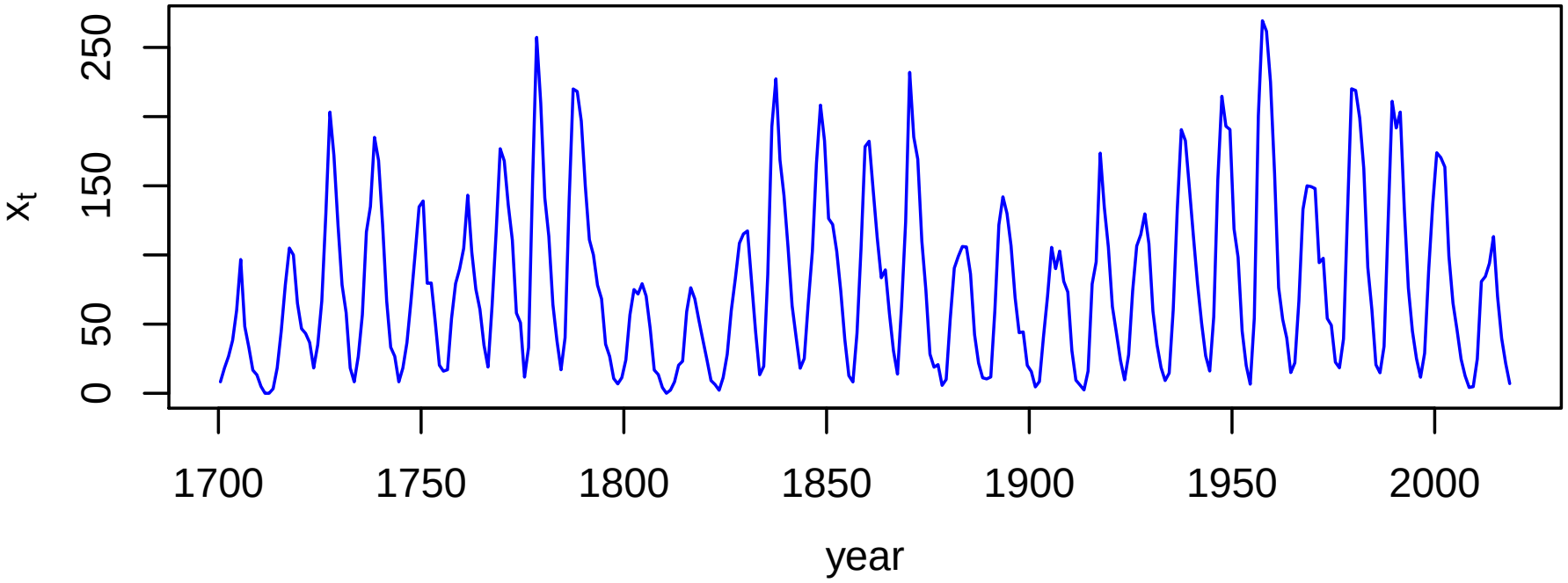
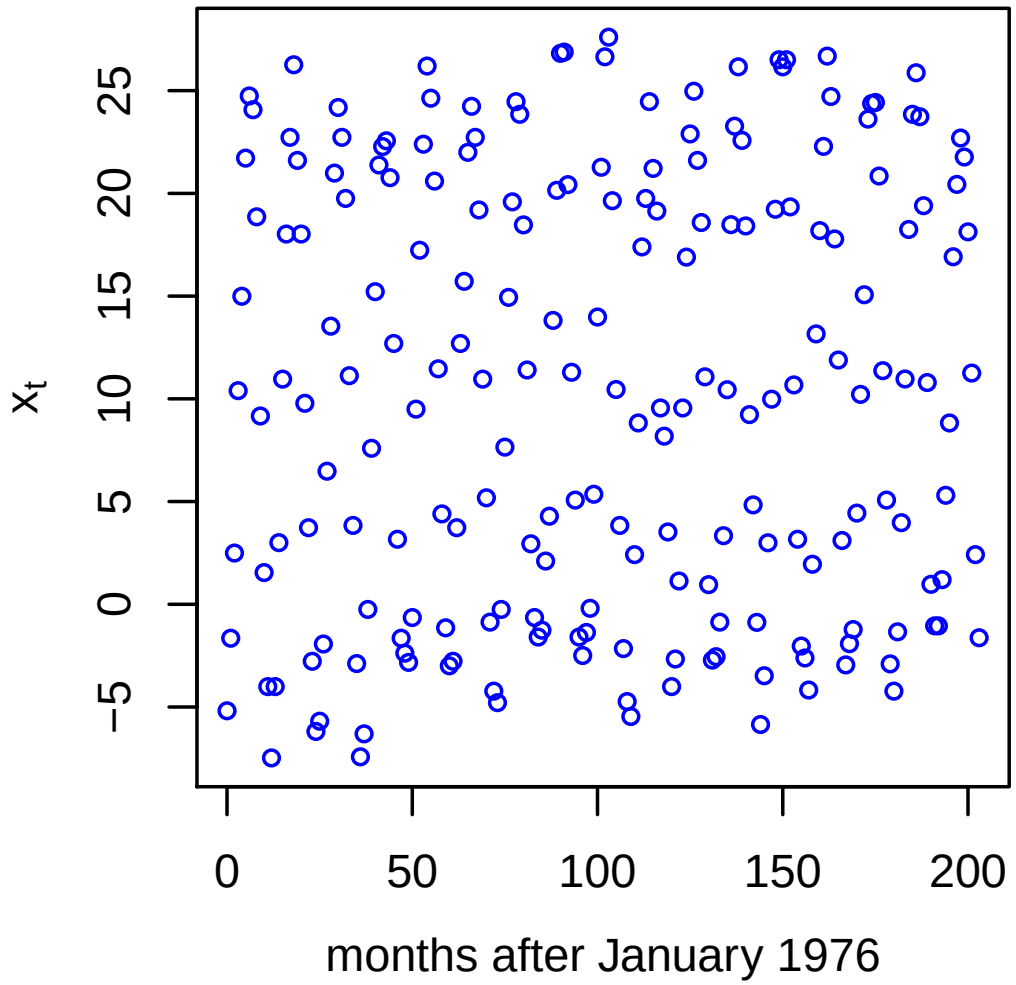


Sunspots (1700–2018)

- index set is discrete parameter and equally spaced a year apart
- univariate time series
- x_t can assume nonnegative rational values (average of daily counts)
- of interest to understand cycles for comparison with physical theories
- forecasting future values also of interest (in principle, if we knew enough about the sun, should be able to predict the number of spots)

Monthly Soil Temperature Versus Month Count



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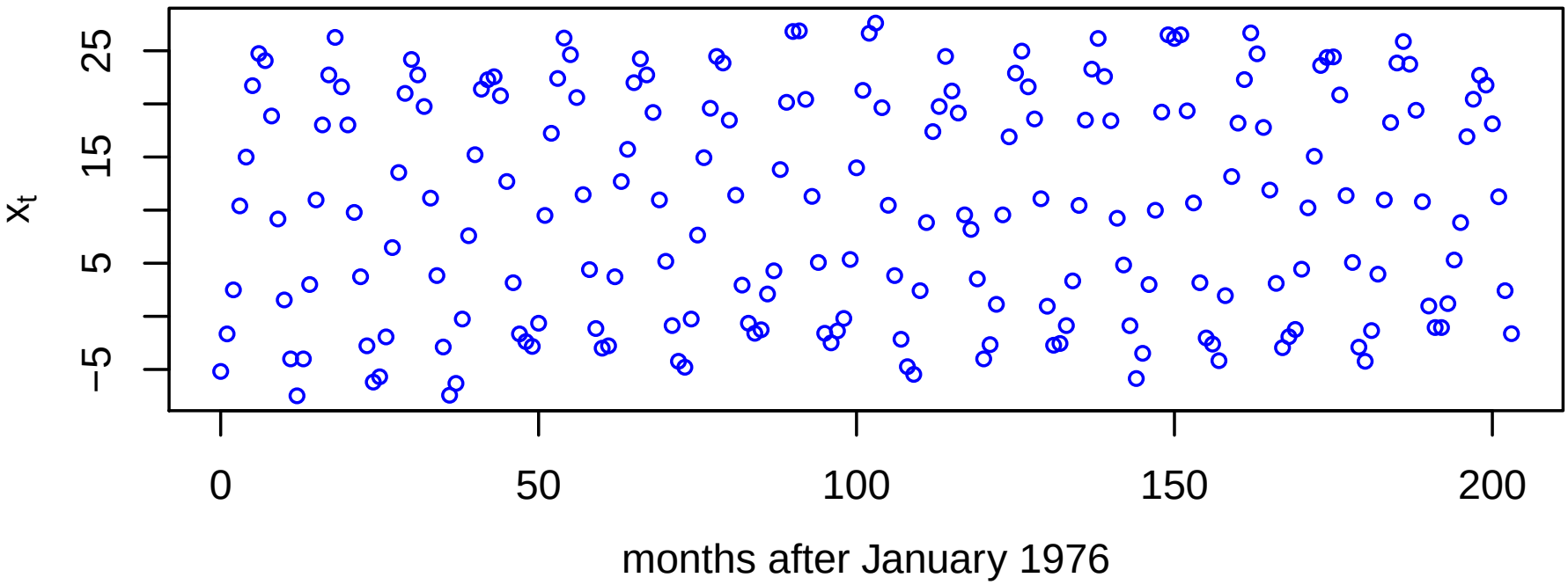

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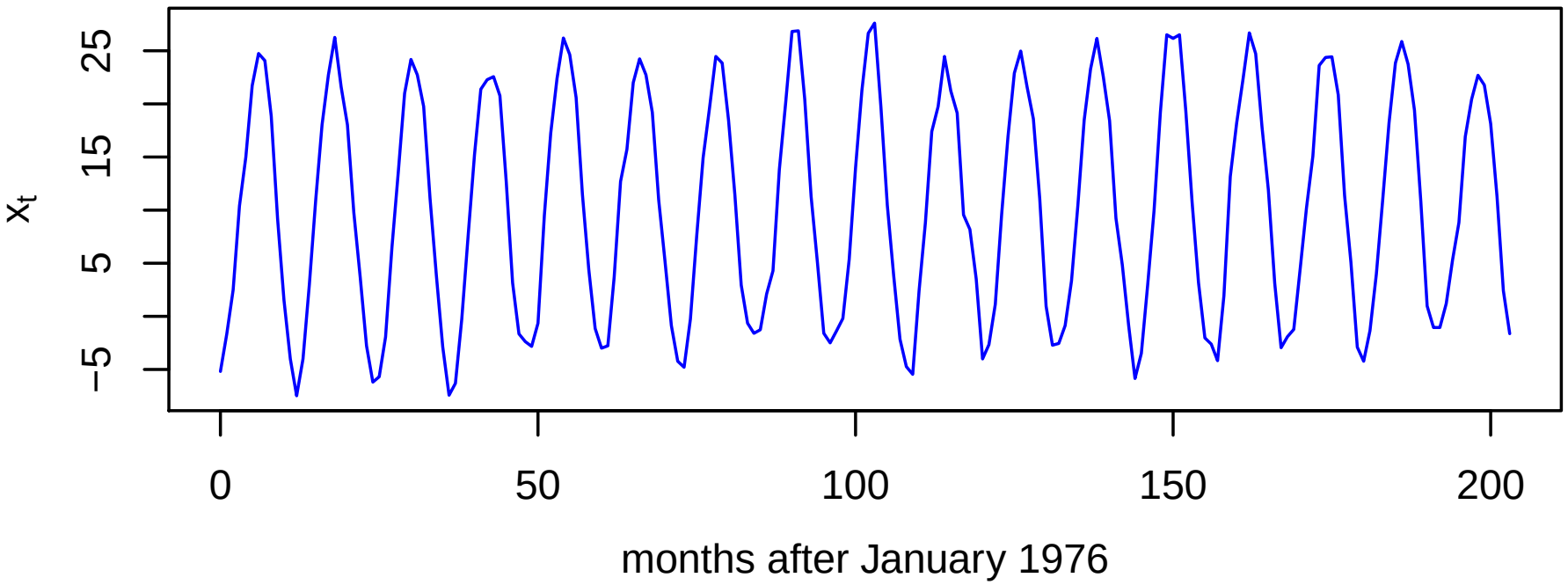

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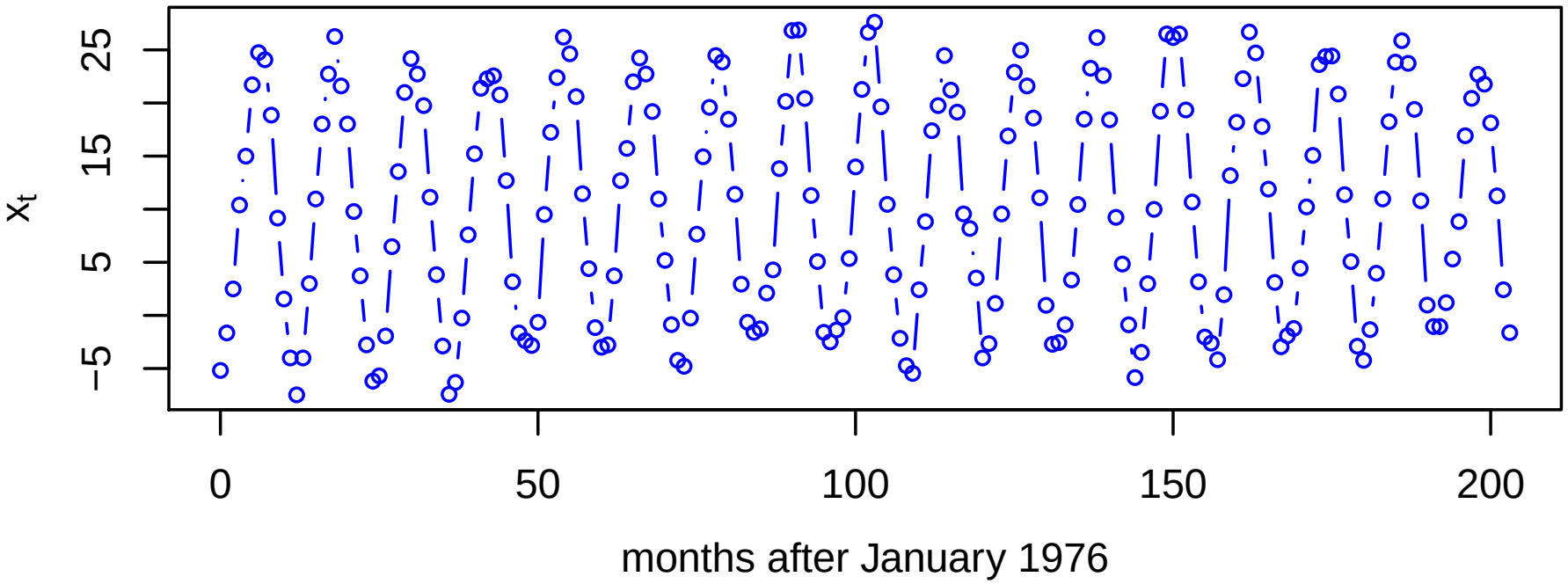
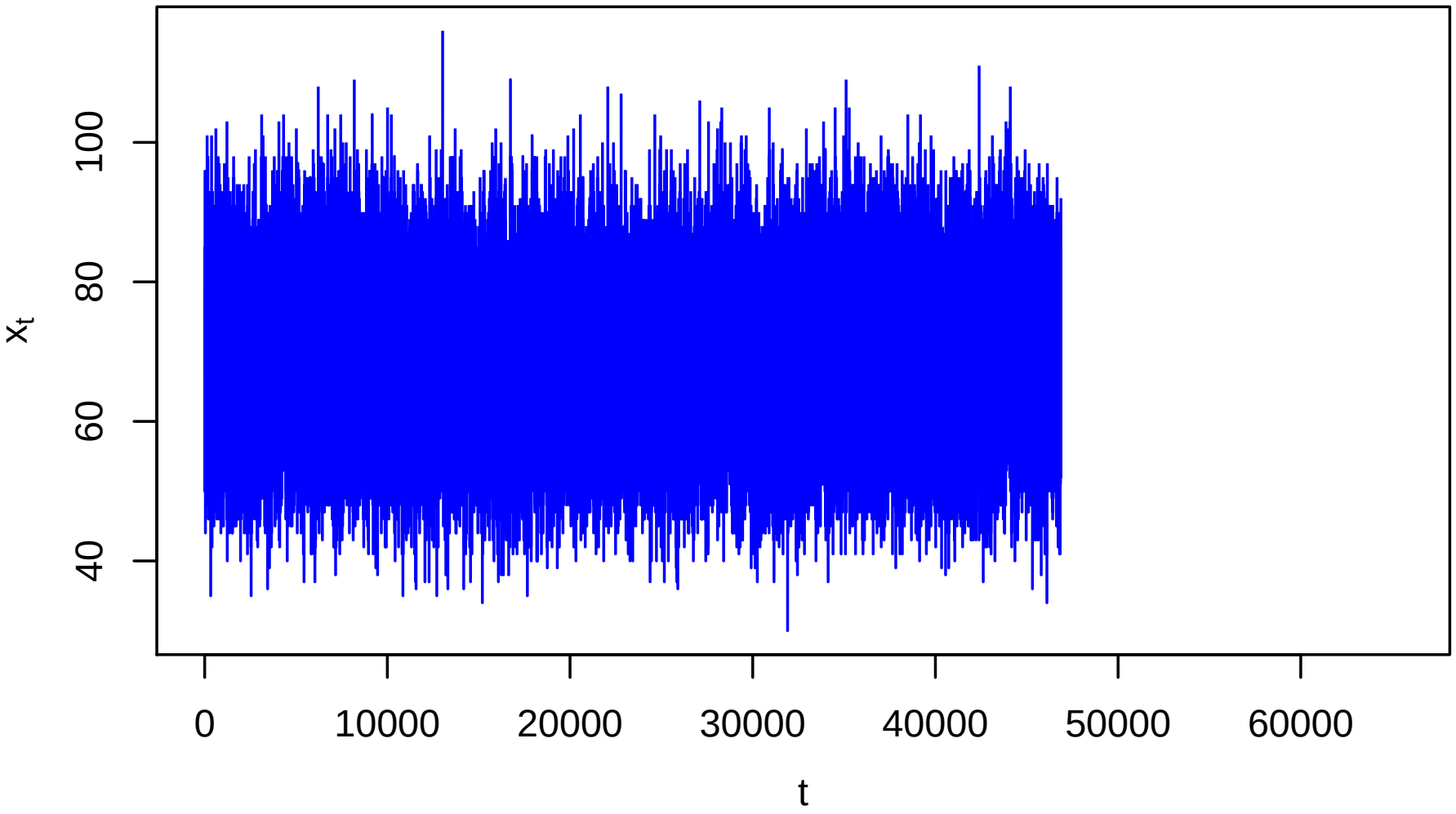


Monthly Soil Temperatures in Mitchell, Nebraska

- index set is discrete parameter & equally spaced (ignoring fact that months have different lengths)
- univariate time series
- x_t is average monthly temperature in centigrade (at depth of 20 cm) – values can't be below $-273.15°$ (absolute zero)
- strong seasonal component, but examining series for a long-term trend might be of interest

Galactic X-Ray Source GX 5–1



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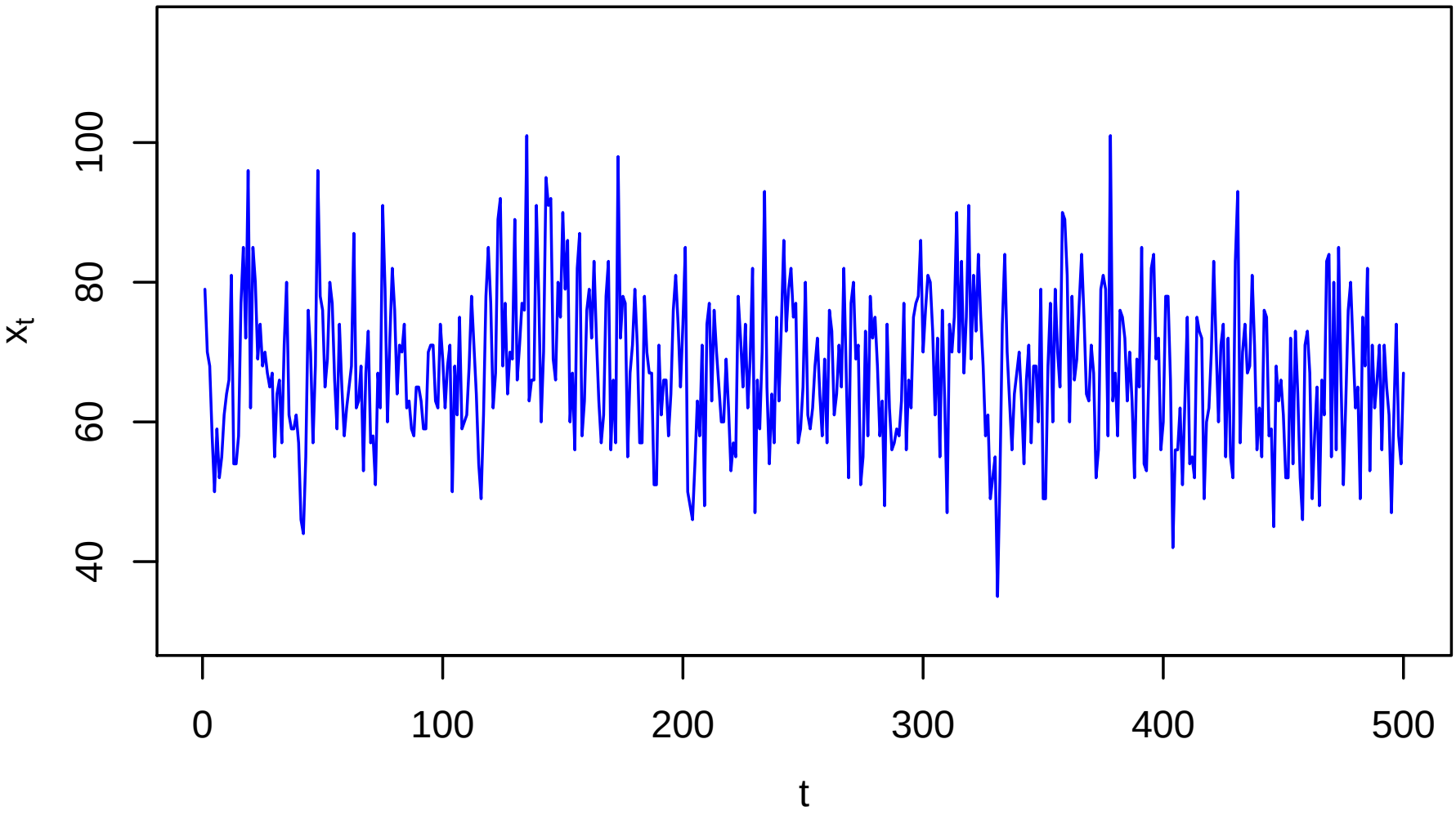

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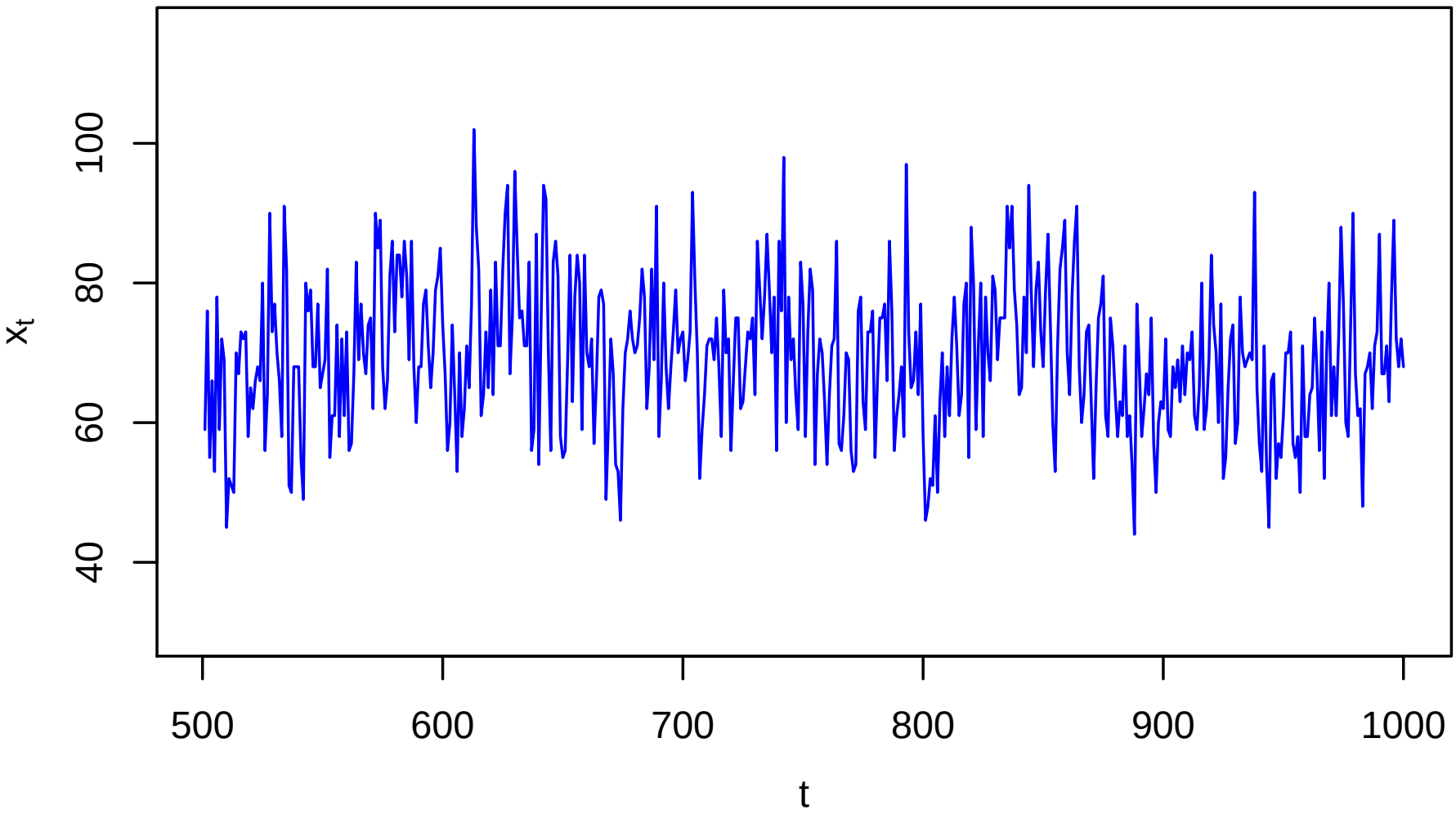

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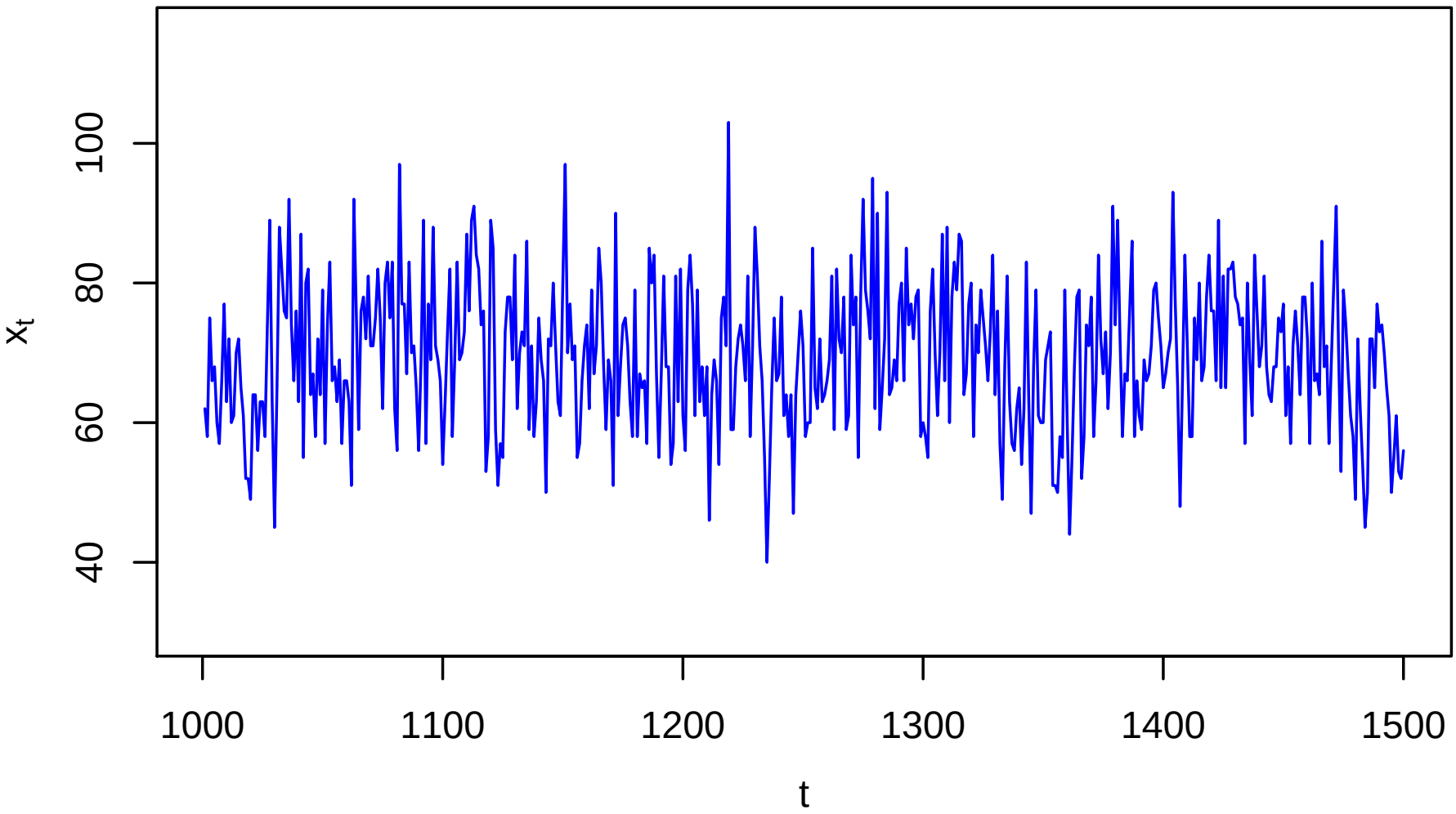

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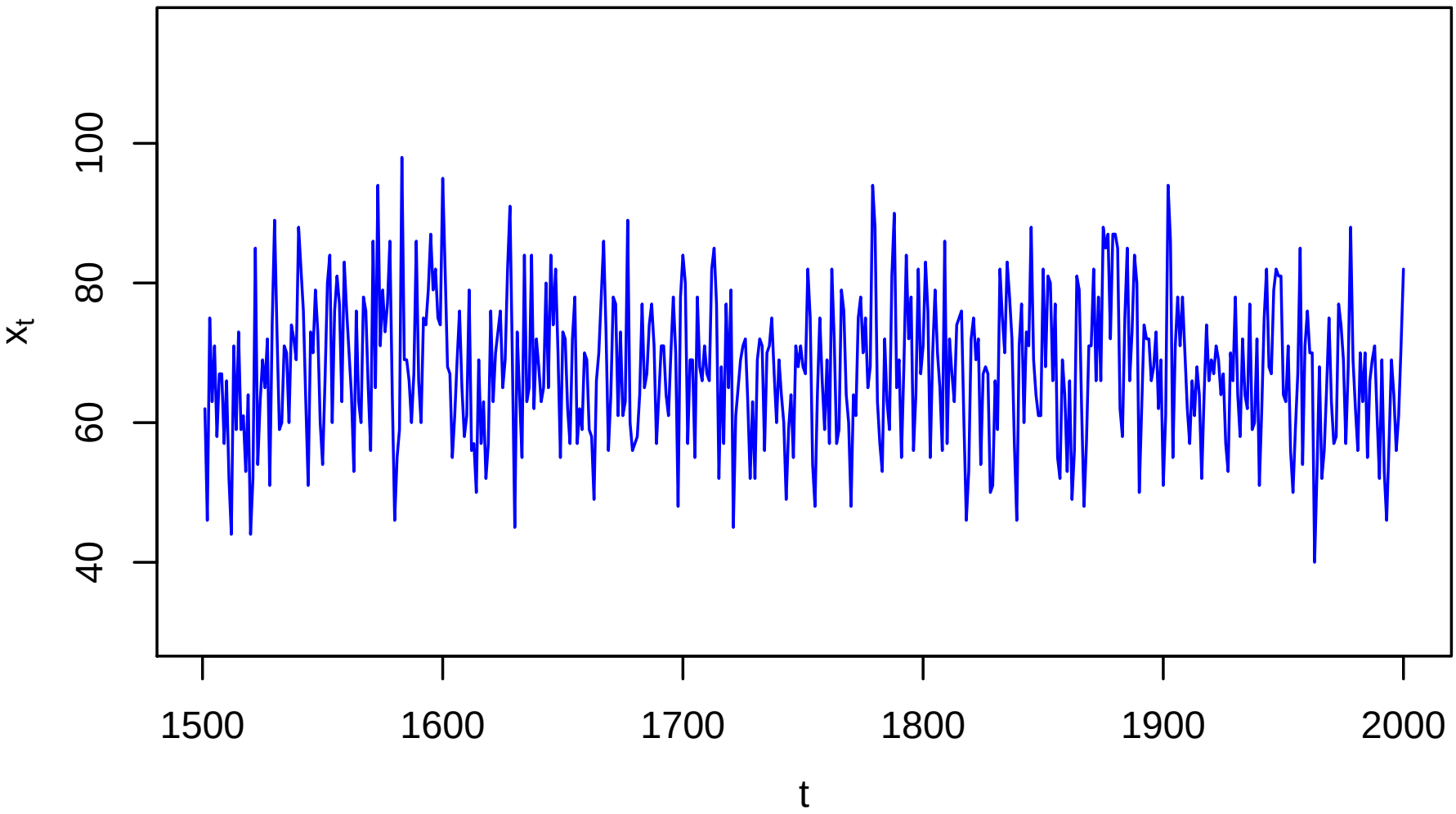
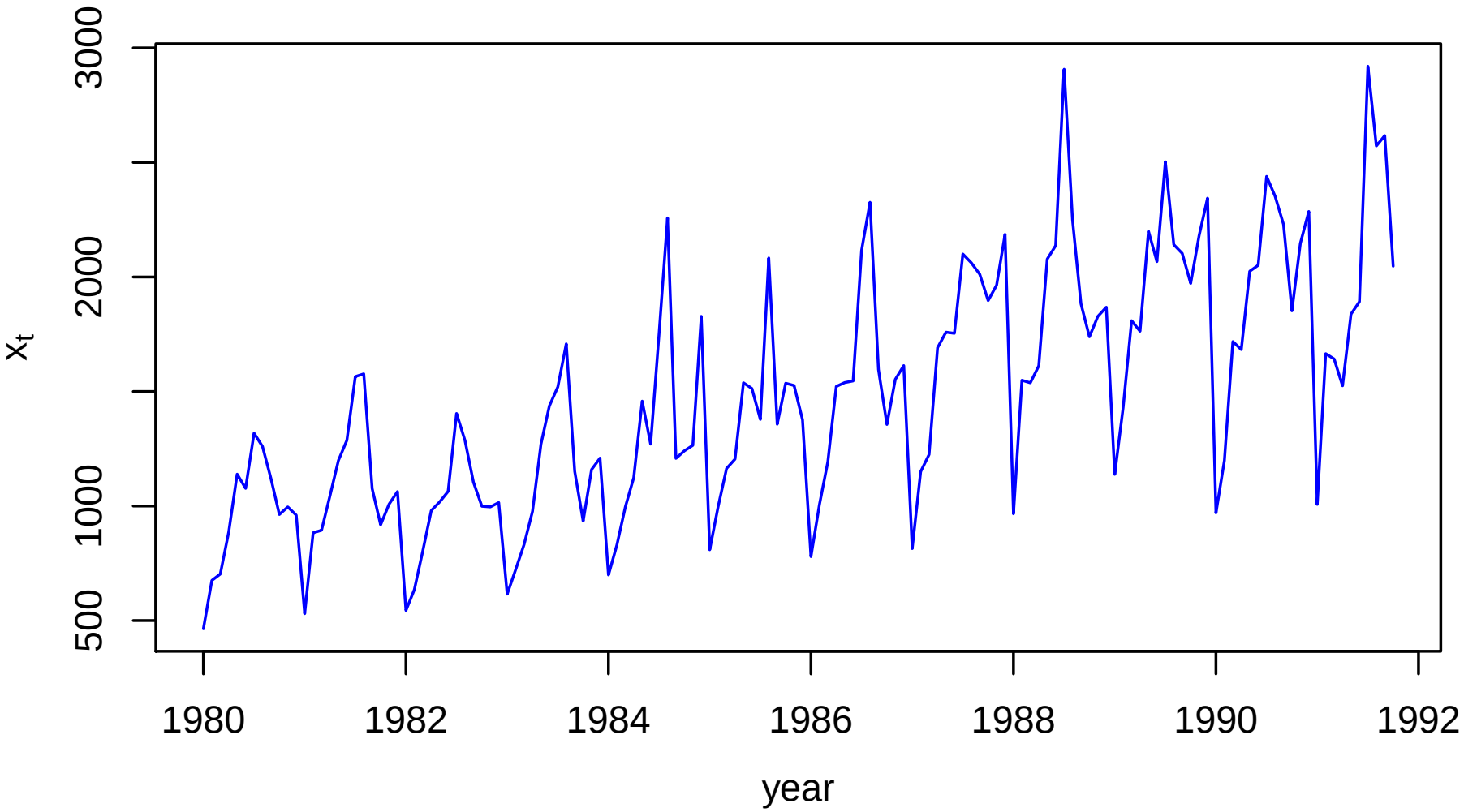


Galactic X-Ray Source GX 5–1

- index set is discrete parameter and equally spaced (128 observations per second measured by Ginga satellite observatory)
- univariate time series
- x_t can assume nonnegative integer values (counts within 1/128-sec intervals)
- understanding patterns of interest for comparison with physical theories

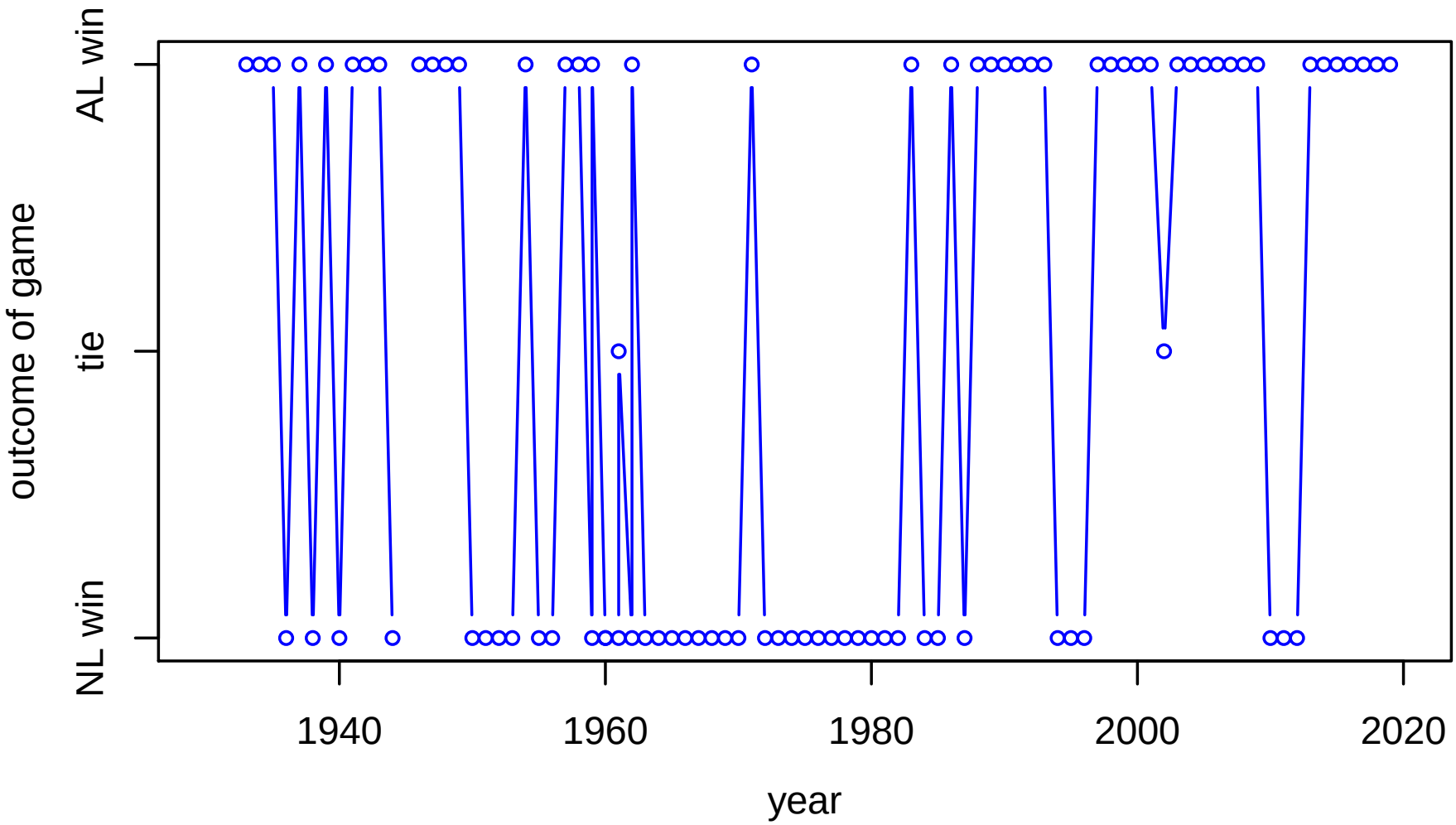
Australian Monthly Red Wine Sales



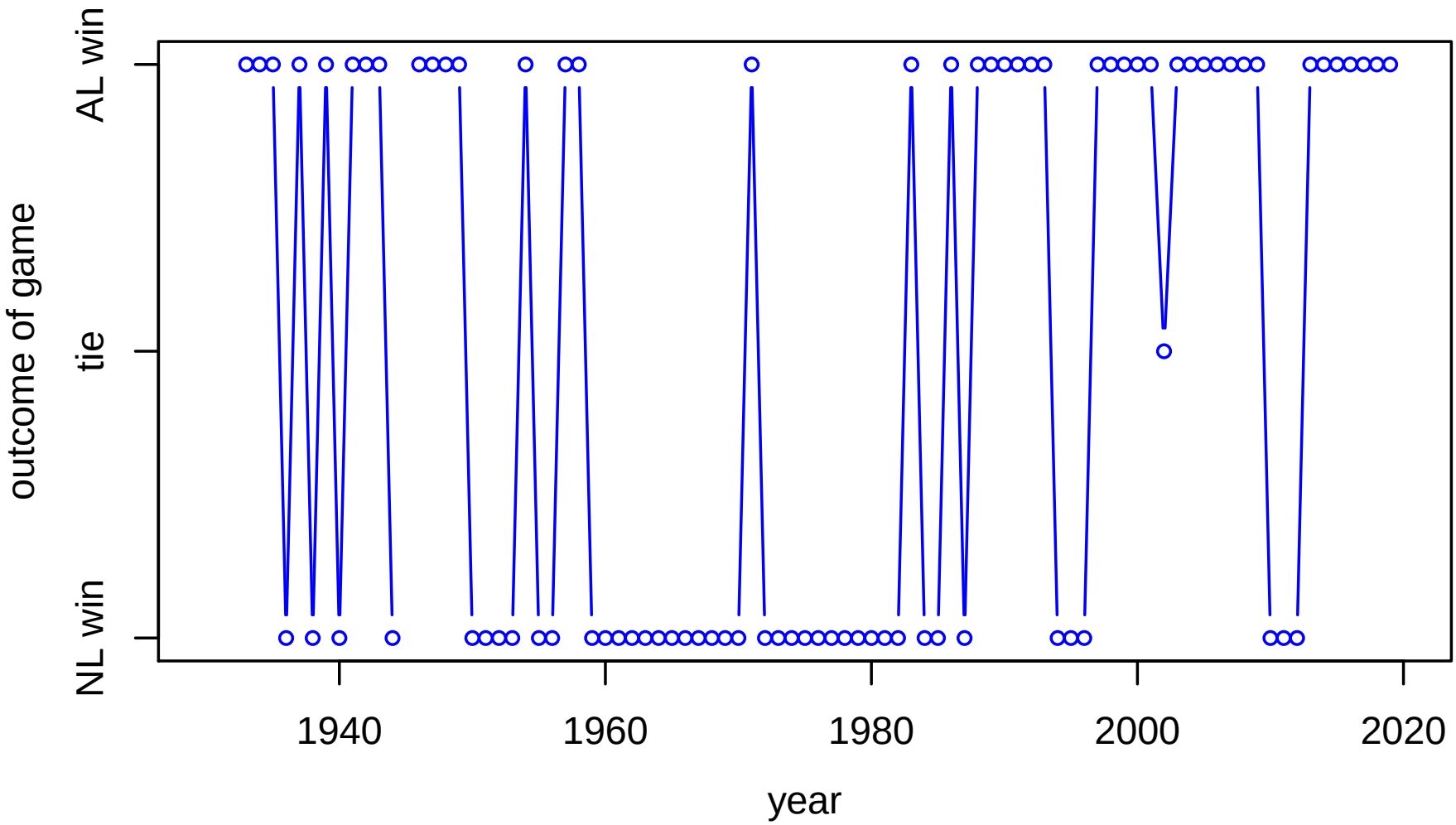
Australian Monthly Red Wine Sales

- index set is discrete parameter and equally spaced (again ignoring fact that months don't have same number of days)
- univariate time series
- x_t can assume nonnegative real values (kiloliters)
- might be of interest to compare seasonal patterns with what goes on in the Northern Hemisphere (does Christmas in summer rather than in winter matter?)
- also of interest to forecast this series (perhaps looking at external factors such as average temperature/rainfall in growing areas, economic conditions, etc.)

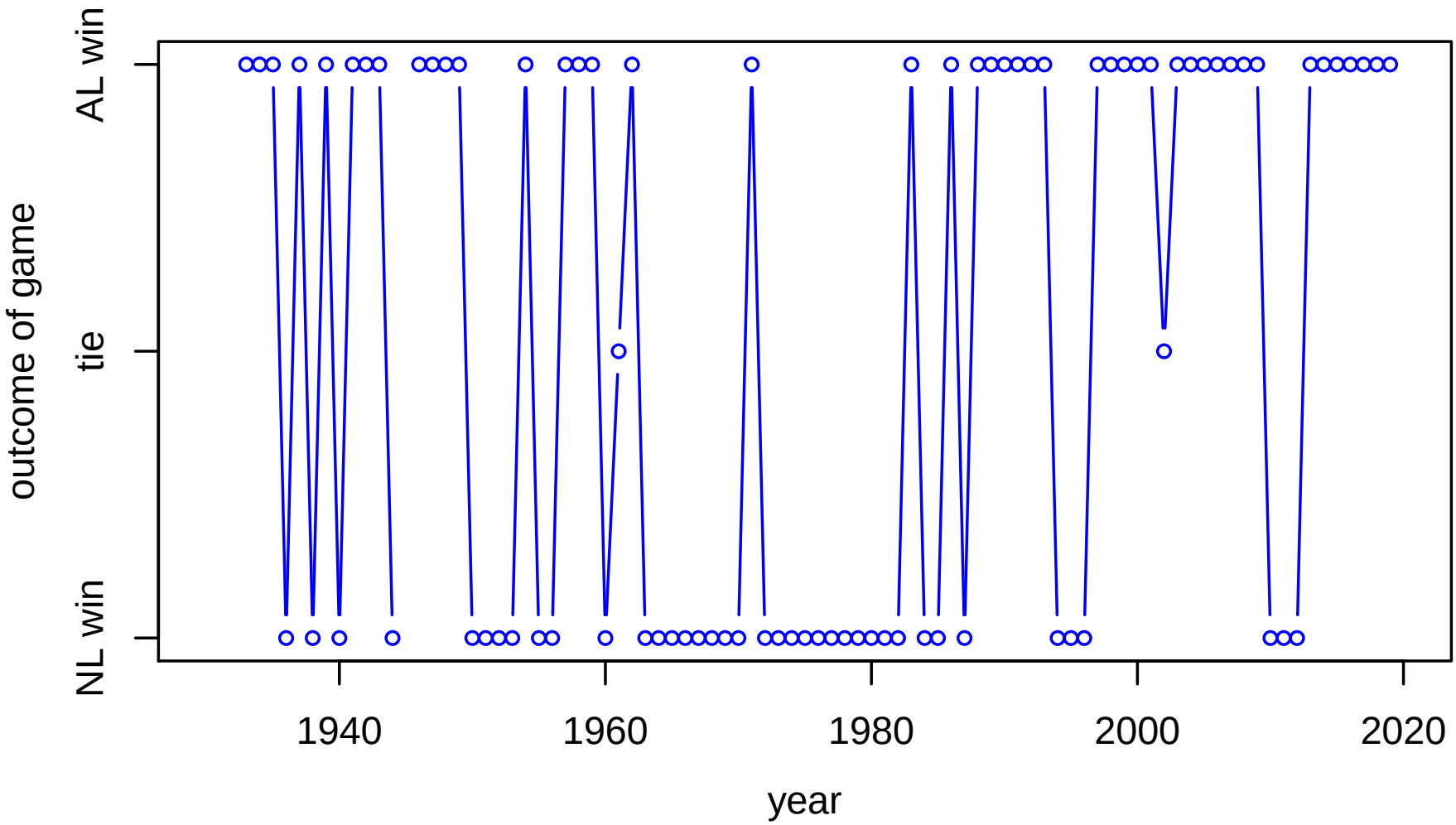
All-Star Baseball Games (45 AL; 43 NL; 2 ties)



All-Star Baseball Games (First in Each Year)

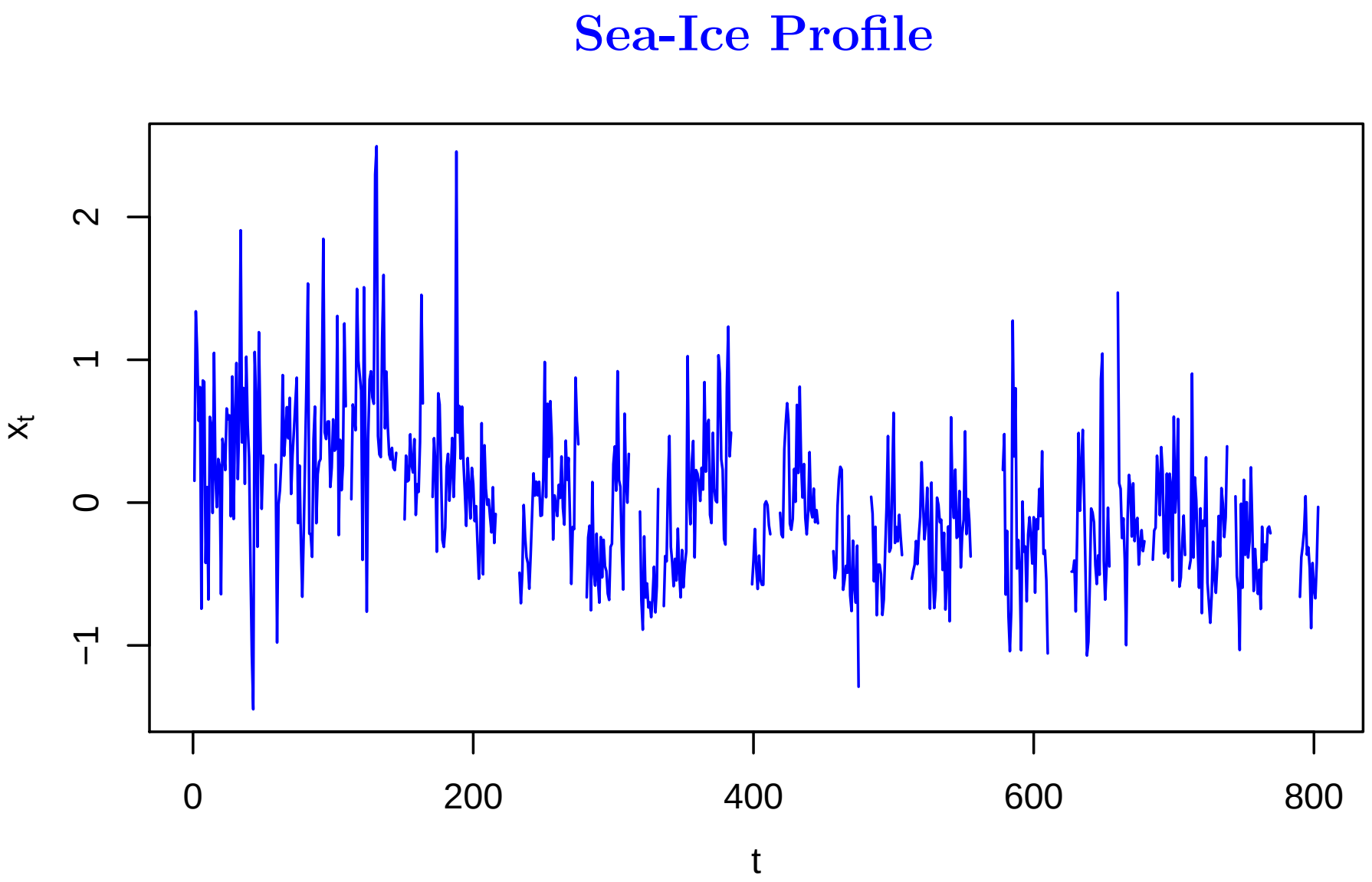


All-Star Baseball Games (Last in Each Year)



All-Star Baseball Games (1933–2019)

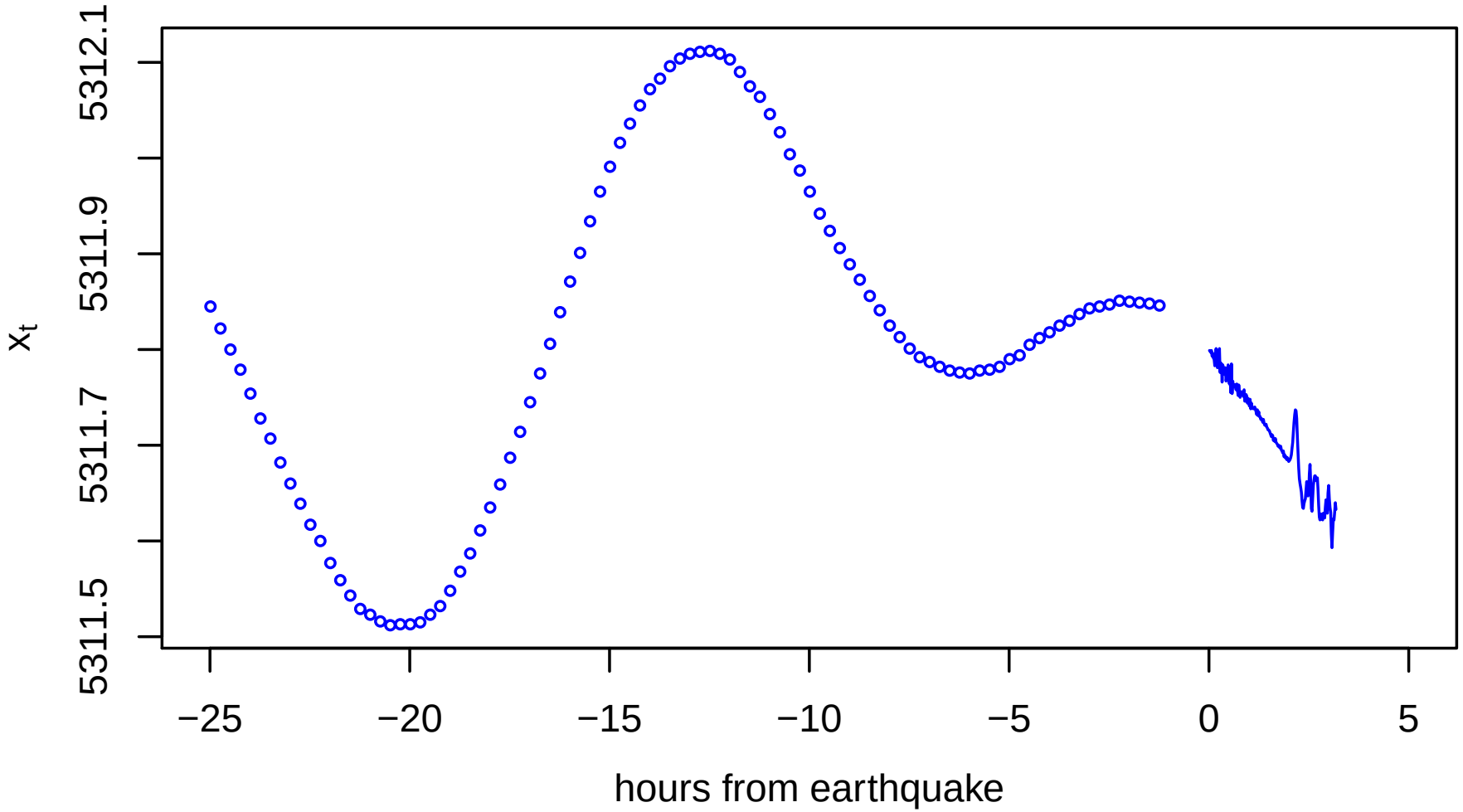
- index set is discrete parameter and equally spaced (almost!), but gappy (no game in 1945)
  - 'almost' because there were two games in each of 1959–1962
- univariate time series
- x_t is categorical, but, if two ties eliminated, could regard as binary-valued
- 45 American League (AL) wins & 43 National League (NL) wins, but win/loss patterns seem to have streaks inconsistent with assumption of independence of outcomes (more later!)
- gamblers would be interested in forecasting this series!


Sea-Ice Profile
x_t
2
1
0
−1
0
200
400
600
800
t

Sea-Ice Profile

- index set is discrete parameter and equally spaced, but gappy
- univariate time series, but here 'time' is distance along a transect
- x_t is real-valued (meters)
- characterizing roughness of series is of interest for comparison with physical models
- filling in the missing observations is also of interest

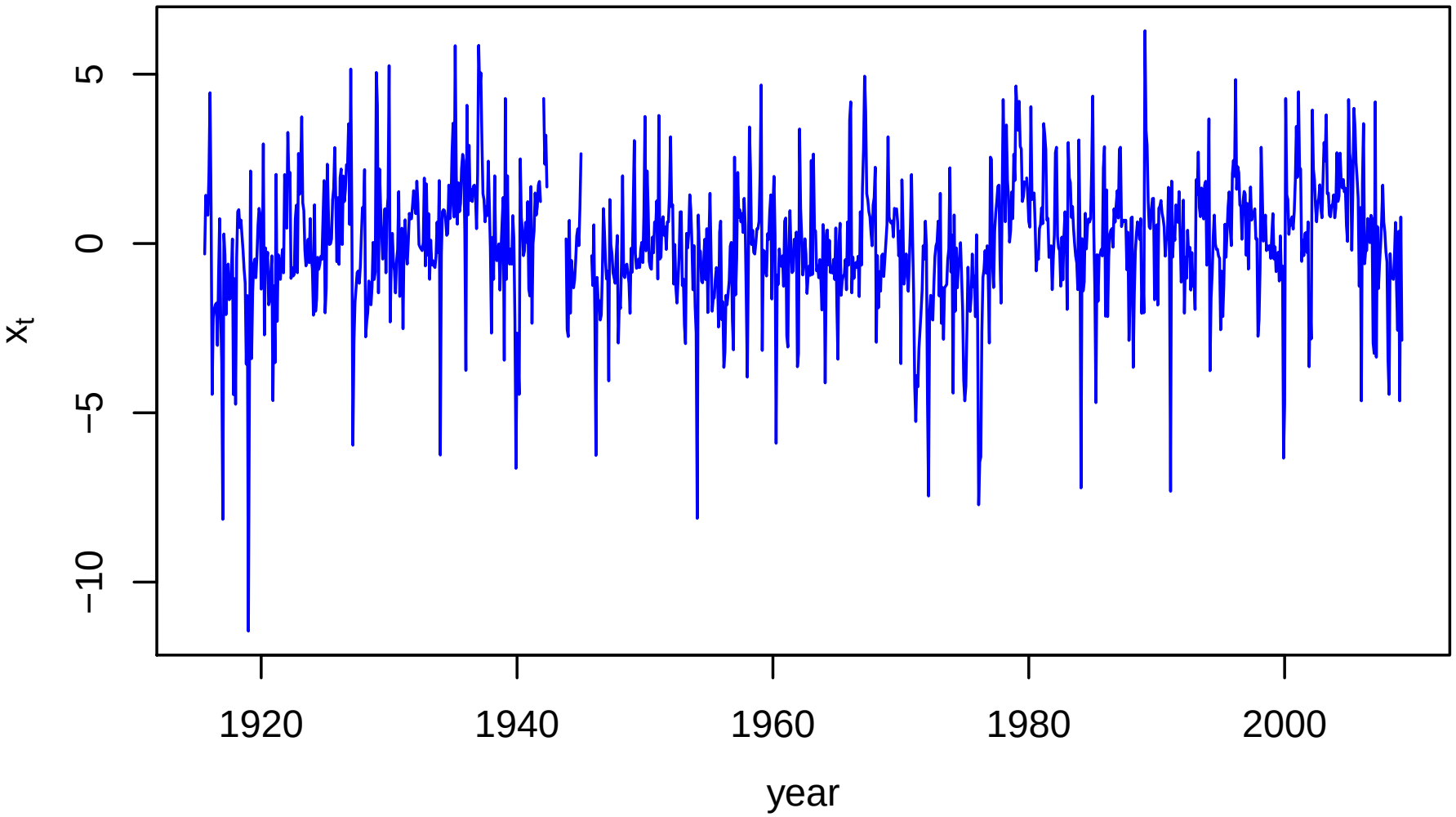
Pressure Measurements from DART® Buoy

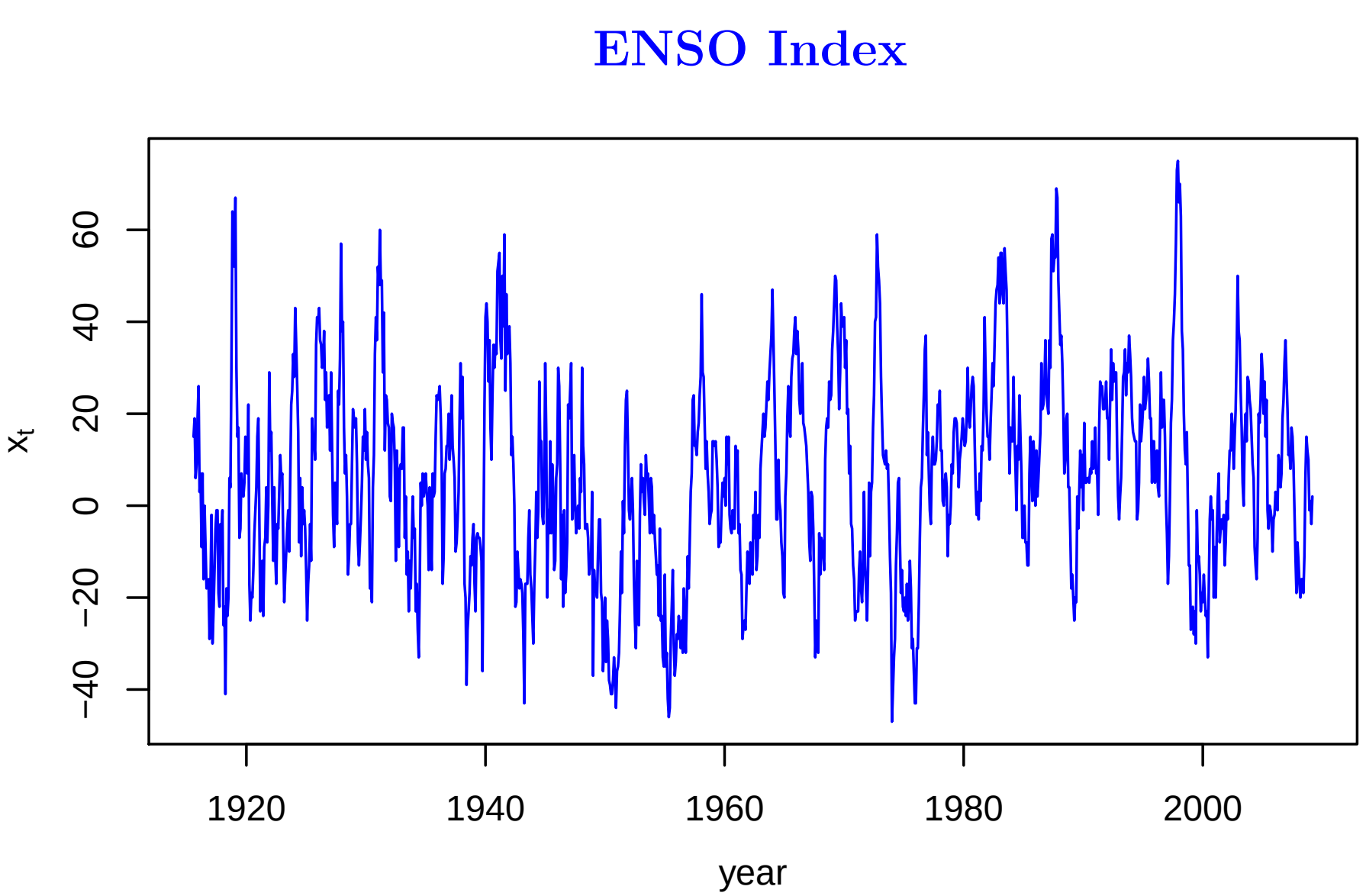


Pressure Measurements from DART® Buoy

- index set of underlying measurements is discrete parameter and equally spaced (one value every 15 seconds), but these values only become available after a delay of up to 2 or 3 years
- one 15-sec value is available every 15 minutes during ambient conditions (four values per hour)
- when triggered by a tsunami-generating earthquake, four-point (1 minute) averages are available at 1 minute intervals
- univariate time series
- x_t is real-valued (height of water column above instrument)
- of interest to remove variations due to tides ('detiding')
- after detiding, of interest to characterize nature of residuals from fits of detided data to numerical models

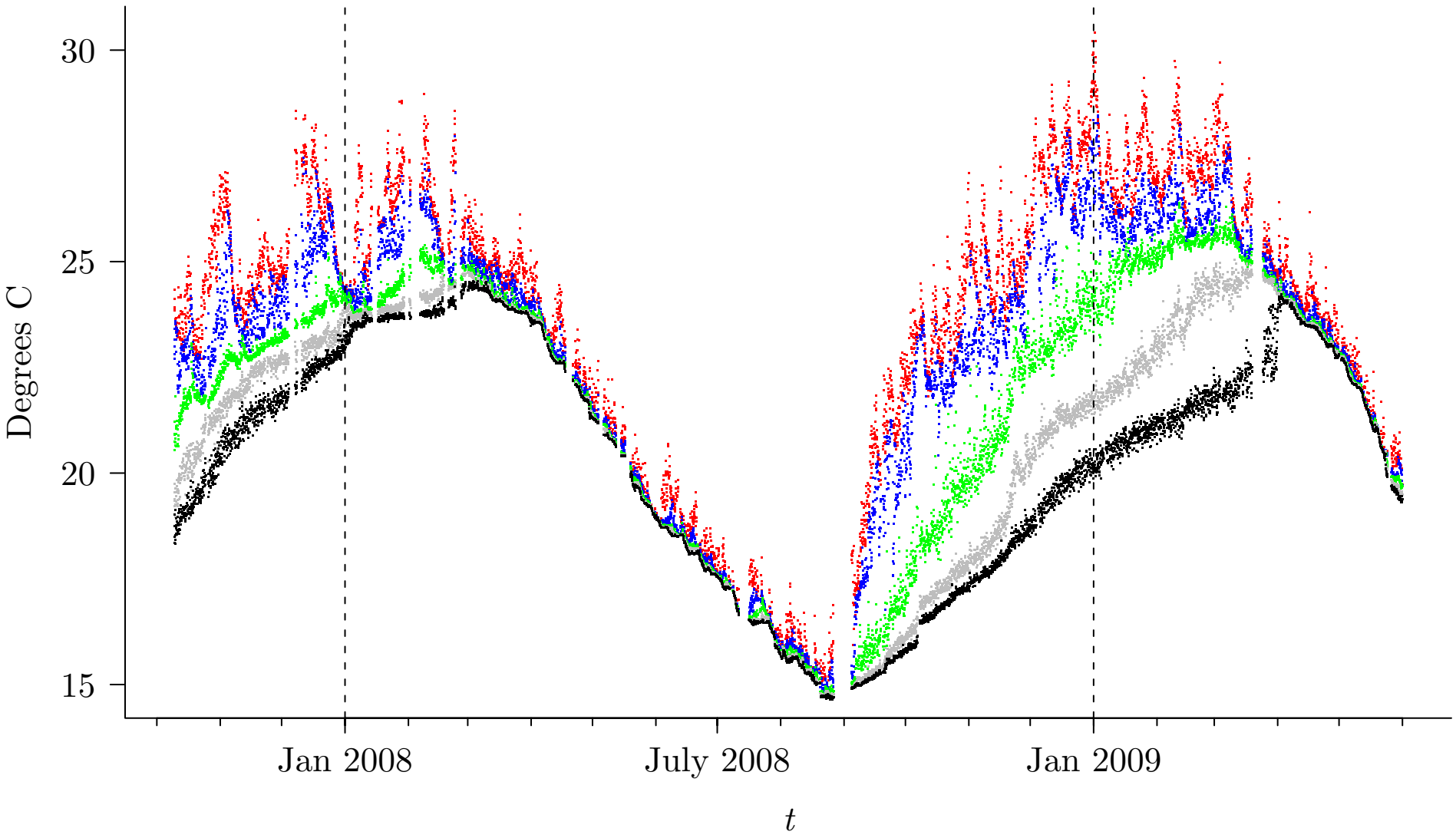
St Paul Temperatures




ENSO Index
x_t
60
40
20
0
−20
−40
1920
1940
1960
1980
2000
year

St Paul Temperatures and ENSO Index

- St Paul is island in Bering Sea (west of mainland Alaska and north of chain of Aleutian Islands; population 479 as of 2010)
- ENSO is abbreviation for 'El Niño/Southern Oscillation', which is a cyclic warming/cooling of eastern and central Pacific ('El Niño' is oceanic component; 'Southern Oscillation' is atmospheric component; ENSO is coupled system)
- index set is discrete parameter and equally spaced (one value every month), but temperature observations are gappy
- bivariate time series
- x_t is real-valued
- of interest to deduce the relationship between the two variates


x_t at Depths of 1, 5, 10, 15 & 20 Meters
30
25
20
15
Degrees C
Jan 2008
July 2008
Jan 2009
t

Temperature Measurements at Wivenhoe Dam

- index set is discrete parameter and equally spaced (almost), but gappy
- multivariate time series, with depth relationship between variates
- x_t can assume real values (degrees centigrade, but negative values highly unlikely!)
- daily, subannual and annual patterns are of interest for monitoring water quality
- of interest to use components of this series as covariates to predict, e.g., algae blooms

Uses/Objectives for Time Series Analysis

- forecasting/prediction (possible due to nonindependence of adjacent values in series – not living in 'independent and identically distributed' (IID) world!)
- compare/contrast two series (how are they related? – e.g., input/output to some system)
- data description (how does series vary?) and exploration (is there anything unusual?)
- process control (e.g., reservoir draining in response to rainfall)
- hypothesis testing
- will advocate data description/statistical approach, but other approaches are in use (see, e.g., article on 'technical analysis' in Wikipedia and critique by J.A. Paulos in his book *A Mathematician Plays the Stock Market*, 2003)

Reference

- G.U. Yule (1927), 'On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Numbers,' *Phil. Trans.* (A), **226**, pp. 267–98