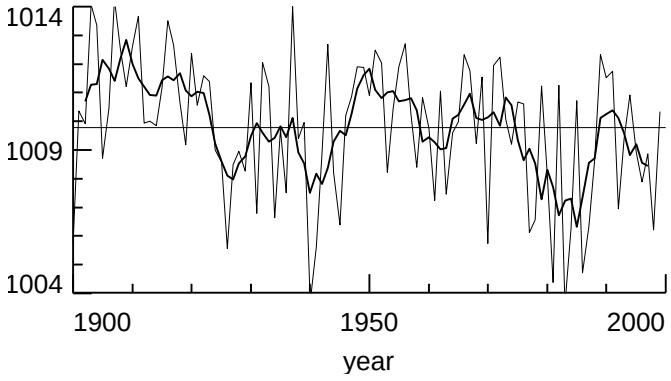


Figure 1: Plot of the NP index (thin curve) and a five year running average of the index (thick). The thin horizontal line depicts the sample mean (1009.8) for the index.

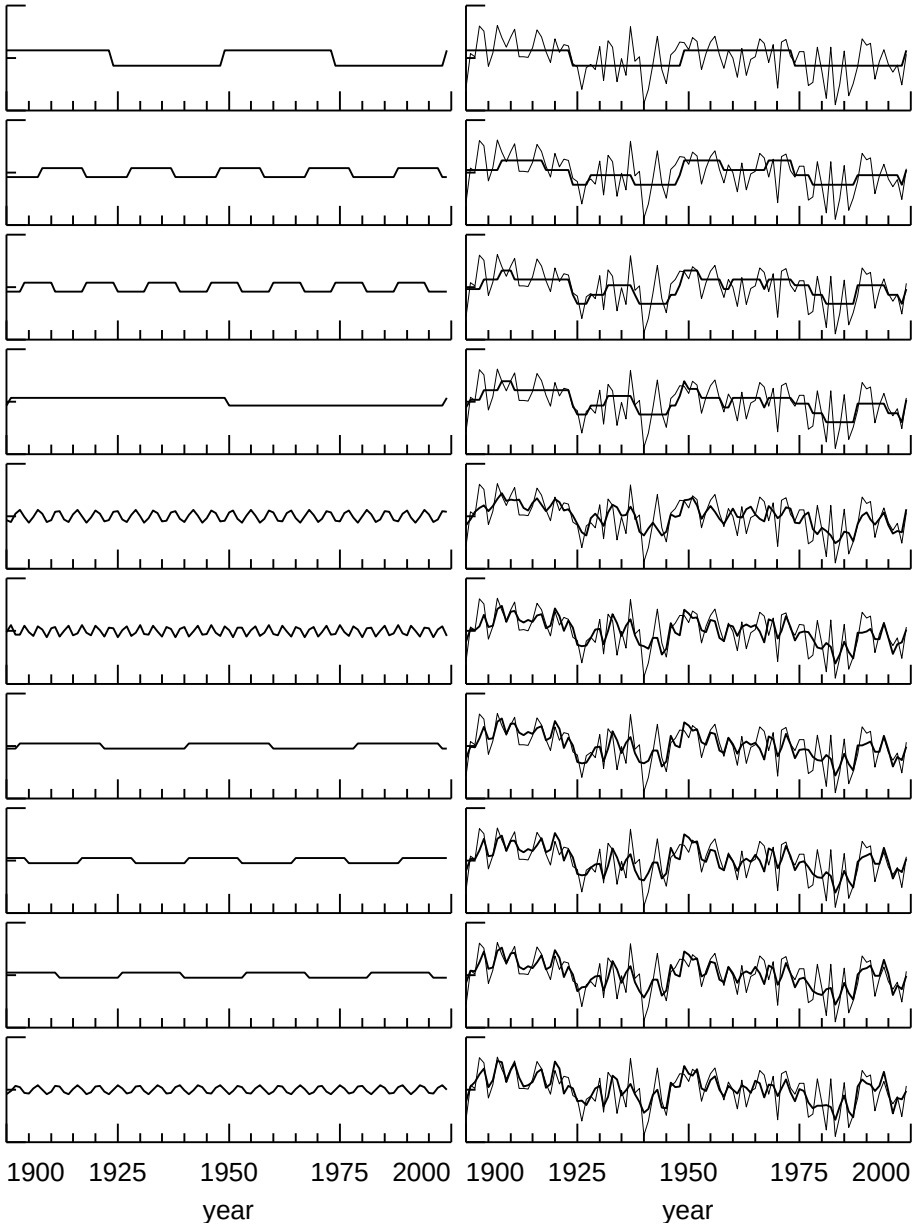


Figure 2: Matching pursuit consisting of a dictionary of sinusoids and square wave oscillators (SWOs) as applied to the NP index. The left-hand column shows the vector chosen at steps 1 to 10 (top to bottom) of the matching pursuit algorithm, while the right-hand column shows the corresponding cumulative approximations plotted over the NP index. The first two vectors selected by matching pursuit are SWOs with periods of, respectively, 50 and 20 years.

| model | parameter | 95% CI | $\sigma$ | 95% CI |
|---|---|---|---|---|
| AR | $\hat{\phi} = 0.21$ | $[0.02, 0.40]$ | $\hat{\sigma}_\epsilon = 2.37$ | $[2.01, 2.67]$ |
| FD | $\hat{\delta} = 0.17$ | $[0.02, 0.32]$ | $\hat{\sigma}_\varepsilon = 2.35$ | $[2.00, 2.66]$ |
| SWO | $\hat{\beta} = -10.09$ | $[-14.51, -5.67]$ | $\hat{\sigma}_e = 2.21$ | $[1.88, 2.50]$ |

Table 1: Autoregressive (AR), fractionally differenced (FD) and square wave oscillator (SWO) process parameter estimates for the NP index.

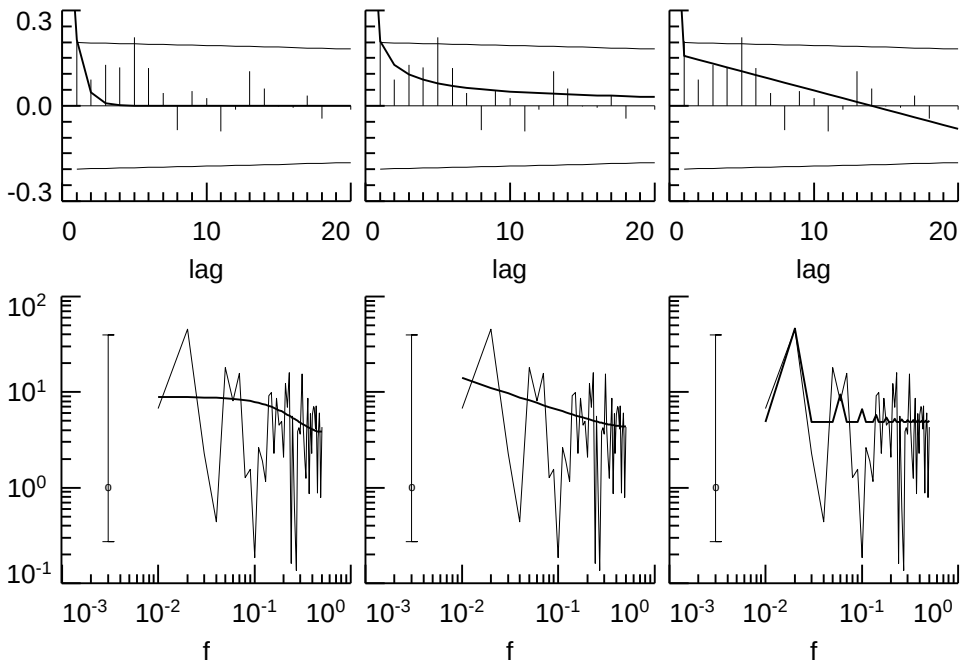


Figure 3: Sample autocorrelation sequence (ACS) and periodogram for the NP index, along with theoretical ACSs and spectral density functions (SDFs) for fitted AR, FD and SWO models (left, middle and right plots, respectively).

| $j$ | model | $T_j$ | $Q_j(0.90)$ | $Q_j(0.95)$ | $Q_j(0.99)$ | $\alpha = 0.05$ test result | $\hat{\alpha}$ |
|---|---|---|---|---|---|---|---|
| 1 | AR | 0.30 | 0.38 | 0.39 | 0.42 | fail to reject | 0.67 |
| | FD | 0.28 | " | " | " | fail to reject | 0.78 |
| | WN | 0.39 | " | " | " | reject | 0.05 |
| 2 | AR | 0.10 | 0.17 | 0.19 | 0.23 | fail to reject | $\gg 0.1$ |
| | FD | 0.07 | " | " | " | fail to reject | $\gg 0.1$ |
| | SWO | 0.10 | " | " | " | fail to reject | $\gg 0.1$ |
| | WN | 0.21 | " | " | " | reject | $\approx 0.03$ |
| 3 | AR | 4.65 | 7.74 | 9.45 | 13.31 | fail to reject | 0.32 |
| | FD | 3.12 | " | " | " | fail to reject | 0.54 |
| | SWO | 2.83 | " | " | " | fail to reject | 0.59 |
| | WN | 12.63 | " | " | " | reject | 0.01 |
| 4 | AR | 4.97 | 7.74 | 9.45 | 13.31 | fail to reject | 0.29 |
| | FD | 3.34 | " | " | " | fail to reject | 0.50 |
| | SWO | 3.00 | " | " | " | fail to reject | 0.56 |
| | WN | 13.31 | " | " | " | reject | 0.01 |

Table 2: Model goodness of fit tests for the NP index.

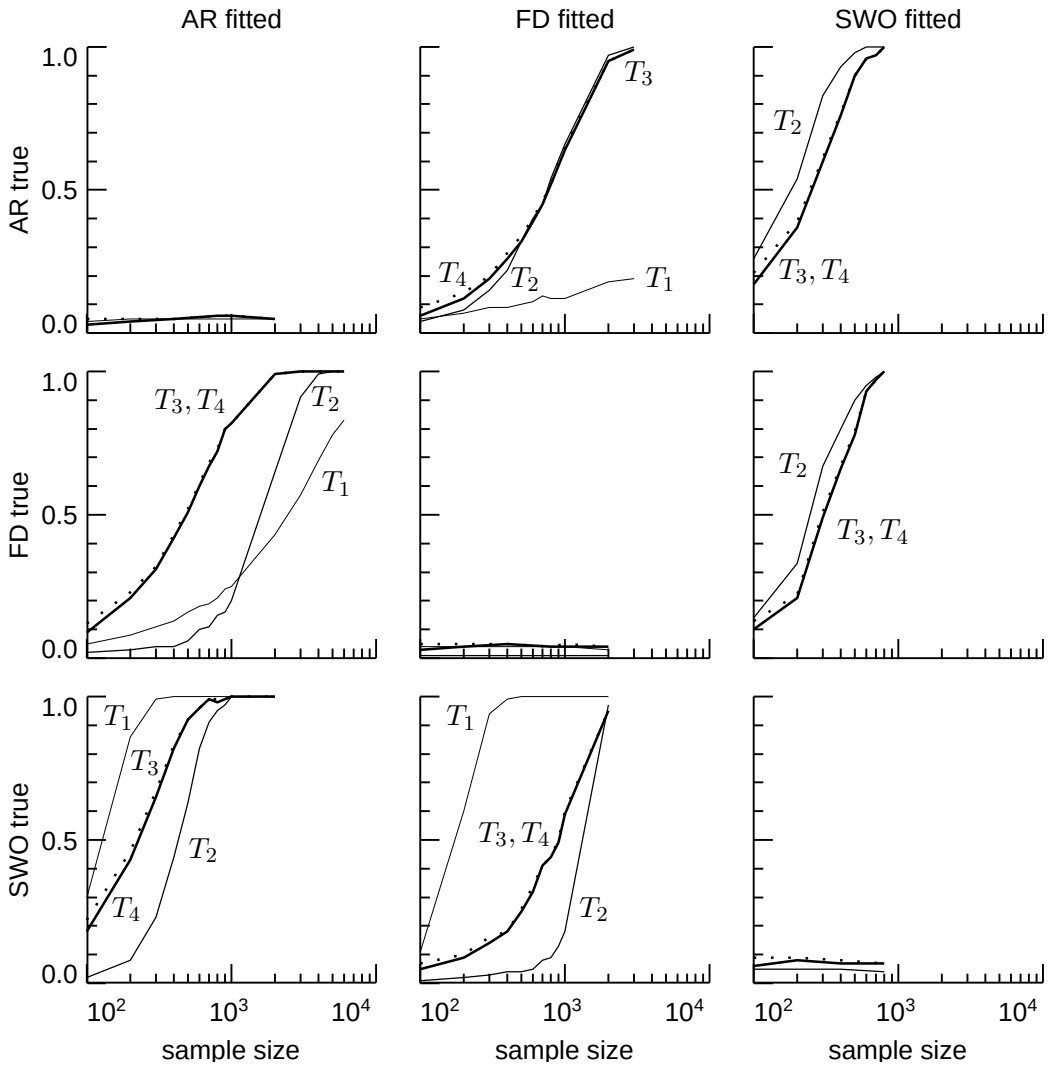


Figure 4: Probability (as a function of sample size) of rejecting the null hypothesis at a 0.05 level of significance that a fitted model $A$ is adequate for a realization of a process $B$ when using the test statistics $T_1$ (thin solid curve), $T_2$ (medium solid), $T_3$ (thick solid) and $T_4$ (dotted). For the plots in the left- to right-hand columns, the fitted models $A$ is, respectively, an FD, AR(1) and SWO model. The same ordering is used for the process $B$ for the plots in the top to bottom rows.

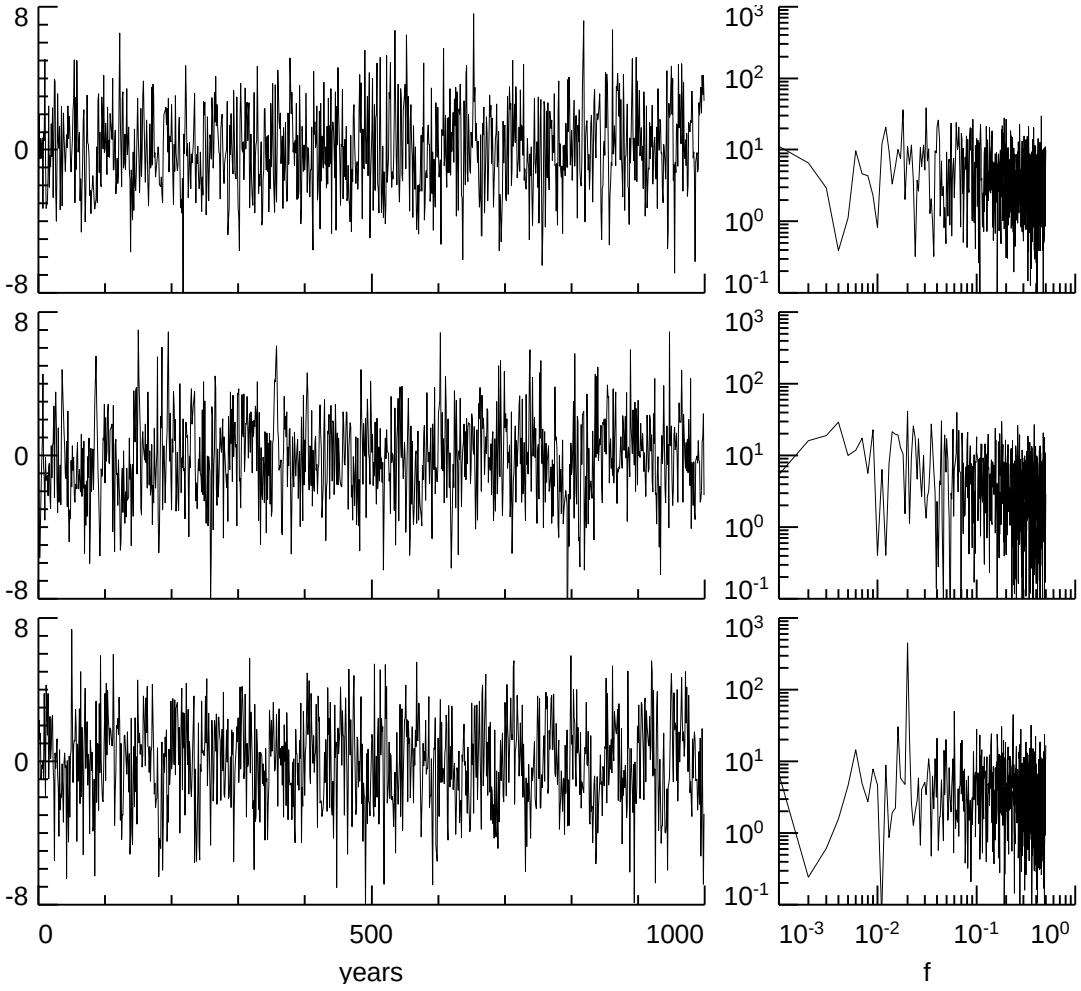


Figure 5: Simulated realizations (left-hand column) of AR(1), FD and SWO processes (top to bottom plots, respectively) with model parameters set to values estimated for the NPI index, along with associated periodograms (right-hand column).

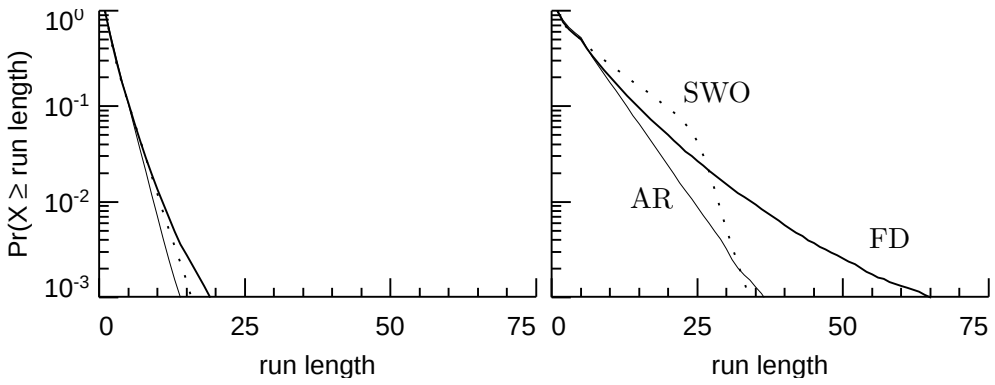


Figure 6: Probability of observing a run that is greater than or equal to a specified run length. The thin, thick and dotted curves denote the AR, FD and SWO processes. The left-hand plot is for processes without smoothing, whereas the right-hand plot is for processes subjected to a five year running average.