Inversion Methods for Determining Tsunami Source Amplitudes from DART Buoy Data

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NOAA-sponsored collaborative effort

overheads for talk available at

http://faculty.washington.edu/dbp/talks.html

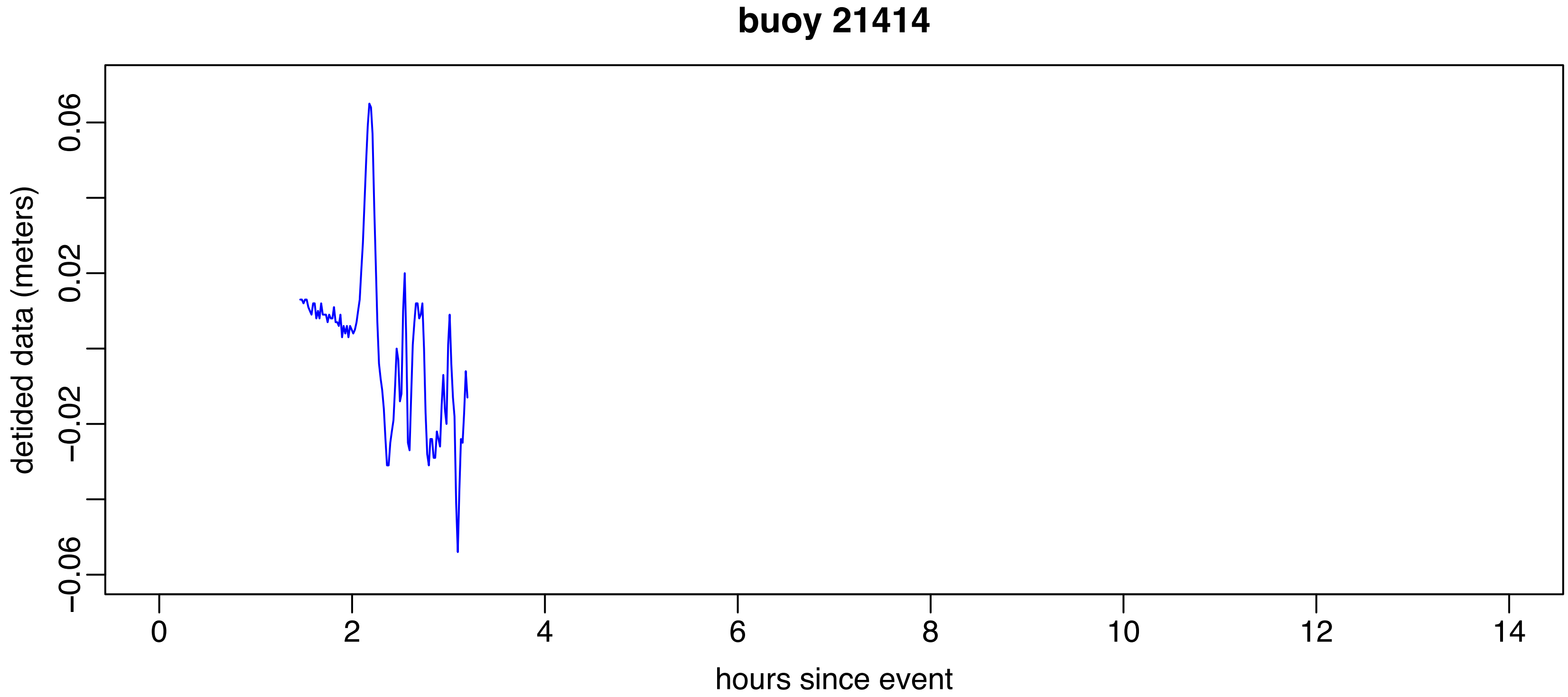
Overview: I

- scientific problem: given data from DART buoys and models for unit magnitude earthquakes from various tsunami source locations, determine actual magnitudes (slips) and location(s) of actual earthquake
- will describe elements of basic inversion algorithm
- start with detided DART buoy data, noting need for detrending
- look at model for single buoy, noting need for interpolation
- introduce least squares criterion by looking at estimation of slip for single source model based upon data from one buoy
- consider assessing statistical variability in estimated slip
- look at effect of using varying amounts of buoy data
- considering adding data from a second DART buoy

Overview: II

- look at using more than one source
- discussion of various 'bells and whistles' currently implemented
  - imposing constraints on slips
  - allowing shifting and stretching of source models
- discussion of work in progress
  - use of statistical tests to select sources
- demo of R implementation of algorithm (if time permits)
- will motivate basic ideas behind algorithm using the Kuril Island event of Nov 2006

Detided DART Buoy Data for Kuril Island Event



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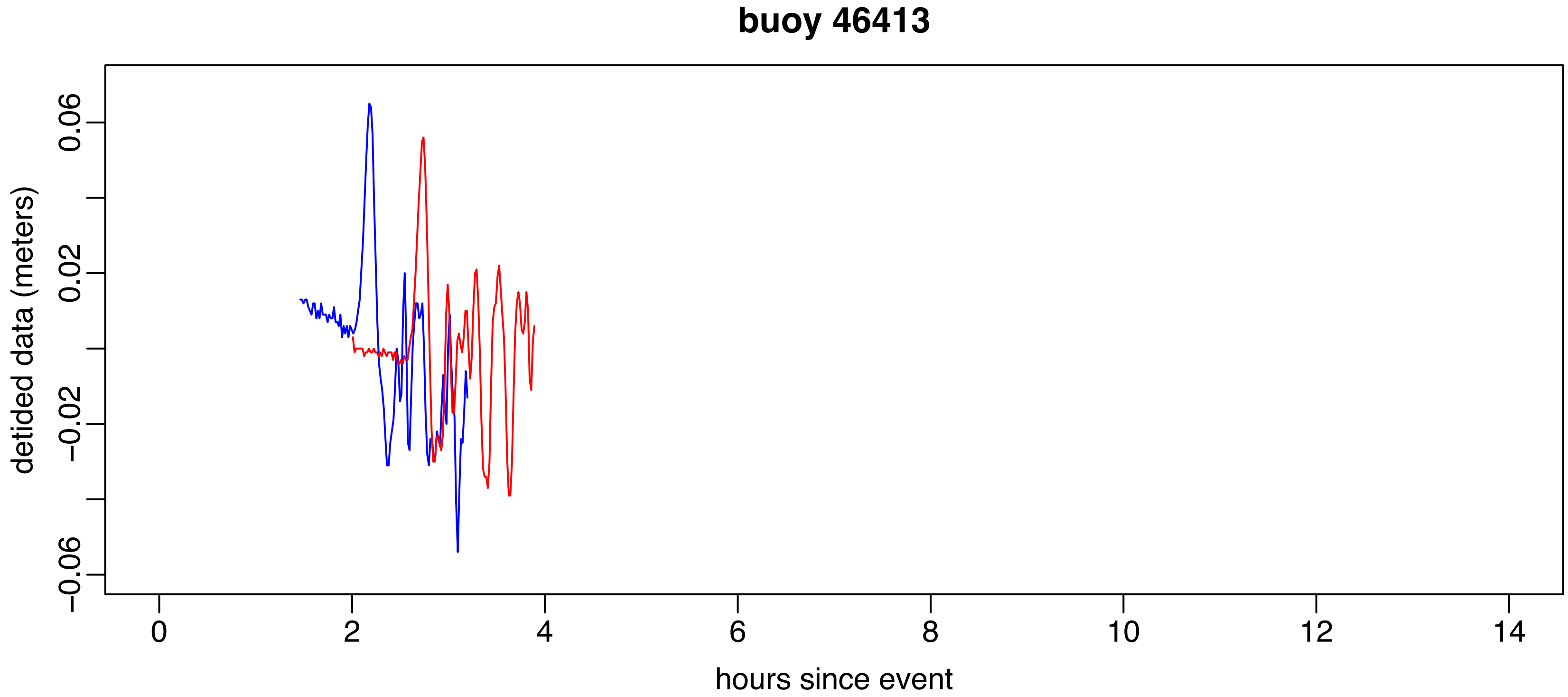

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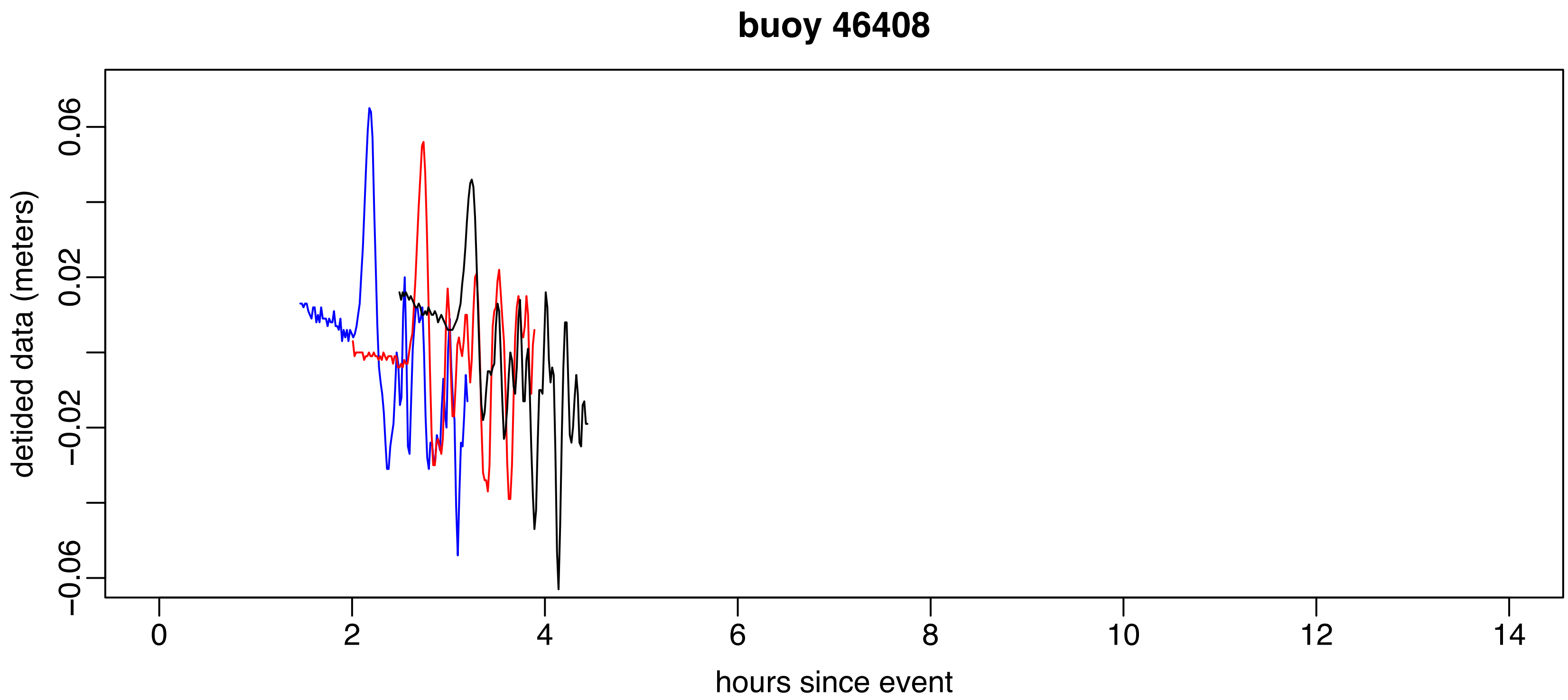

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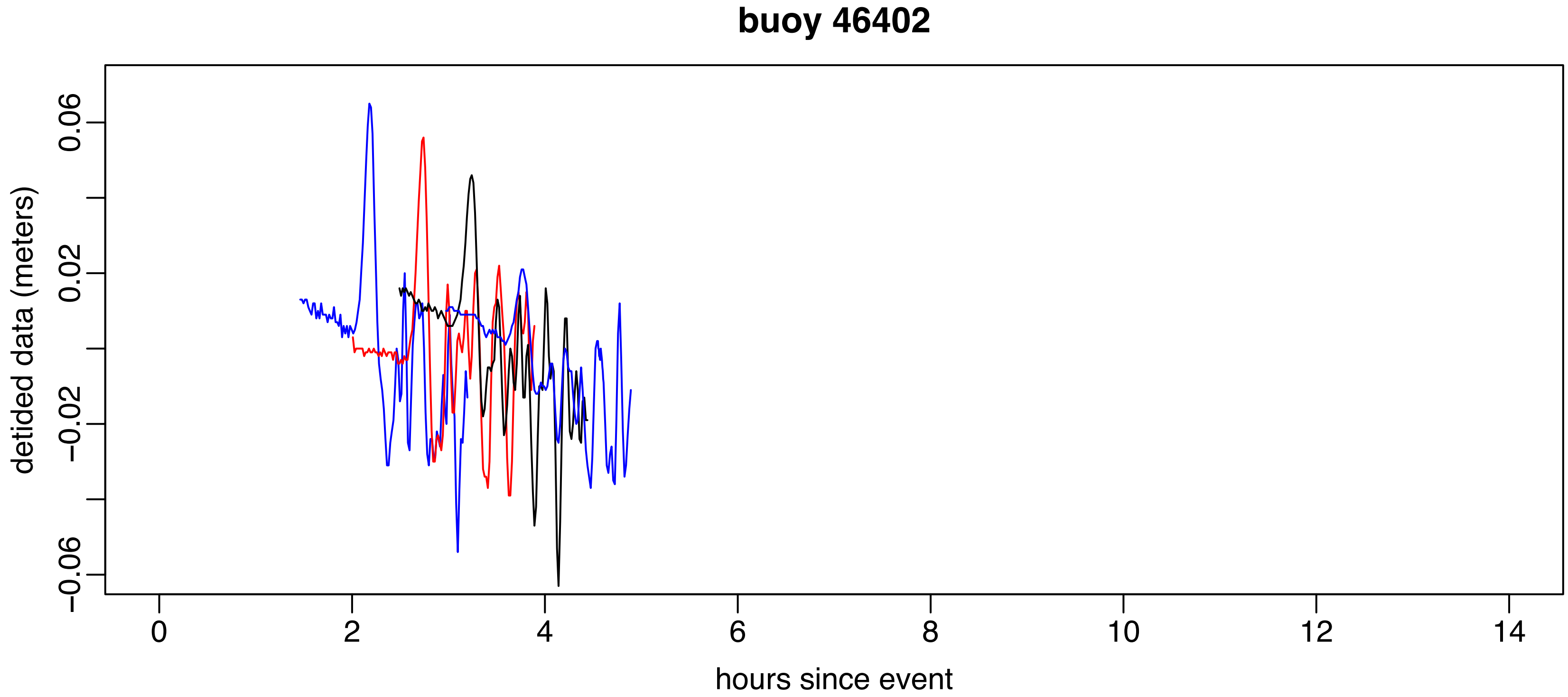

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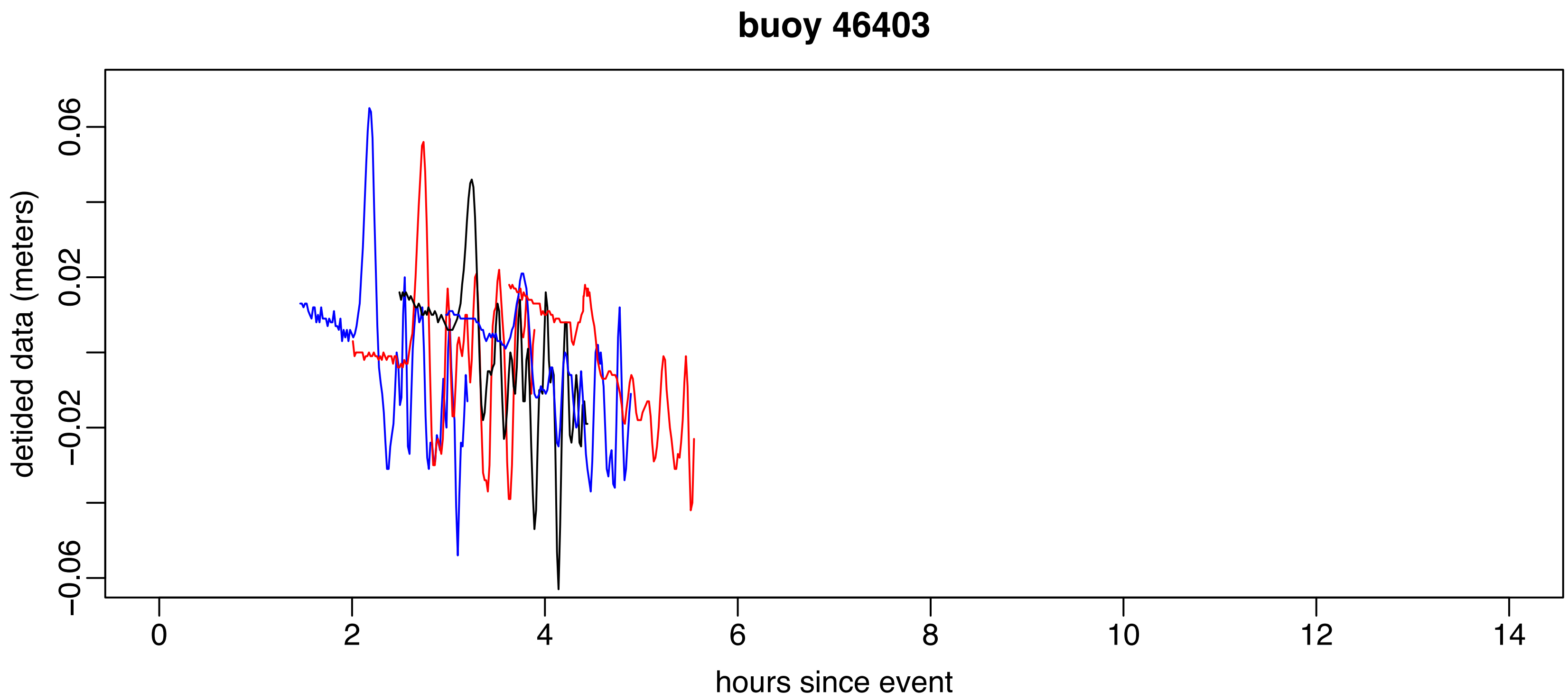

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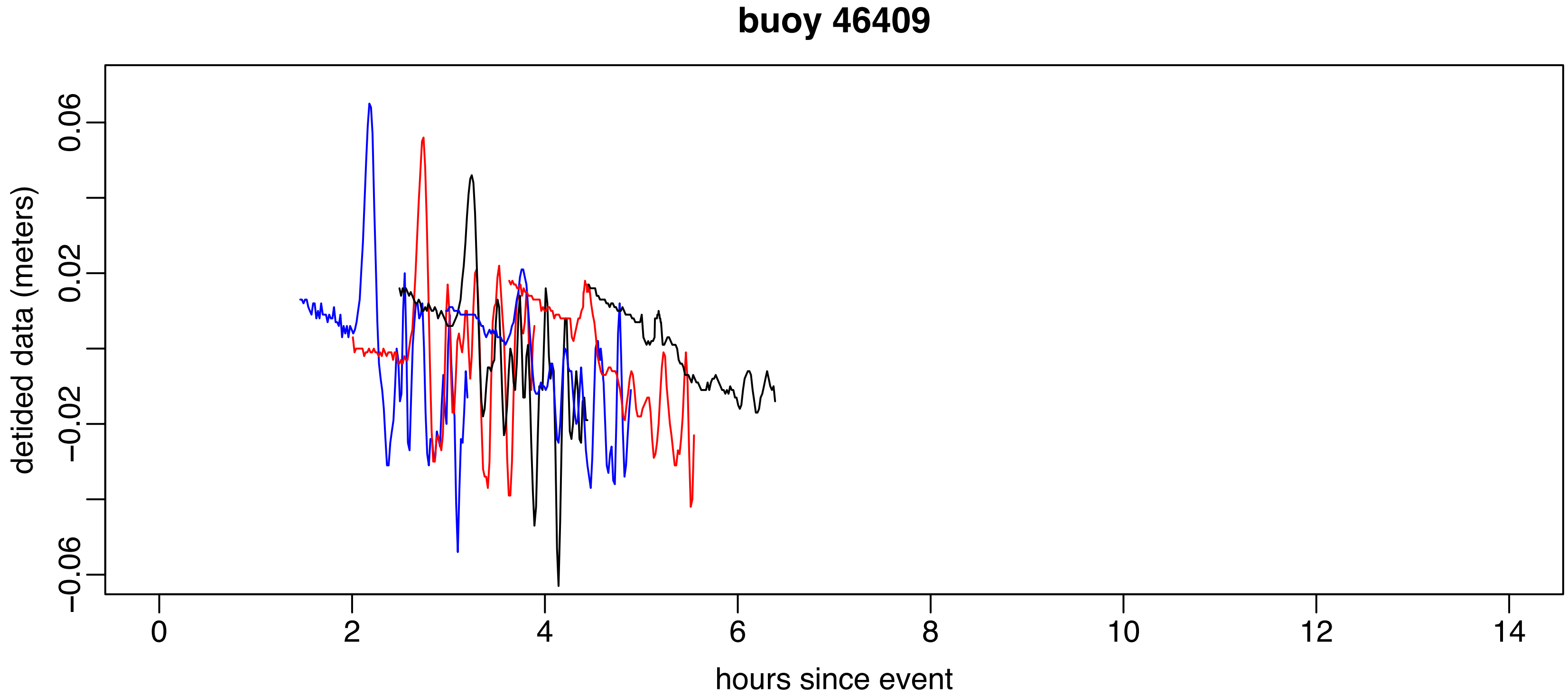

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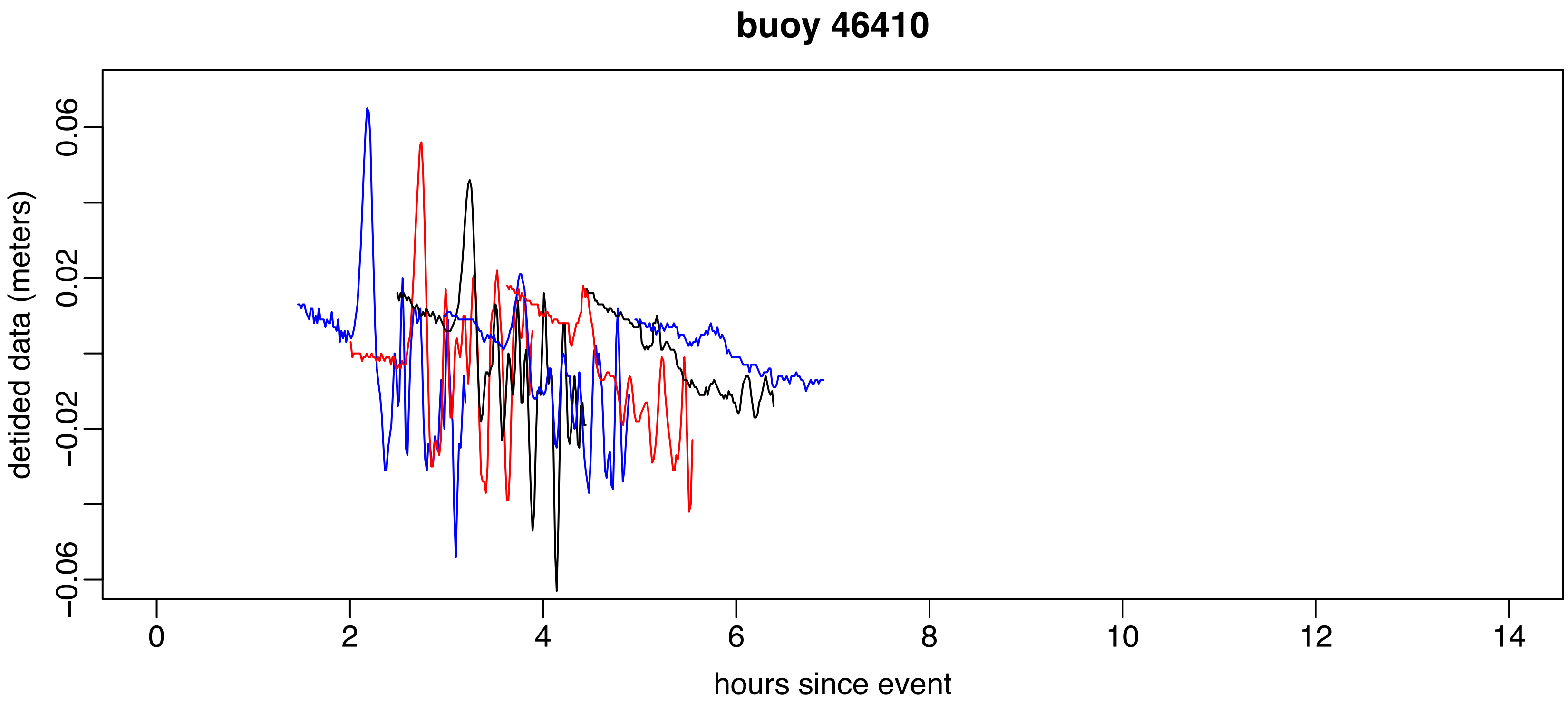

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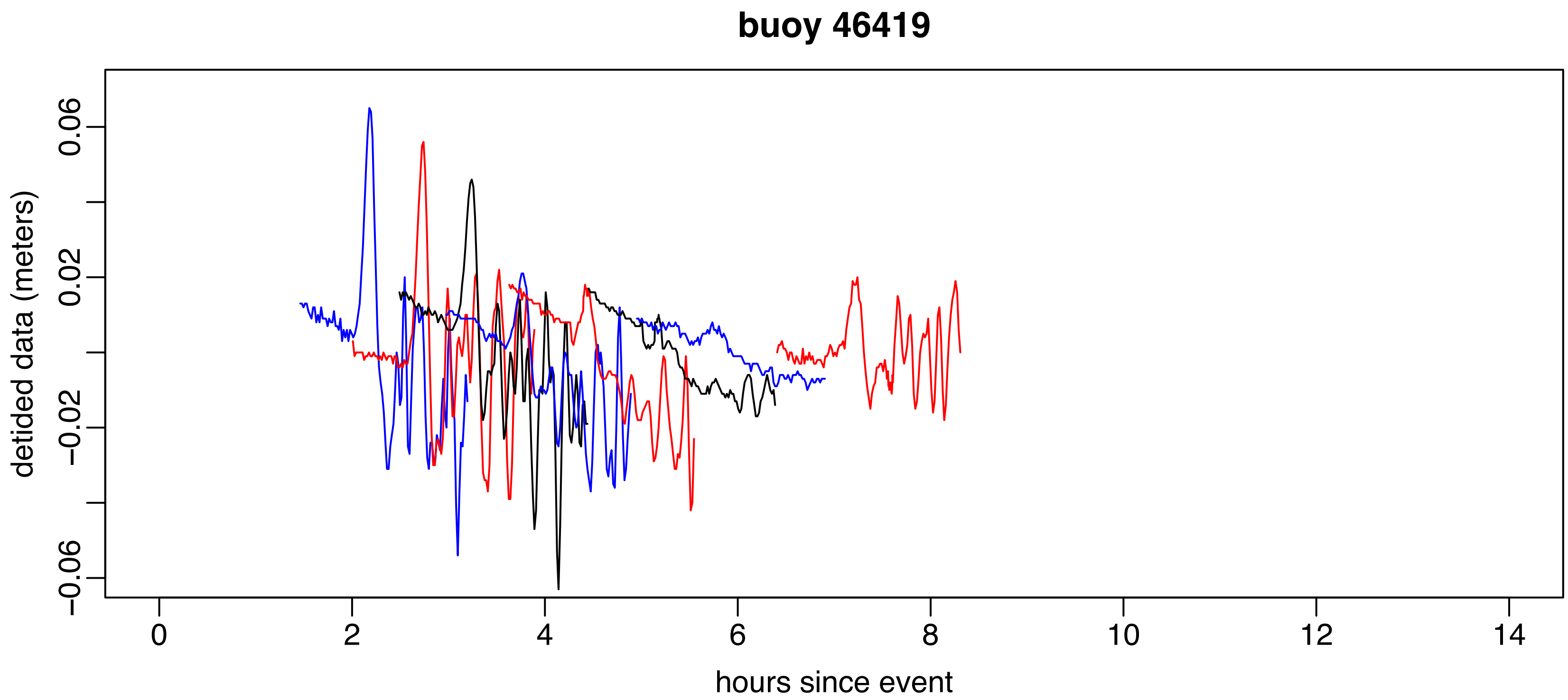

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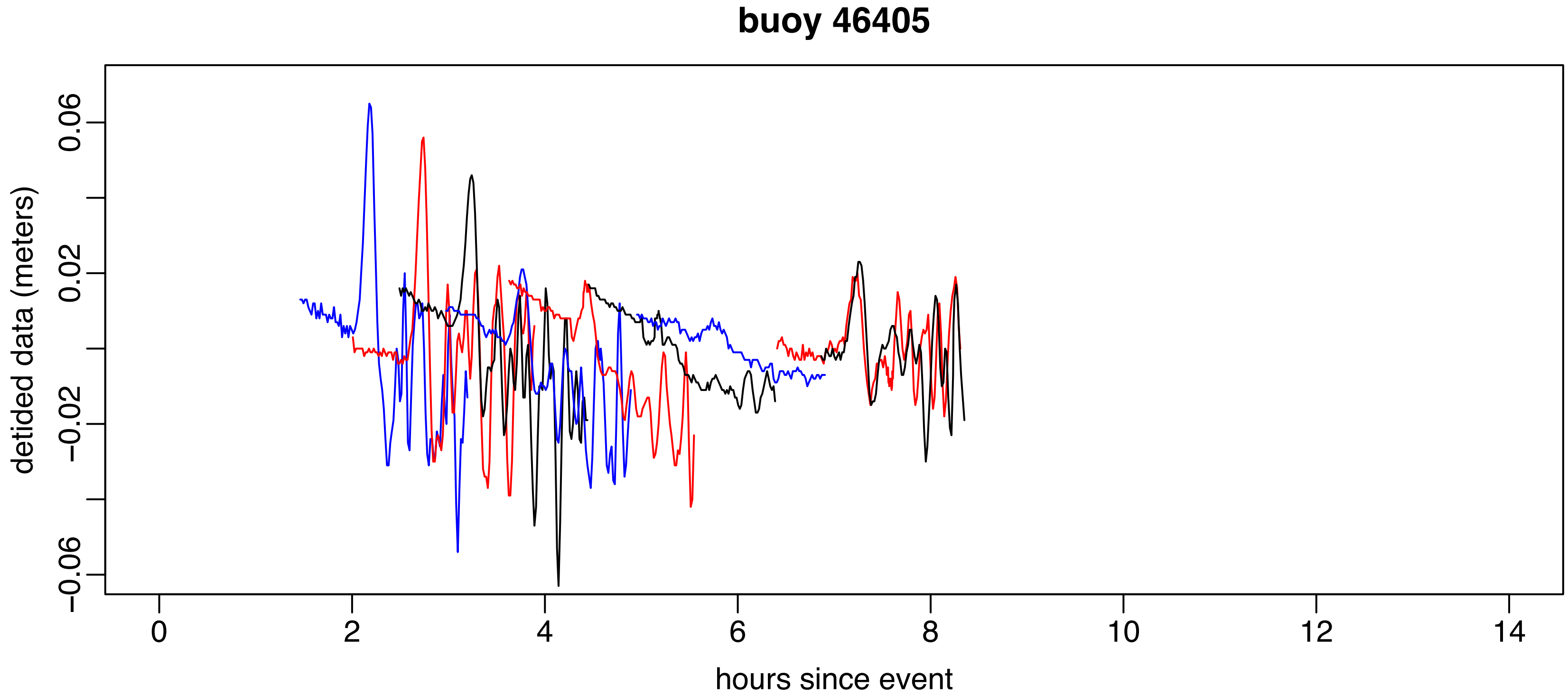

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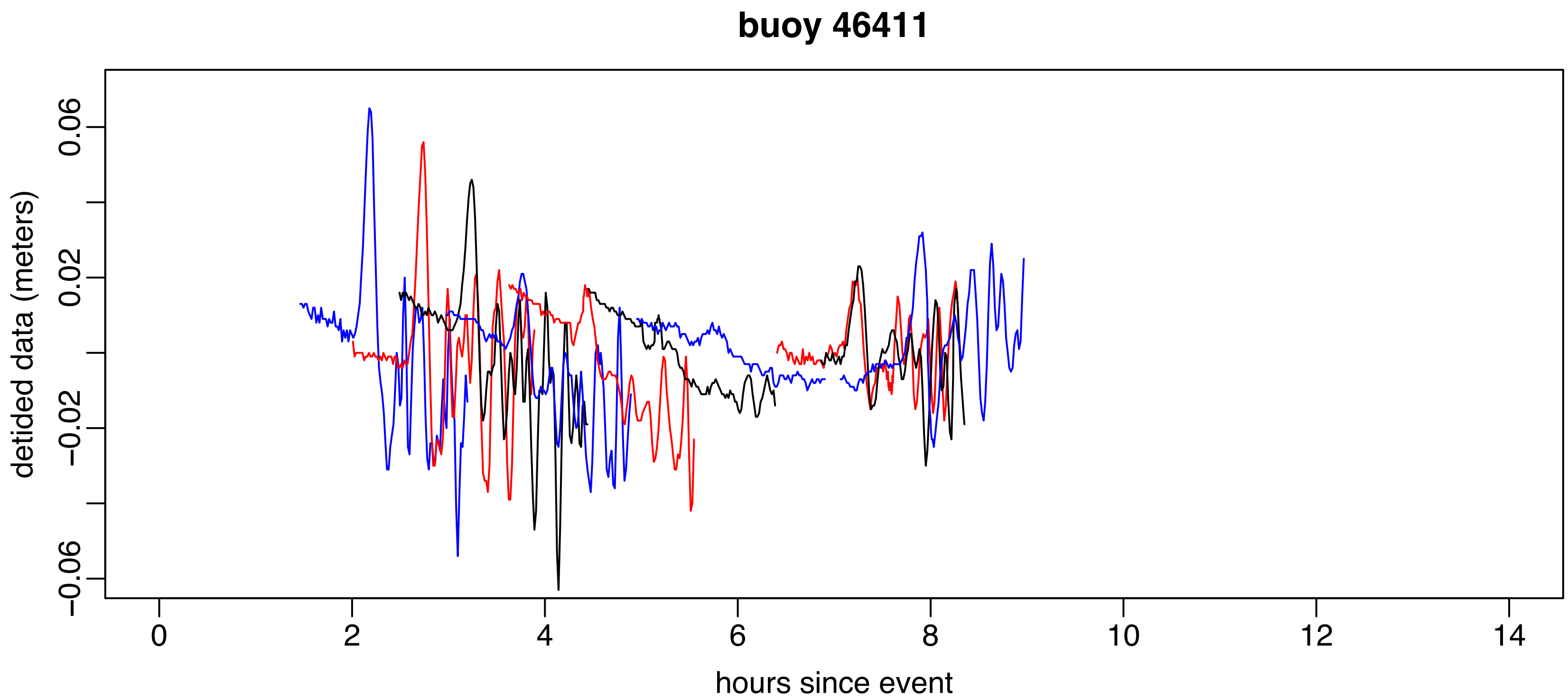

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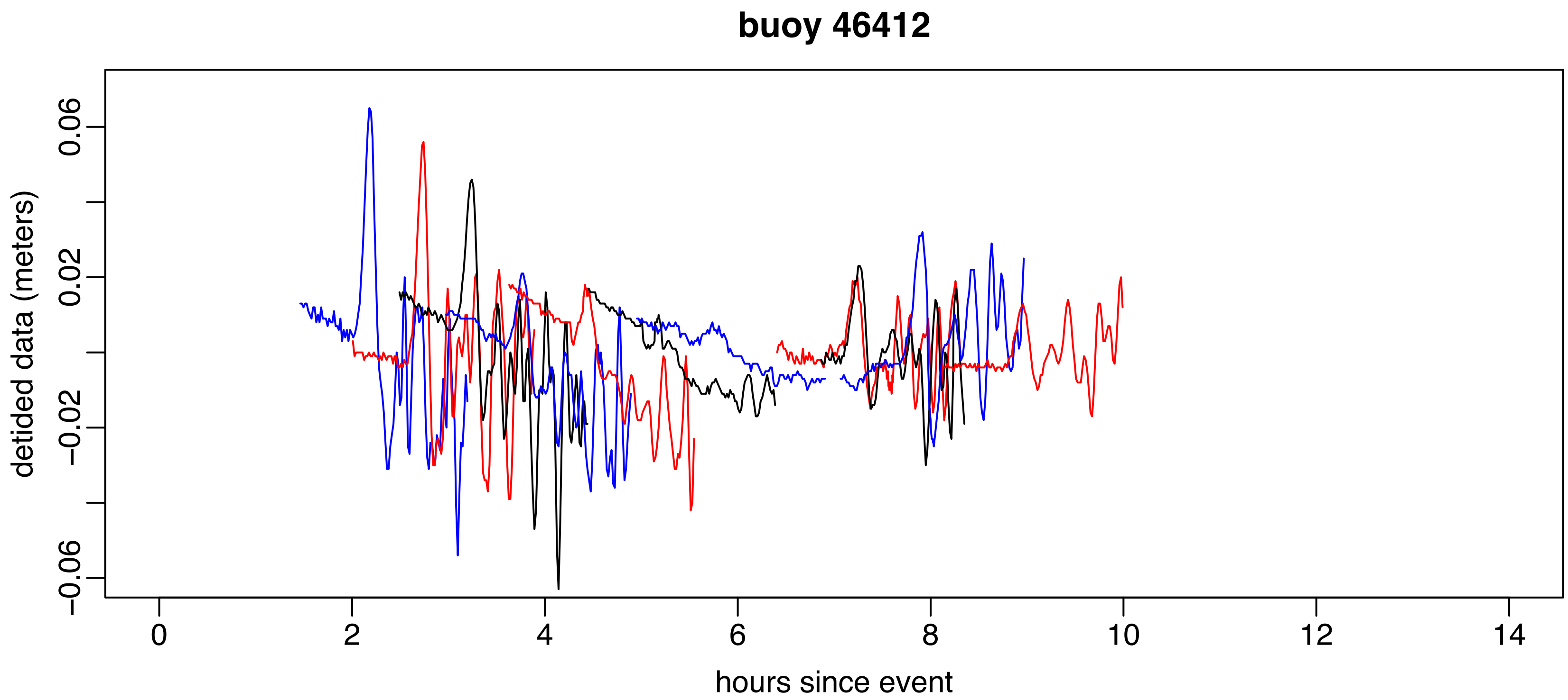

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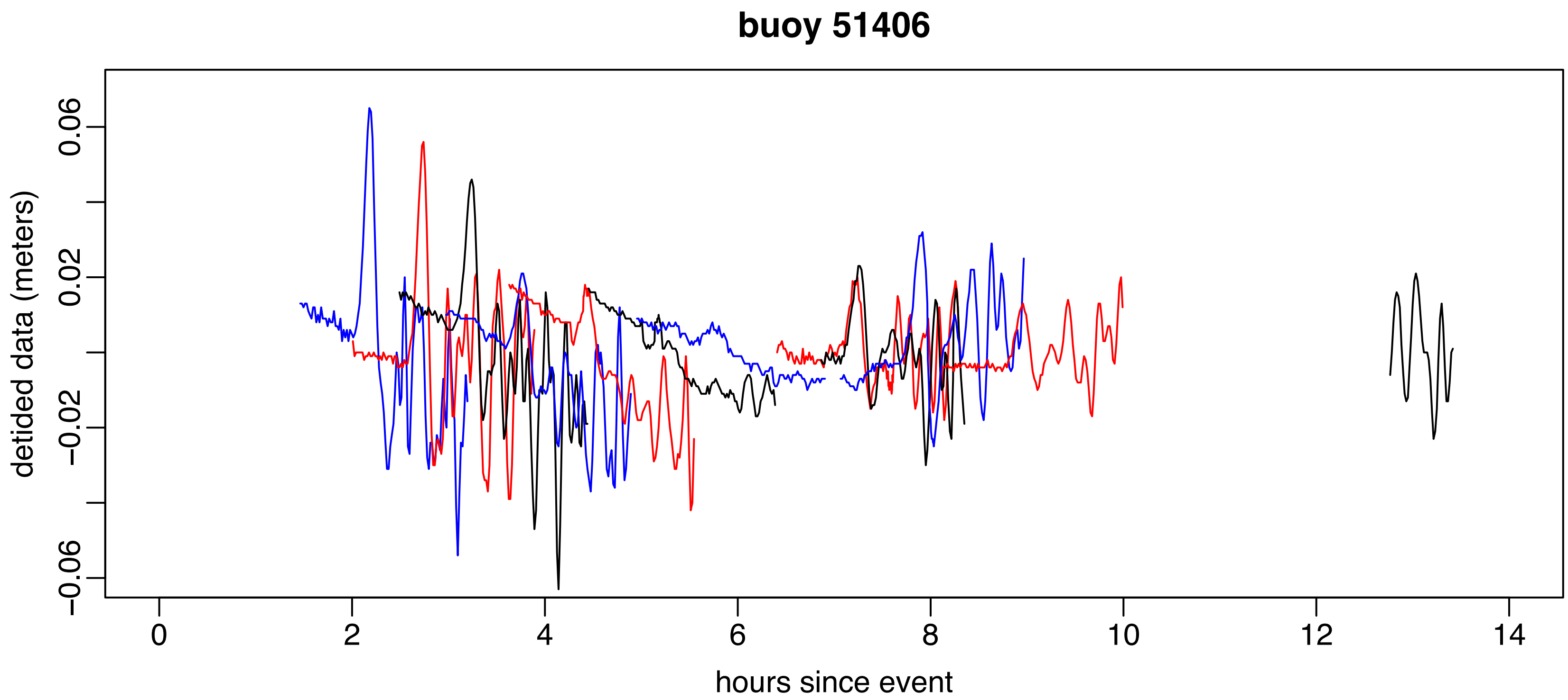
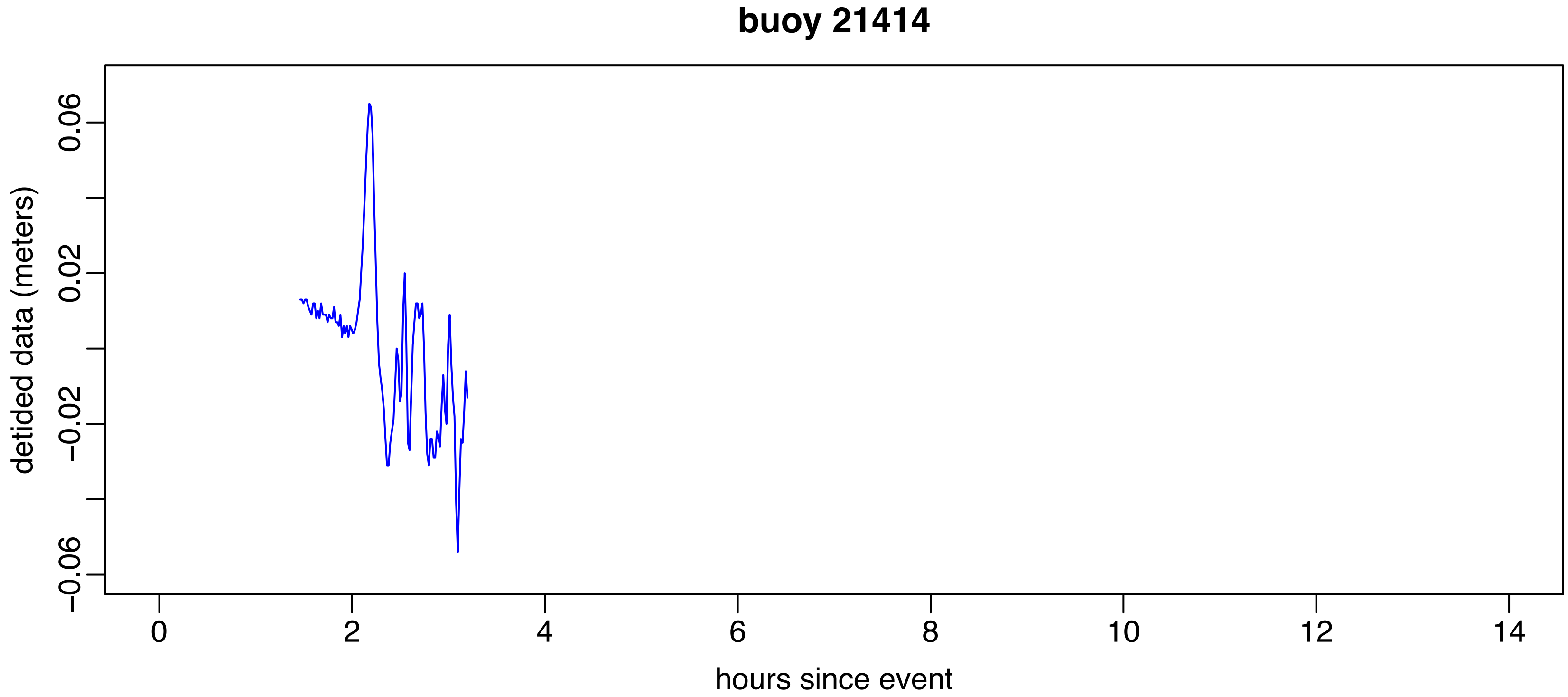


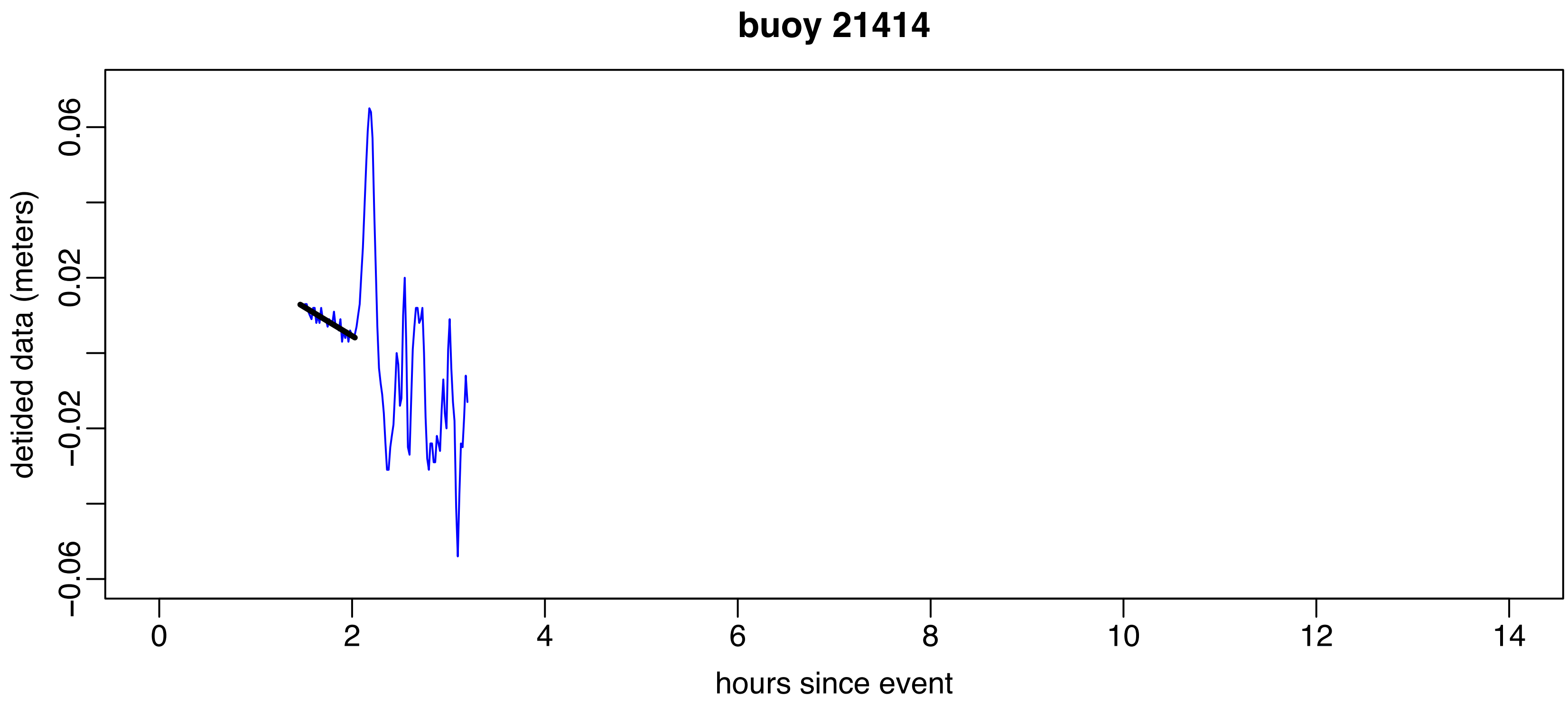
Detrending

- inversion procedure assumes data have been successfully detided, which is not the case here
- data subjected to simple detrending procedure
  - identify region before start of first wave
  - fit line to this data using least squares procedure
  - extend fitted line through all the data
  - subtract extended line from data, yielding detrended data
- detrended data used as input to inversion procedure

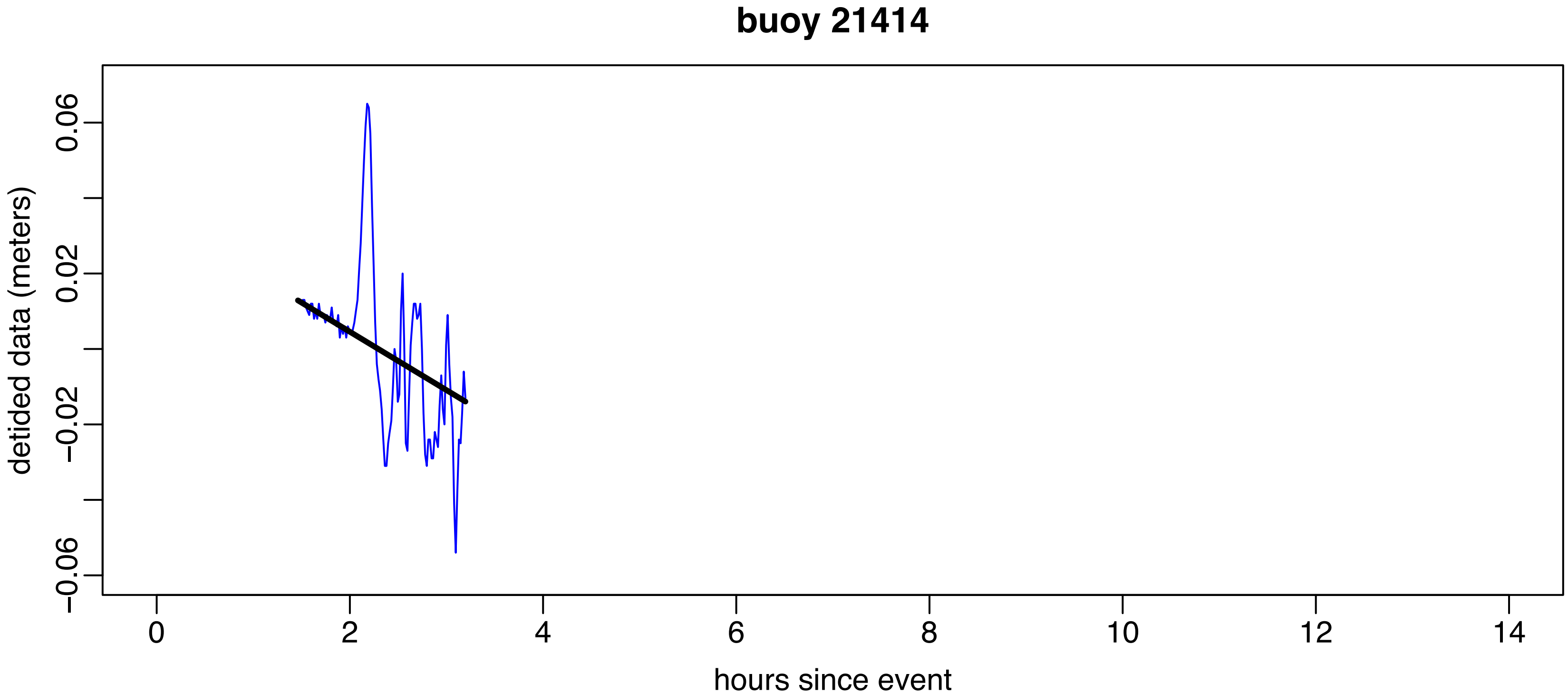
Detided DART Buoy Data for Kuril Island Event



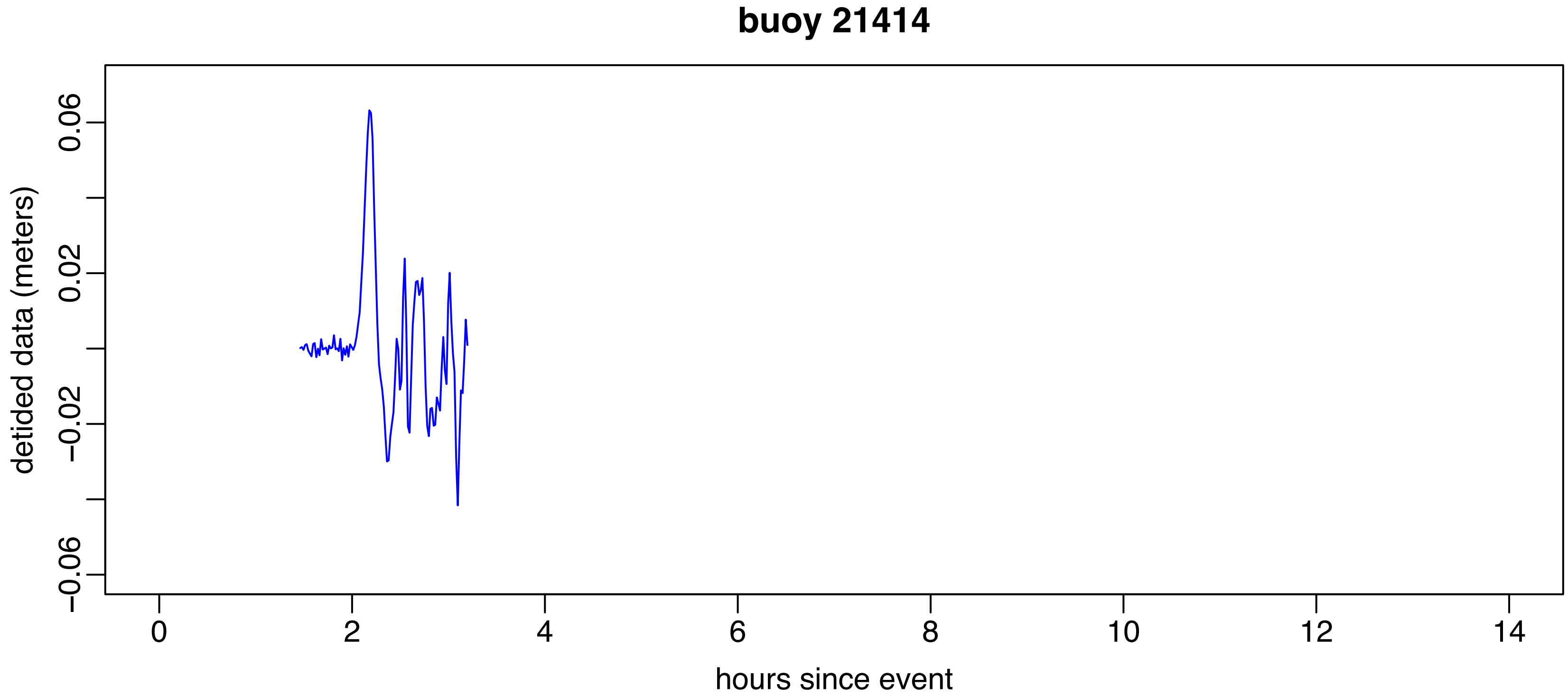
Line Fitted to Bouy Data Before First Wave



Extending Fitted Line Through All Data



Detrended DART Buoy Data for Kuril Island Event



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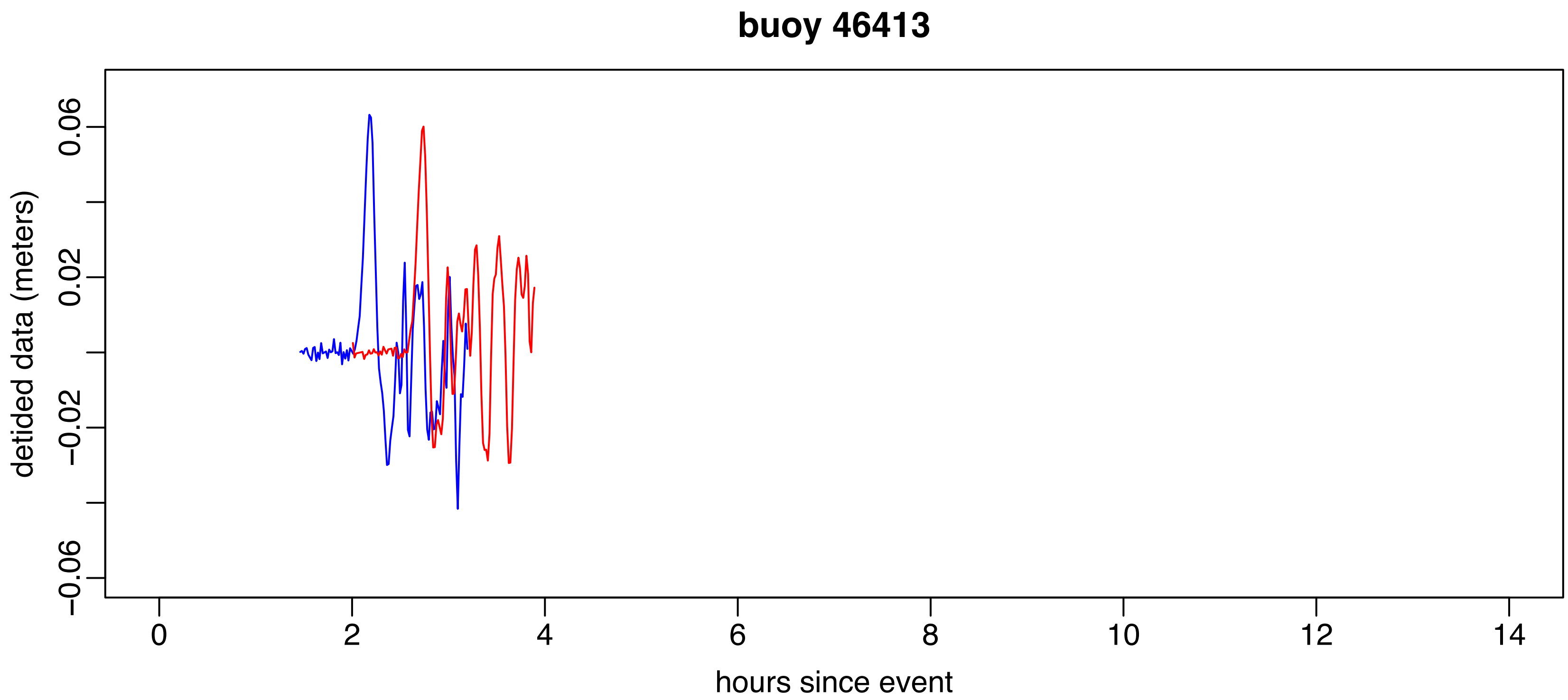

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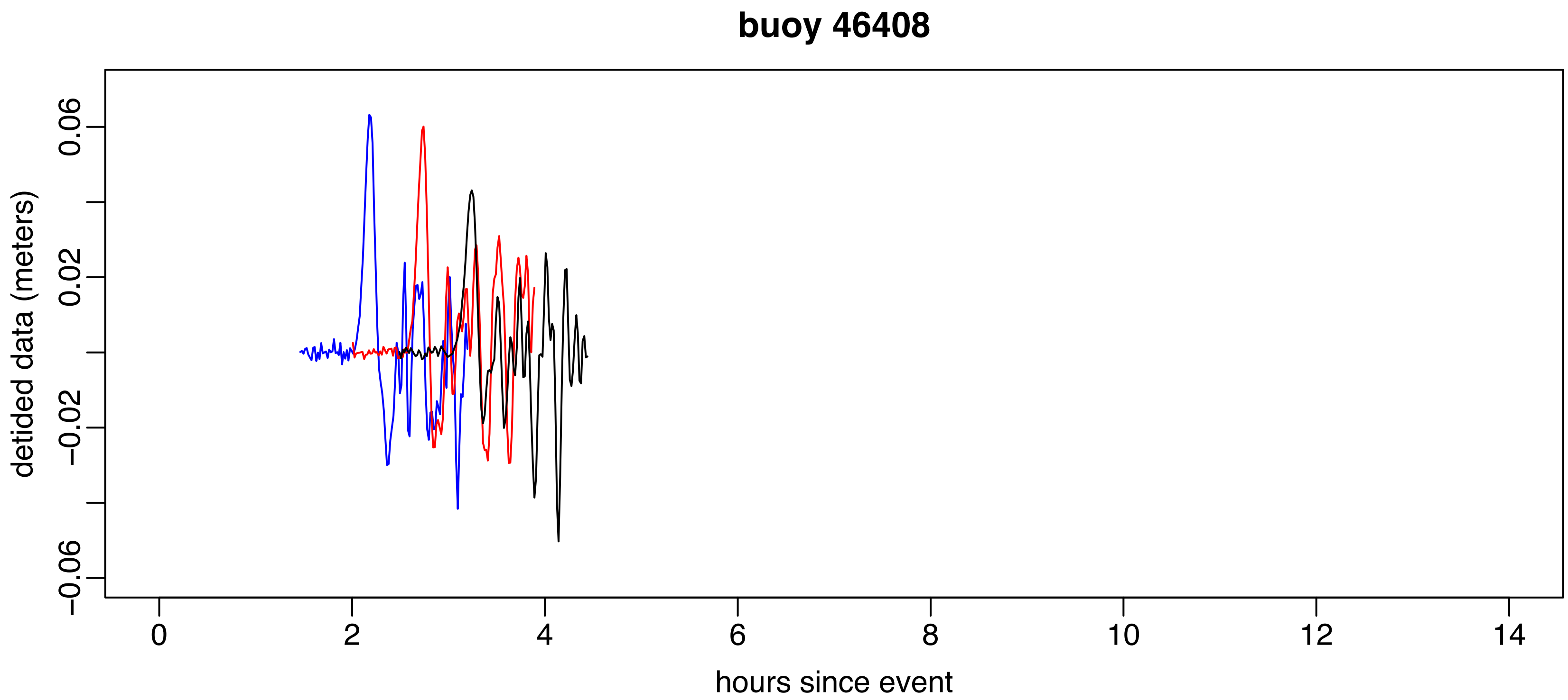

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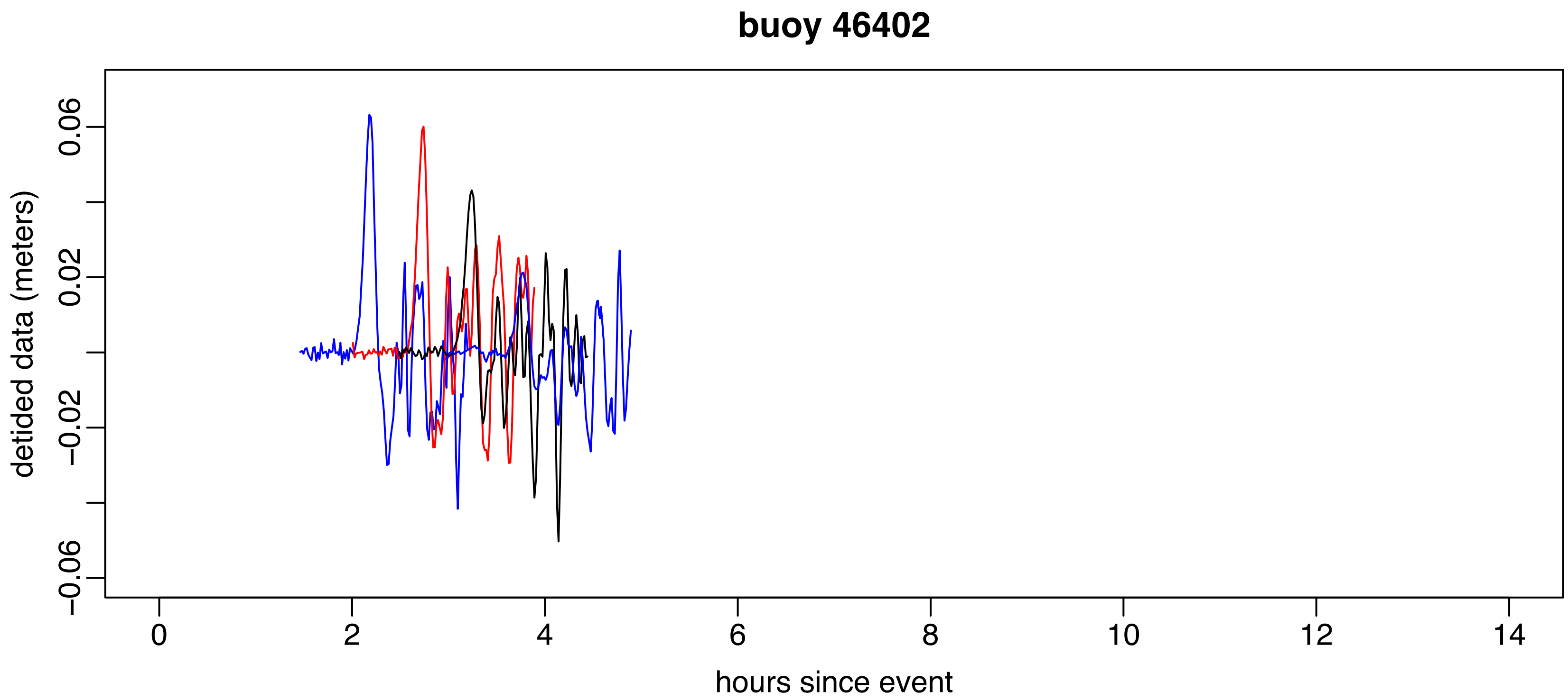

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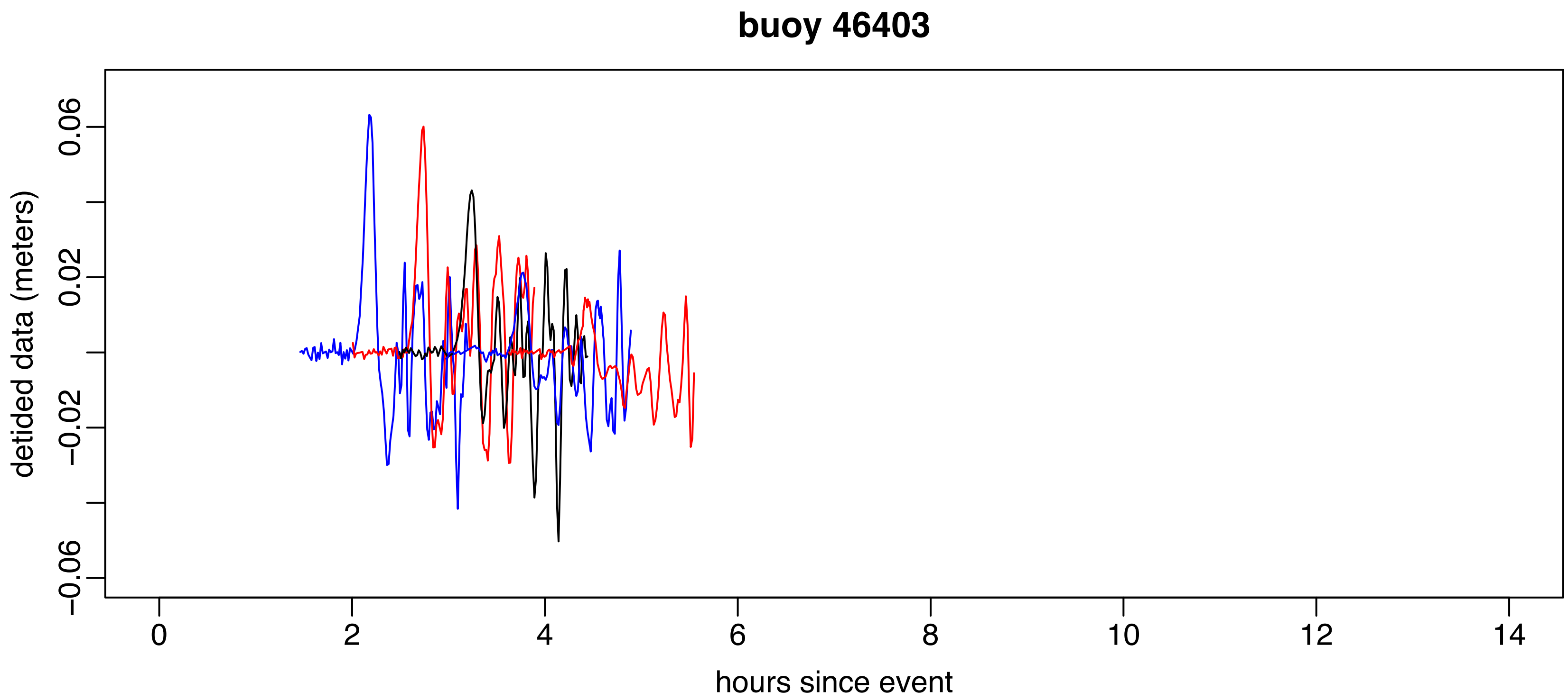

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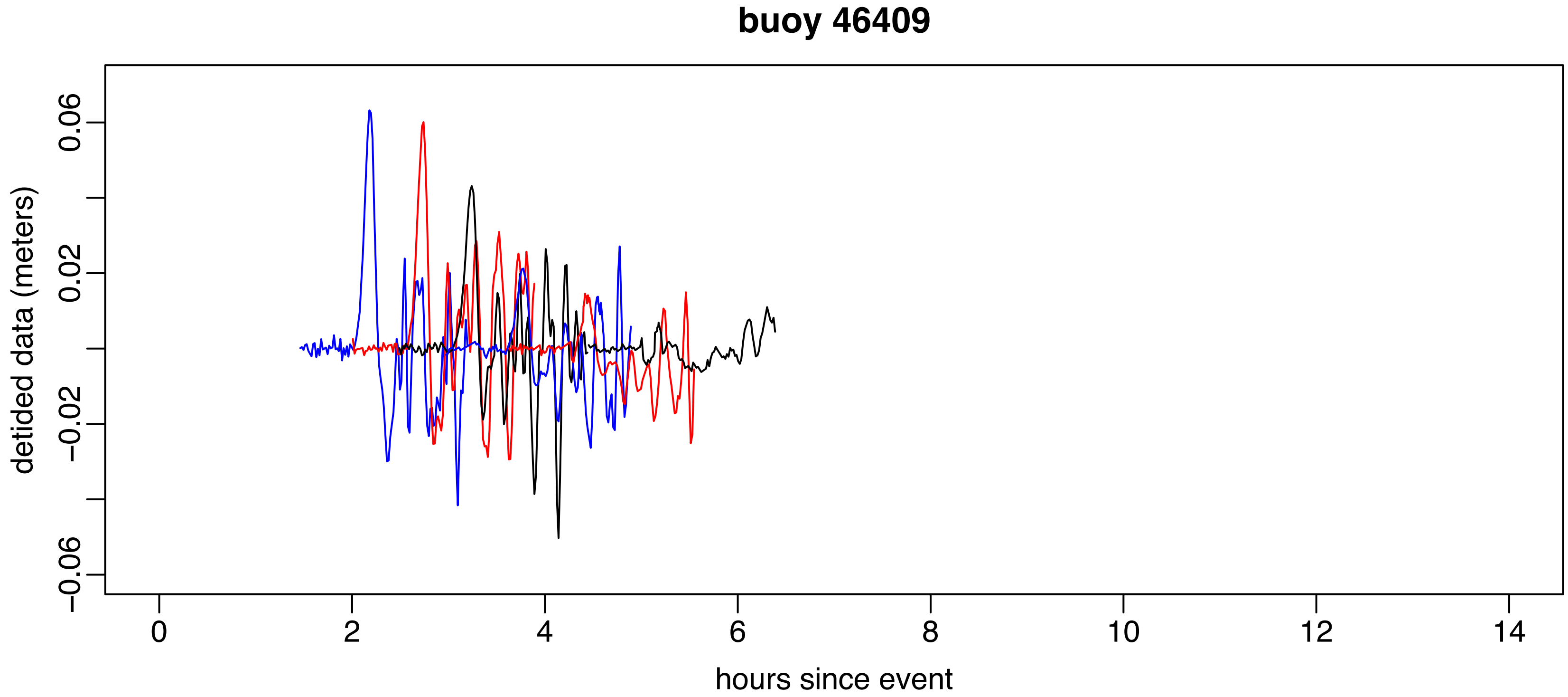

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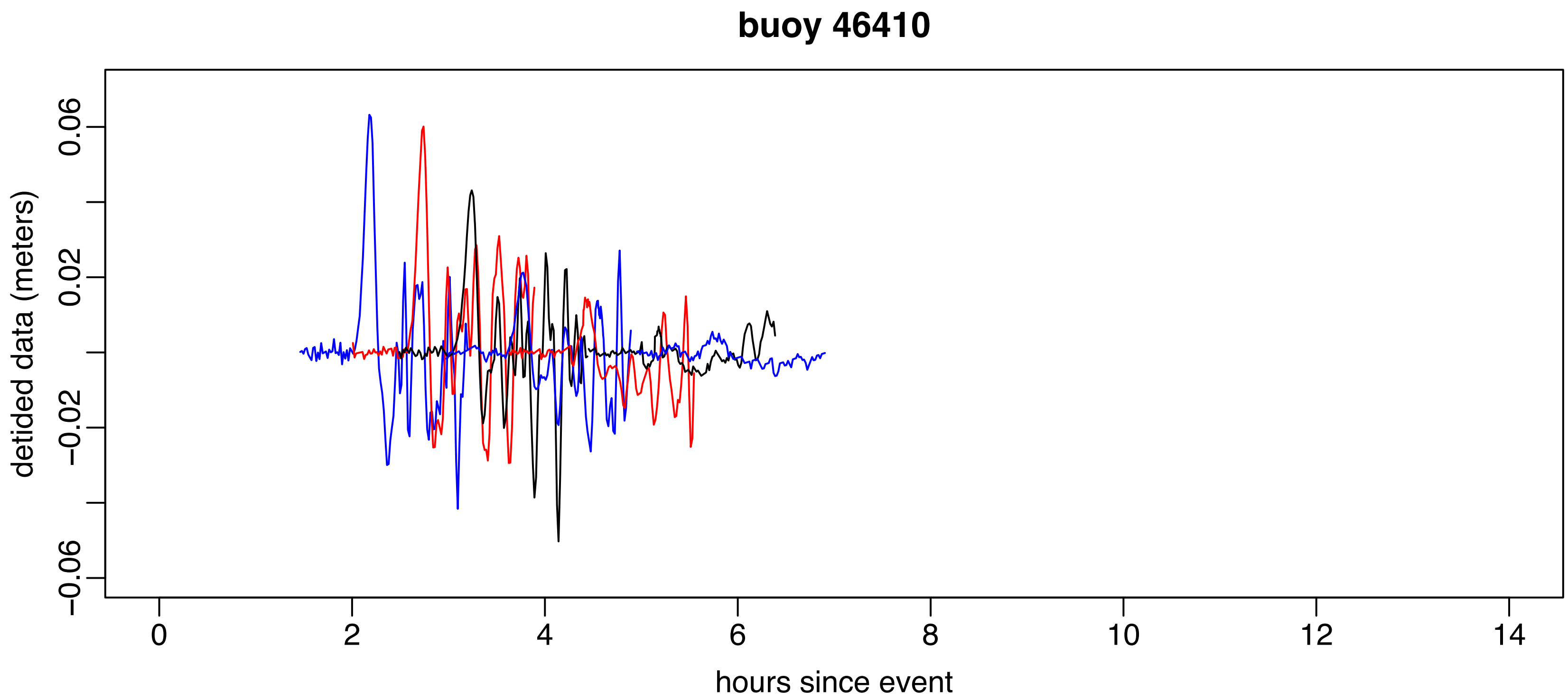

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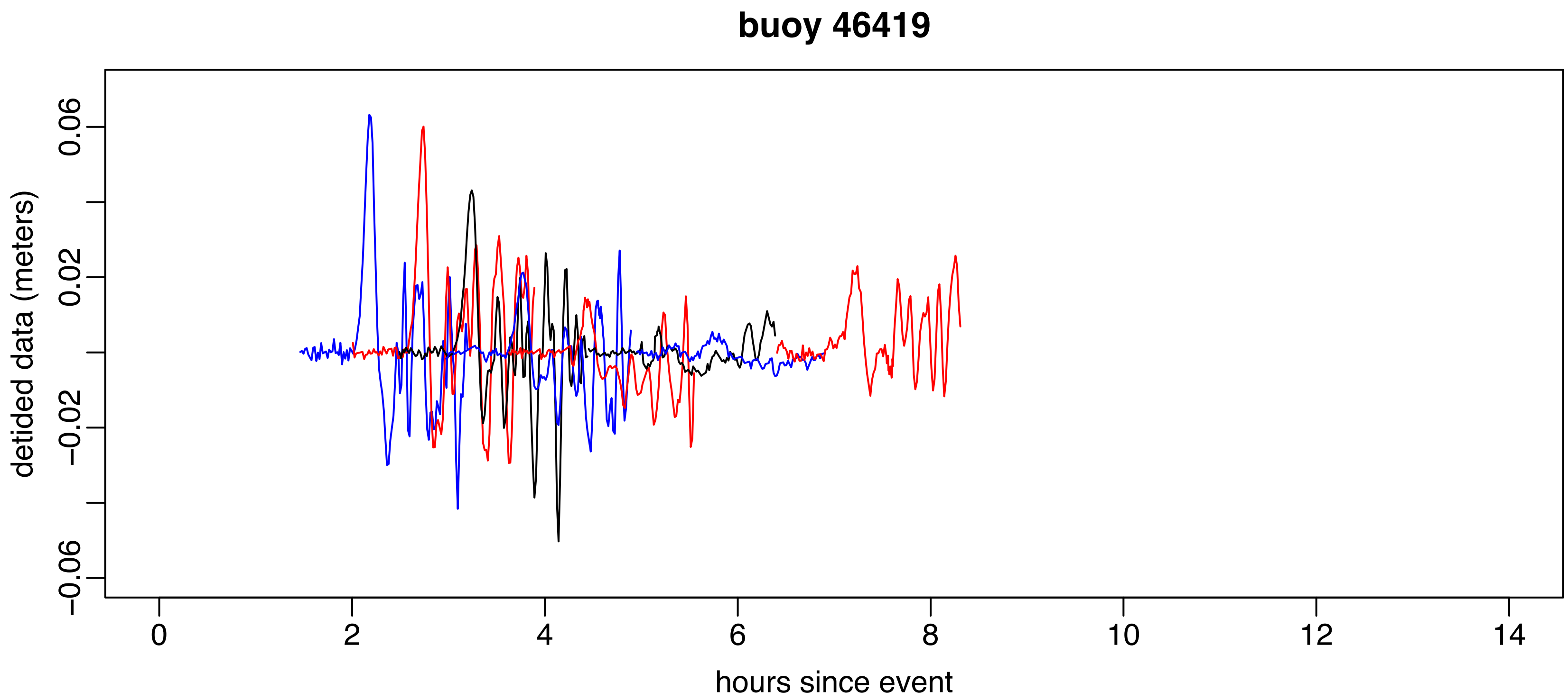

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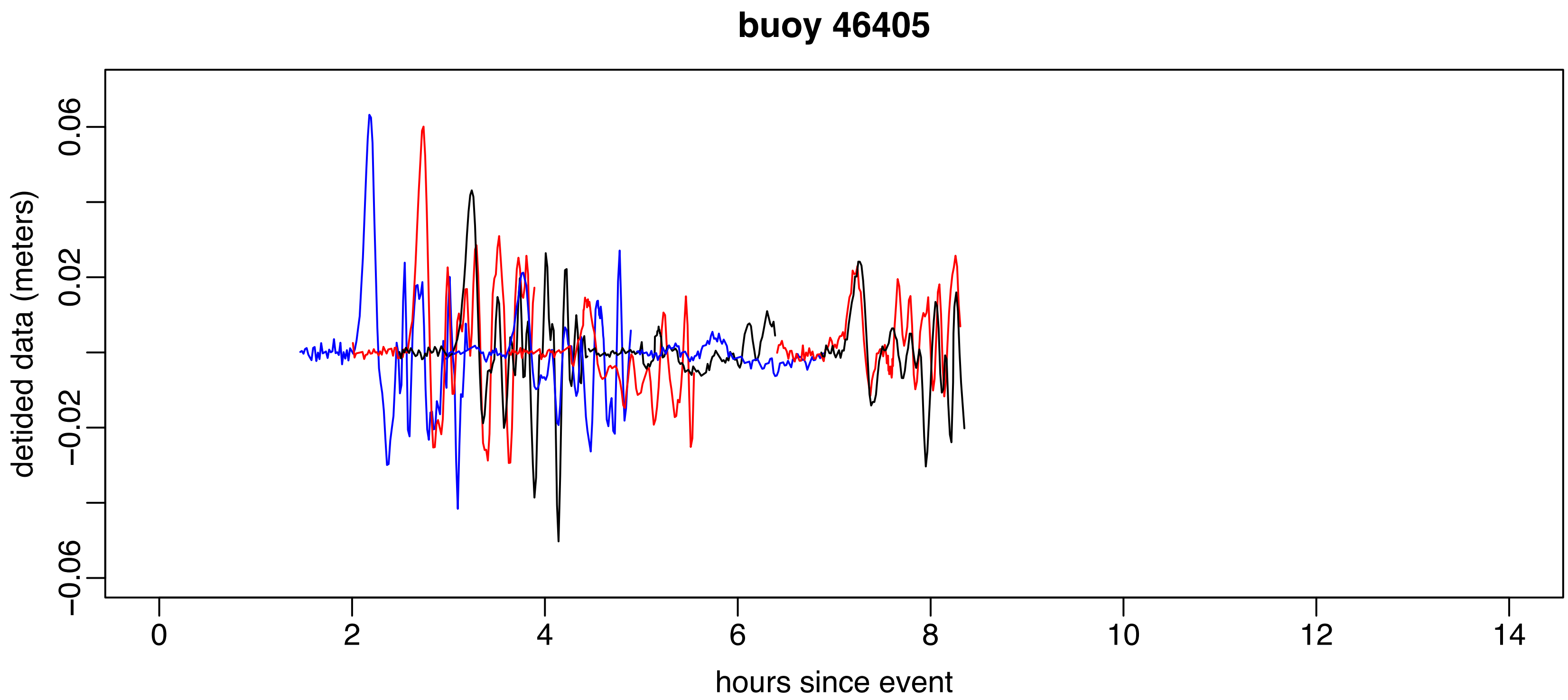

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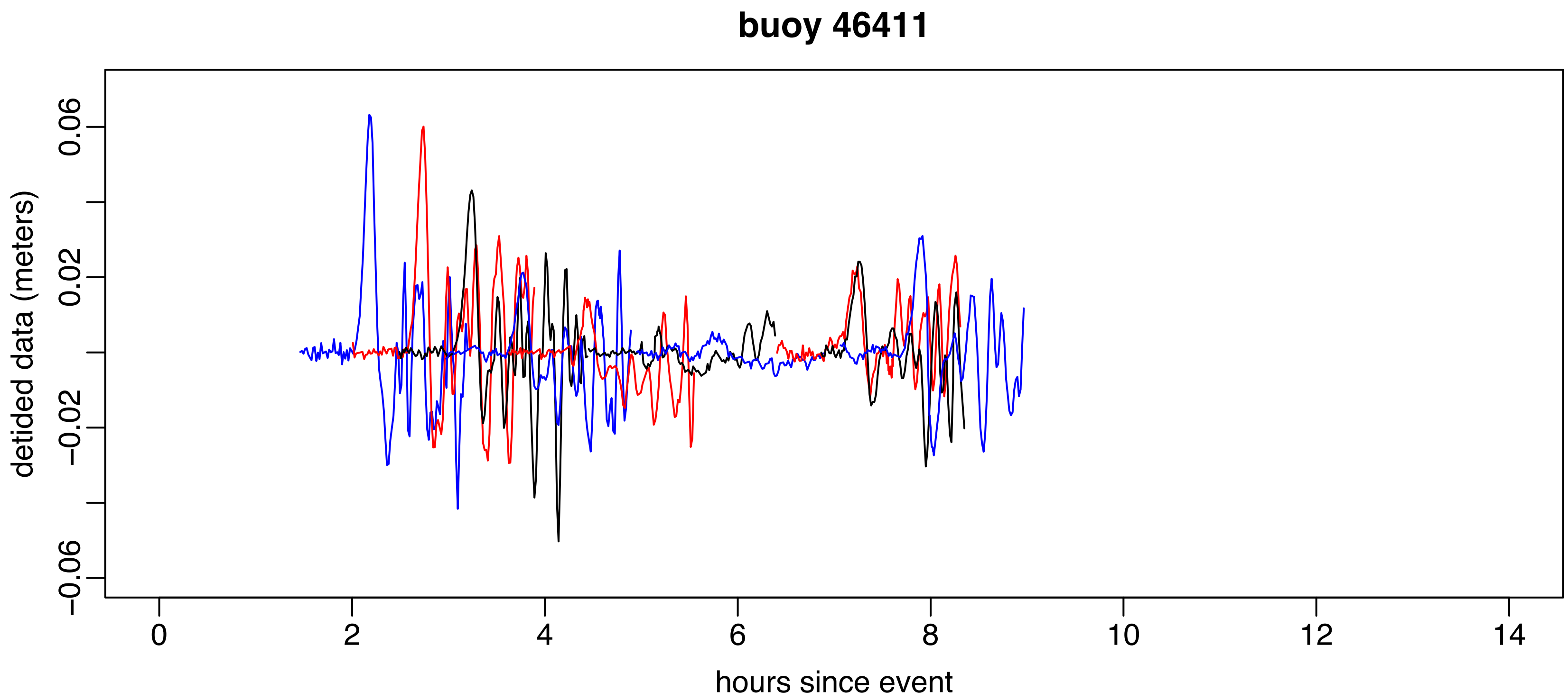

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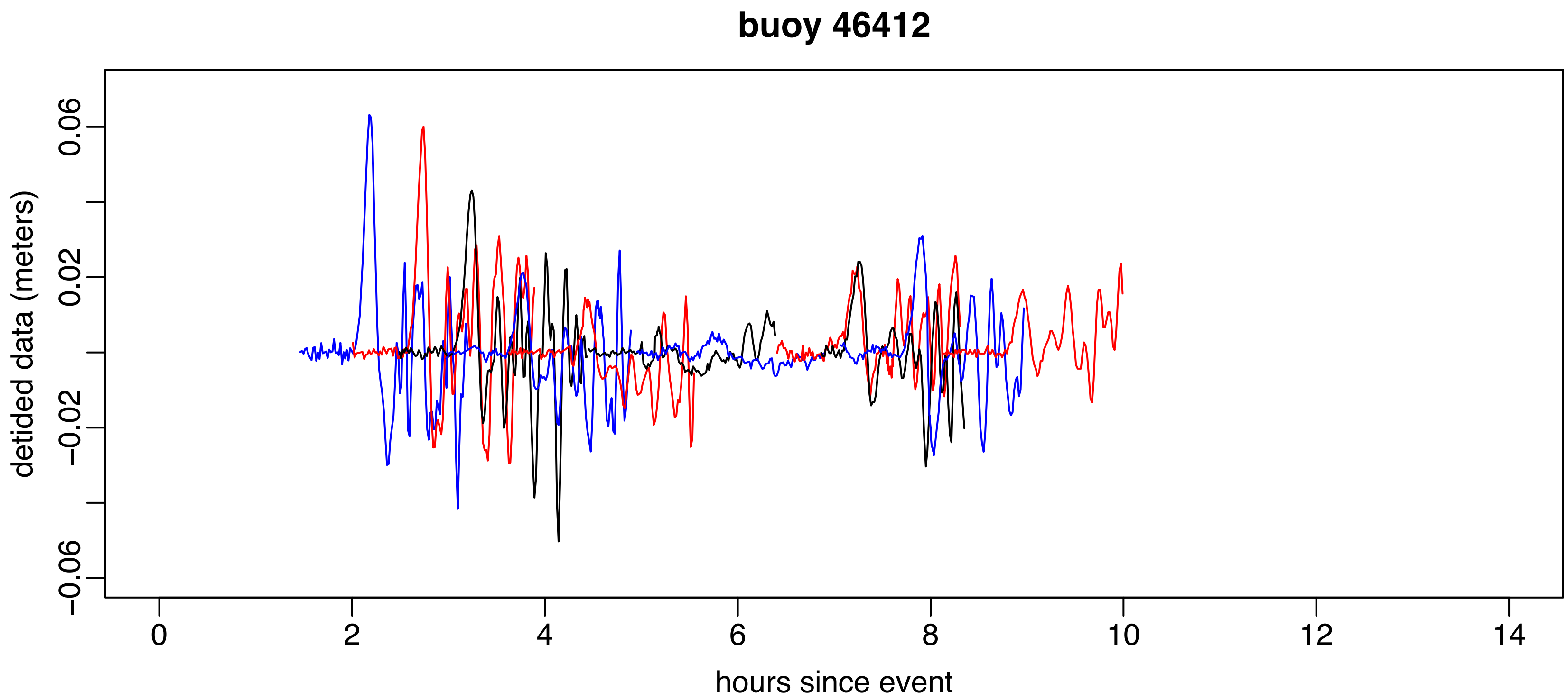

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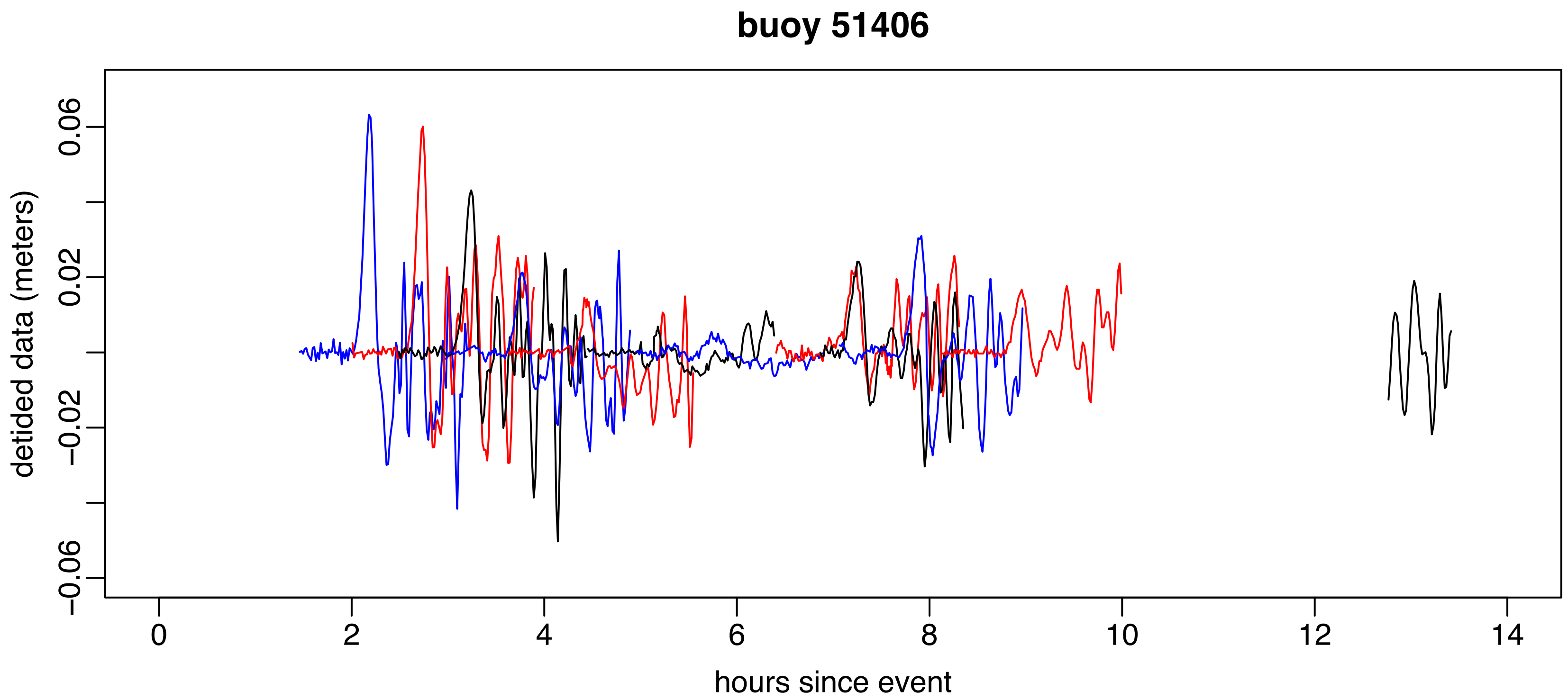
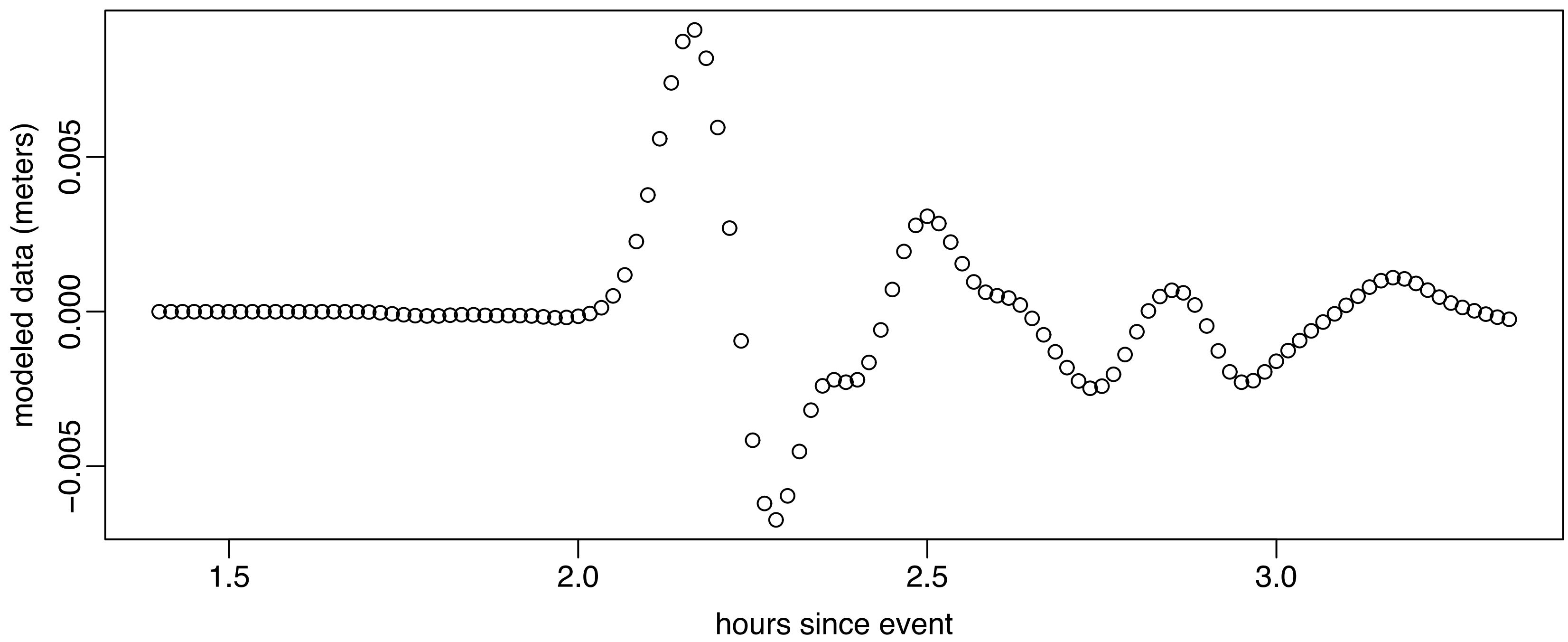


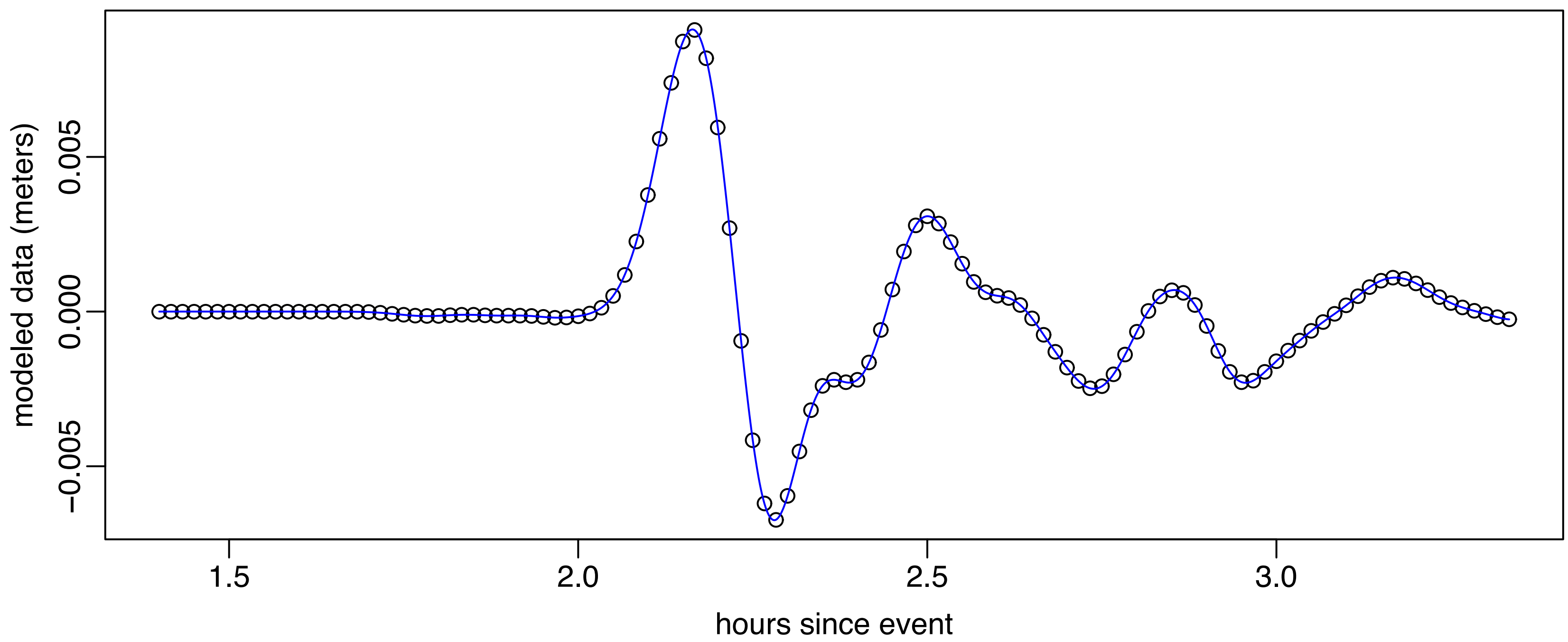
Models for DART Buoys

- consider source locations for Kuril Island event
- for a given source location (e.g., a12), can generate a model of what we would expect to see at each bouy if the earthquake came from just that source
- each model is generated over a grid of discrete times, which might or might not correspond to the times at which DART buoy data are collected
- use cubic spline to interpolate model, so can regard model $g(t)$ and its first derivative $g'(t)$ as being defined for all times t
- as an example, consider model from a12 source for buoy 21414

a12 Source Model for Buoy 21414

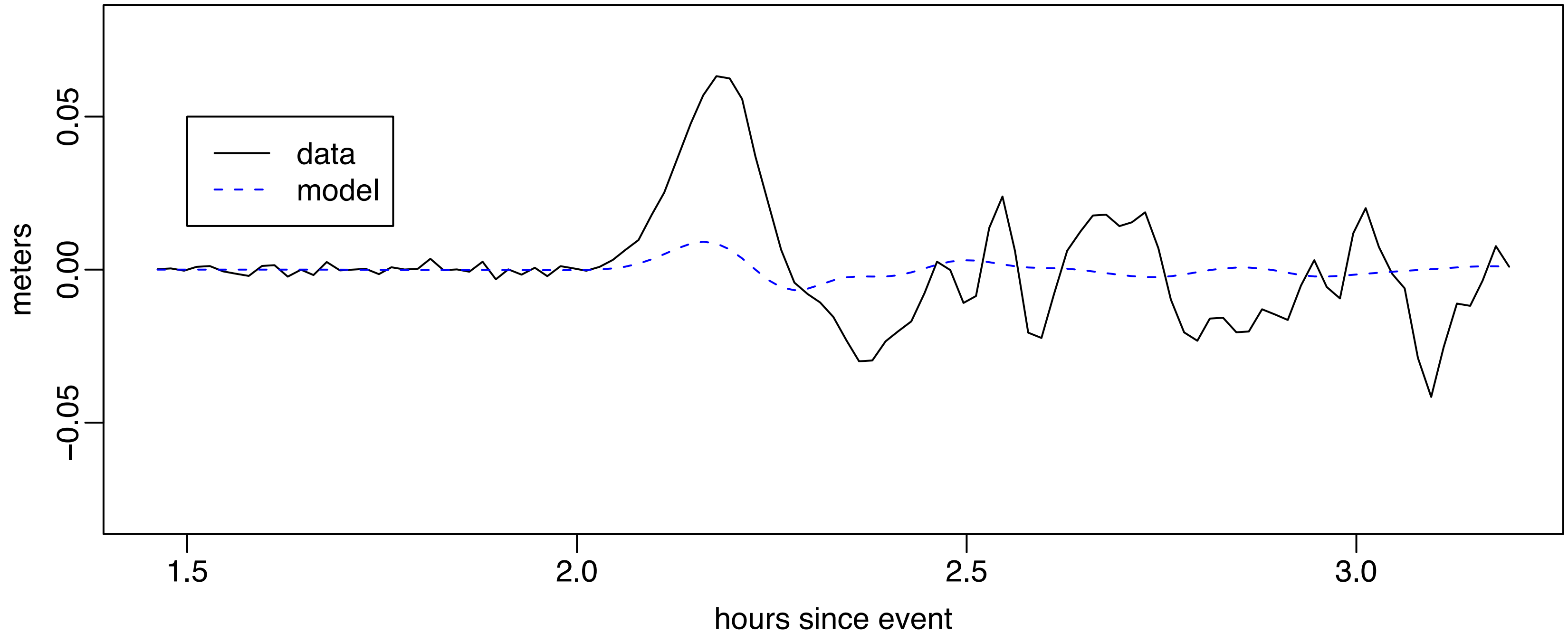


a12 Source Model for Buoy 21414



Fitting Models to DART Buoy Data: I

- model from a12 source for buoy 21414 generated under assumption of a unit magnitude for the earthquake



- poor match, so multiple model $g(t)$ by A to get a better fit, where A is interpreted as earthquake magnitude (the 'slip')

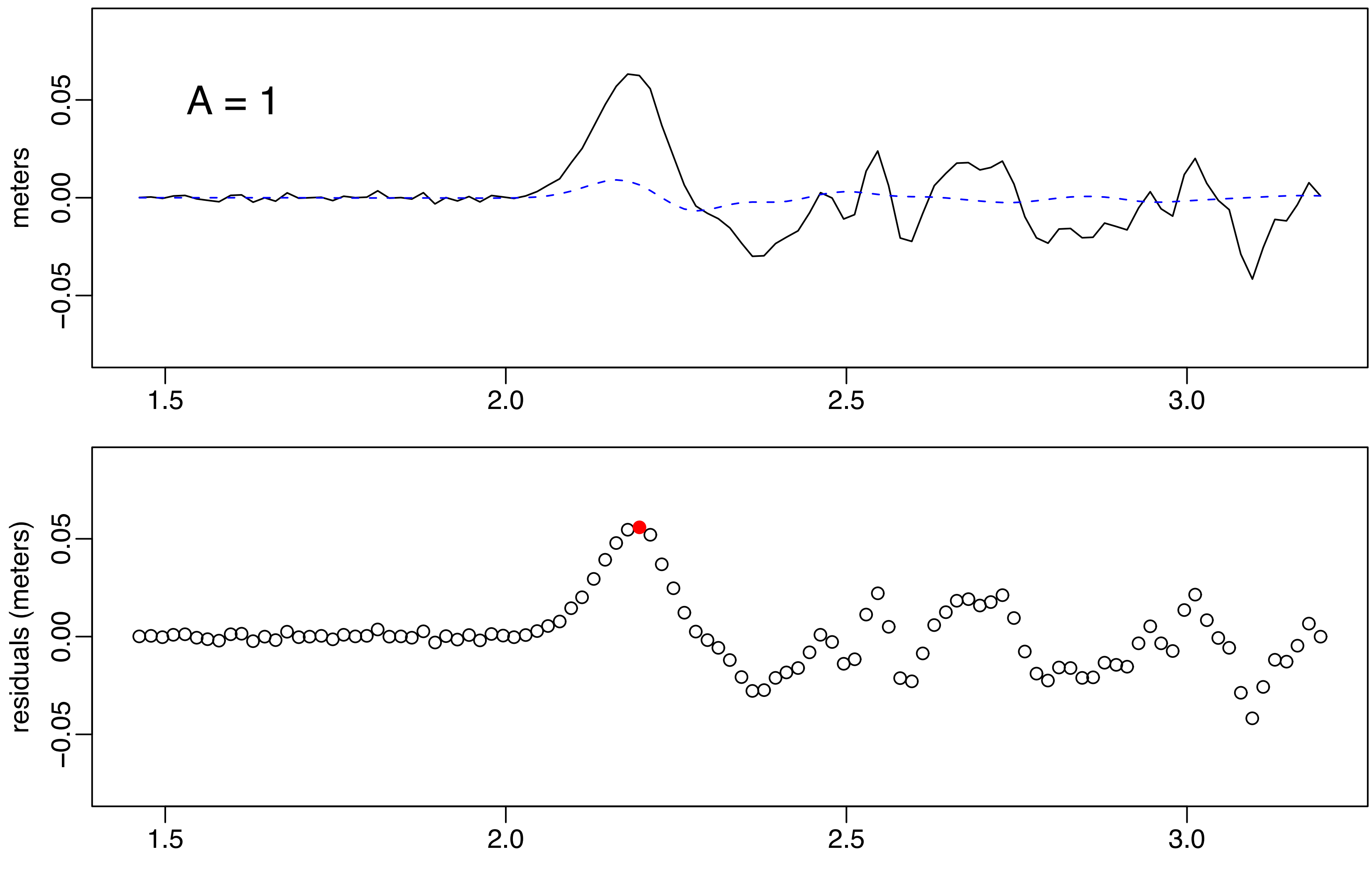
Fitting Models to DART Buoy Data: II

- let x_t represent the DART buoy data at time t
- we entertain the model

$$x_t = A \cdot g(t) + e_t,$$

  where e_t is a residual term (mismatch between data and model)
- if we let A range over a grid of values, then we can compute corresponding residuals $e_t = x_t - A \cdot g(t)$ for any given A
- as an example, let's compute residuals from fit of a12 source to buoy 21414 data for $A = 1.0, 1.2, 1.4, \ldots, 10.0$, marking residual with largest absolute value with a red dot (for discussion later on)

Matching a12 Model to 21414 Data by Varying Slip



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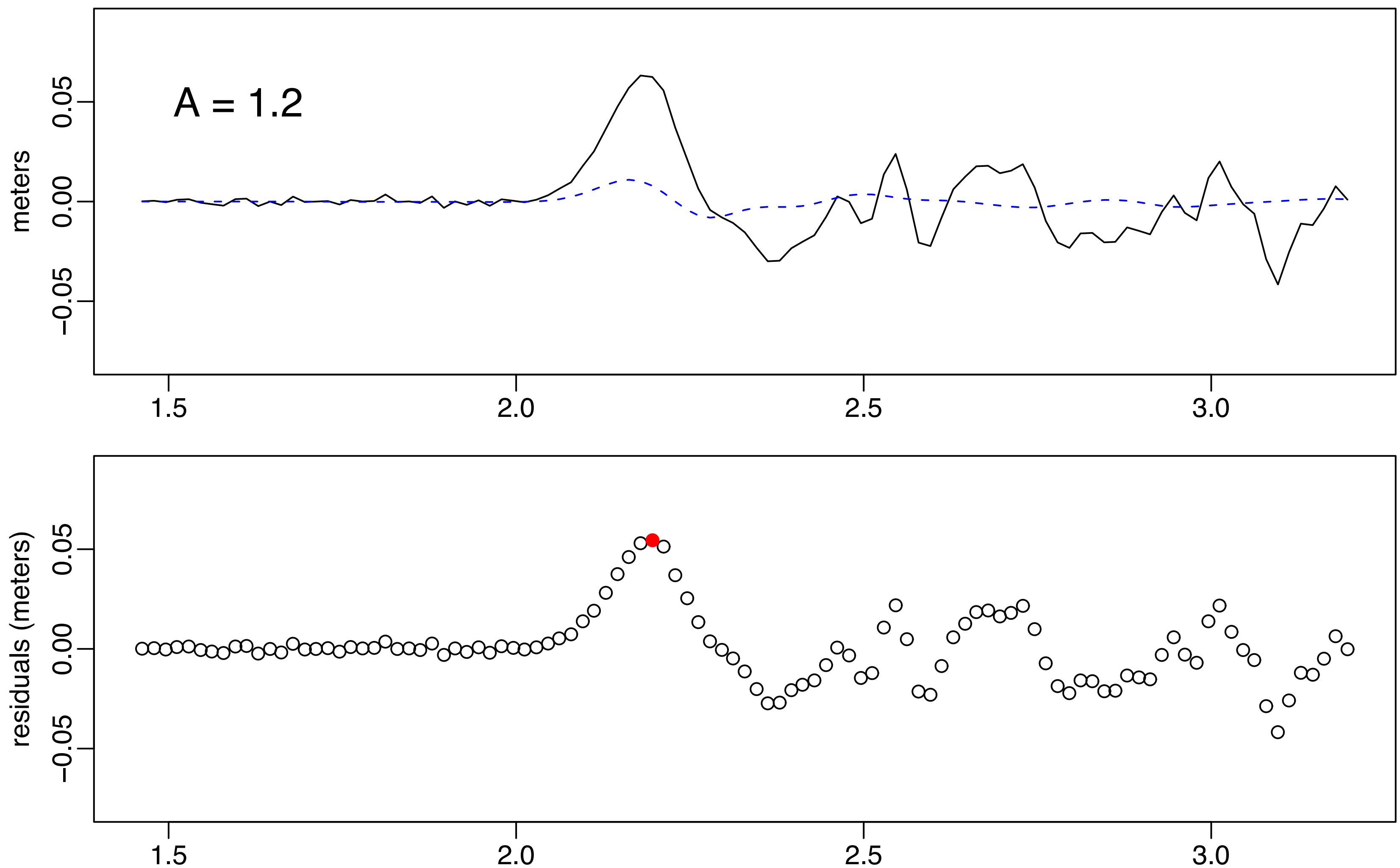

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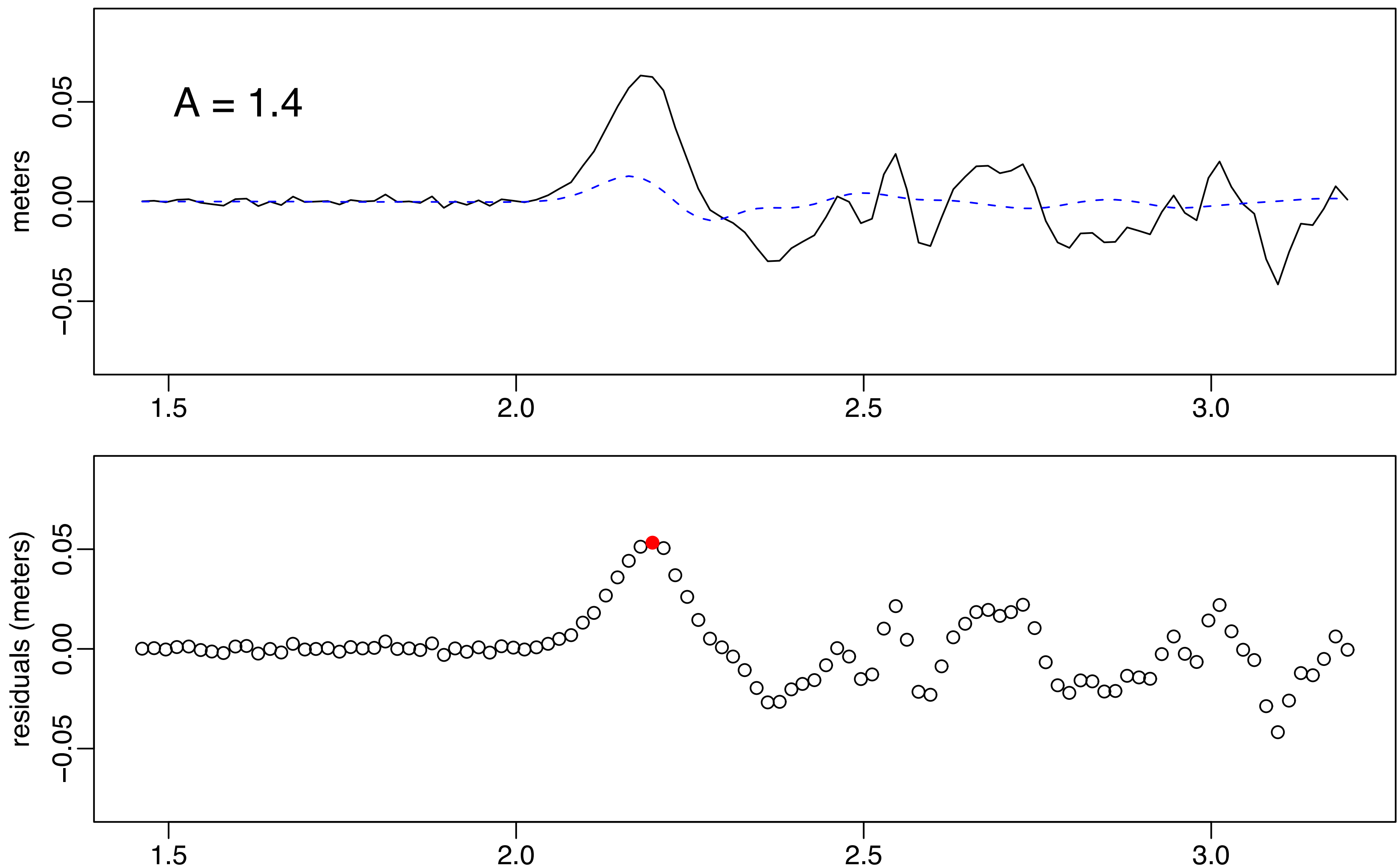

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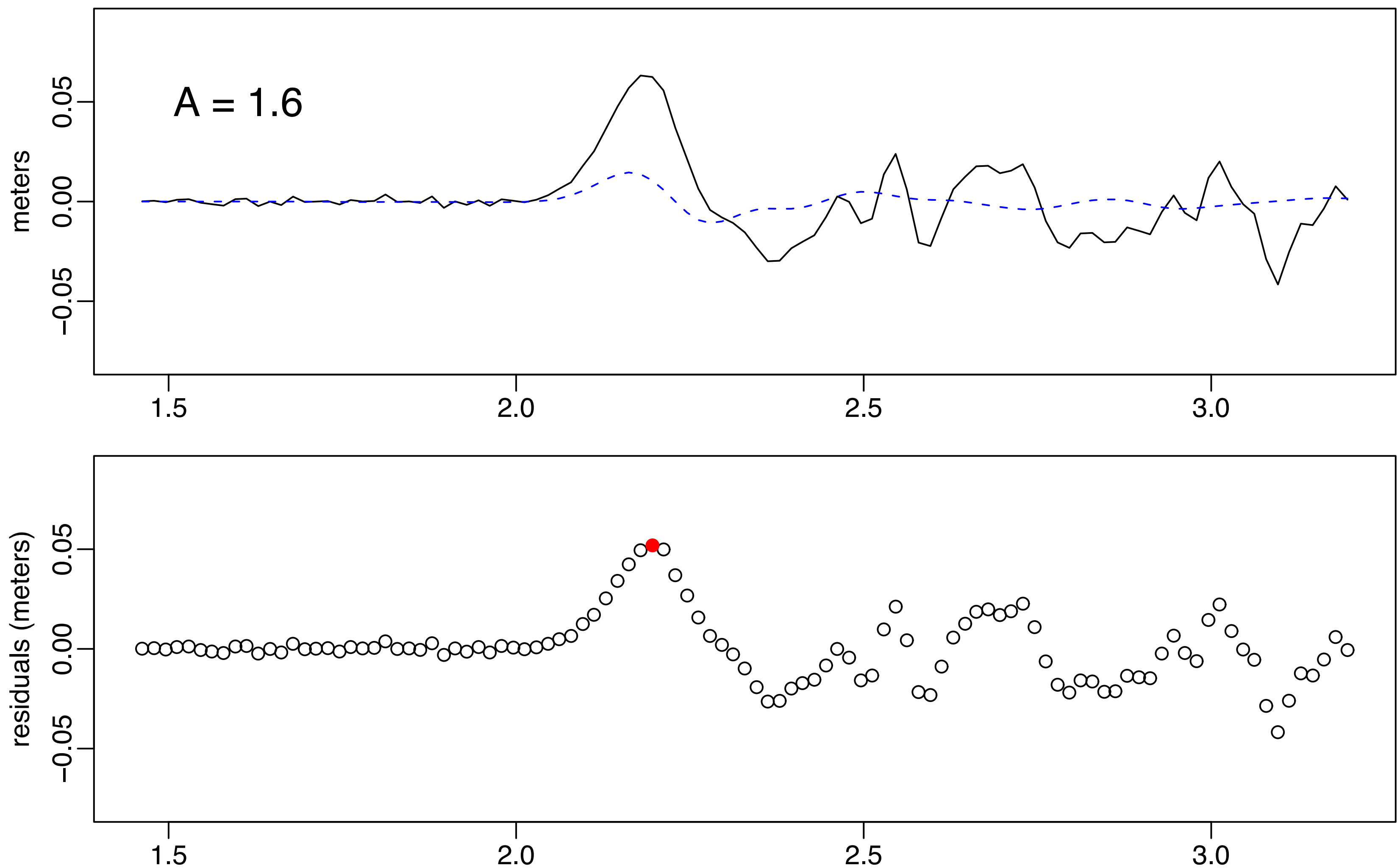

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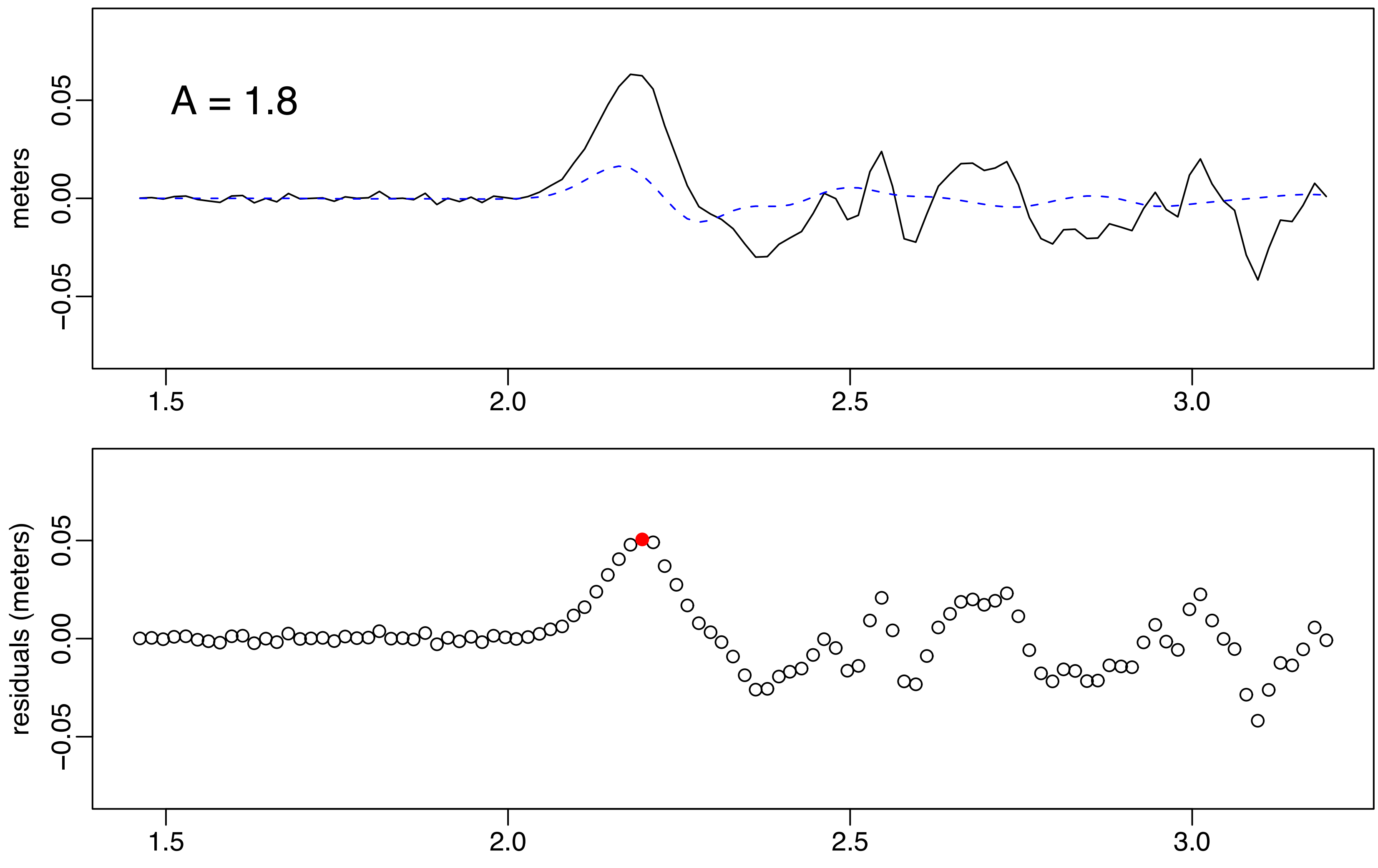

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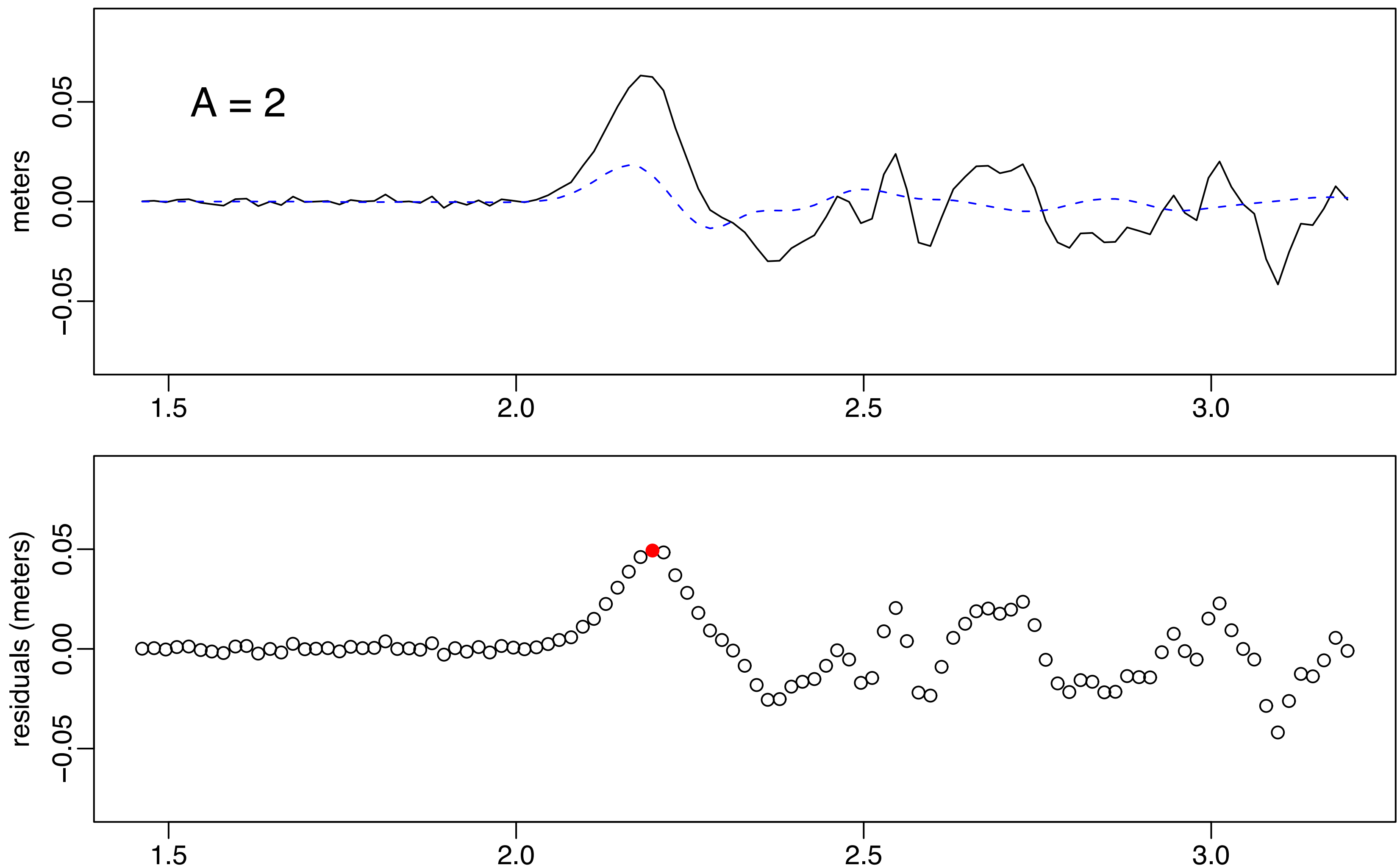

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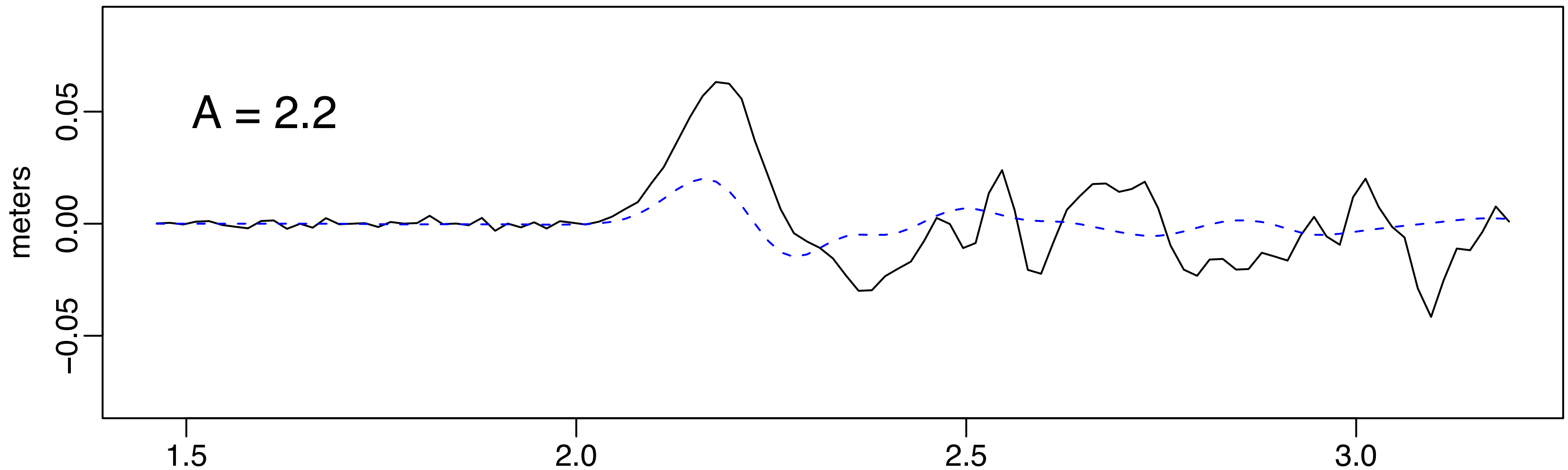

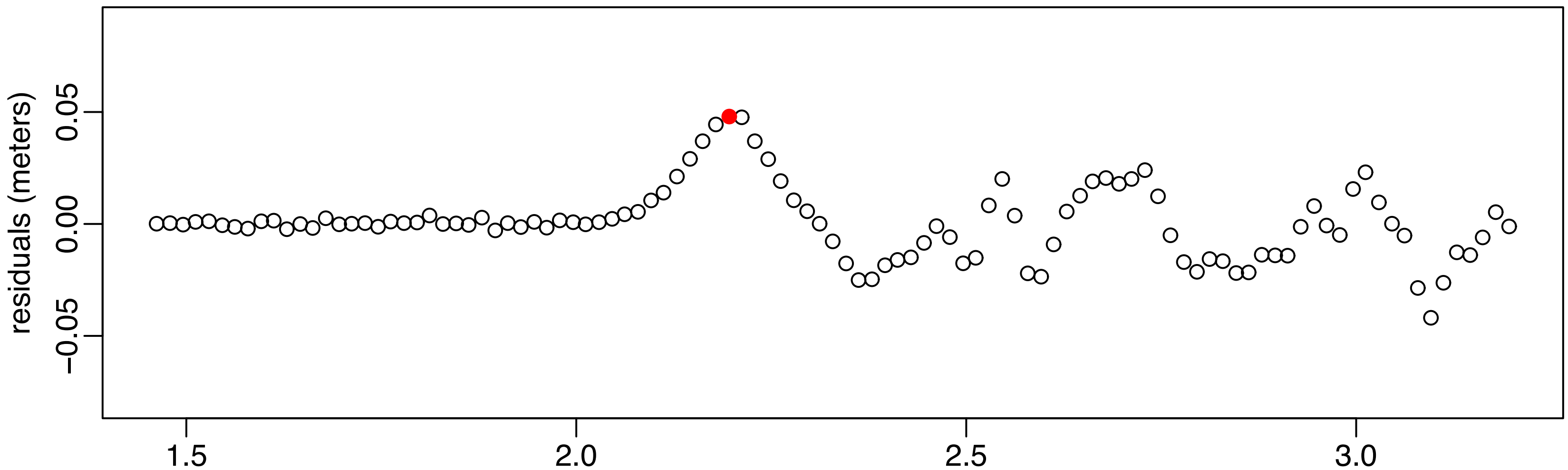

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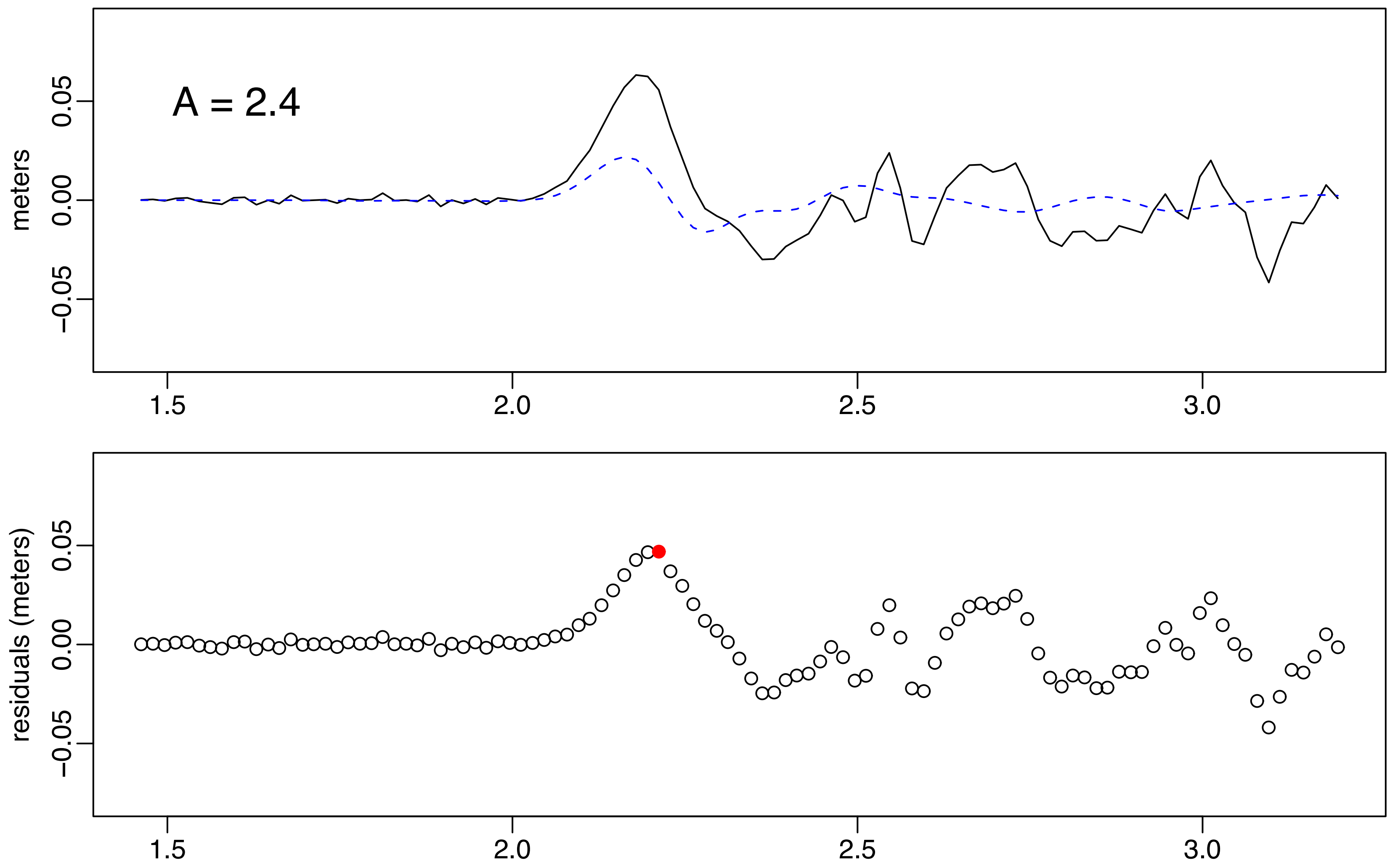

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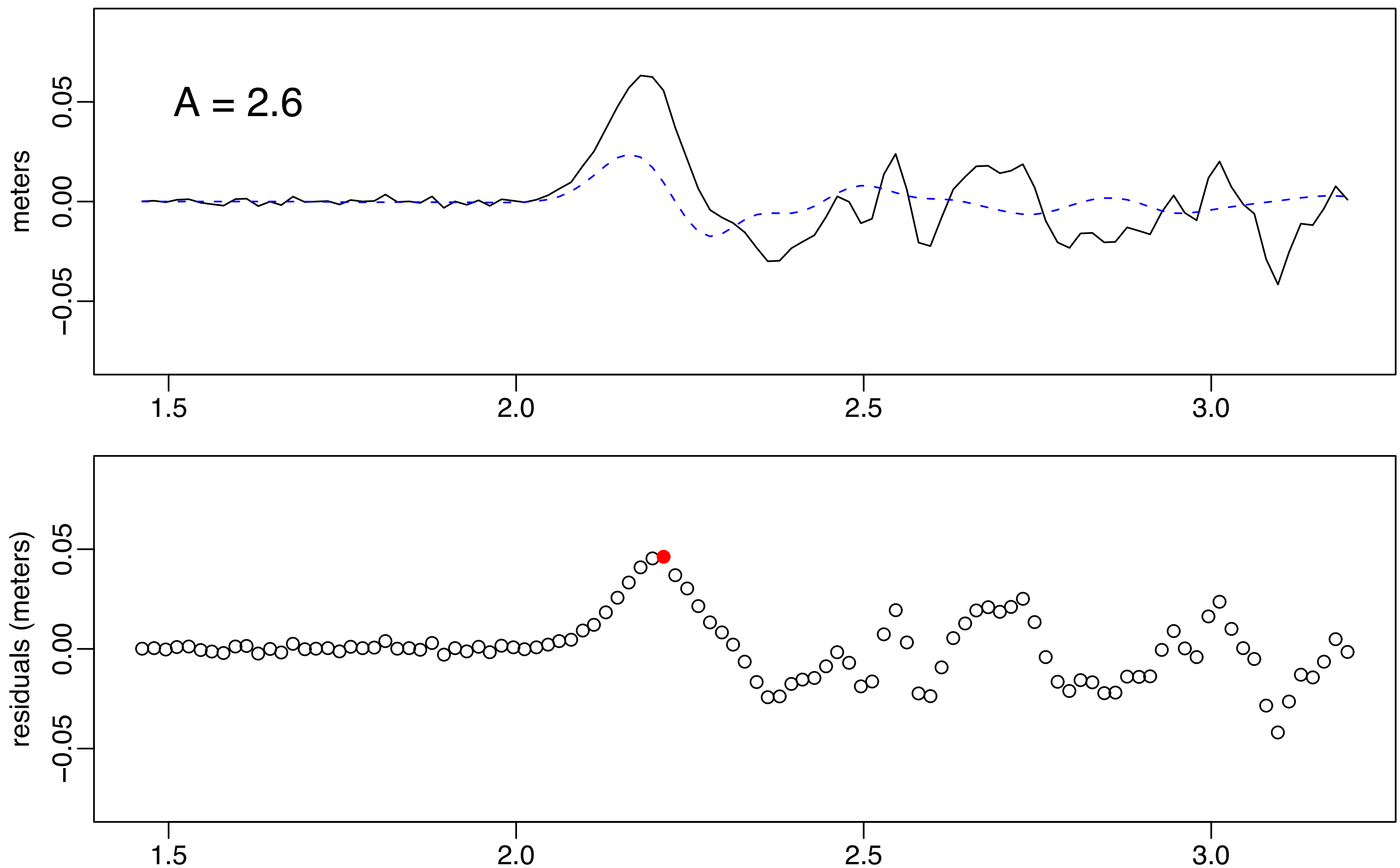

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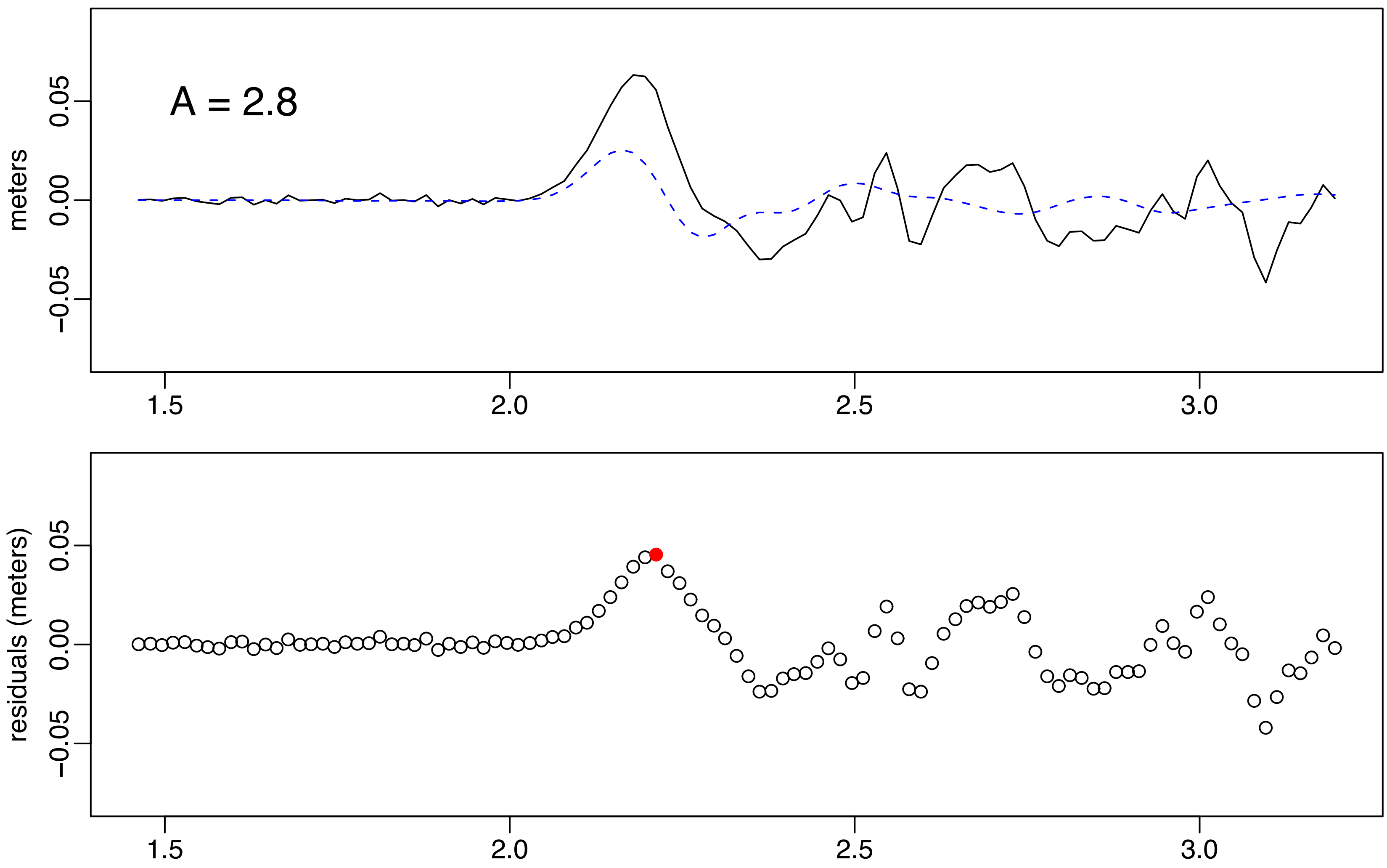

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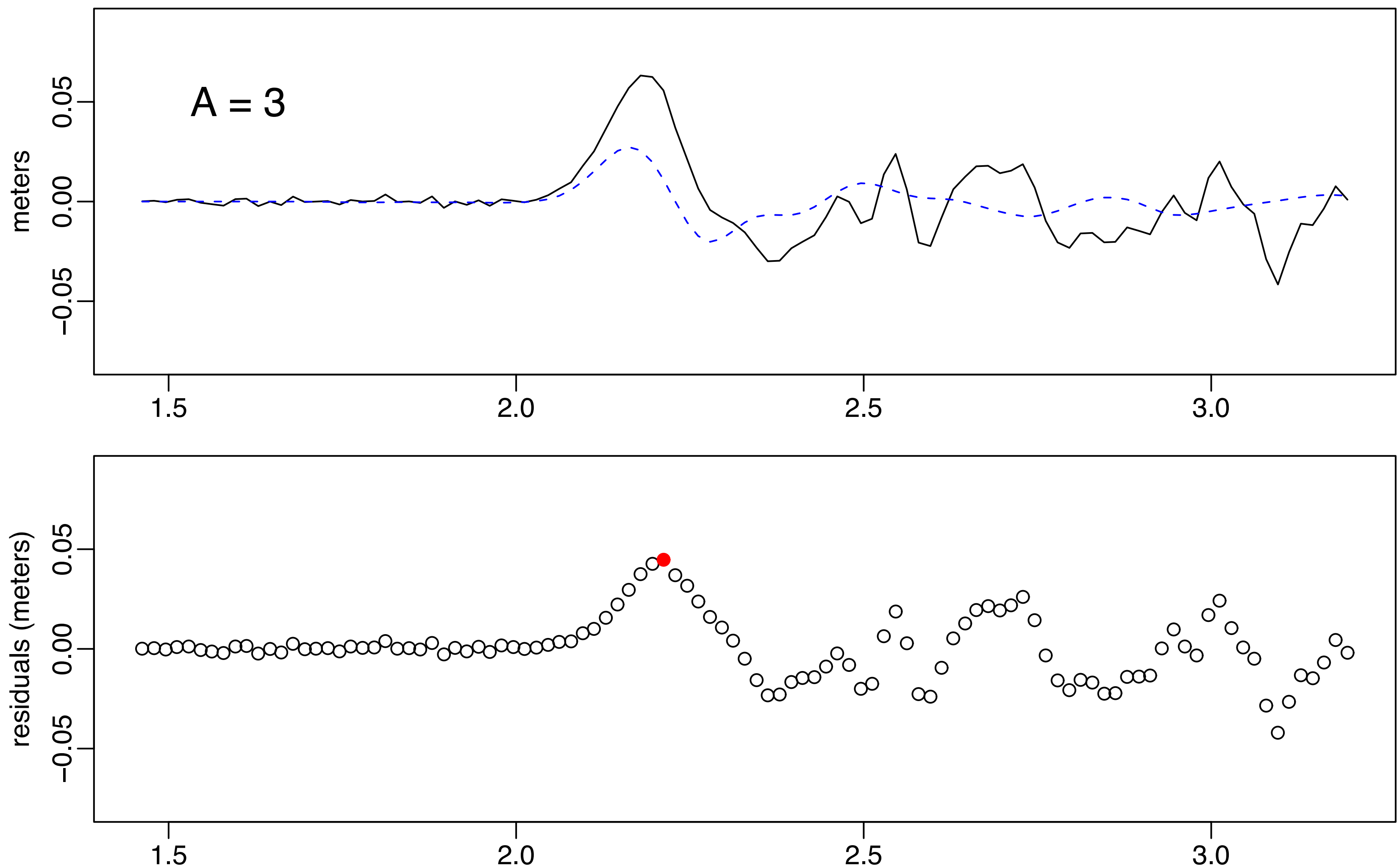

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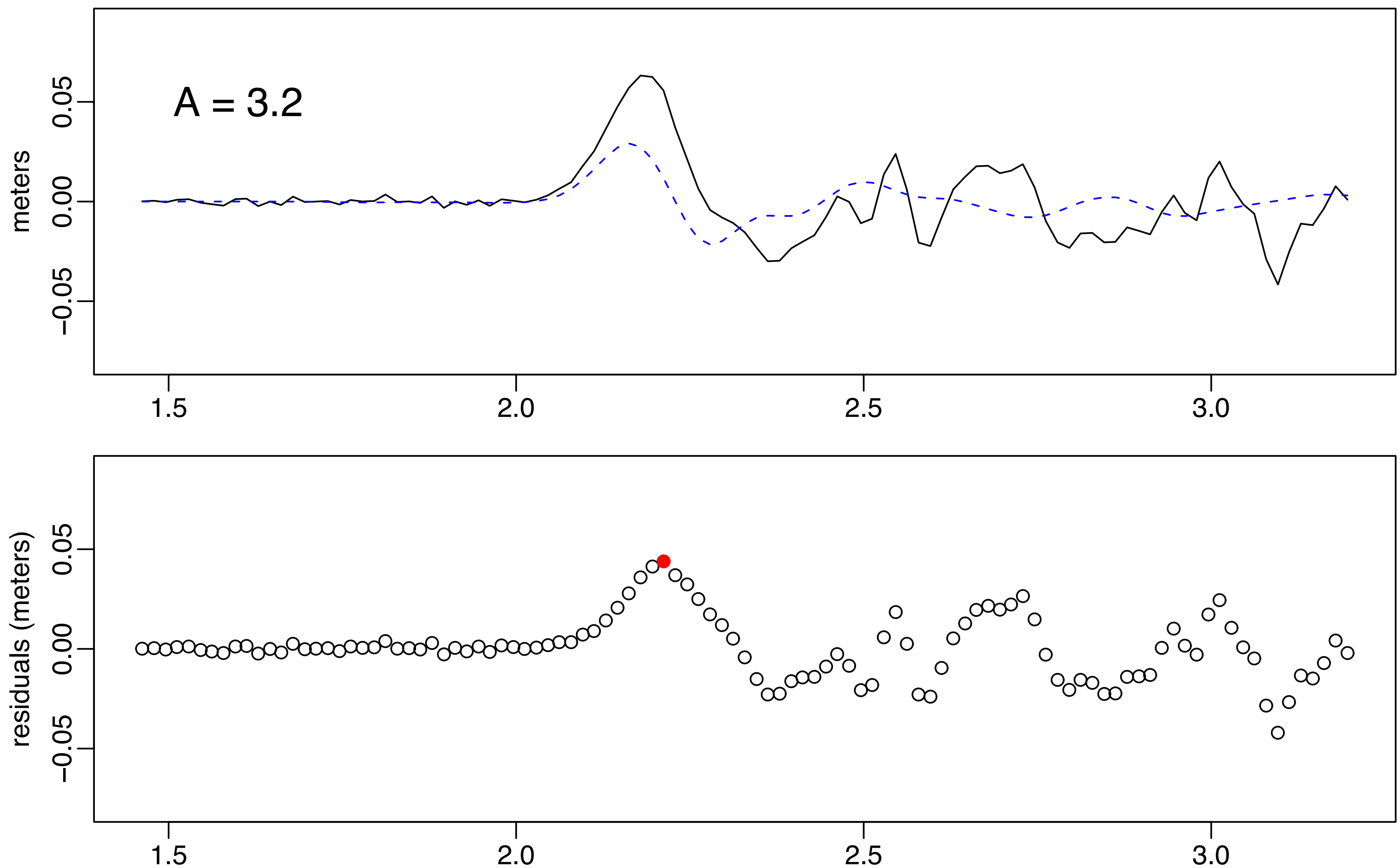

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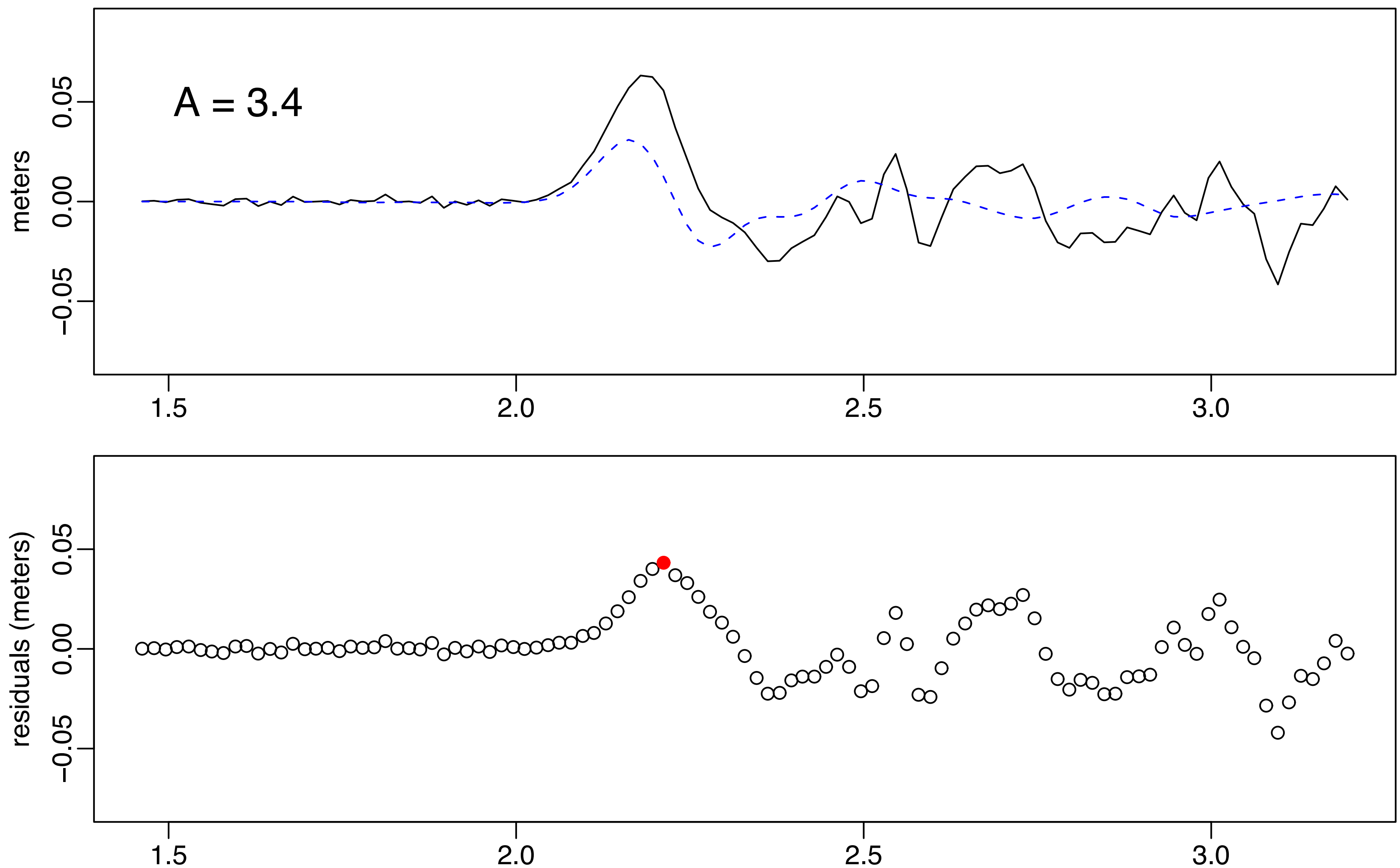

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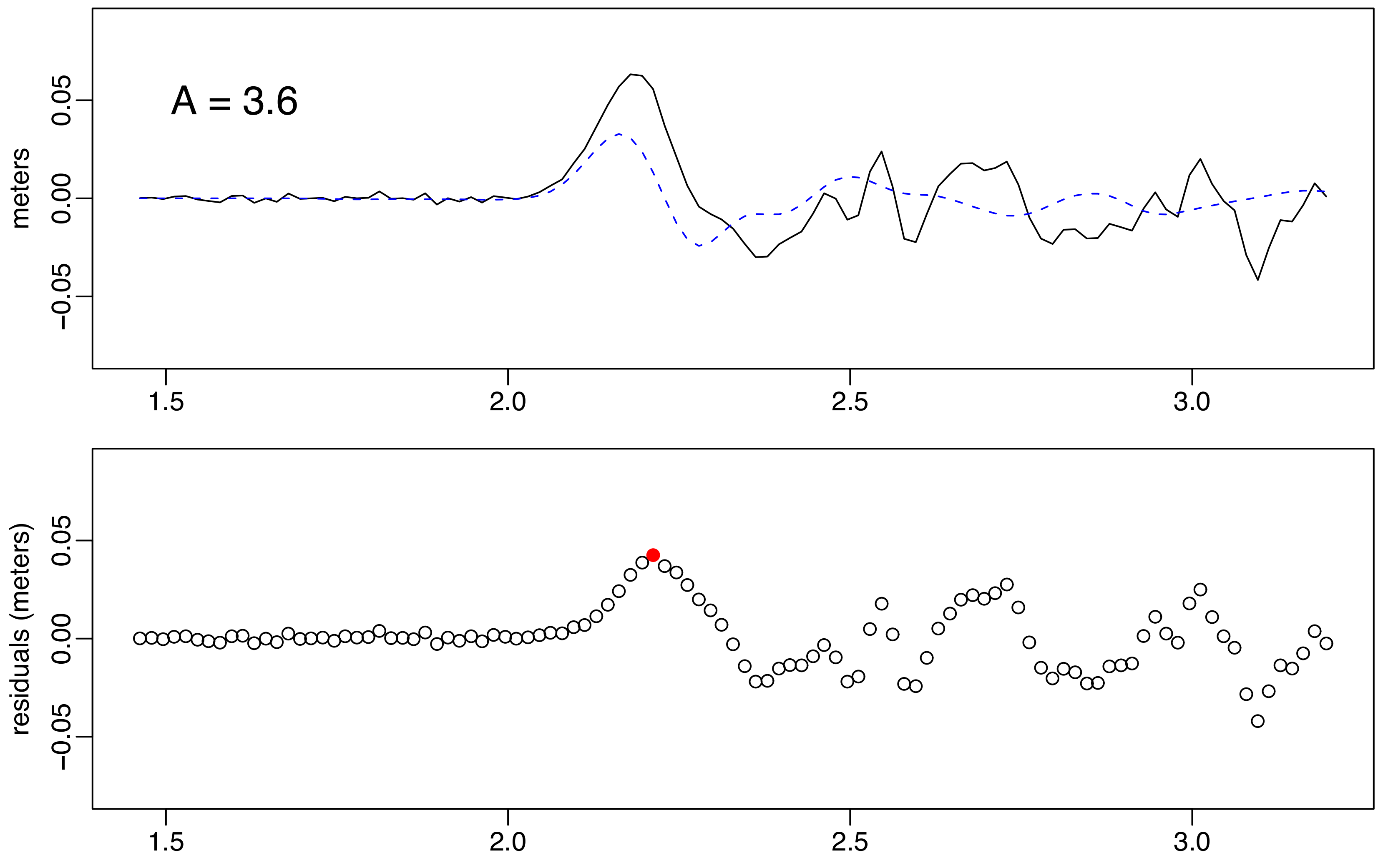

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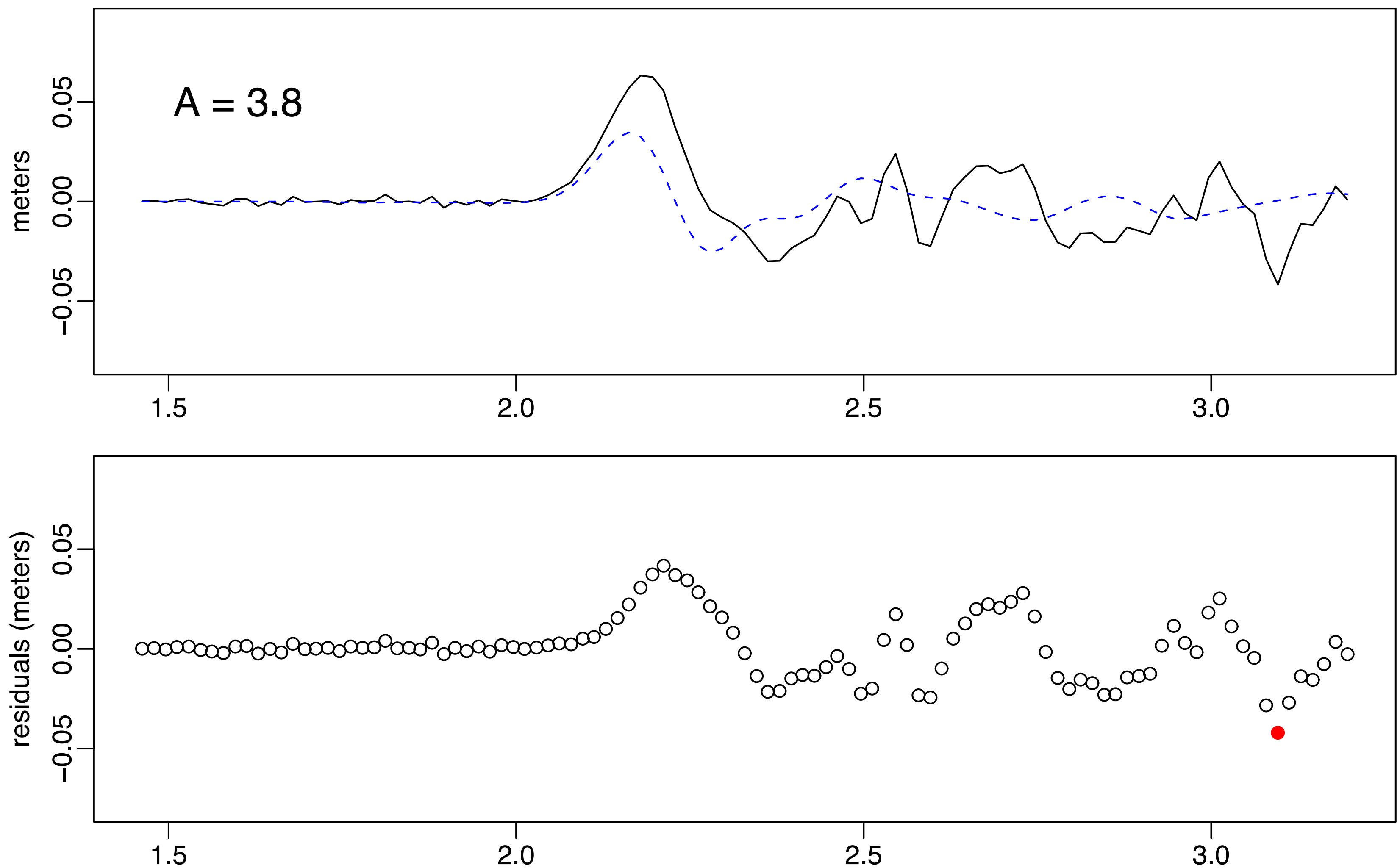

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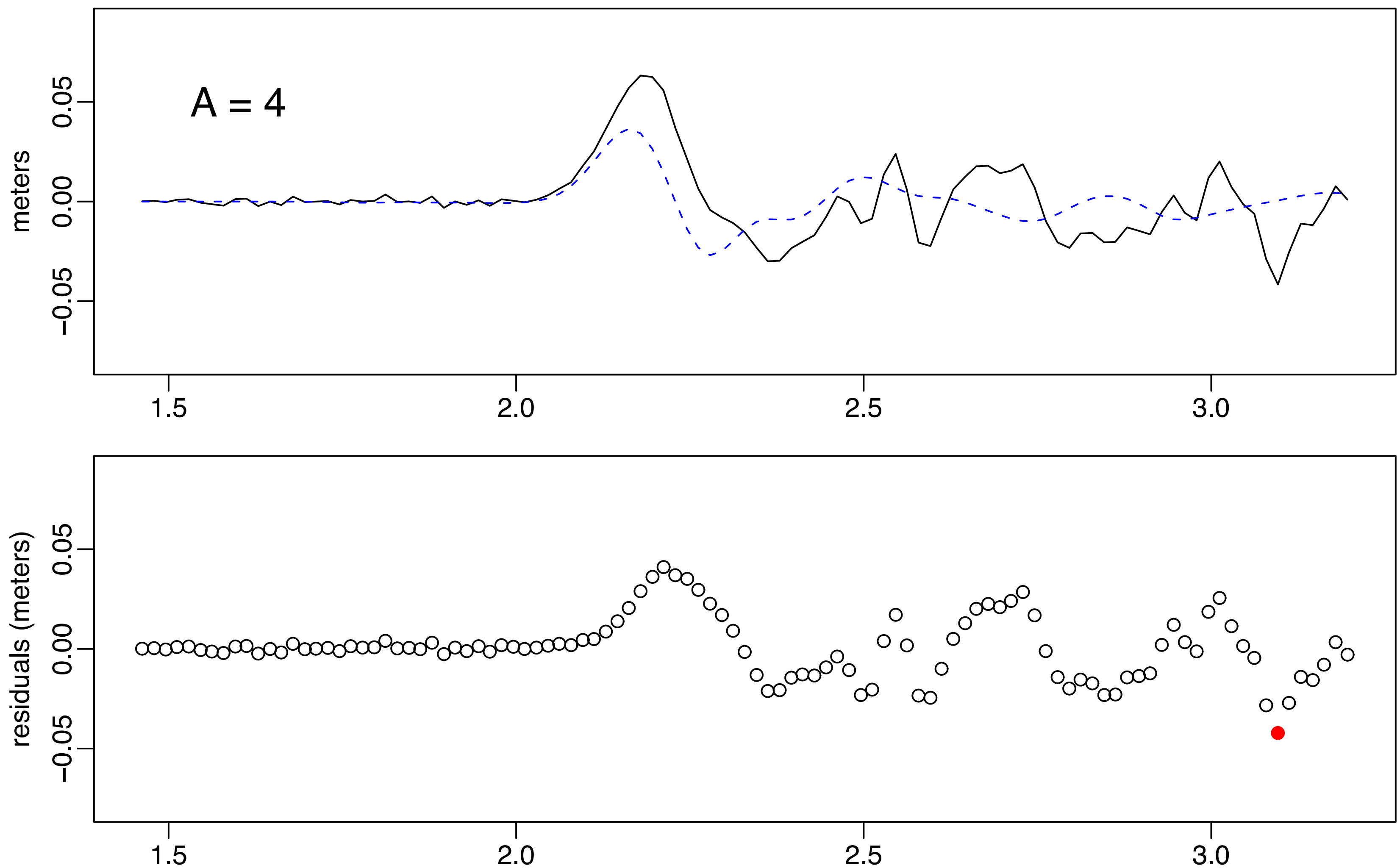

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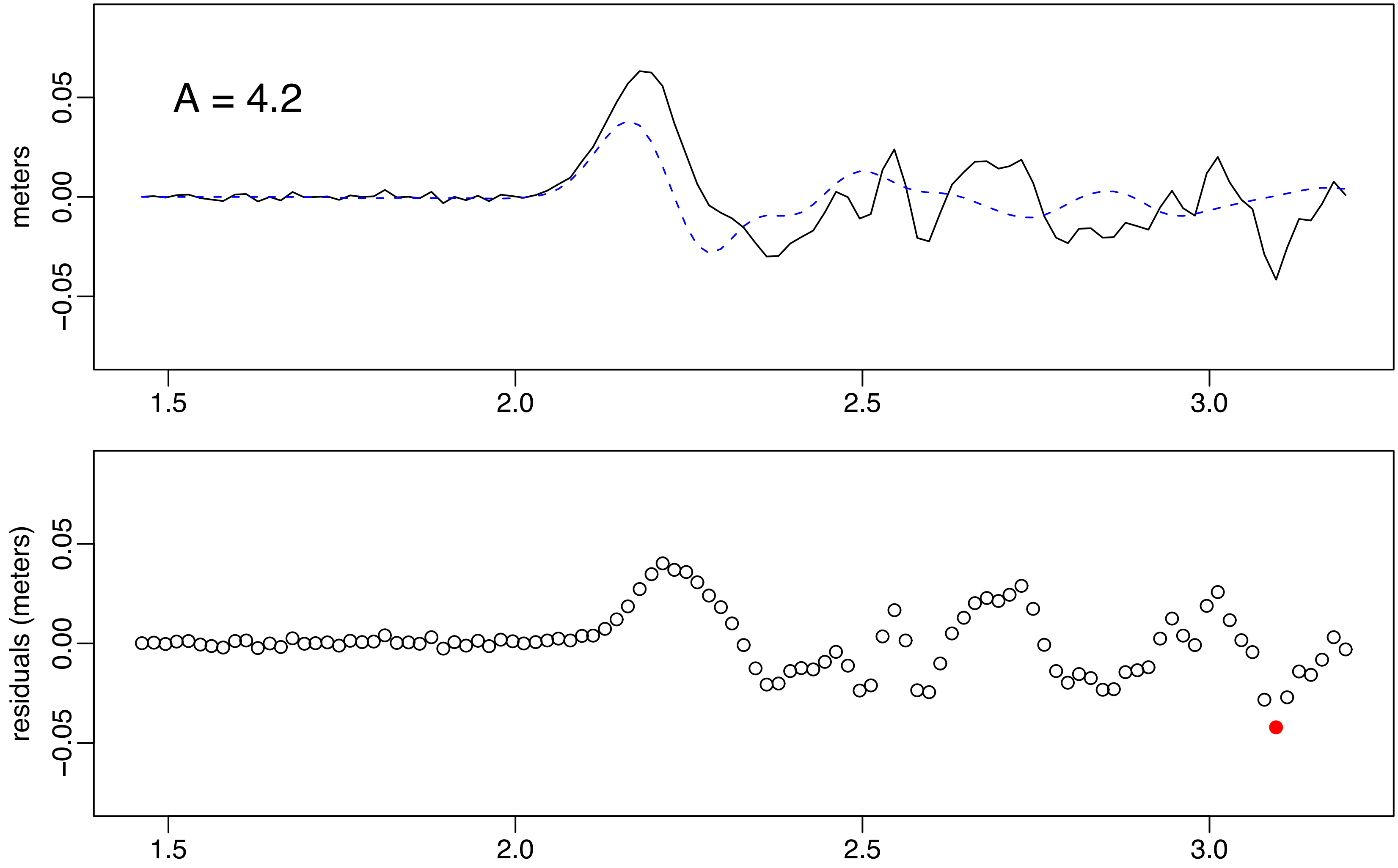

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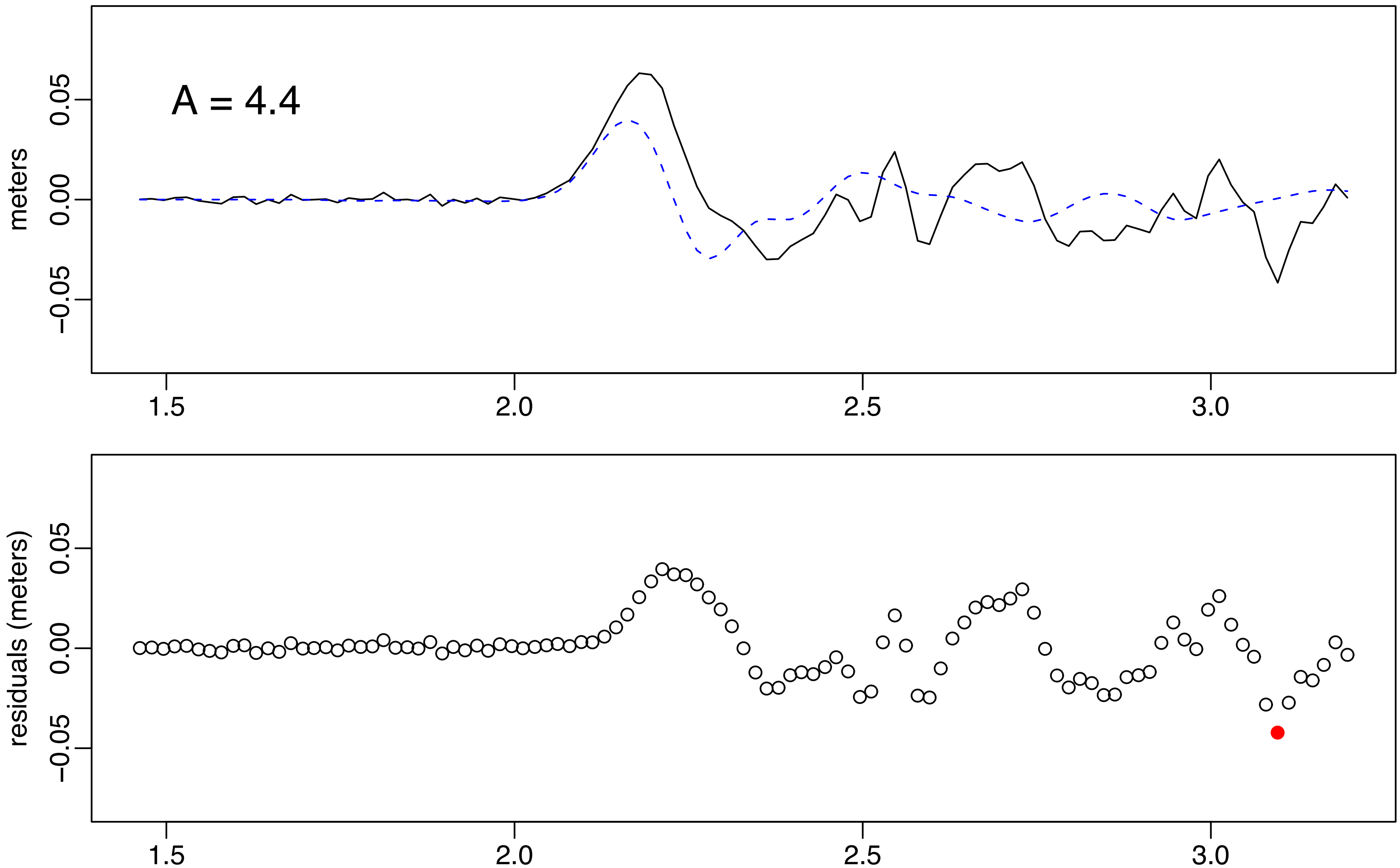

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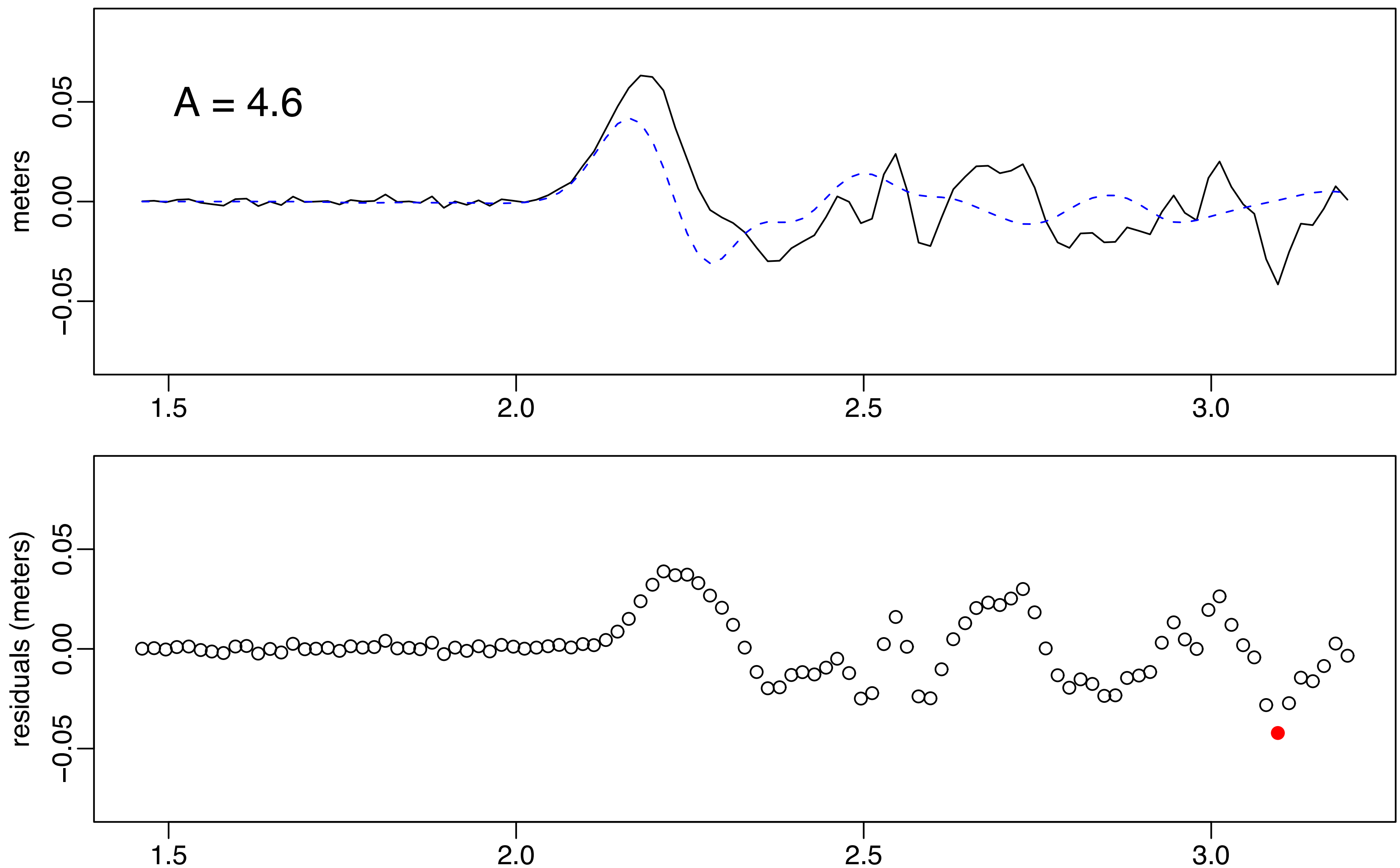

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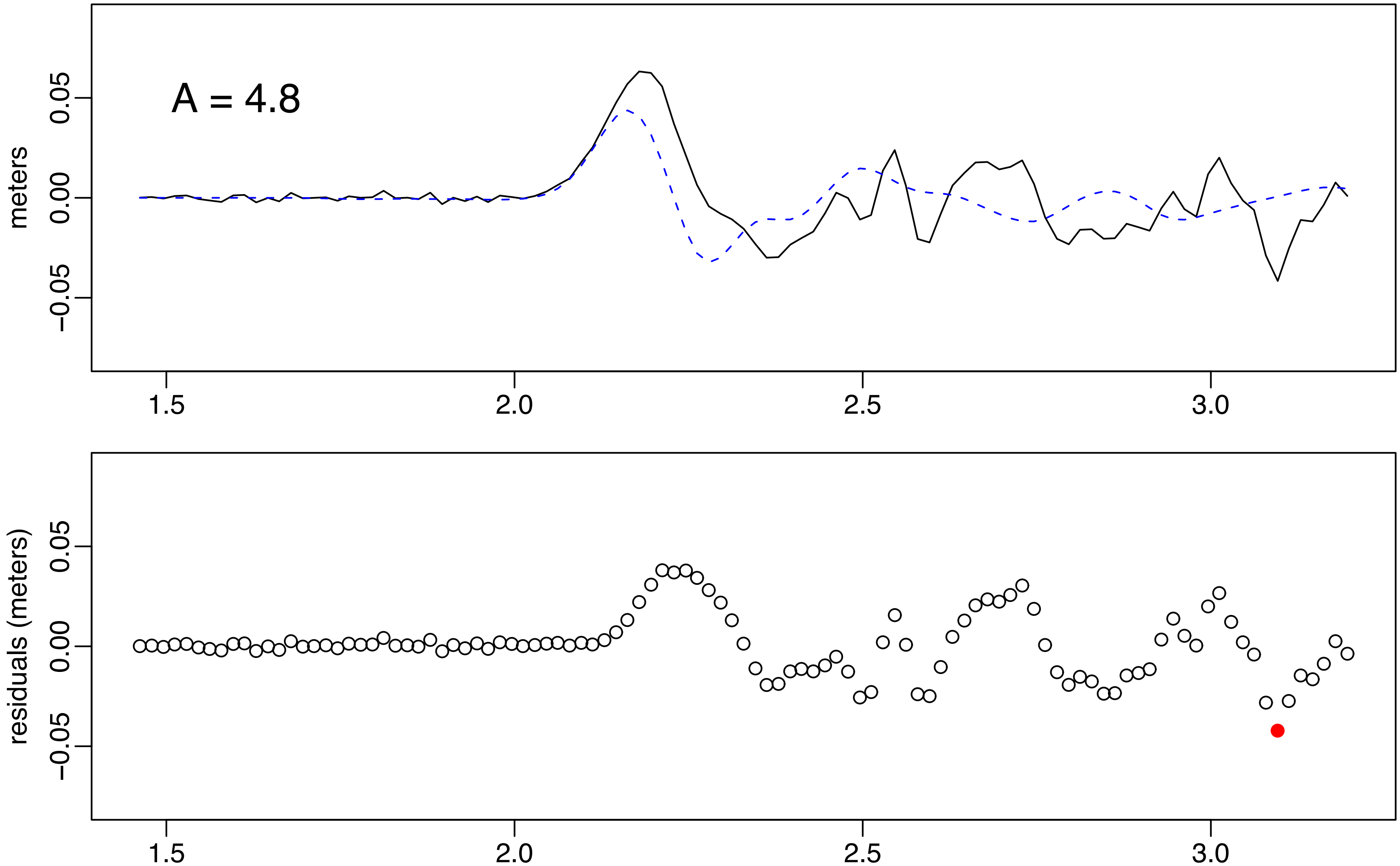

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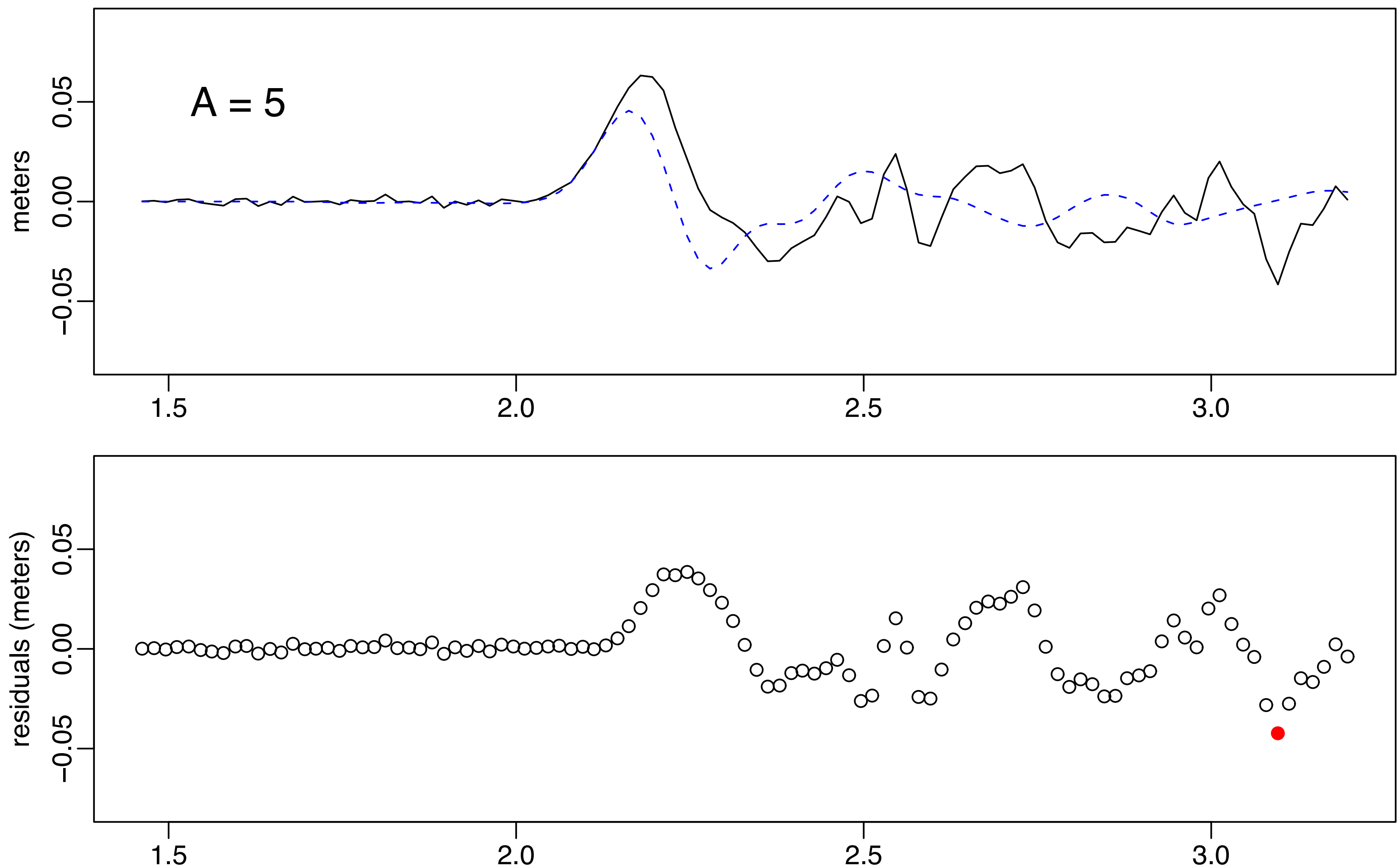

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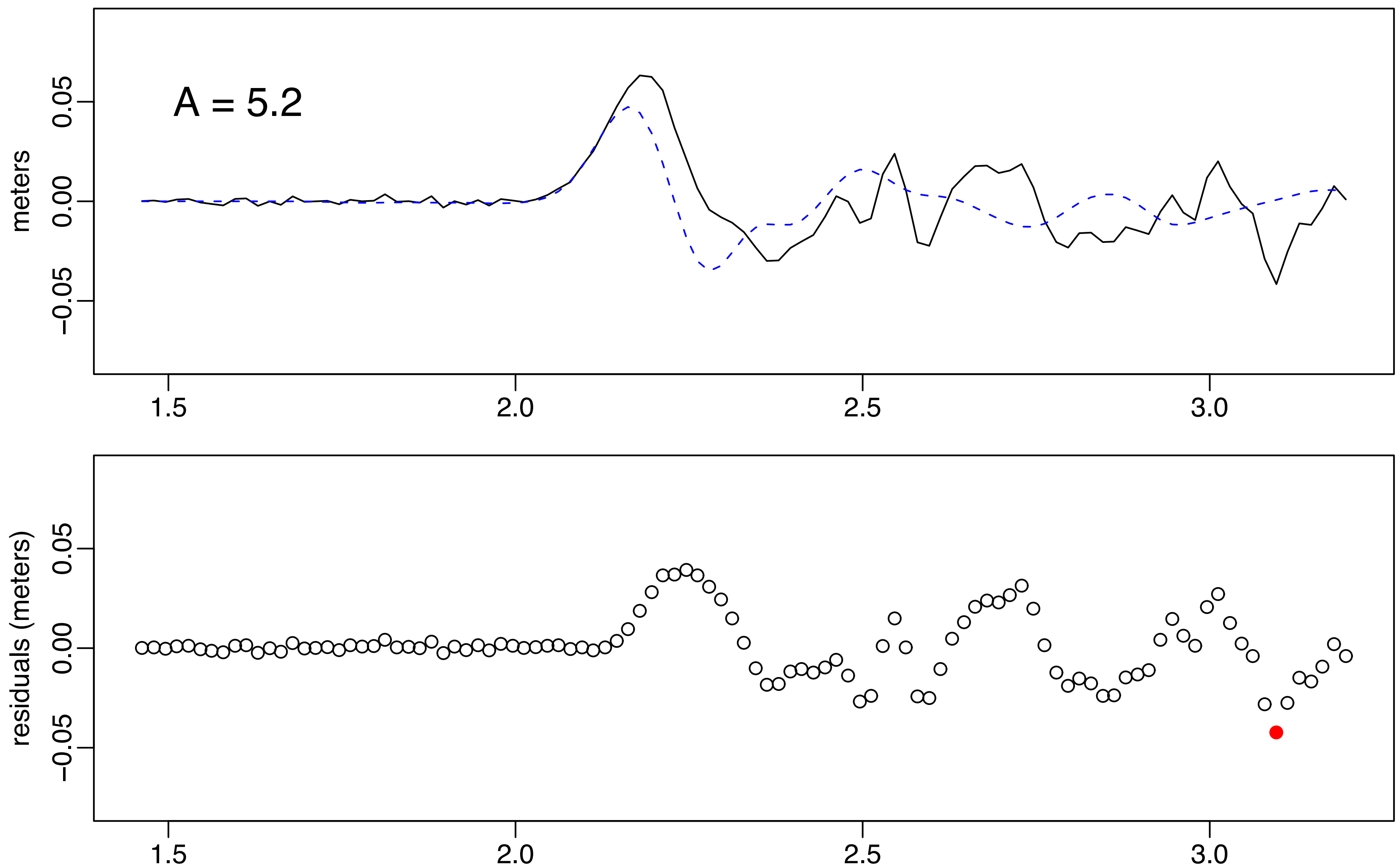

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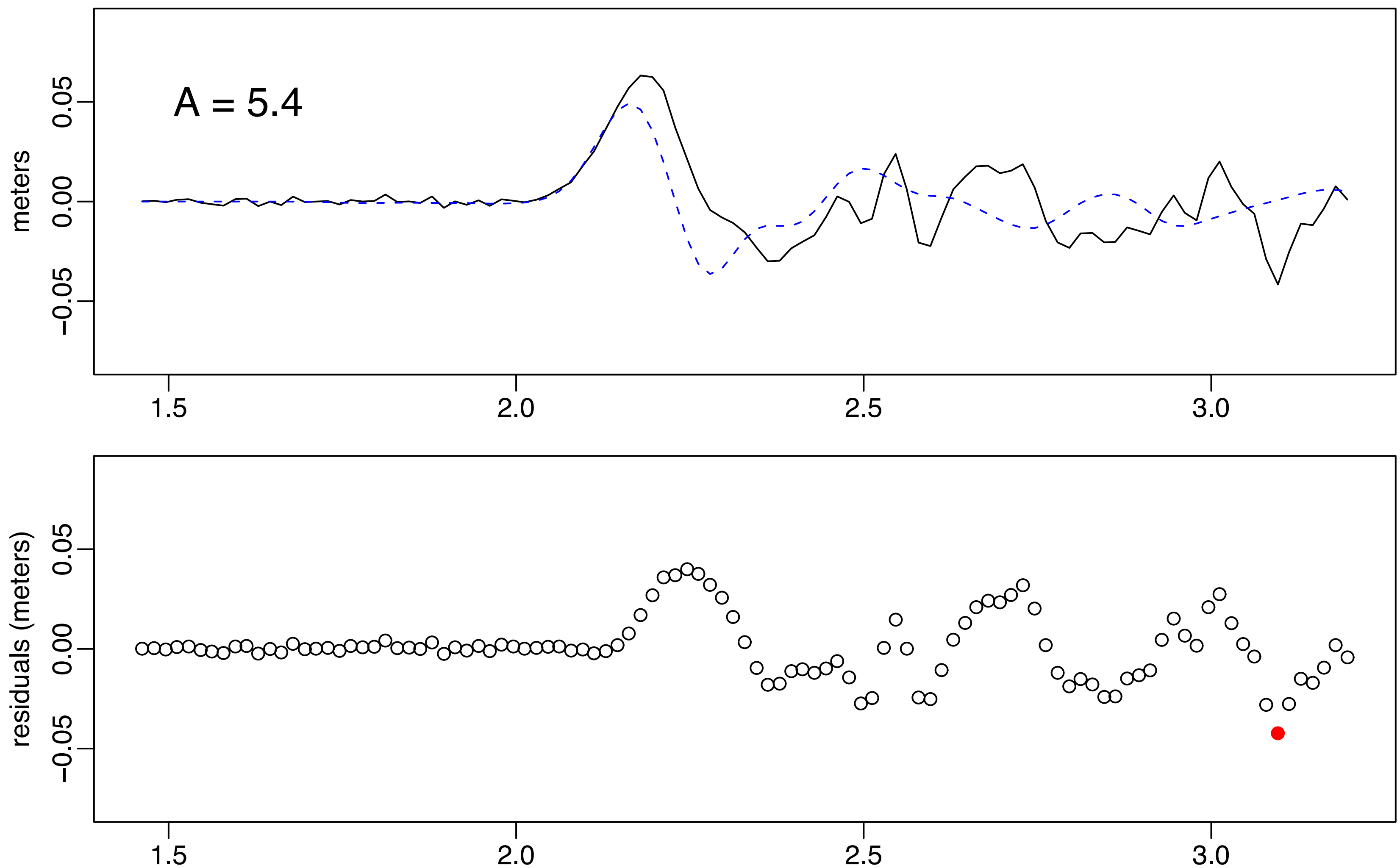

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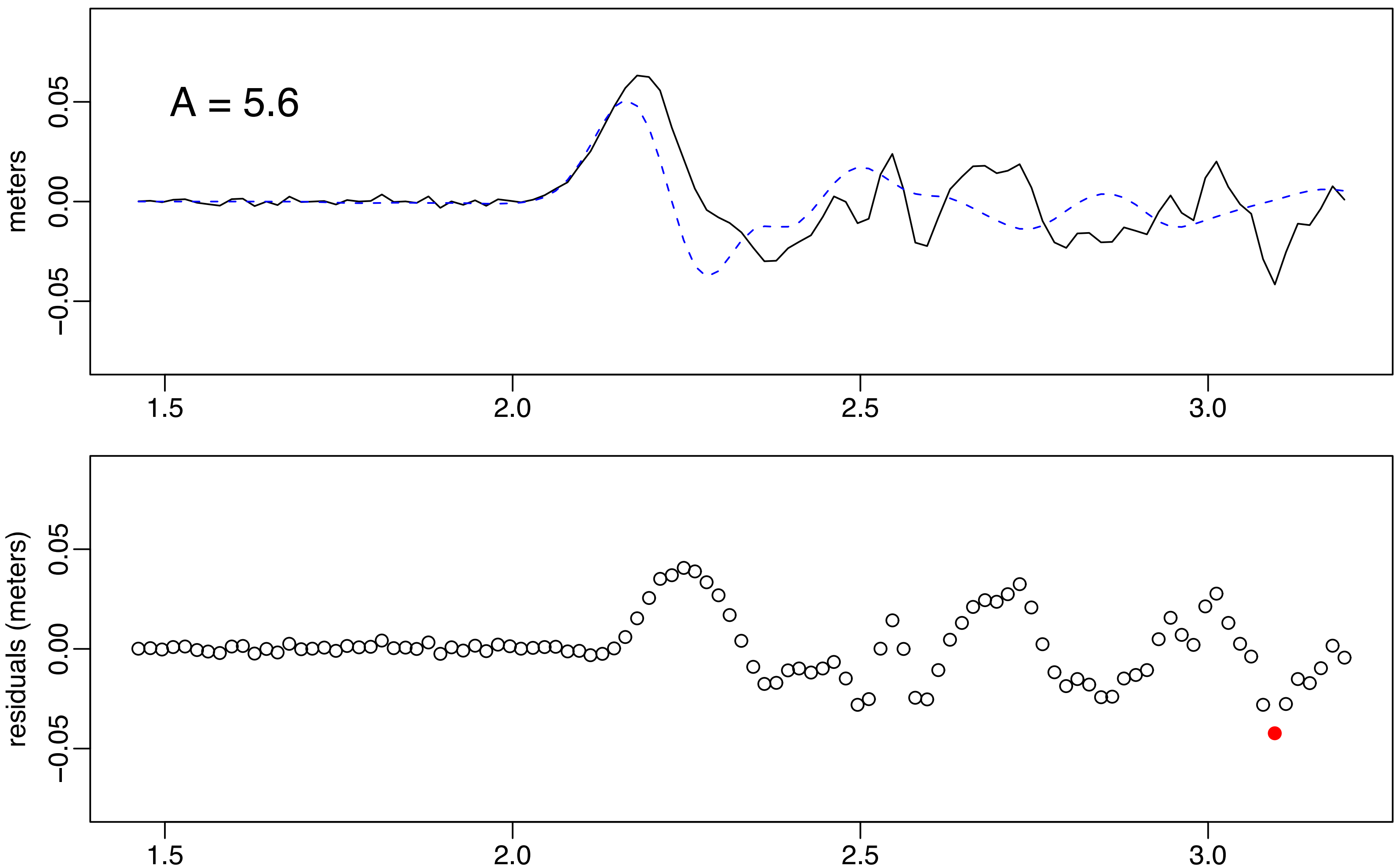

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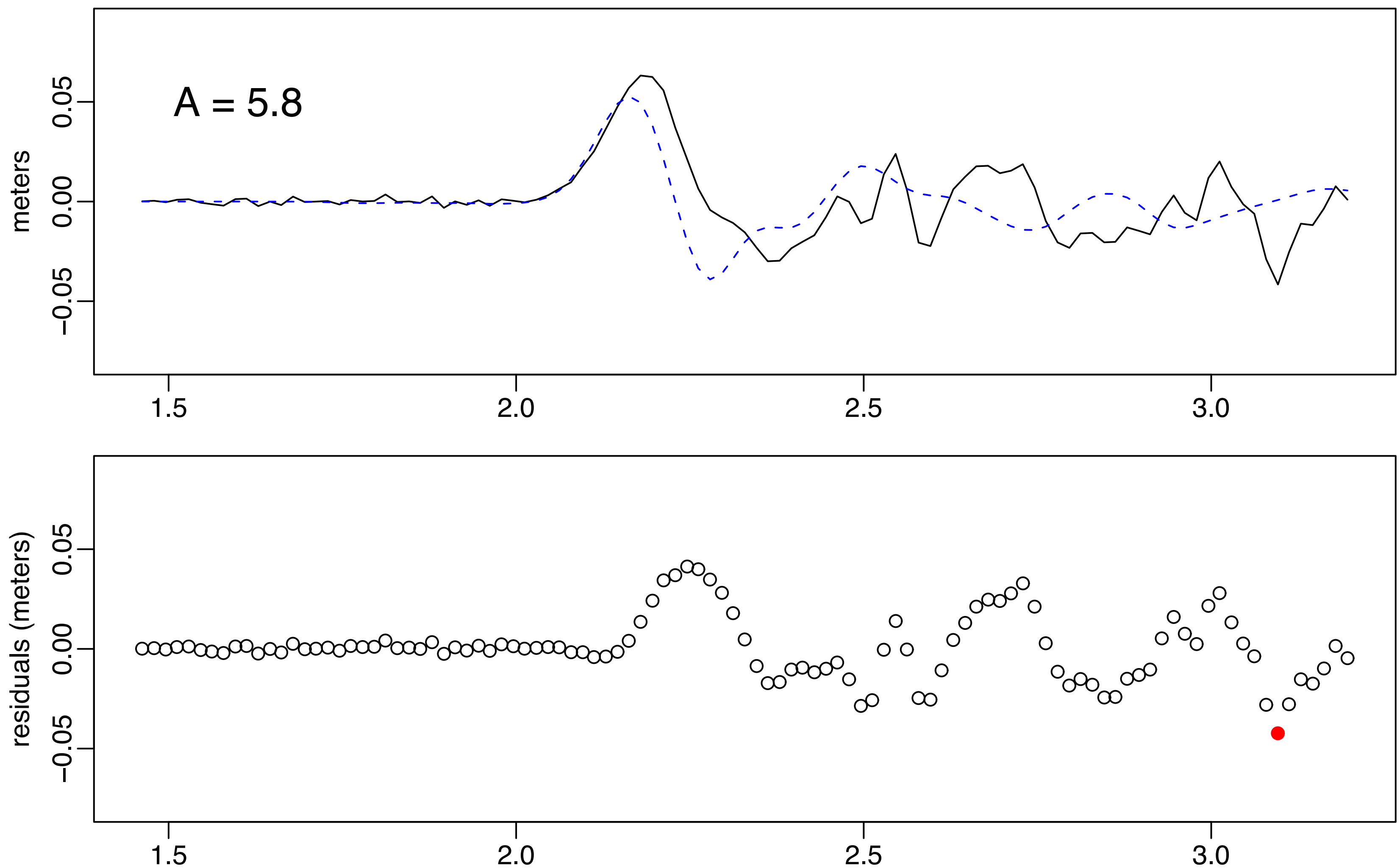

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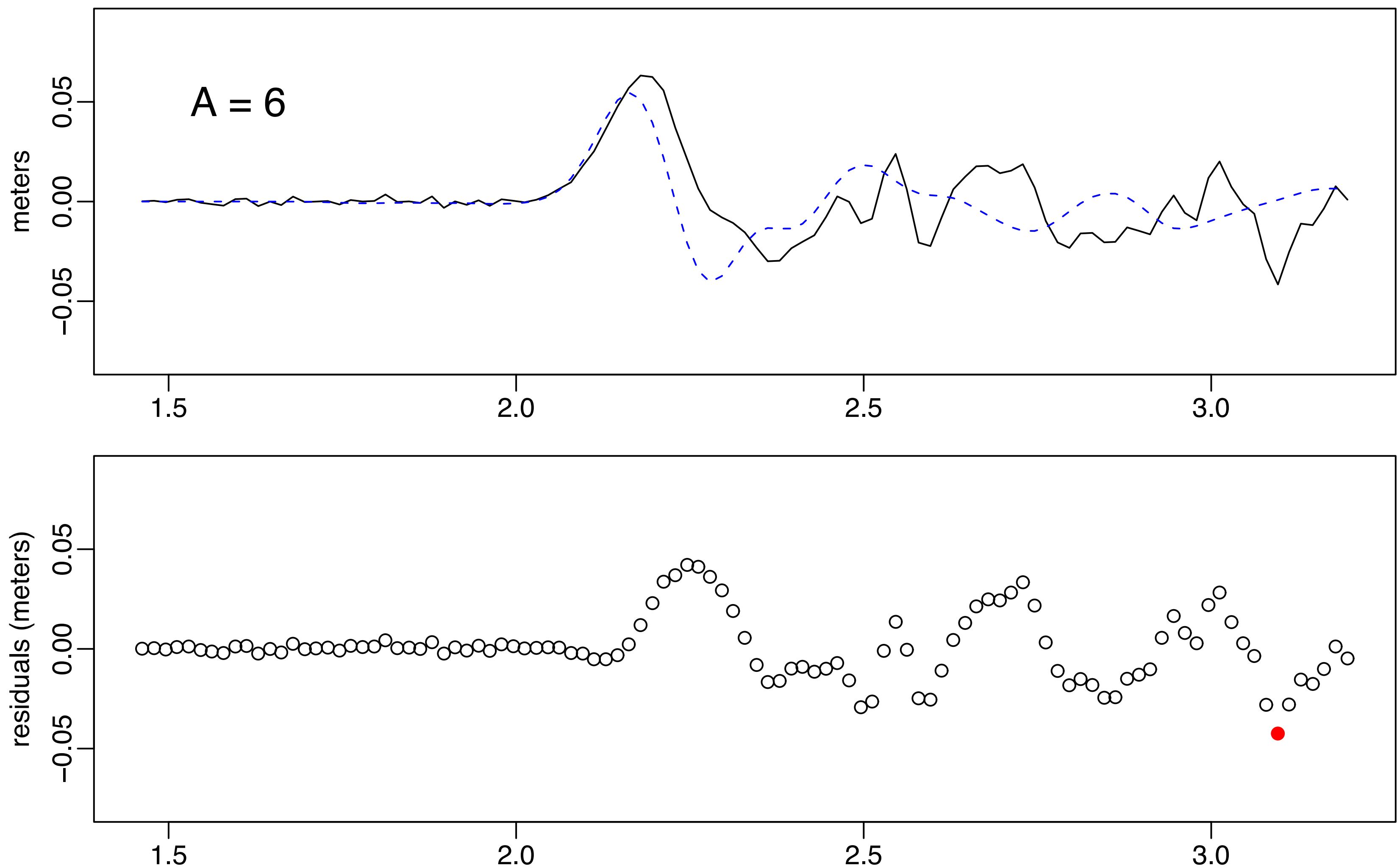

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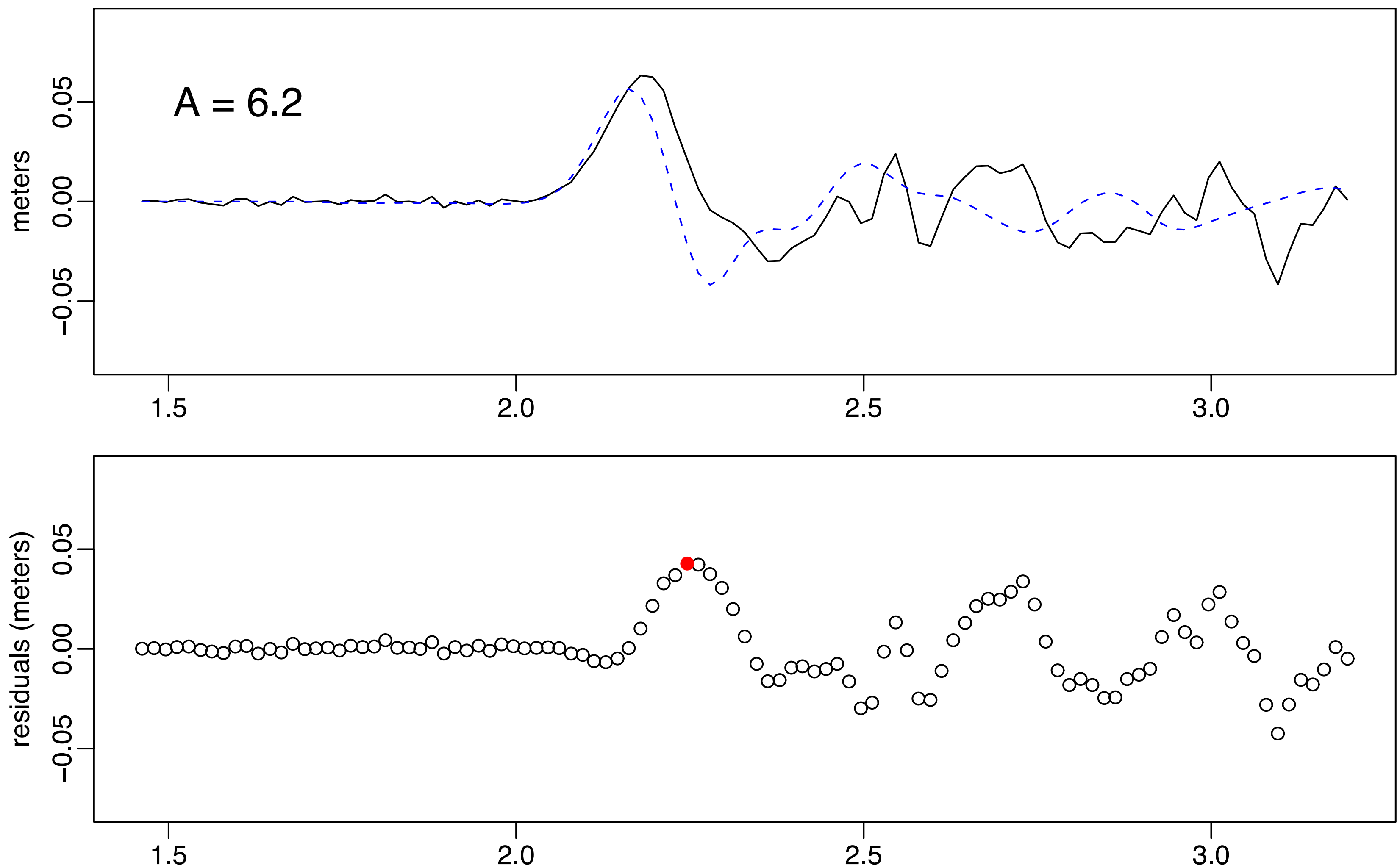

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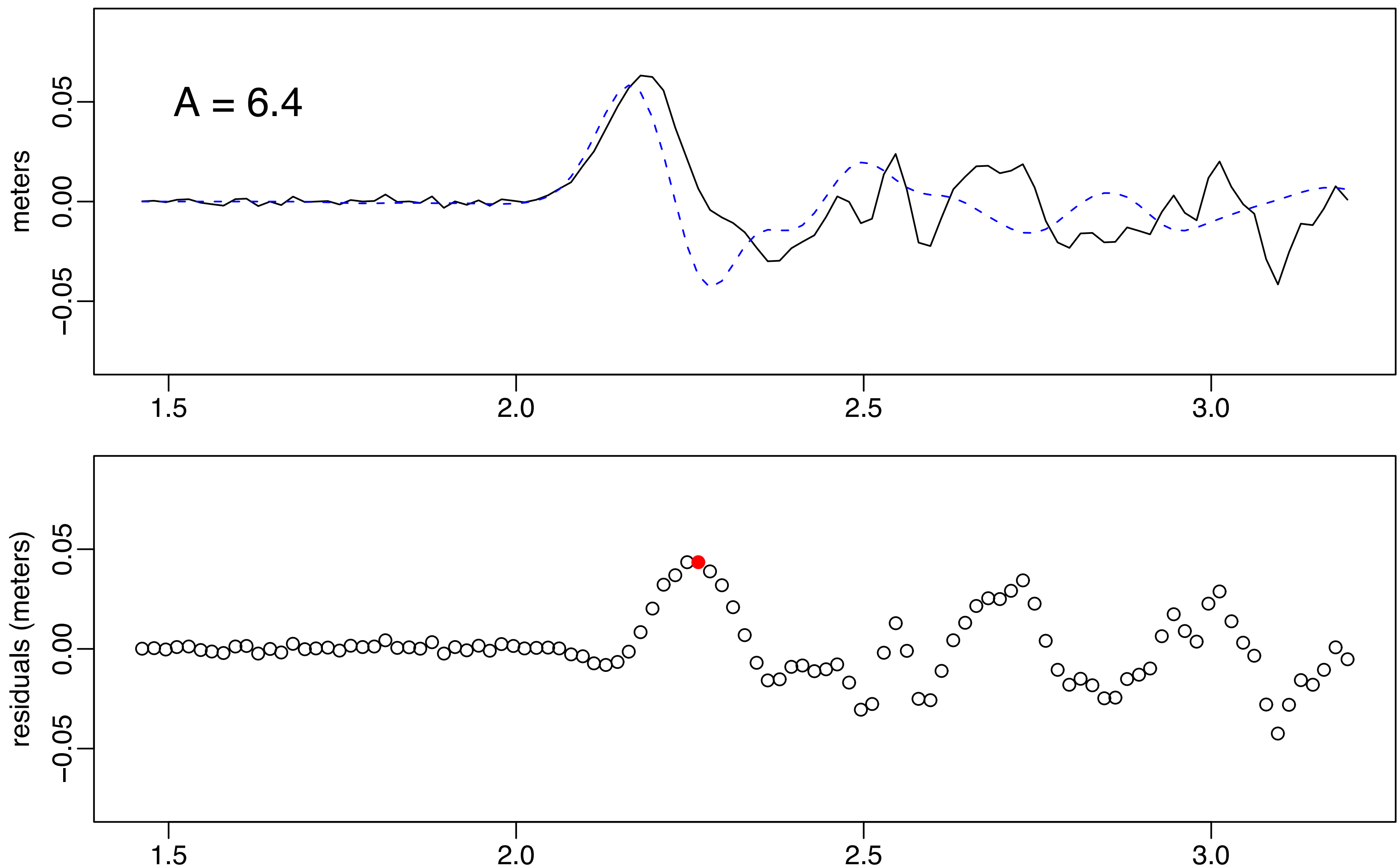

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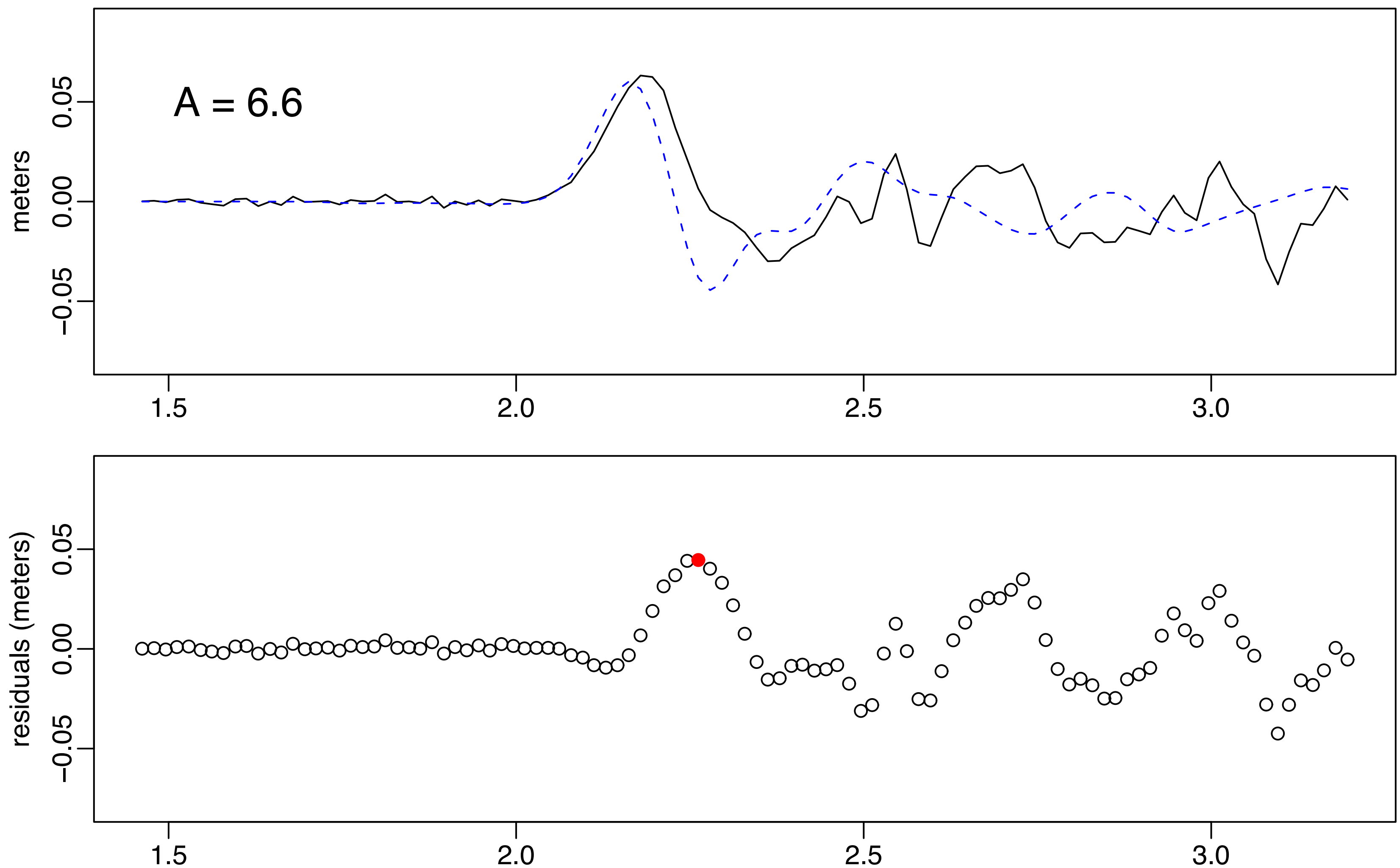

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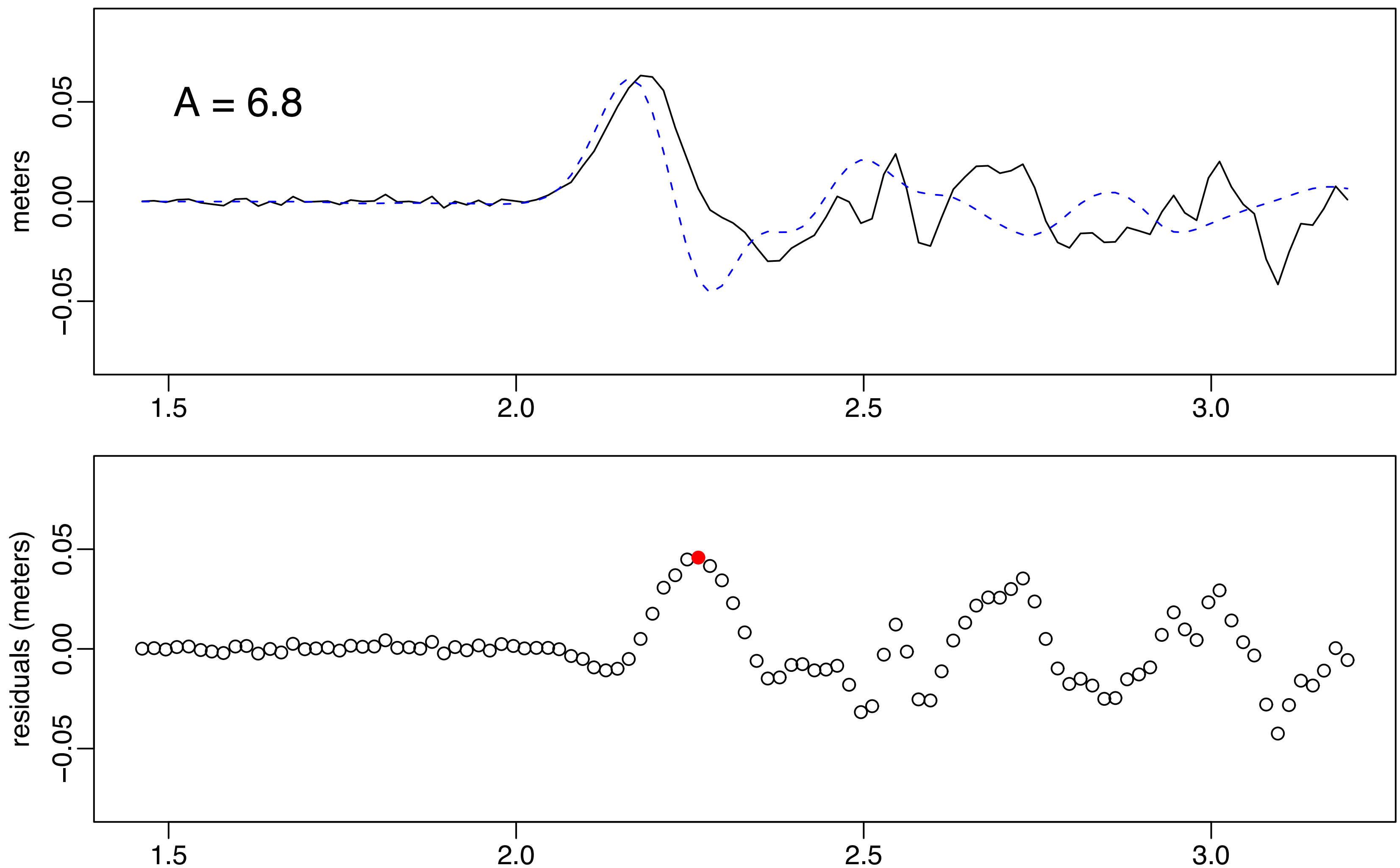

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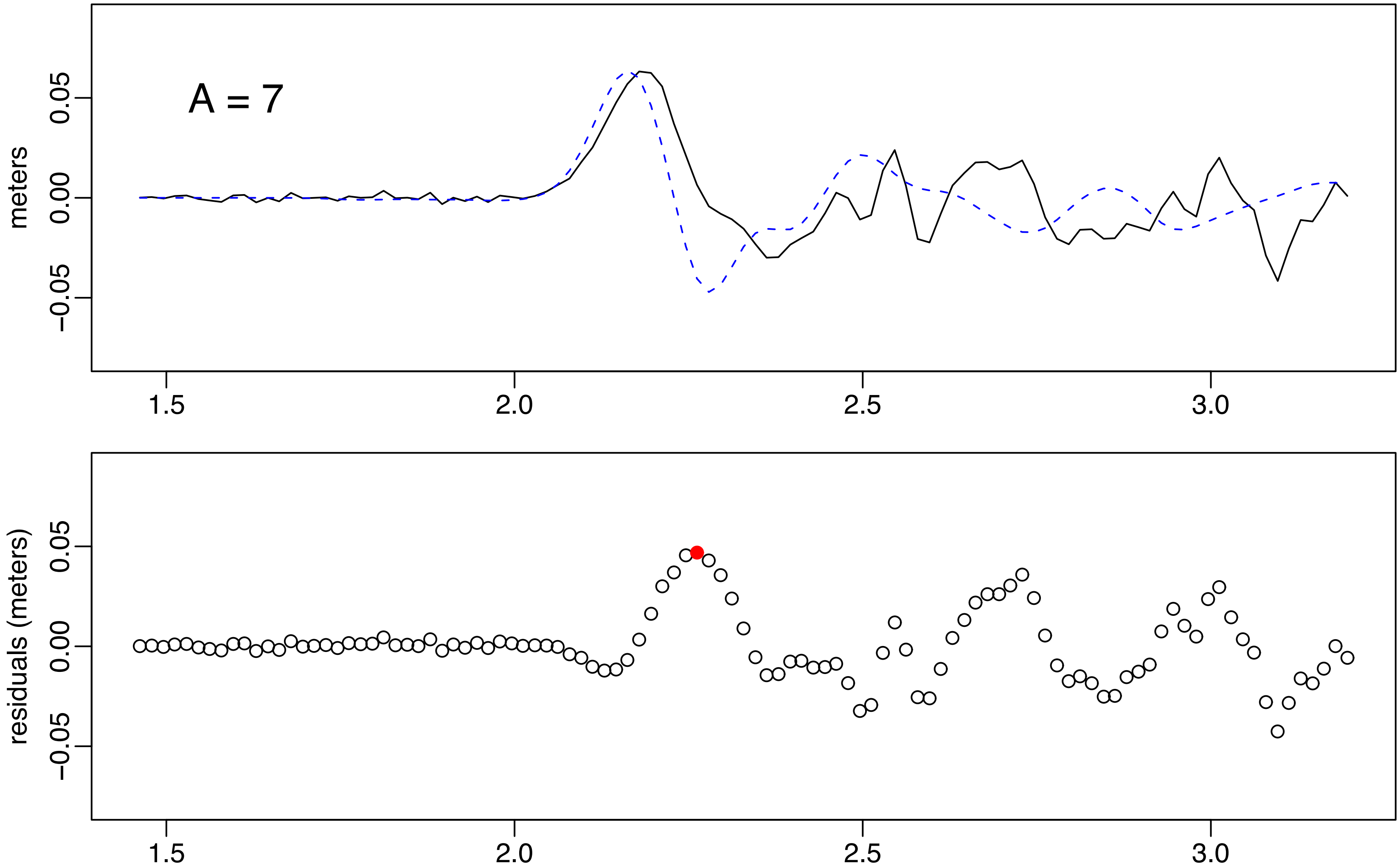

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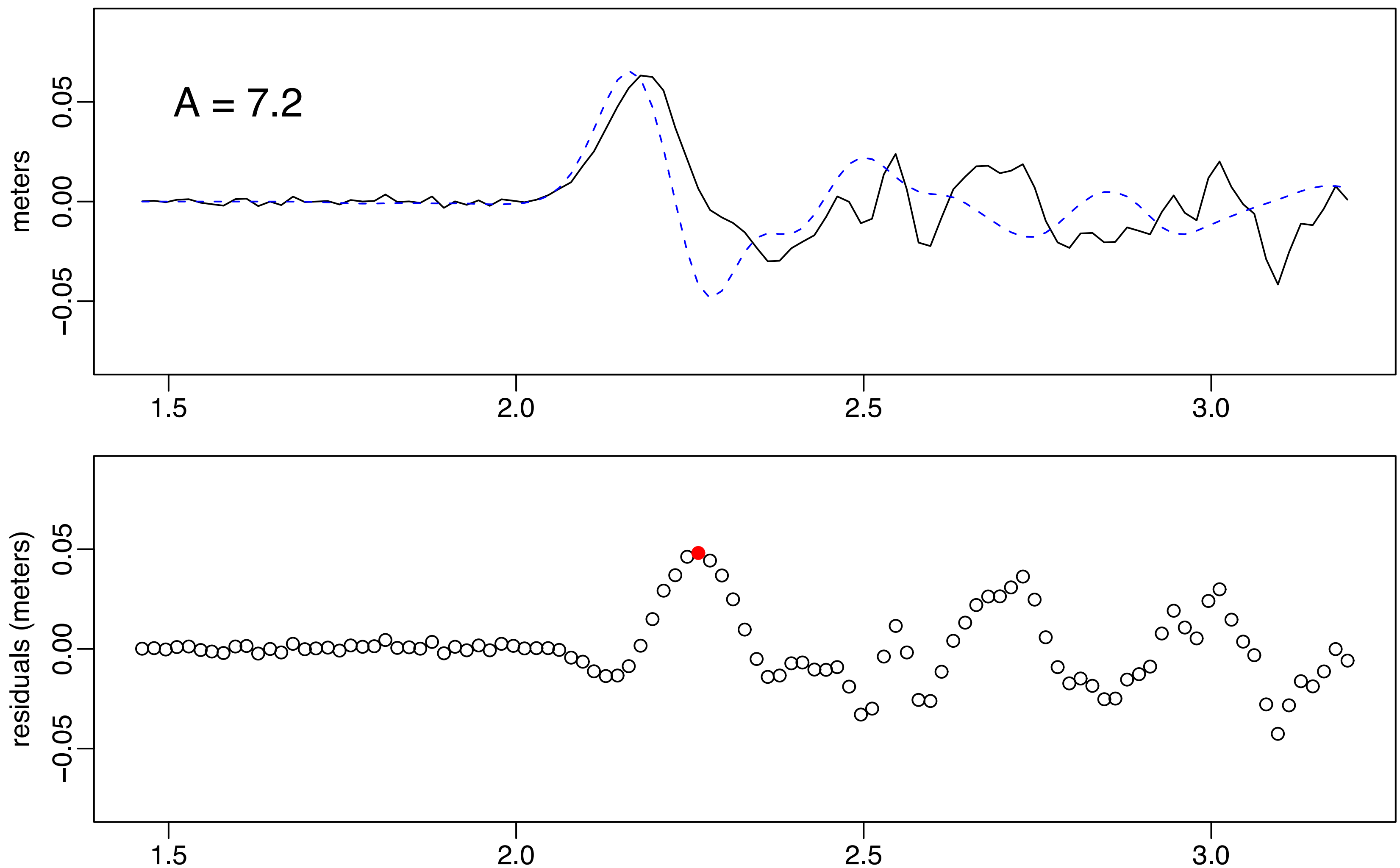

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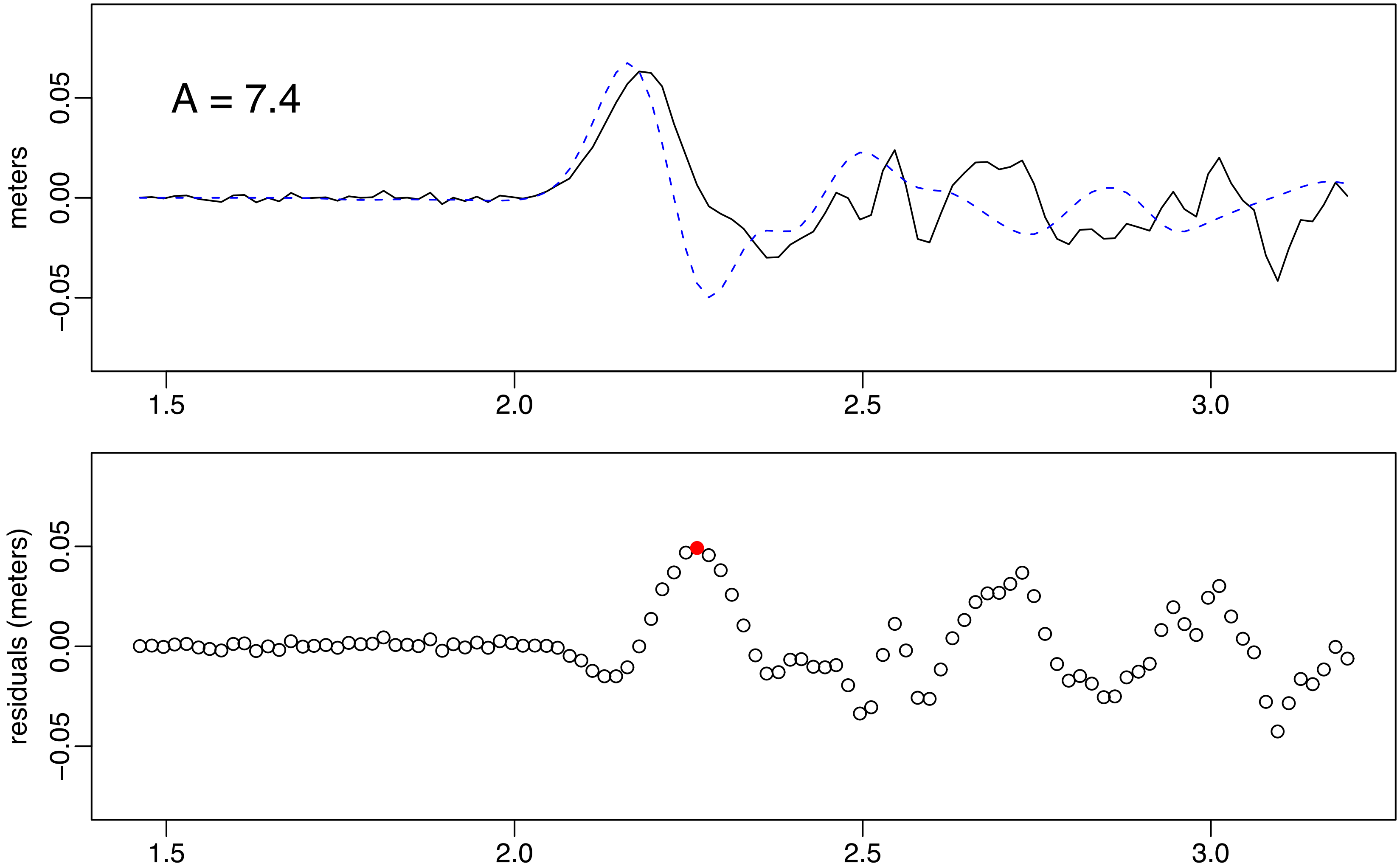

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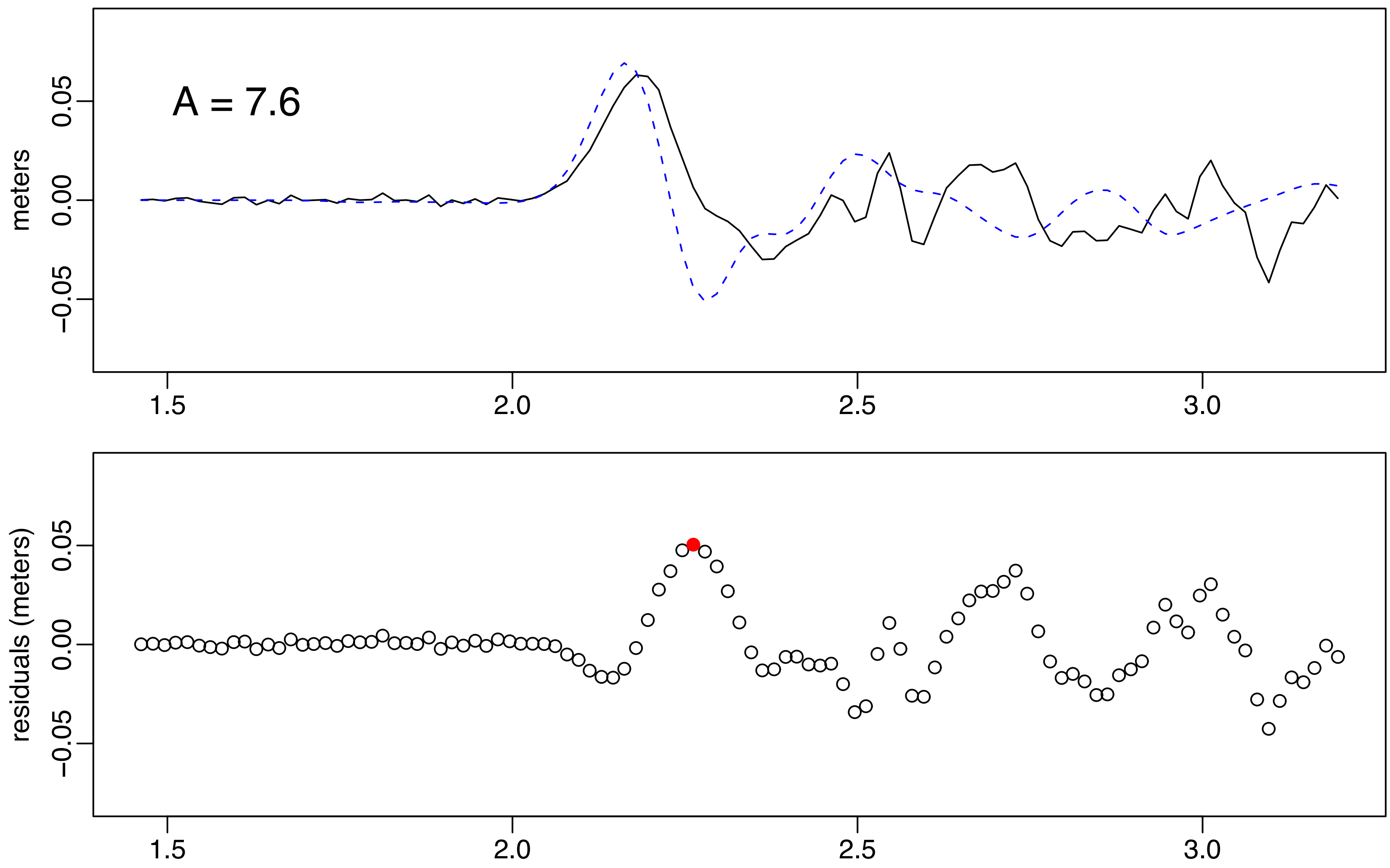

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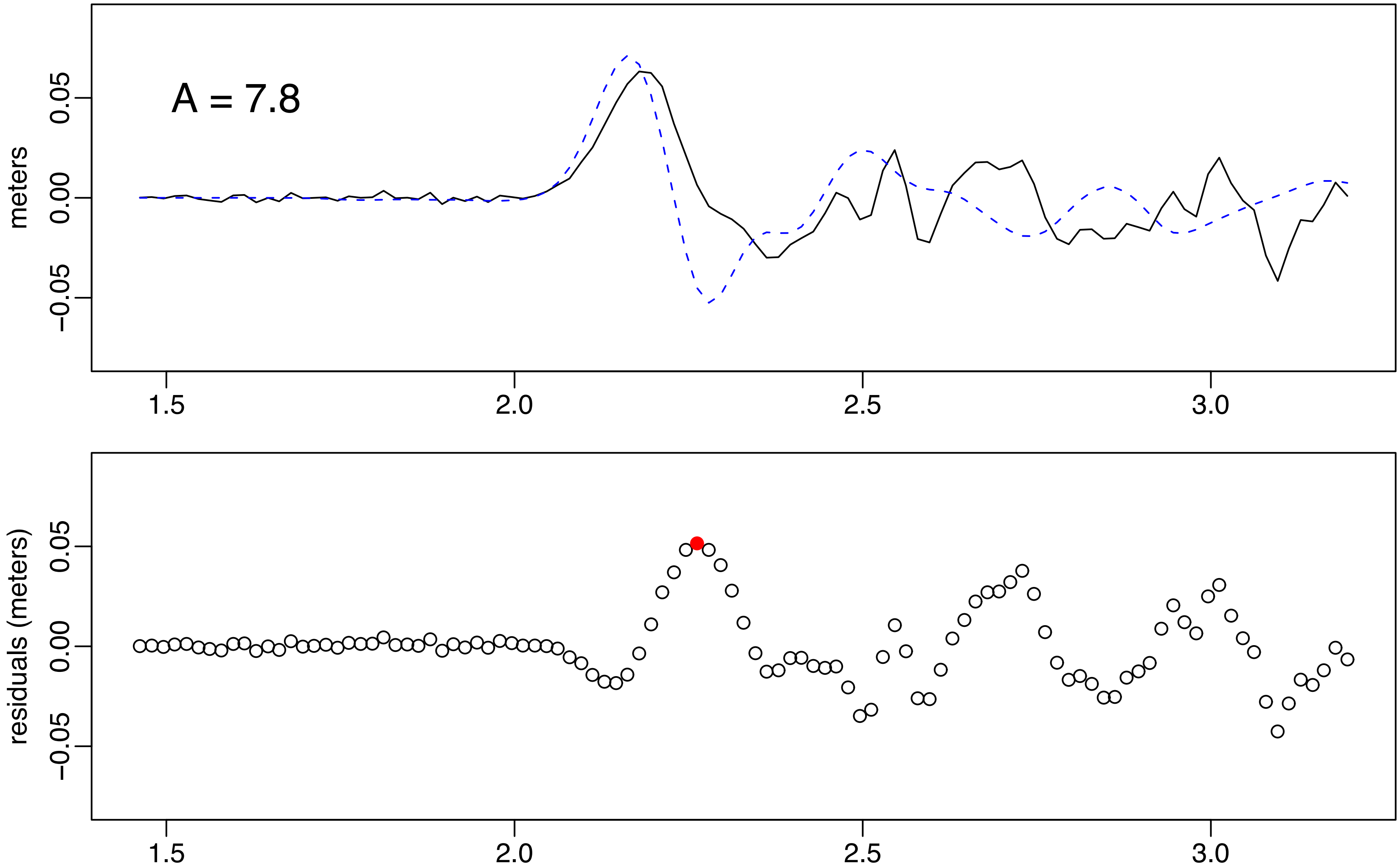

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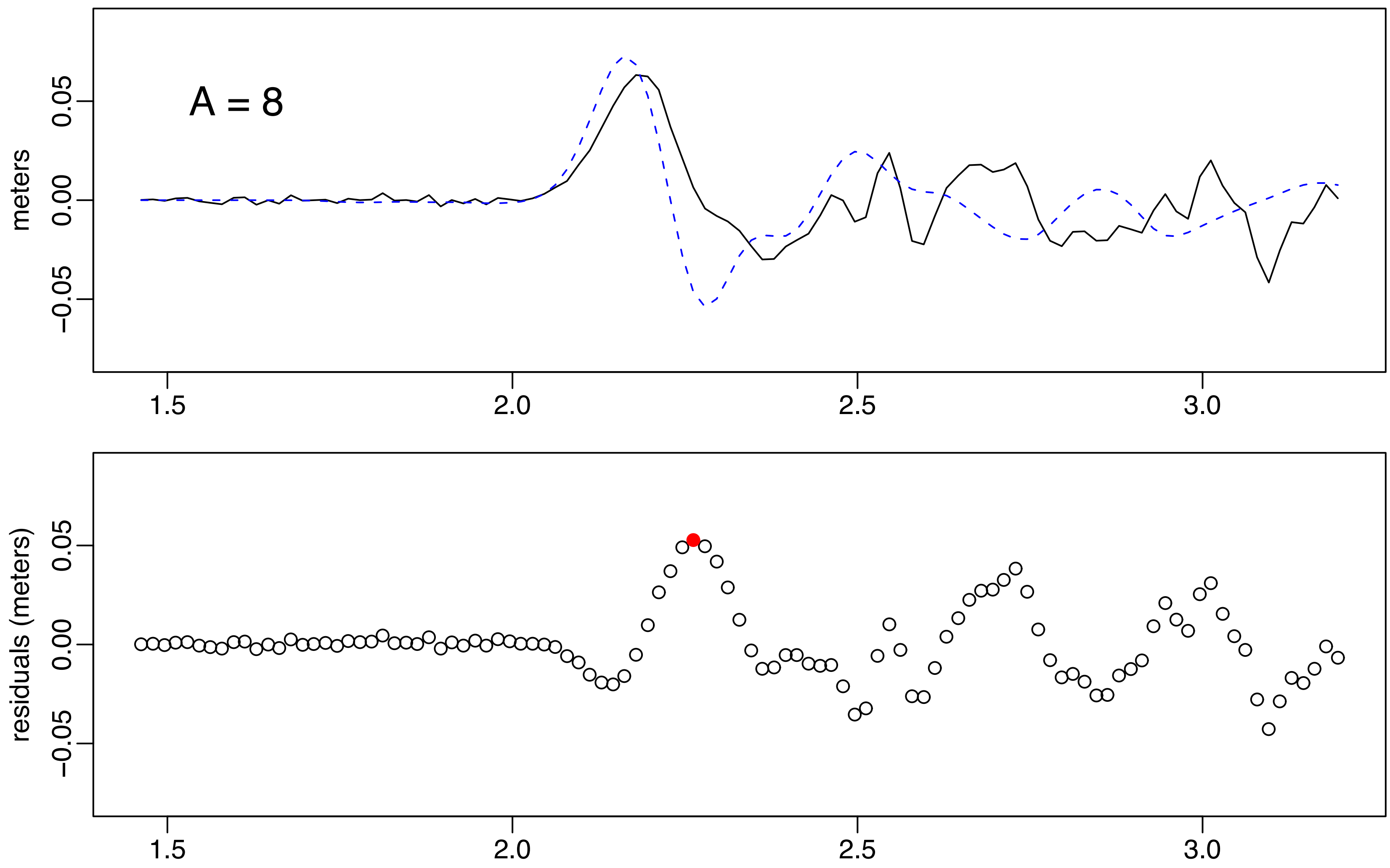

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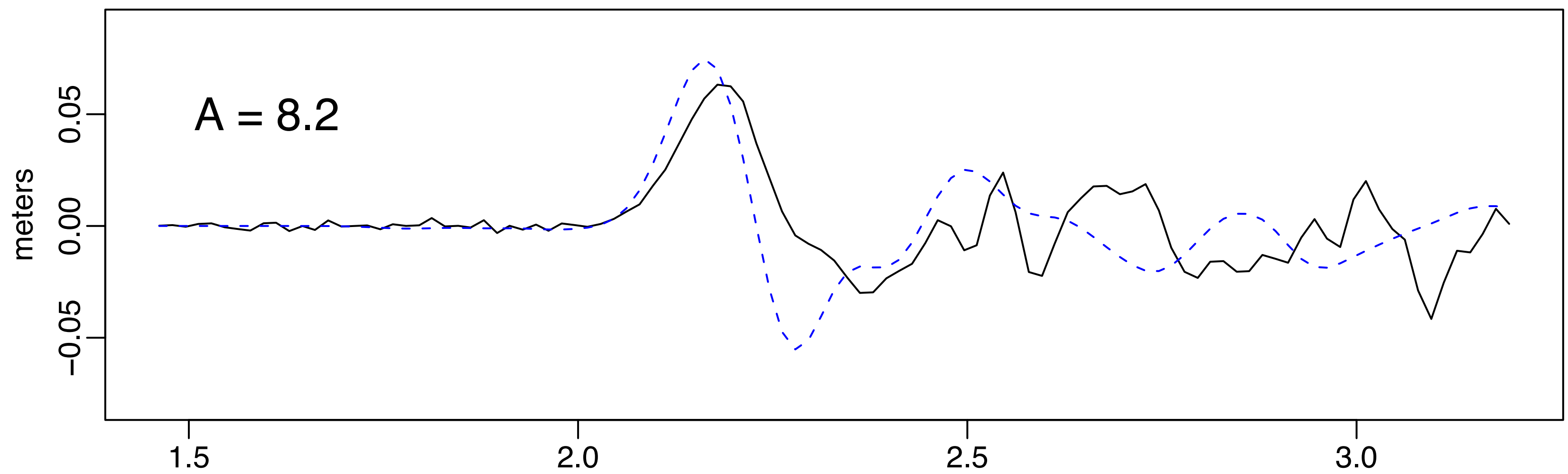

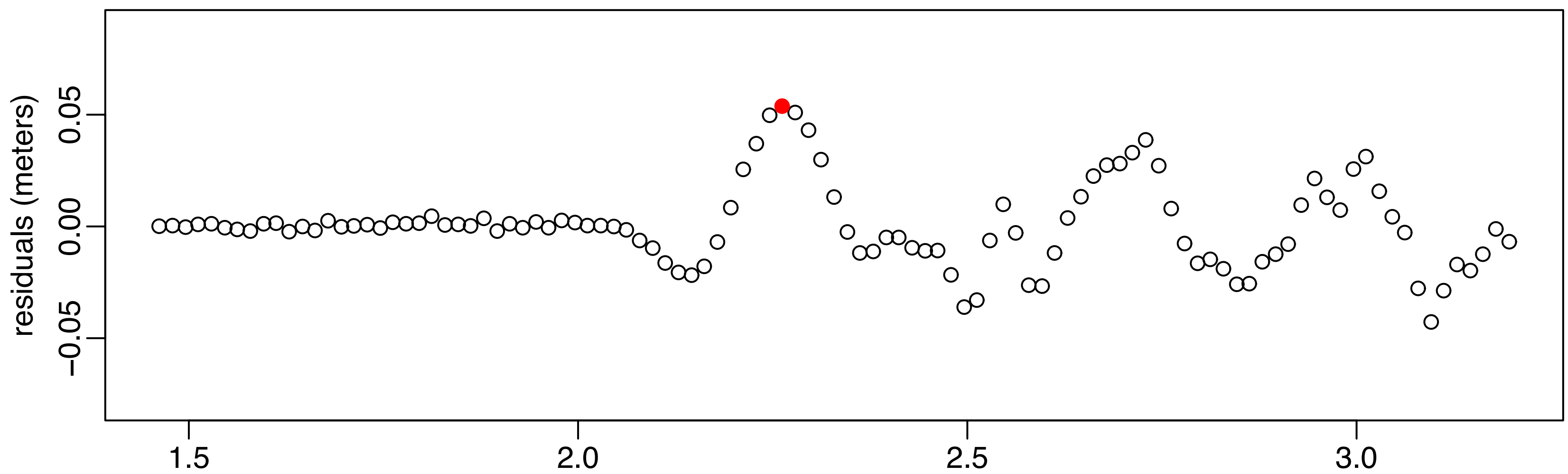

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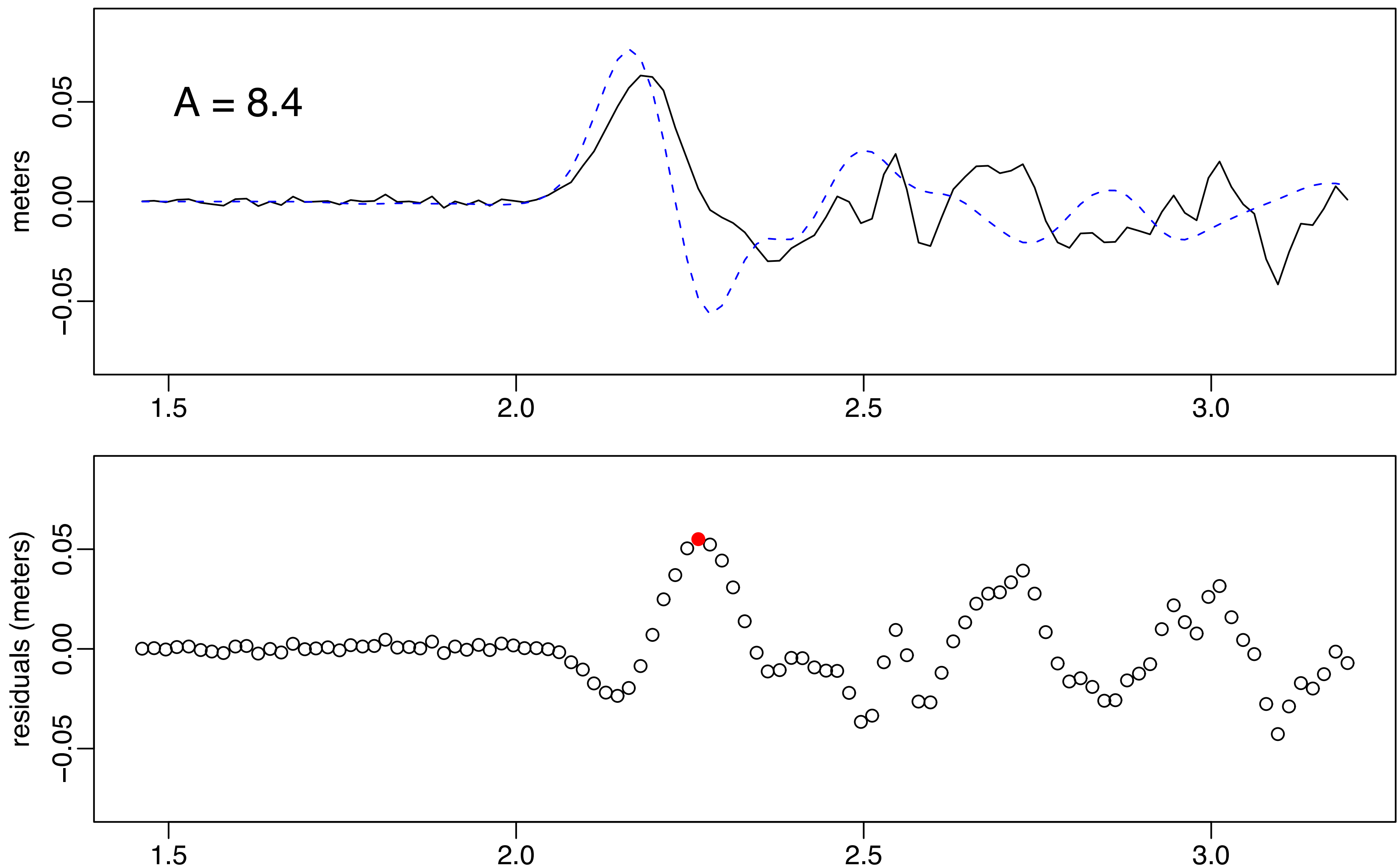

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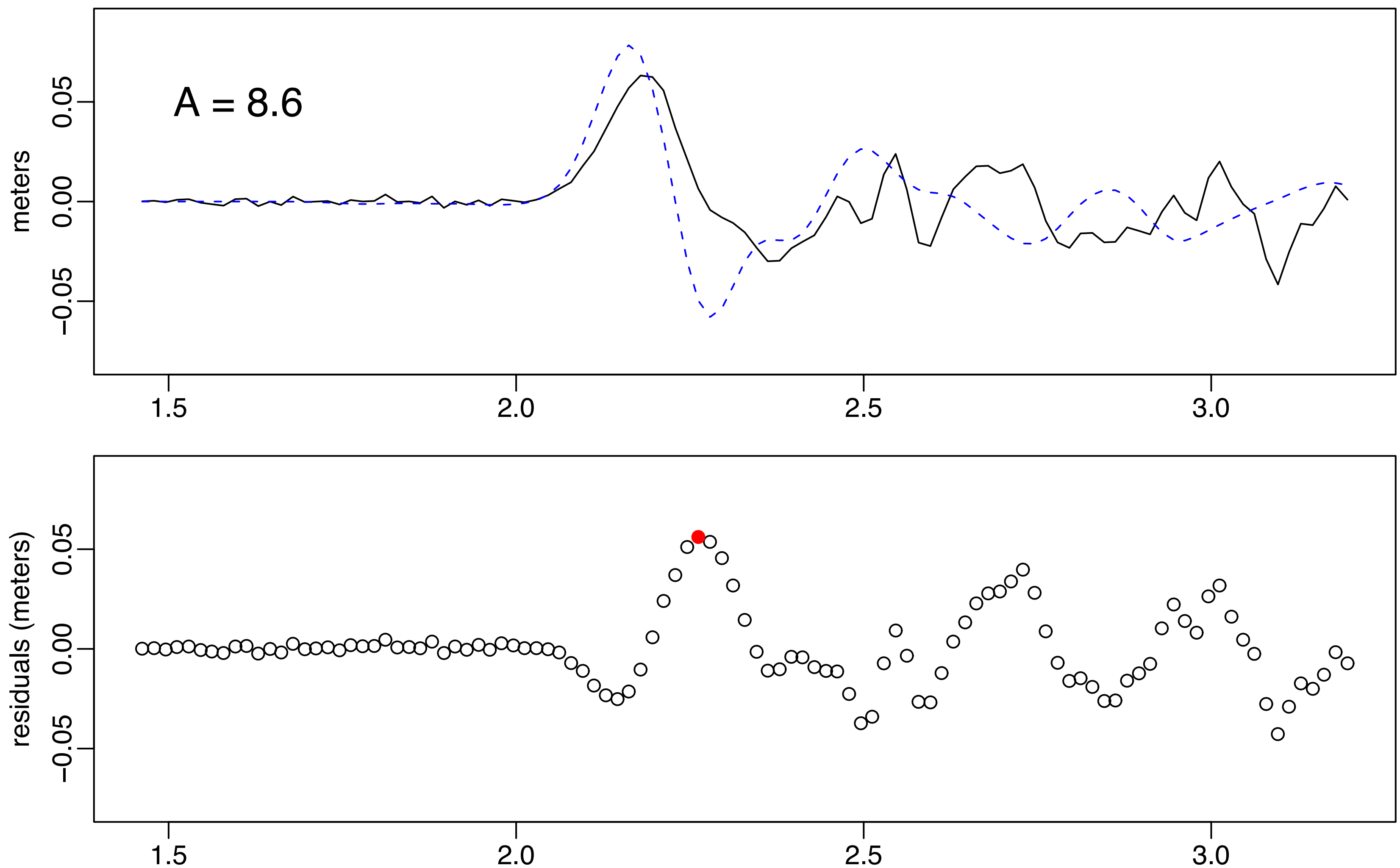

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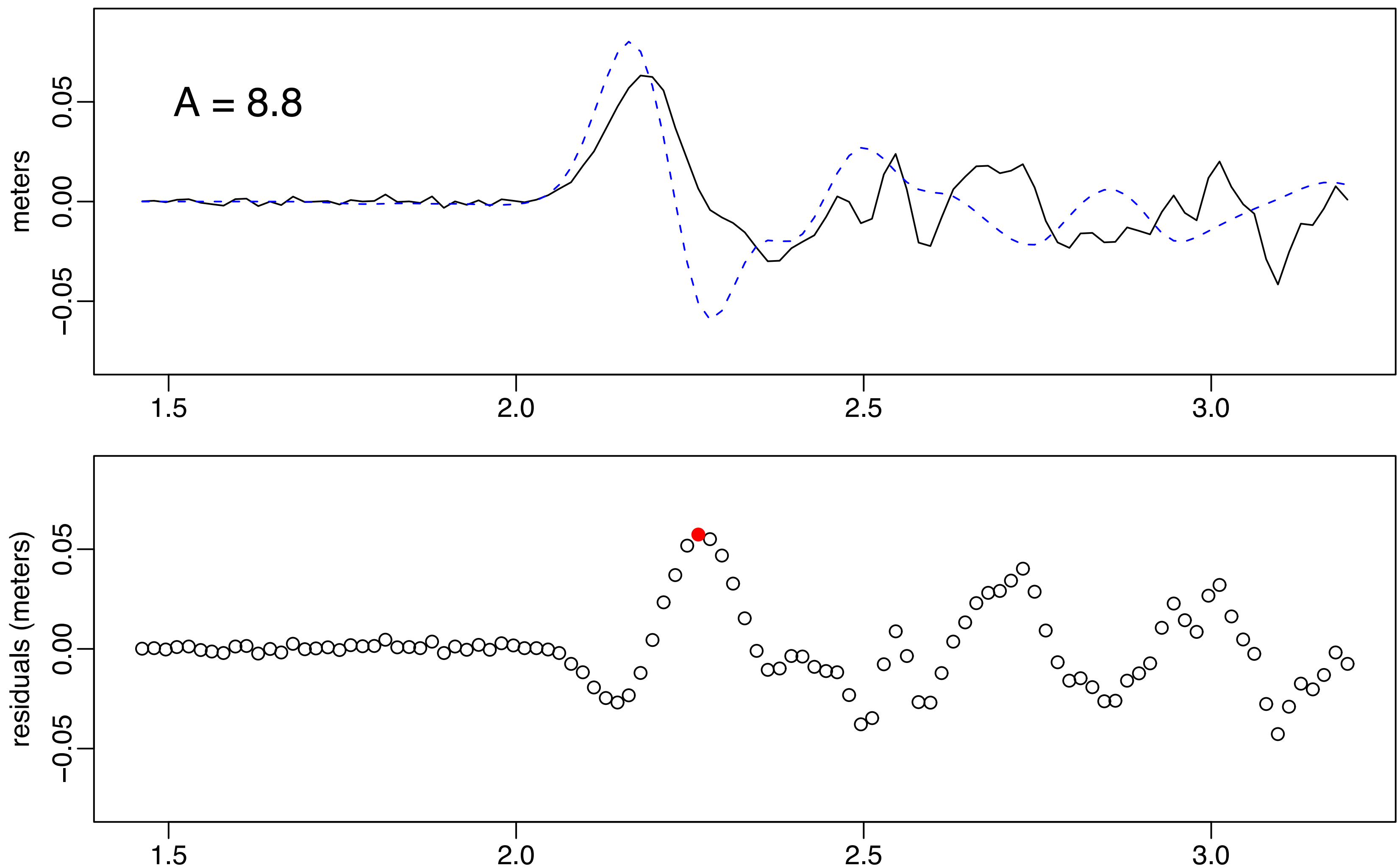

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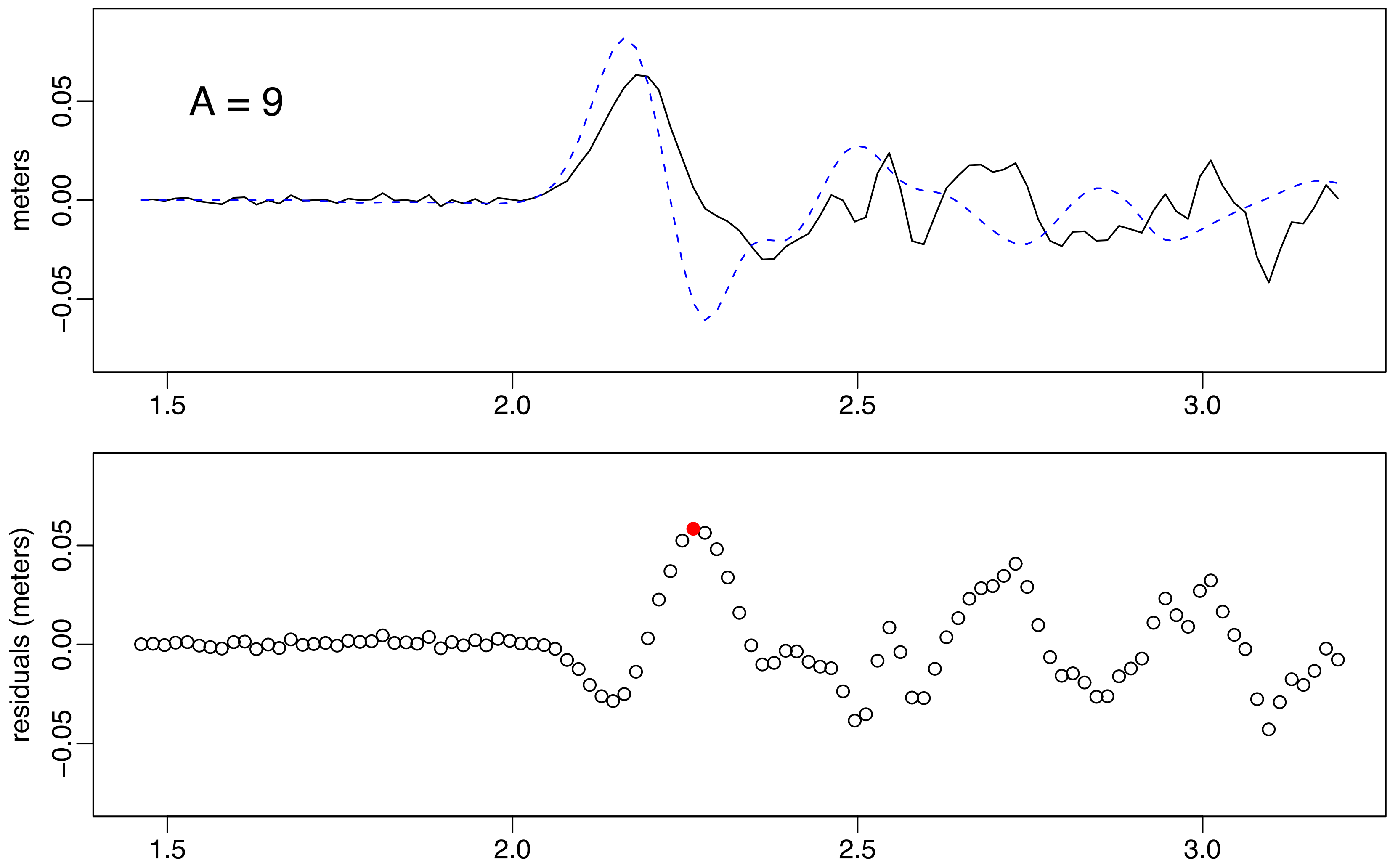

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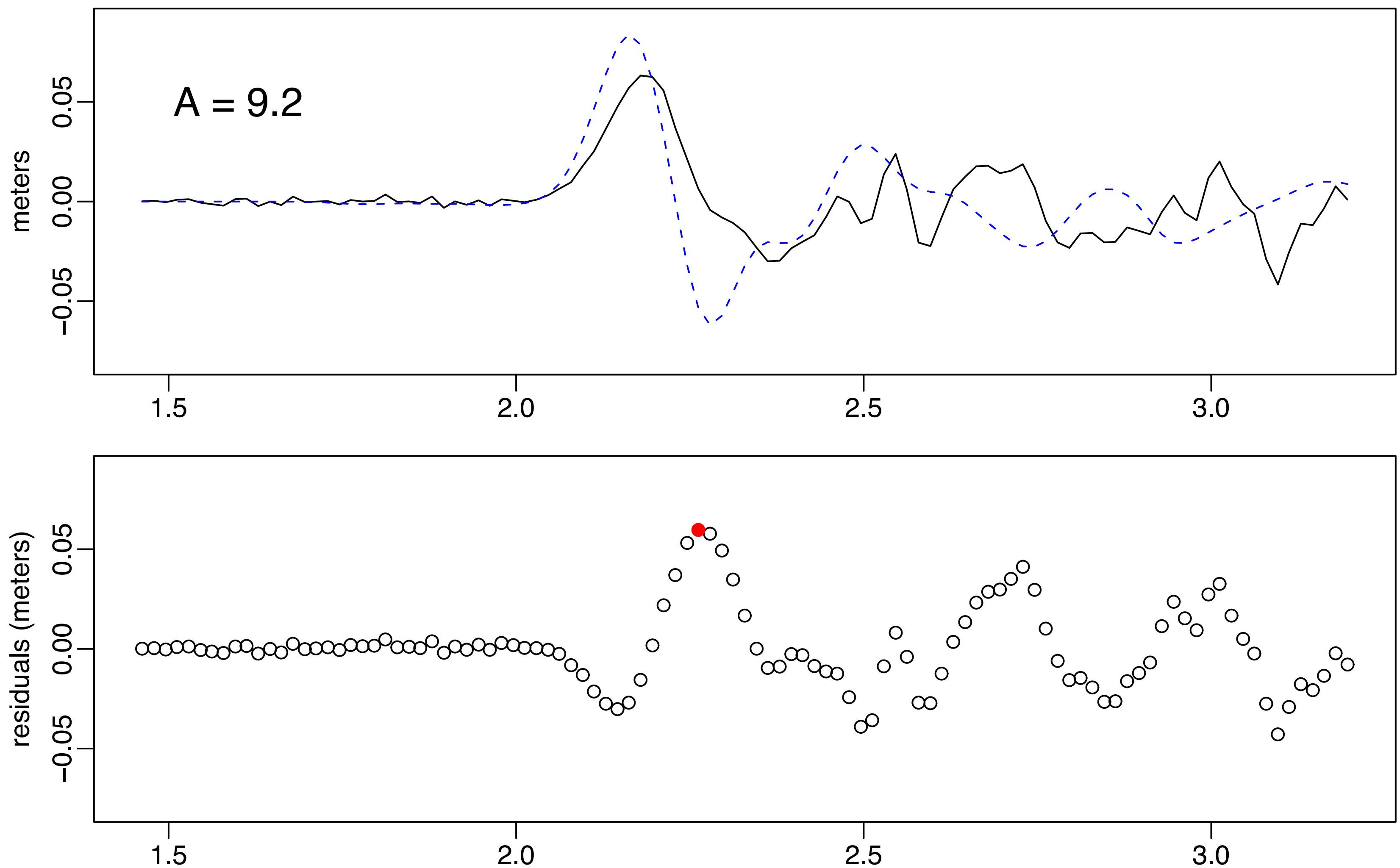

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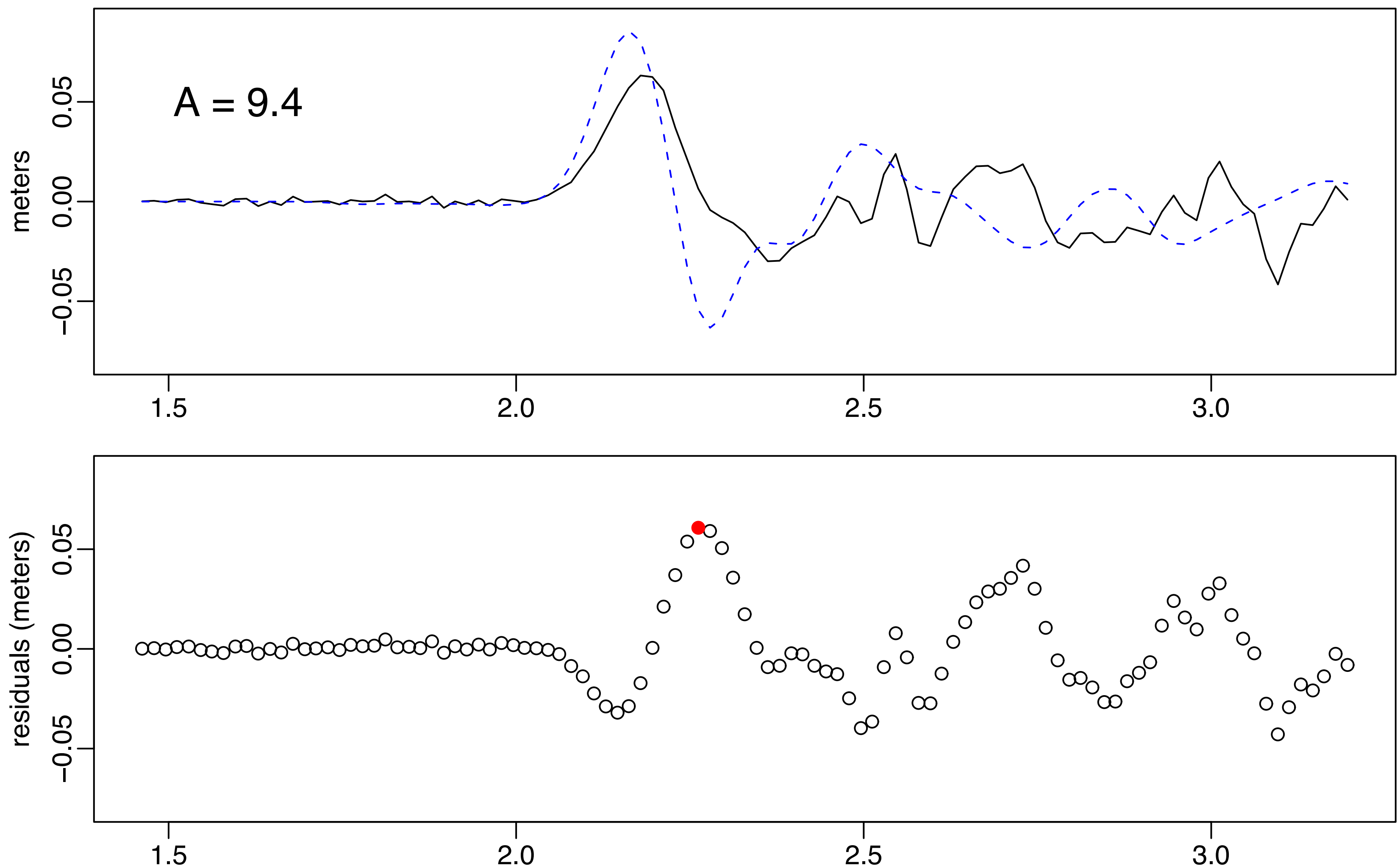

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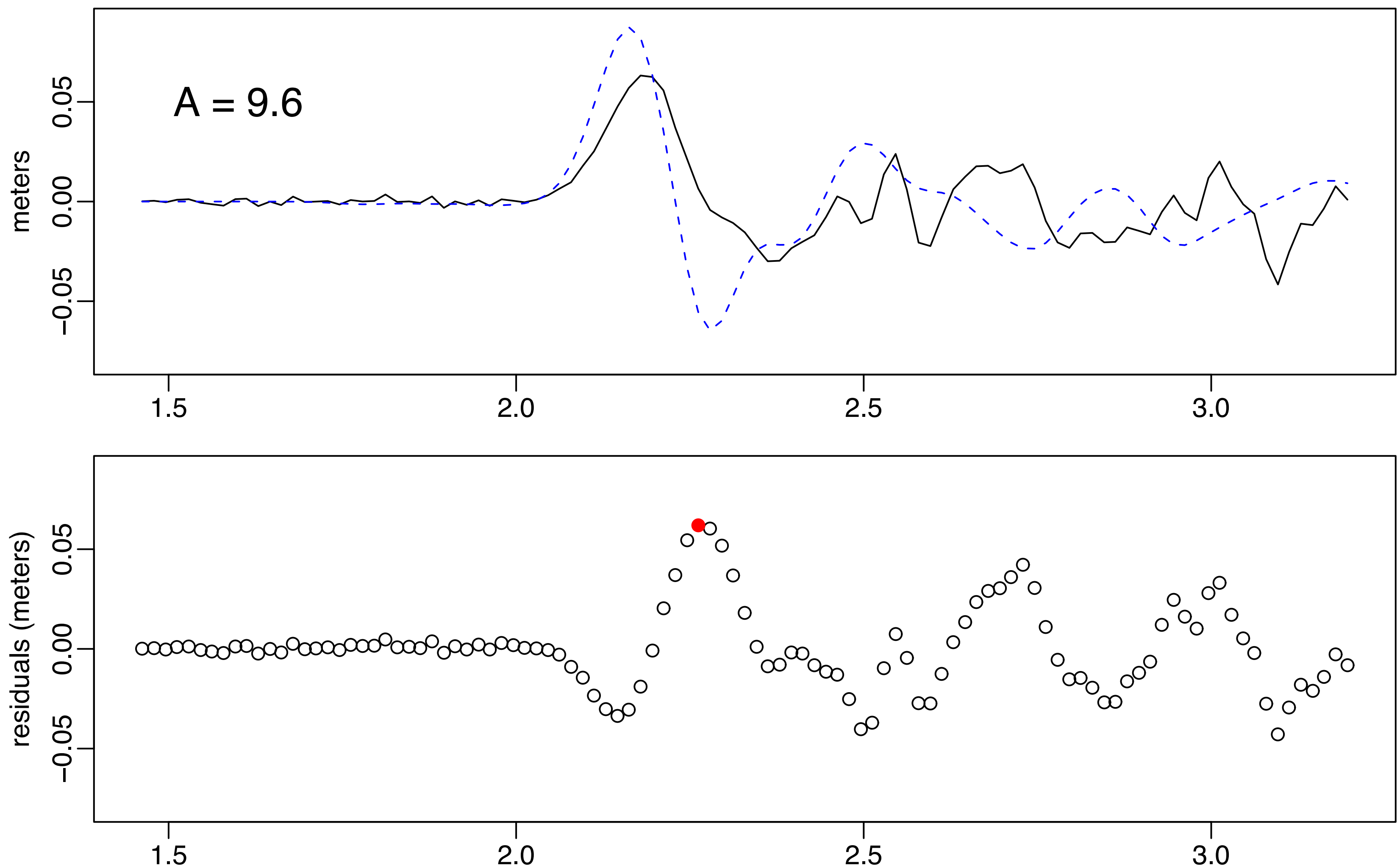

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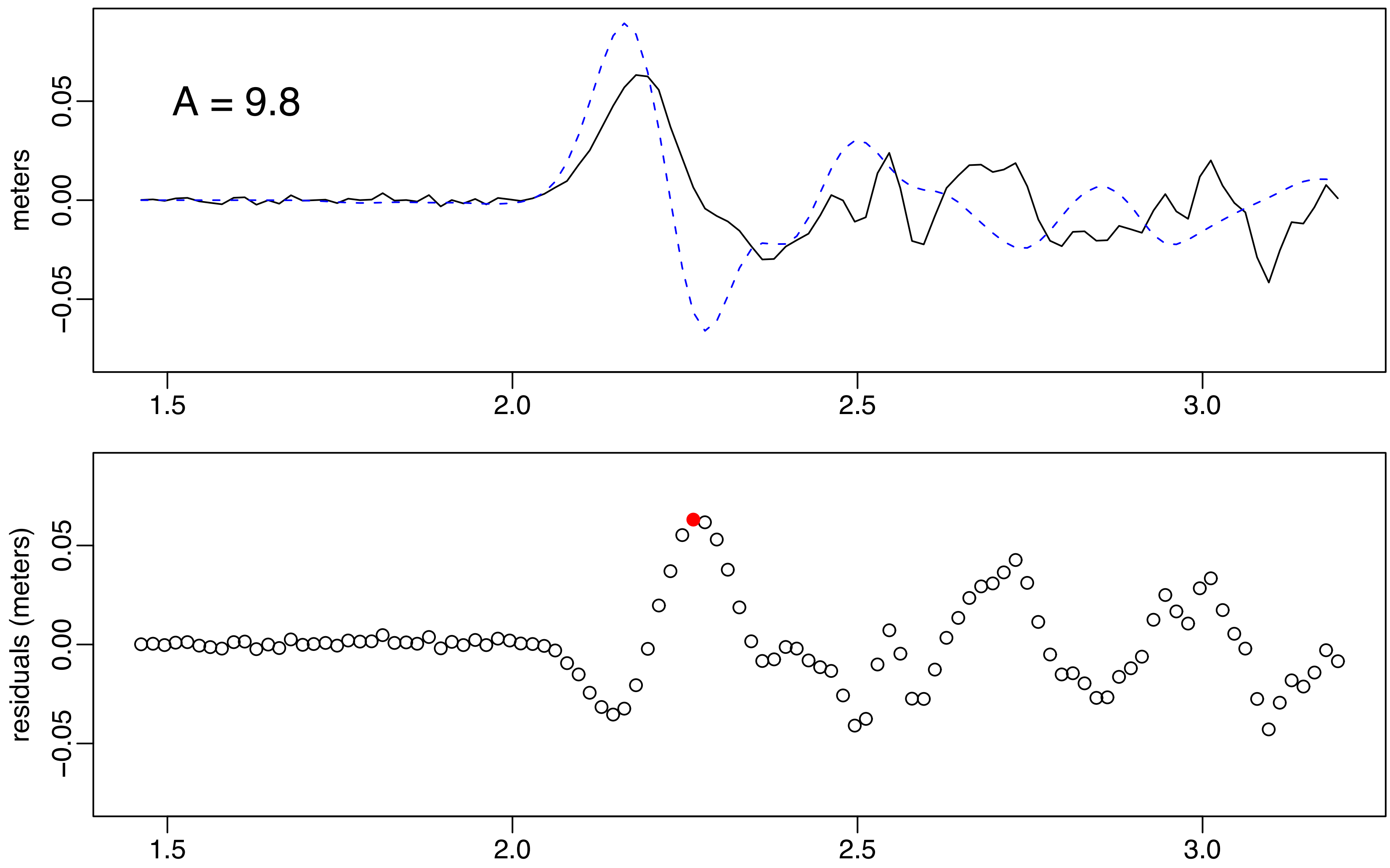

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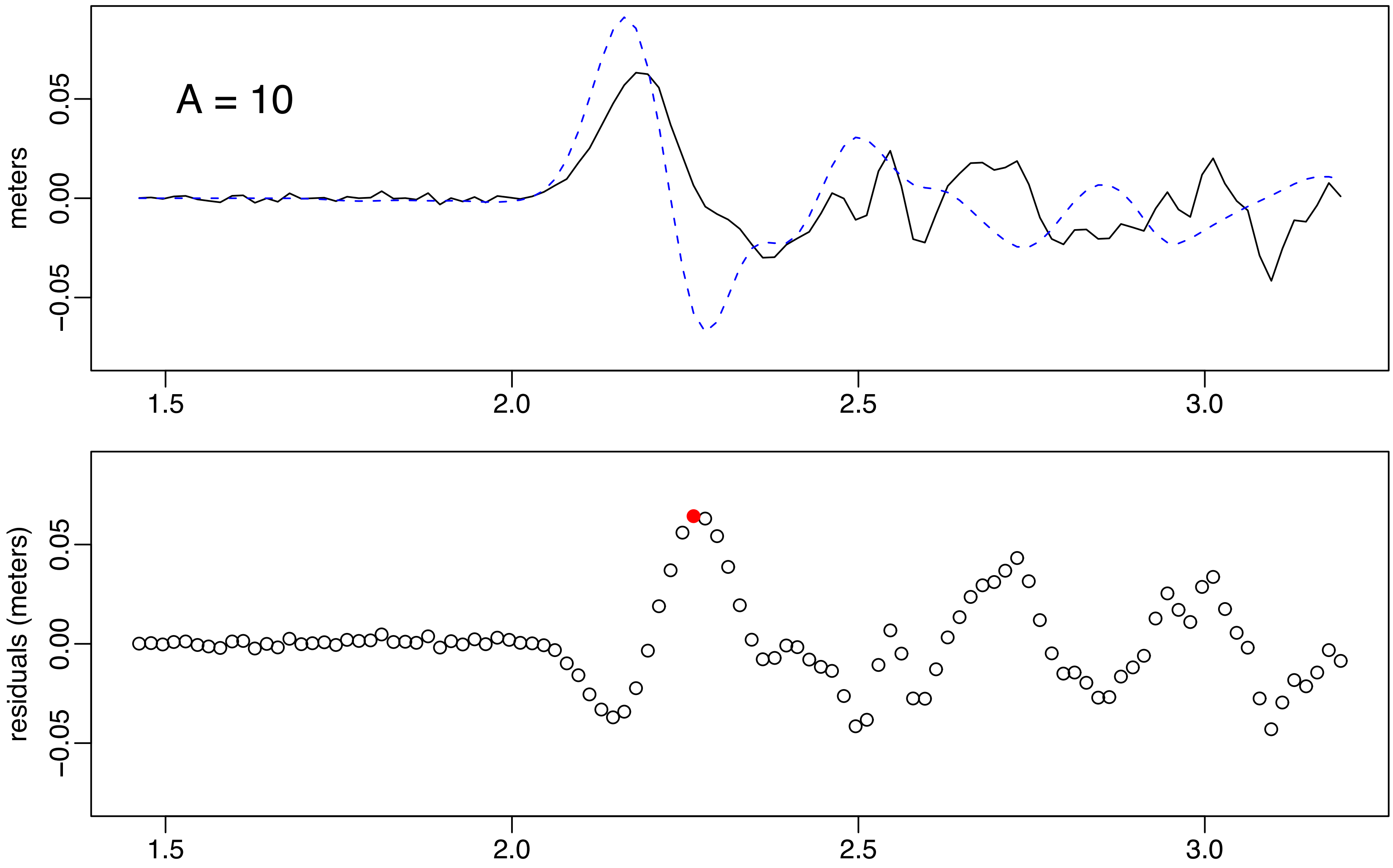


Fitting Models to DART Buoy Data: III

- Q: what is the 'best' choice for A?
- set A such that residuals e_t are 'small' by some measure
- many measures are possible – here are three common ones:
  - make sum of squared residuals as small as possible:

$$\sum_t e_t^2 = \sum_t [x_t - A \cdot g(t)]^2 \equiv f_2(A)$$

  - make sum of magnitudes of residuals as small as possible:

$$\sum_t |e_t| = \sum_t |x_t - A \cdot g(t)| \equiv f_1(A)$$

  - make largest magnitude of residuals as small as possible:

$$\max_t |e_t| = \max_t |x_t - A \cdot g(t)| \equiv f_\infty(A)$$

Fitting Models to DART Buoy Data: IV

- here is a specialized one:
  - make sum of squared residuals at peak and trough as small as possible:

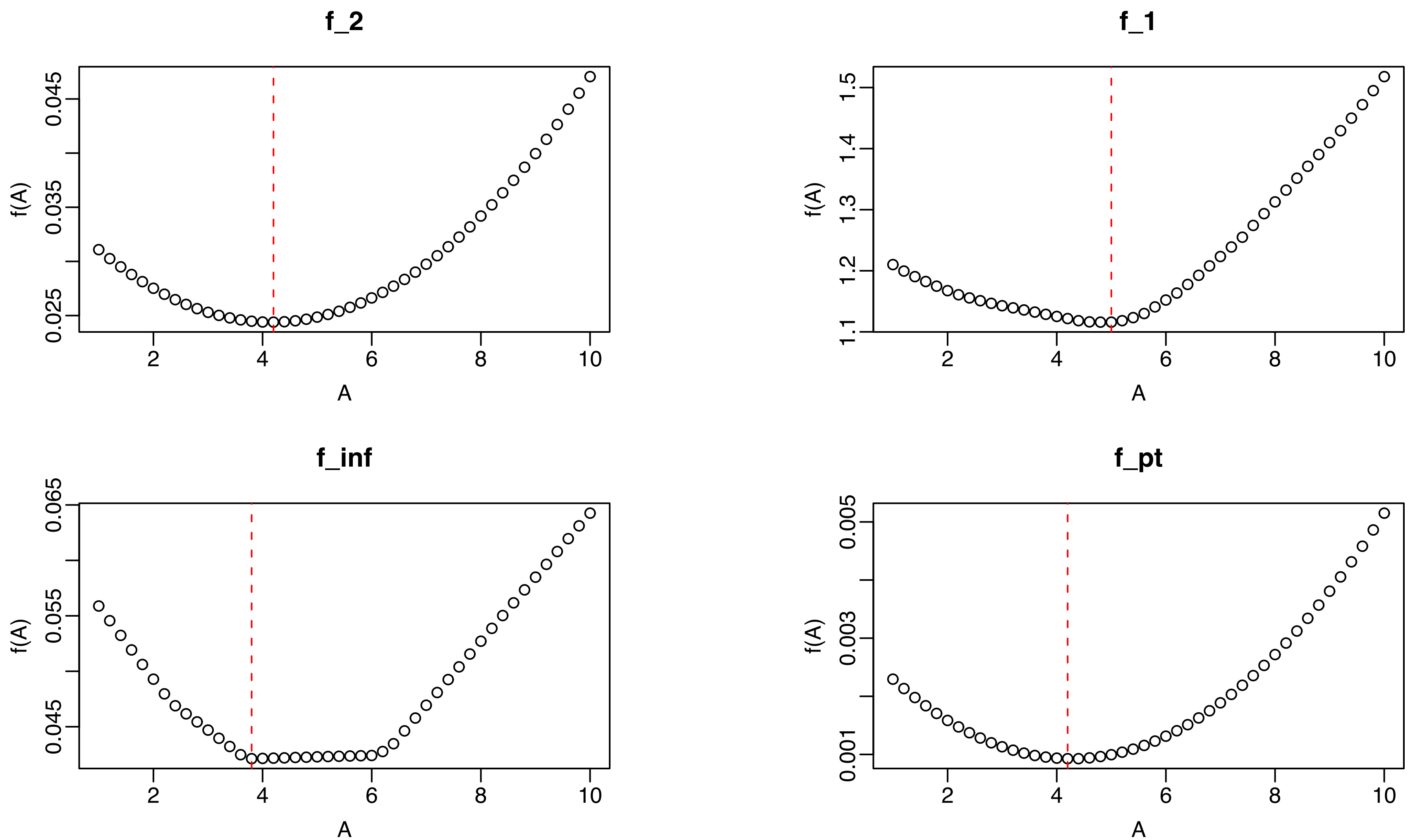
$$e_{t_0}^2 + e_{t_1}^2 = \left[x_{t_0} - A \cdot g(t_0)\right]^2 + \left[x_{t_1} - A \cdot g(t_1)\right]^2 \equiv f_{pt}(A),$$

  where t_0 and t_1 are such that

$$g(t_0) = \max_t \{g(t)\} \text{ and } g(t_1) = \min_t \{g(t)\}$$

- let's look at plots of $f(A)$ versus A for the four measures, where, as before, $A = 1.0, 1.2, 1.4, \ldots, 10.0$ (for explanation of 'bath-tub' appearance of $f_\infty(A)$ vs. A, study evolution of red dots on plots of residuals)

Four Residual Measures $f(A)$ versus Slip A



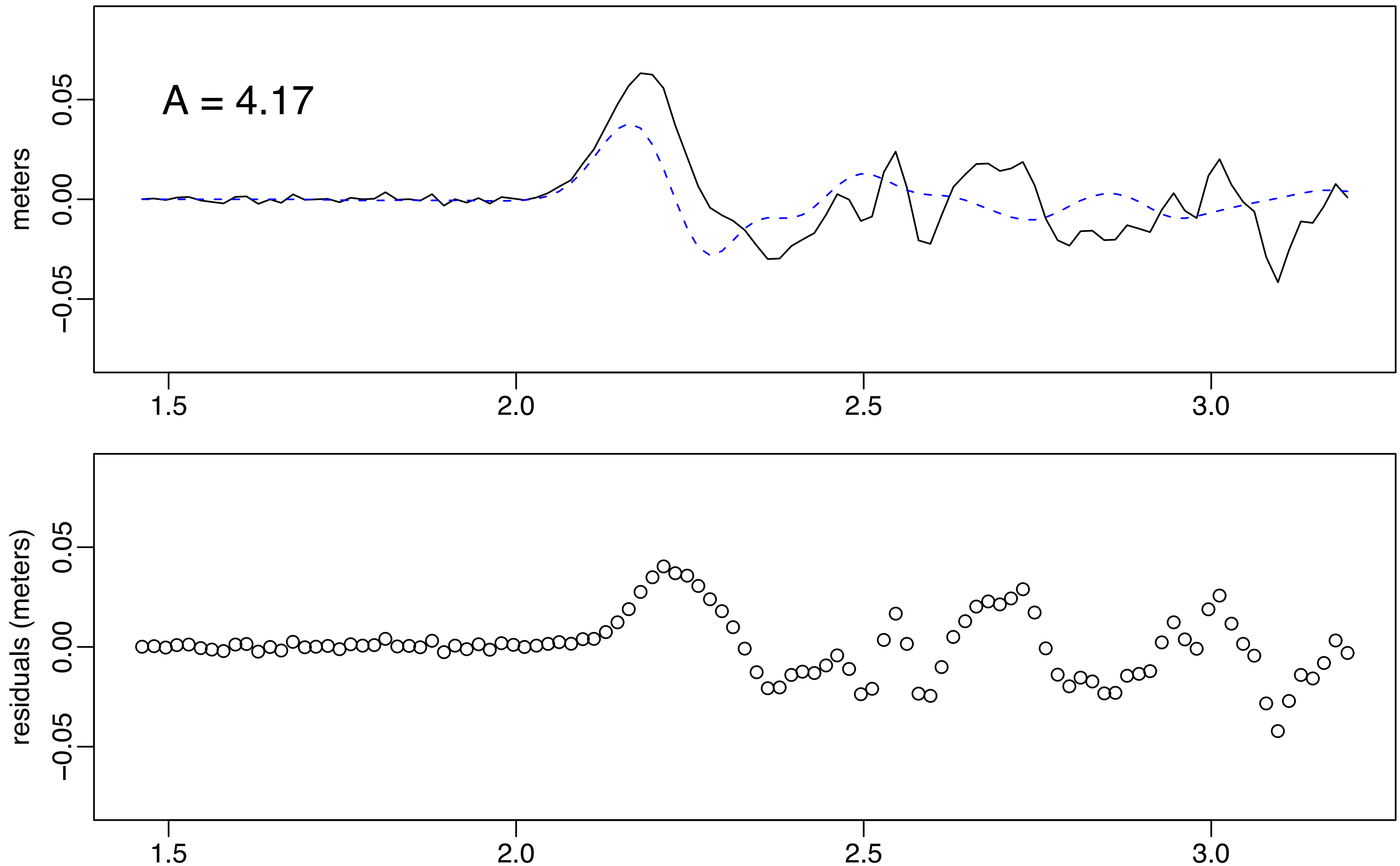
Fitting Models to DART Buoy Data: V

- estimated slips are $\hat{A}_2 = 4.2$, $\hat{A}_1 = 5$, $\hat{A}_\infty = 3.8$ and $\hat{A}_{pt} = 4.2$
- two measures based on least squares, i.e., $f_2(A)$ and $f_{pt}(A)$, have certain advantages, including:
  - no need to do grid search because location of minimum of
$$f(A) \equiv \sum_t \left[x_t - A \cdot g(t)\right]^2$$
    is given by a simple formula:
$$\hat{A}_{ls} = \frac{\sum_t x_t g(t)}{\sum_t \left[g(t)\right]^2}, \quad \text{here yielding } \hat{A}_2 \doteq 4.17 \text{ and } \hat{A}_{pt} \doteq 4.26$$
    (follows from taking equation $f'(A) = 0$ and solving for A)
  - statistical variation in $\hat{A}_{ls}$ easy to quantify

Assessing Variability in $\hat{A}_{ls}$: I

- reformulate model $x_t = A \cdot g(t) + e_t$ in vector notation as $\mathbf{x} = A\mathbf{g} + \mathbf{e}$, where $\mathbf{x}$ is column vector containing the x_t's etc.
- least squares estimate of A is $\hat{A}_{ls} = \mathbf{g}^T\mathbf{x}/\mathbf{g}^T\mathbf{g}$
- need to consider statistical properties of residuals e_t
- if residuals were Gaussian (normally) distributed and uncorrelated with a common variance σ_e^2, then $\hat{A}_{ls}$ is Gaussian distributed with mean A and variance $\sigma_e^2/\mathbf{g}^T\mathbf{g}$
- allows us to compute standard deviations (SDs) and to write $\hat{A}_2 = 4.17 \pm 0.59$ and $\hat{A}_{pt} = 4.26 \pm 2.69$ (note size of SDs)
- assumptions of uncorrelatedness and common variance are dicey, as can be seen from plot of residuals associated with $\hat{A}_2$

Least Squares Estimate $\hat{A}_2$ of Slip for a12 & 21414



Assessing Variability in $\hat{A}_{ls}$: II

- assumption of common variance of e_t's not viable for data before first wave, but, because $g(t) = 0$ there, these data have no effect on estimate $\hat{A}_{ls} = \sum_t x_t g(t) / \sum_t [g(t)]^2$
- while assumption of common variance reasonable for data beginning at first wave, assumption of uncorrelatedness is not
- can model correlation using a first order autoregressive process:

$$e_t = \phi e_{t-1} + w_t,$$

  where w_t is Gaussian white noise
- implies that correlation between e_t and $e_{t+\tau}$ given by $\phi^{|\tau|}$
- estimate of ϕ via correlation between e_t & e_{t+1} yields $\hat{\phi} \doteq 0.86$

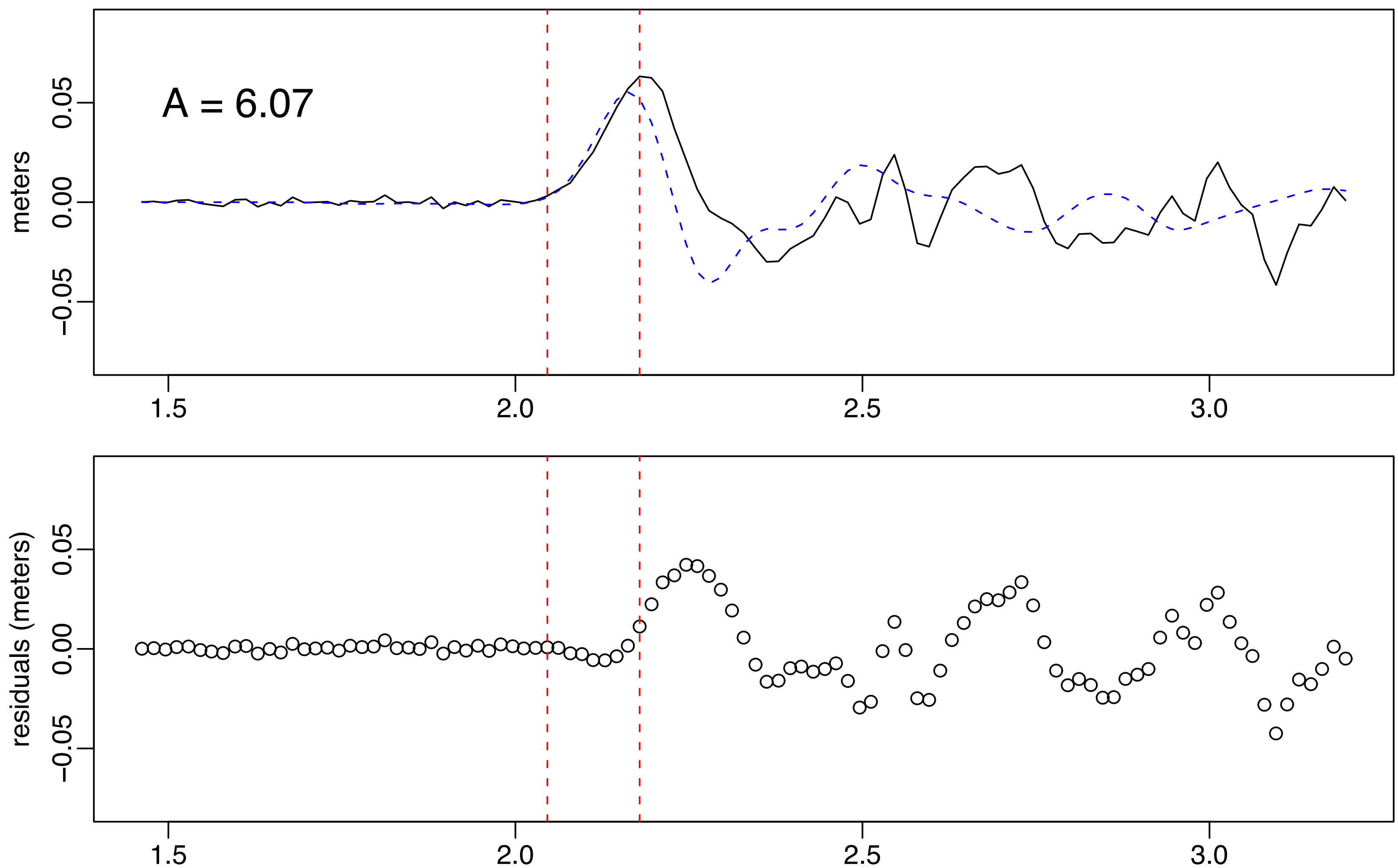
Assessing Variability in $\hat{A}_{ls}$: III

- theory says $\hat{A}_{ls}$ is Gaussian distributed with mean A and variance $\sigma_e^2 \cdot \mathbf{g}^T V \mathbf{g} / (\mathbf{g}^T \mathbf{g})^2$, where V is matrix whose (j, k)th element is $\phi^{|j-k|}$
- yields $\hat{A}_2 = 4.17 \pm 1.33$, which has larger SD than what was obtained under questionable assumptions ($\hat{A}_2 = 4.17 \pm 0.59$)

Least Squares (LS) as Criterion for Estimating Slips

- inversion algorithm uses LS as criterion for estimating slip A
- user must decide amount of DART data to use
- two extremes: all available data or just two data points
- use of more data should yield estimator $\hat{A}$ with smaller variance *if* model is valid over entire range of data
- if model decreases in validity as time increases, should limit data to, say, first quarter wave or first full wave
- real-time constraints also dictate interest in use of limited amount of data
- starting with a quarter wave of data, let's look at LS fits involving varying amounts of data

LS Fit of a12 Model to Selected 21414 Data



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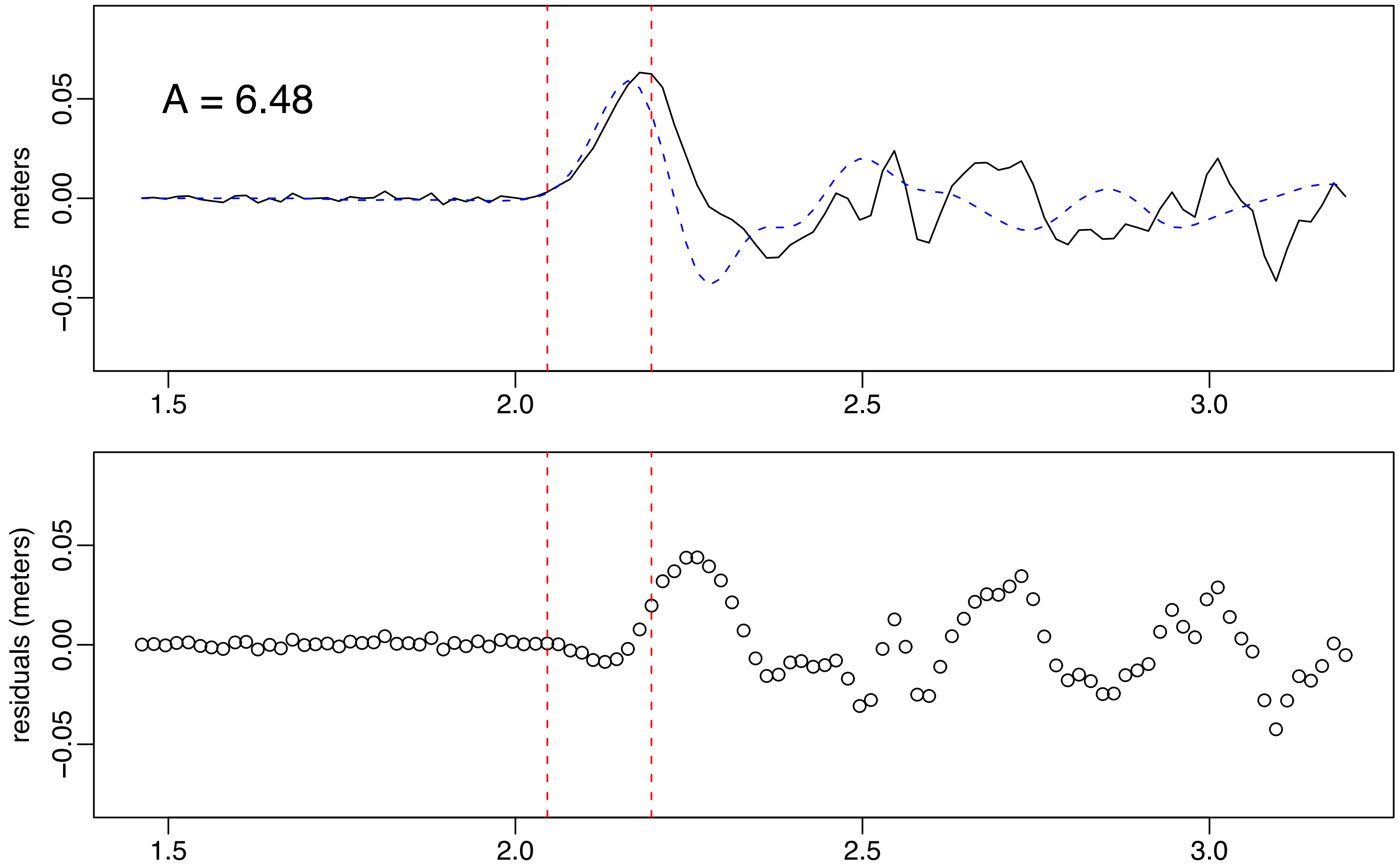

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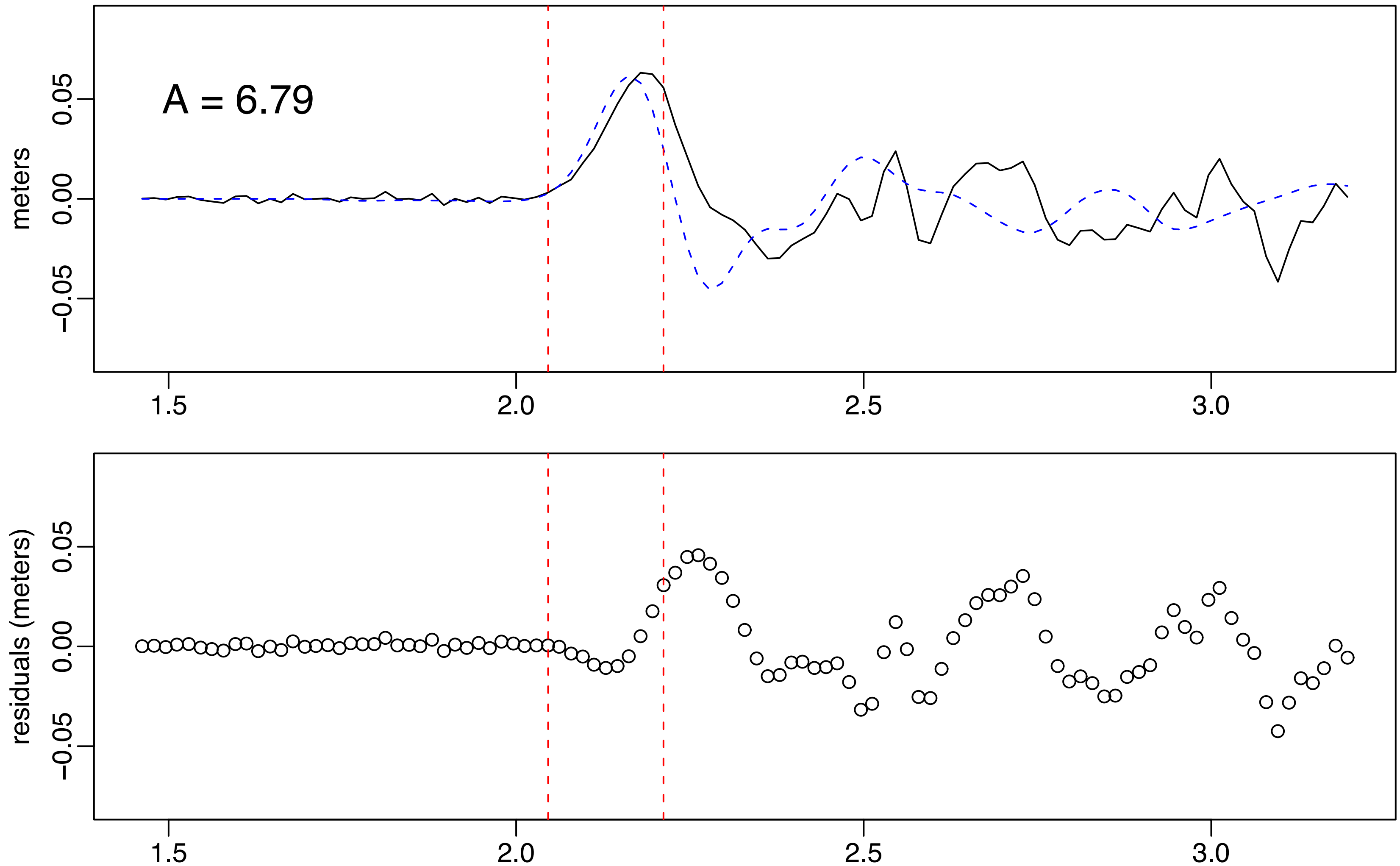

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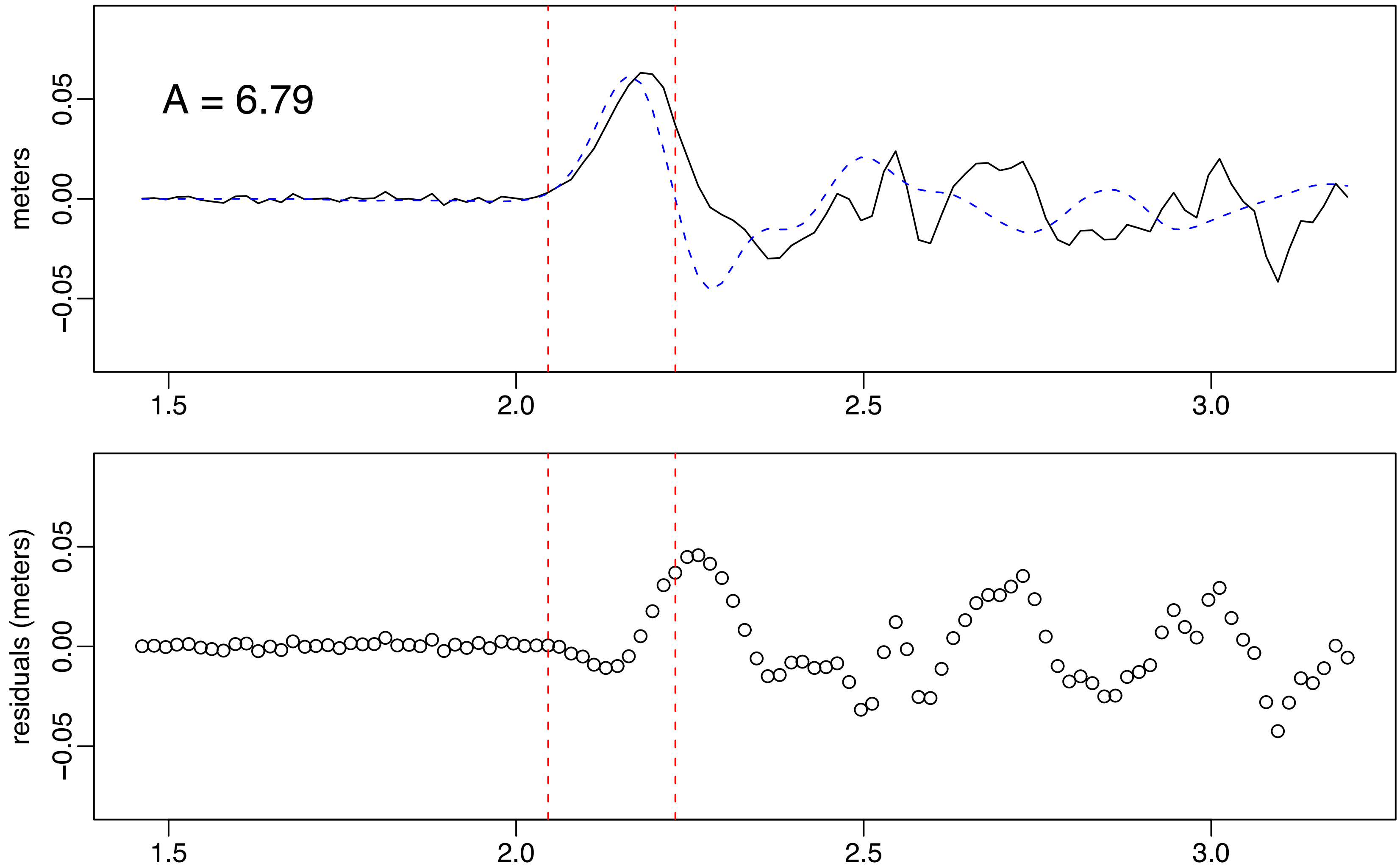

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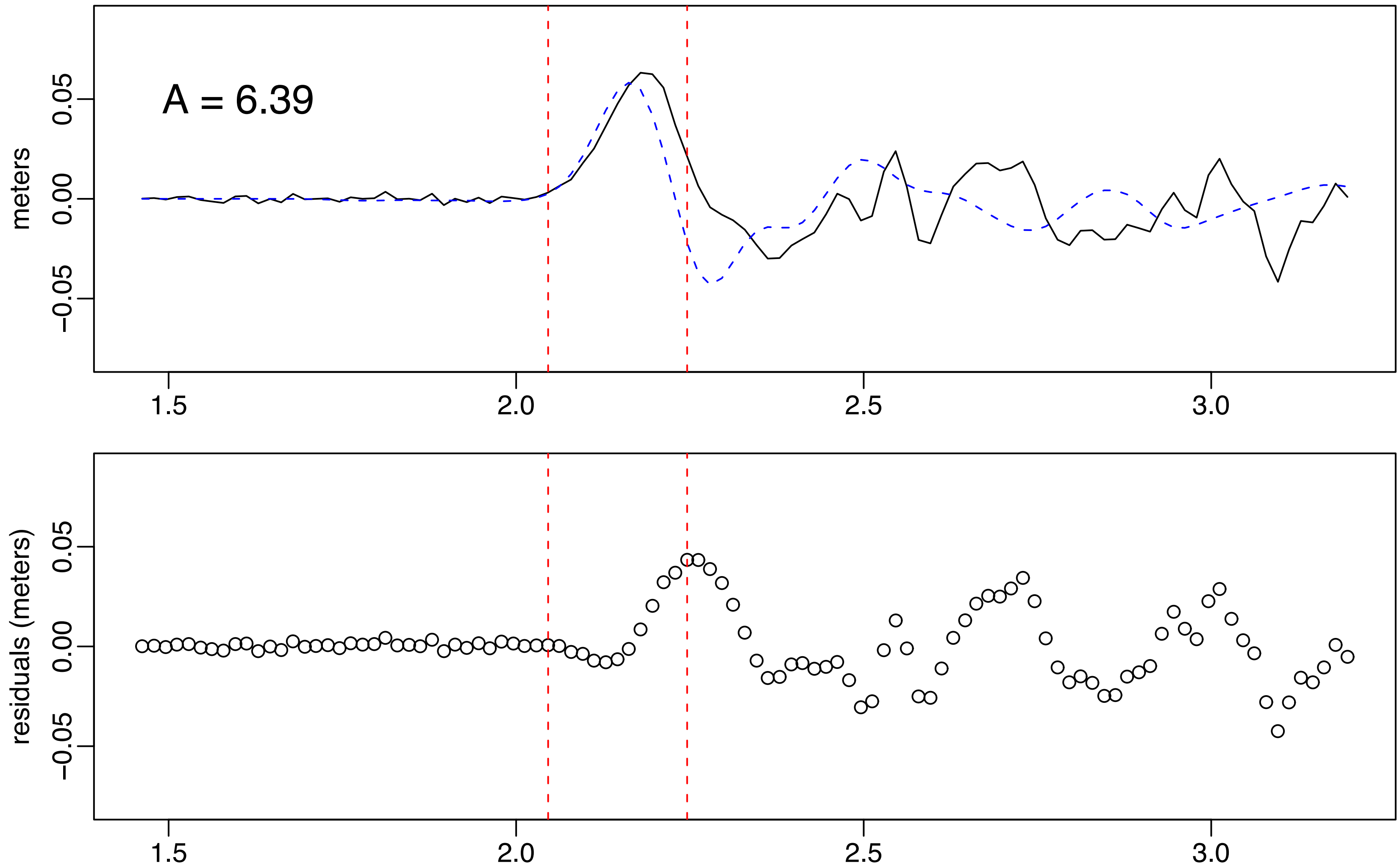

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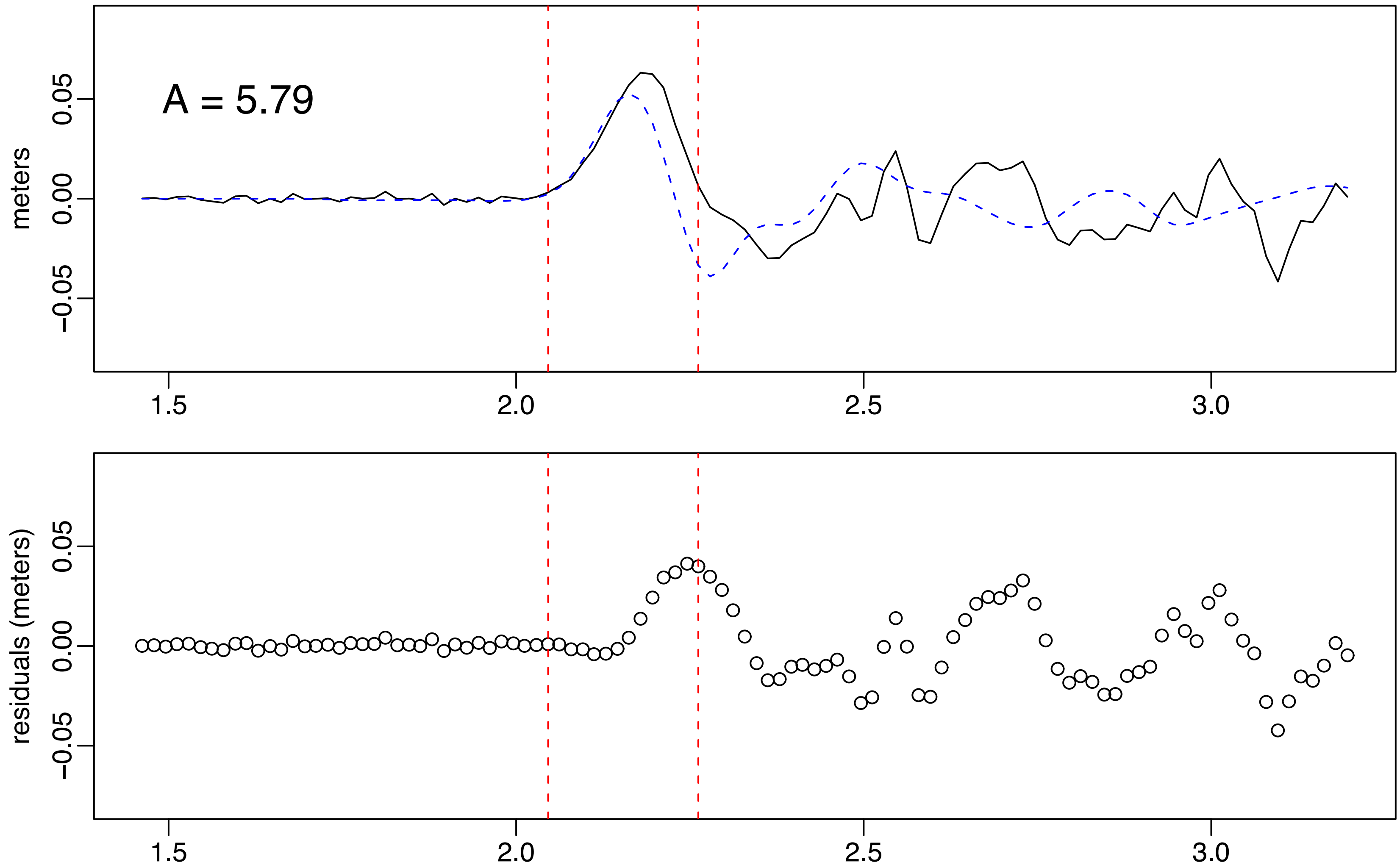

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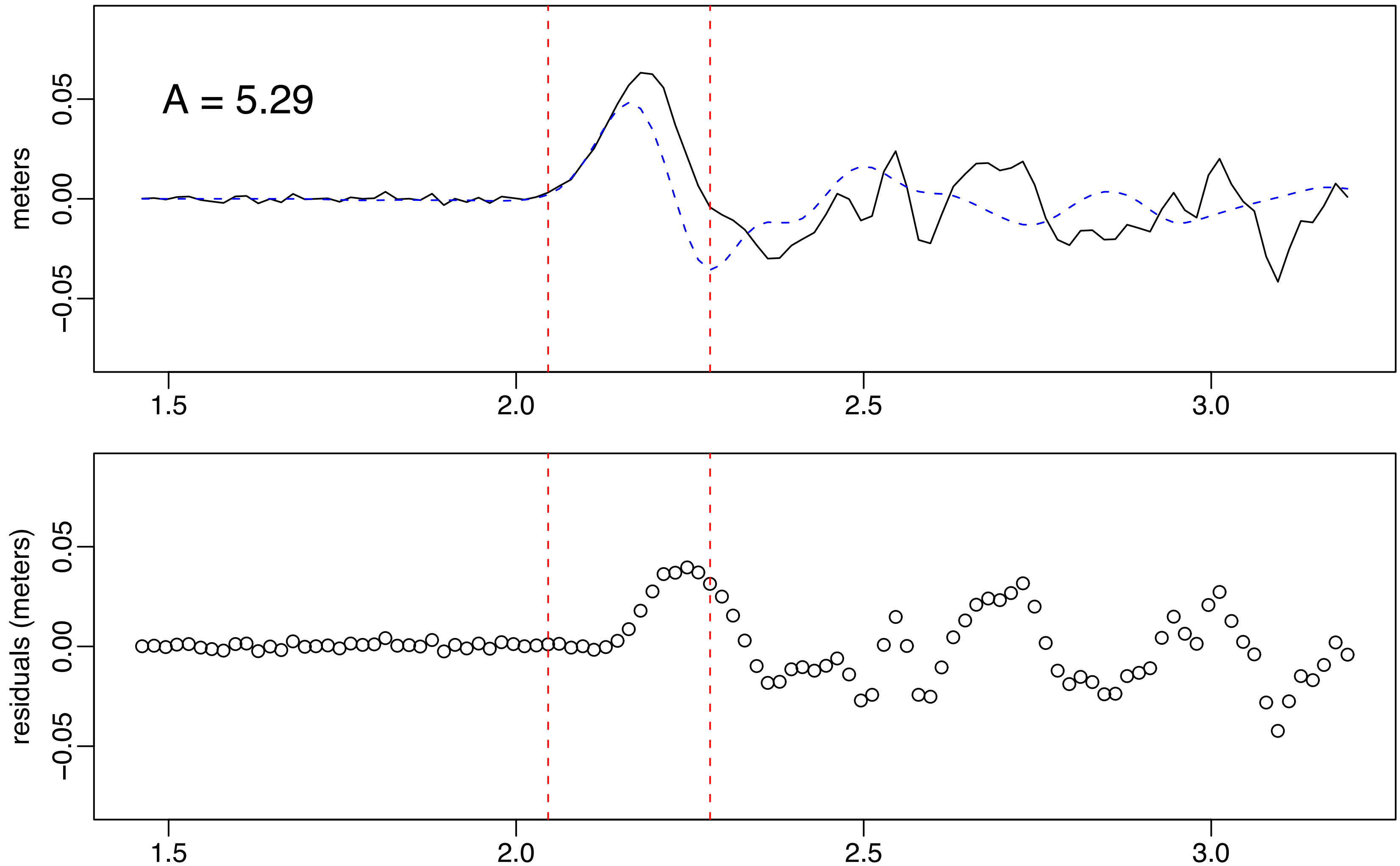

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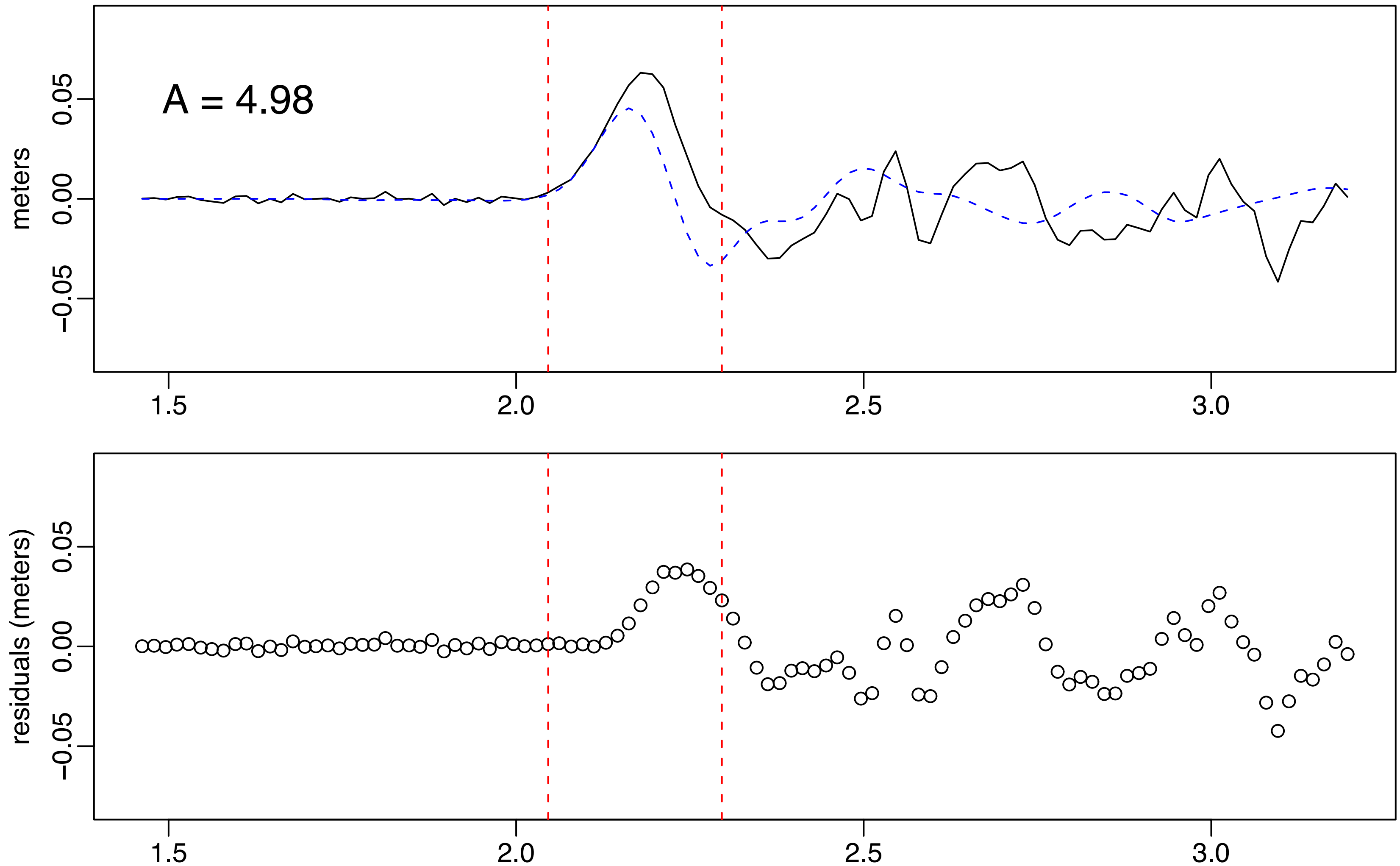

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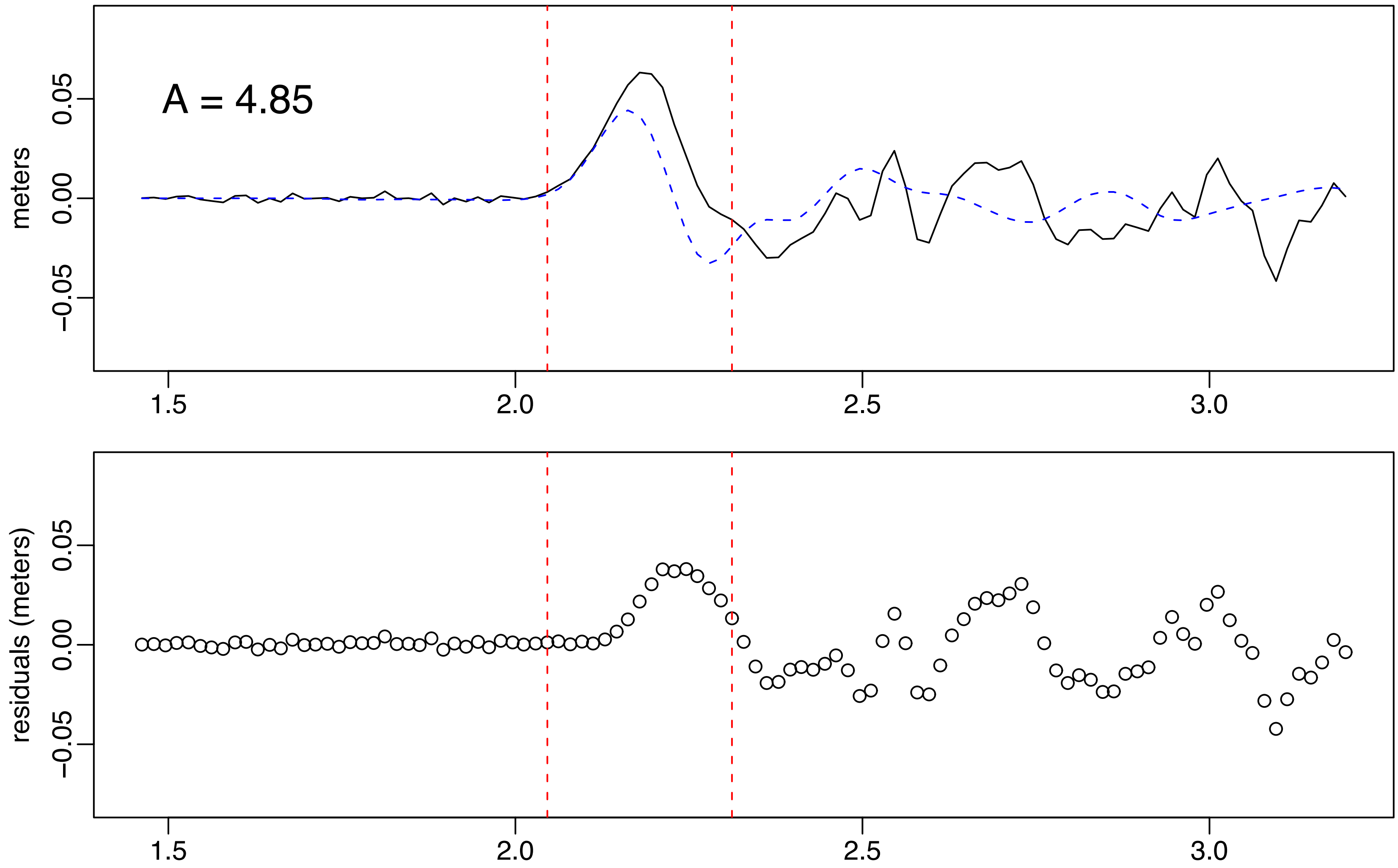

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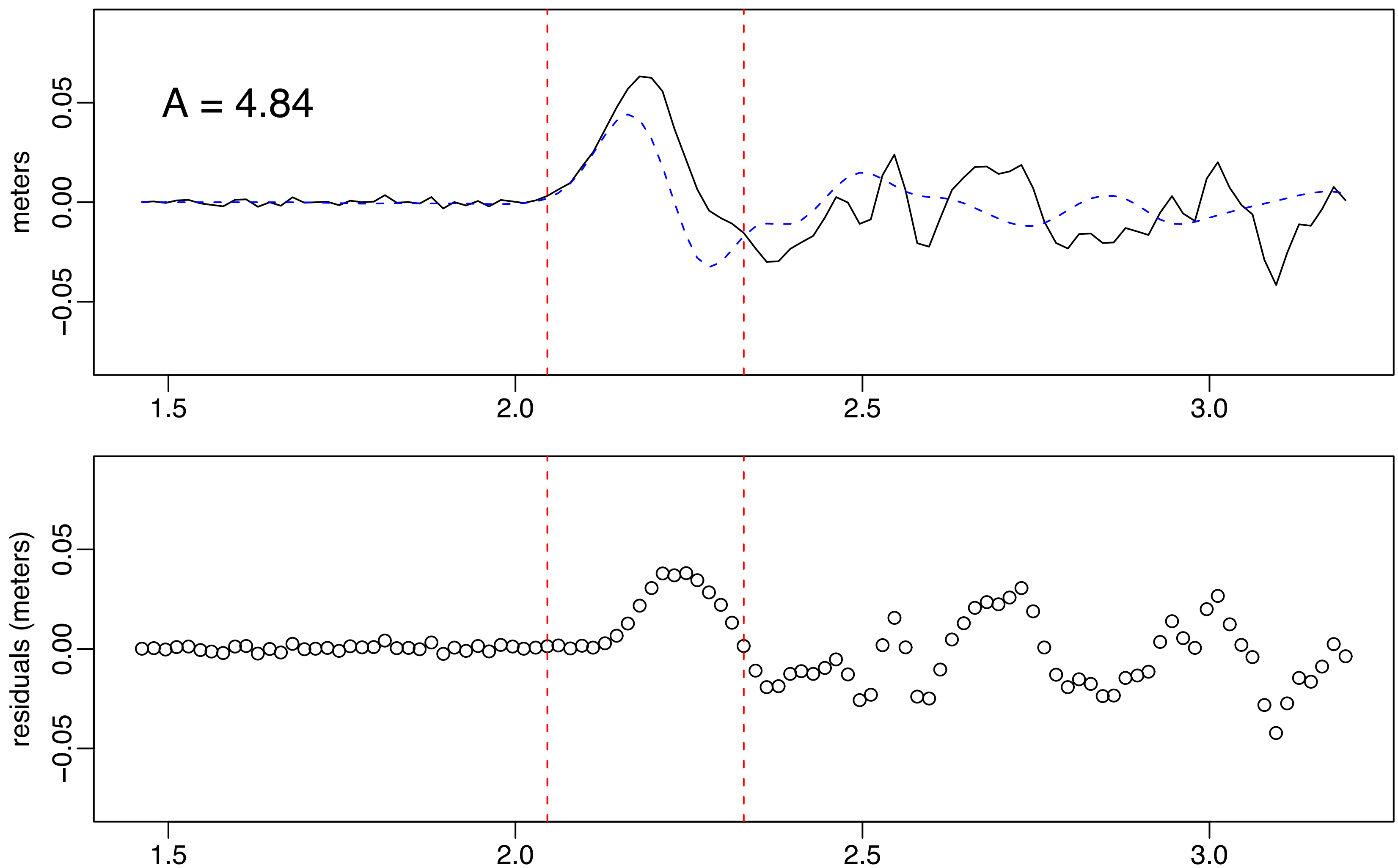

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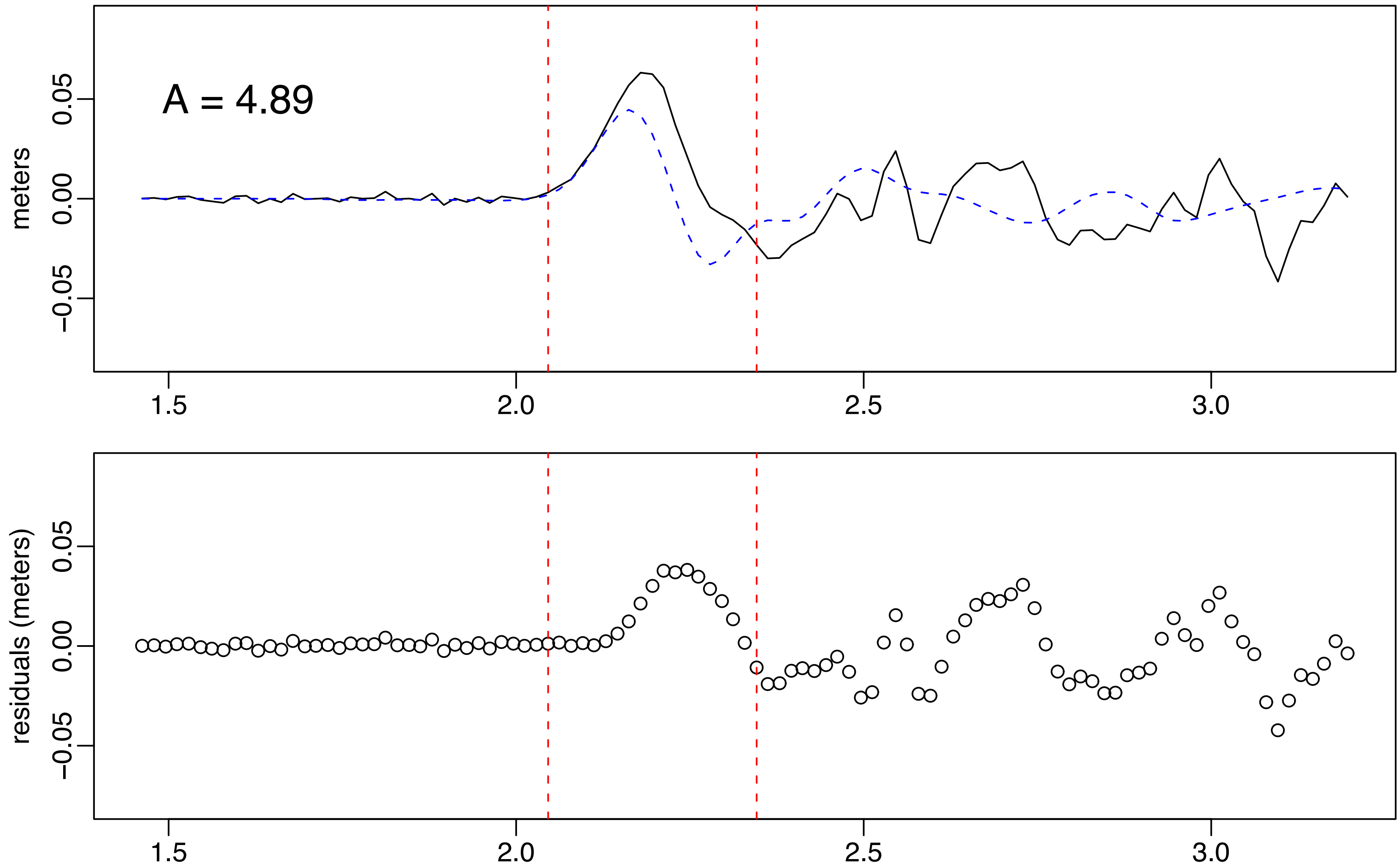

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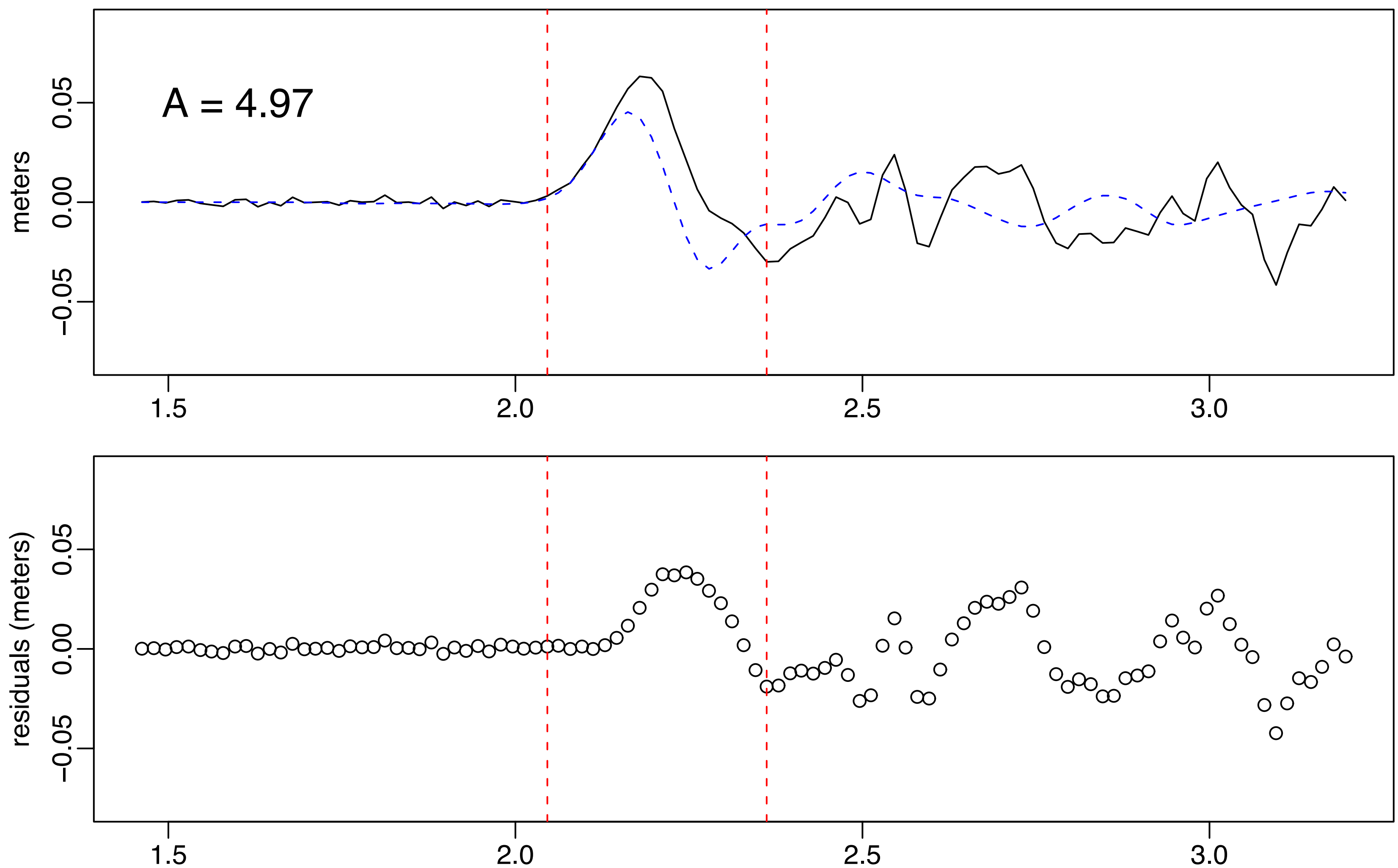

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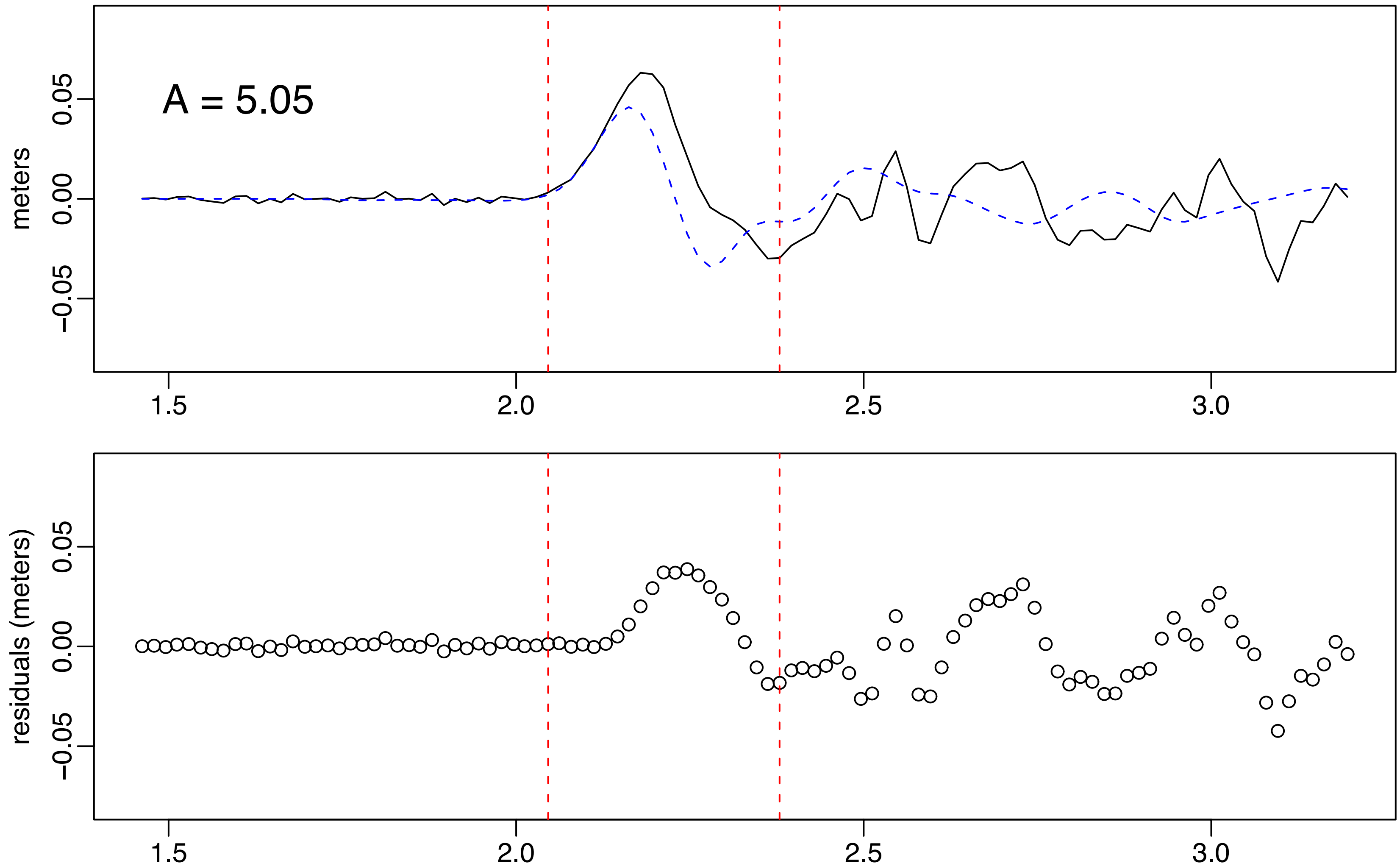

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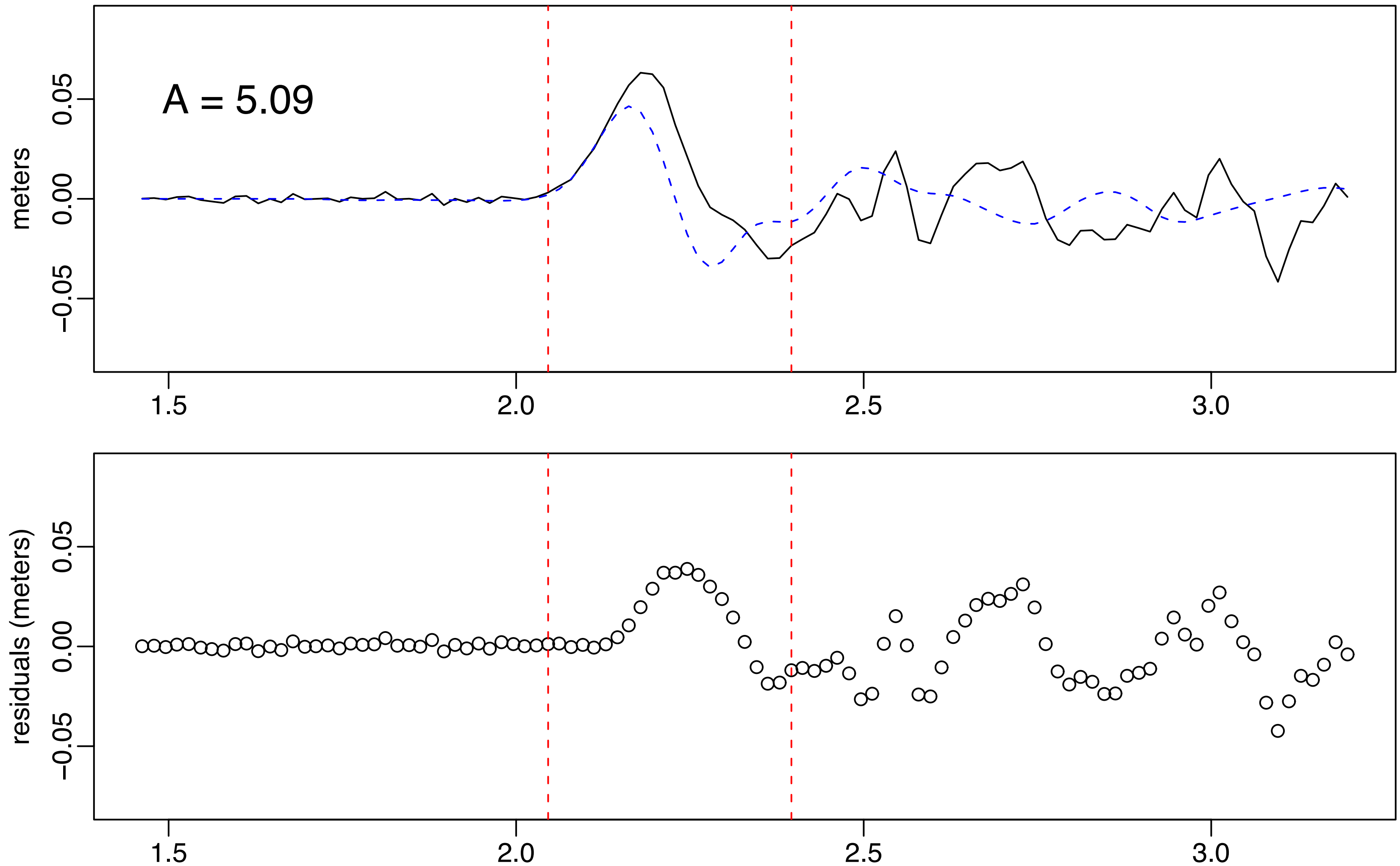

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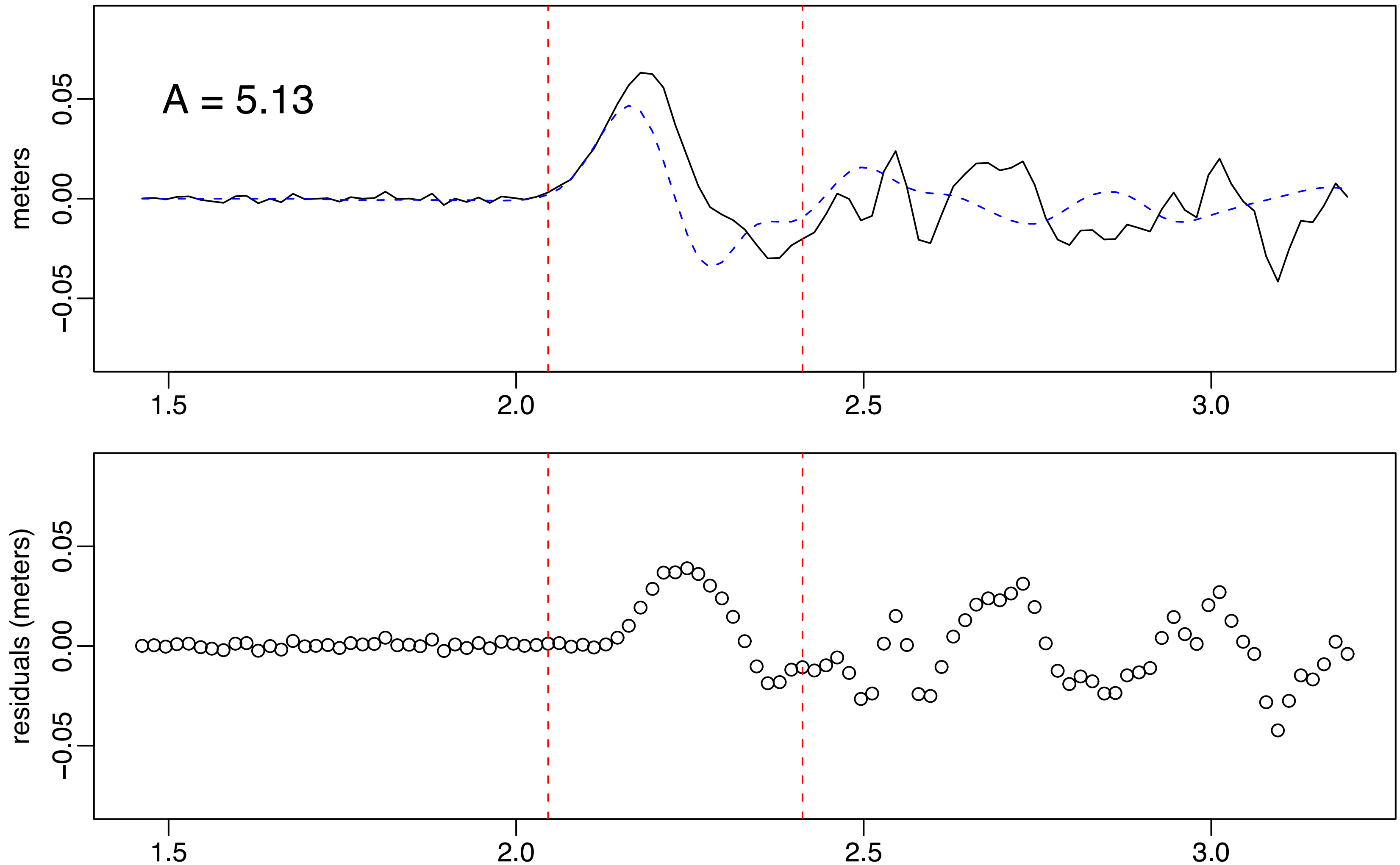

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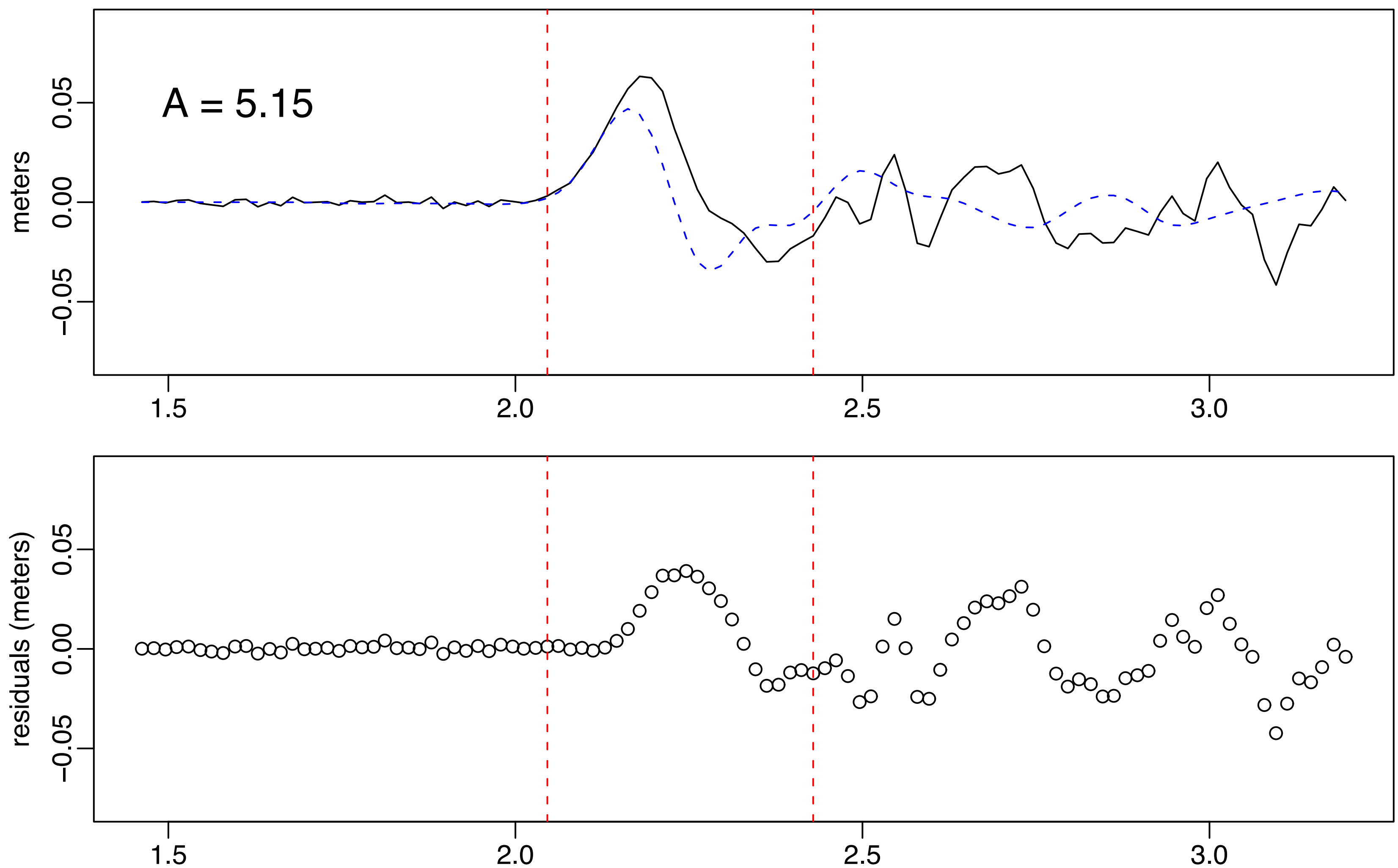

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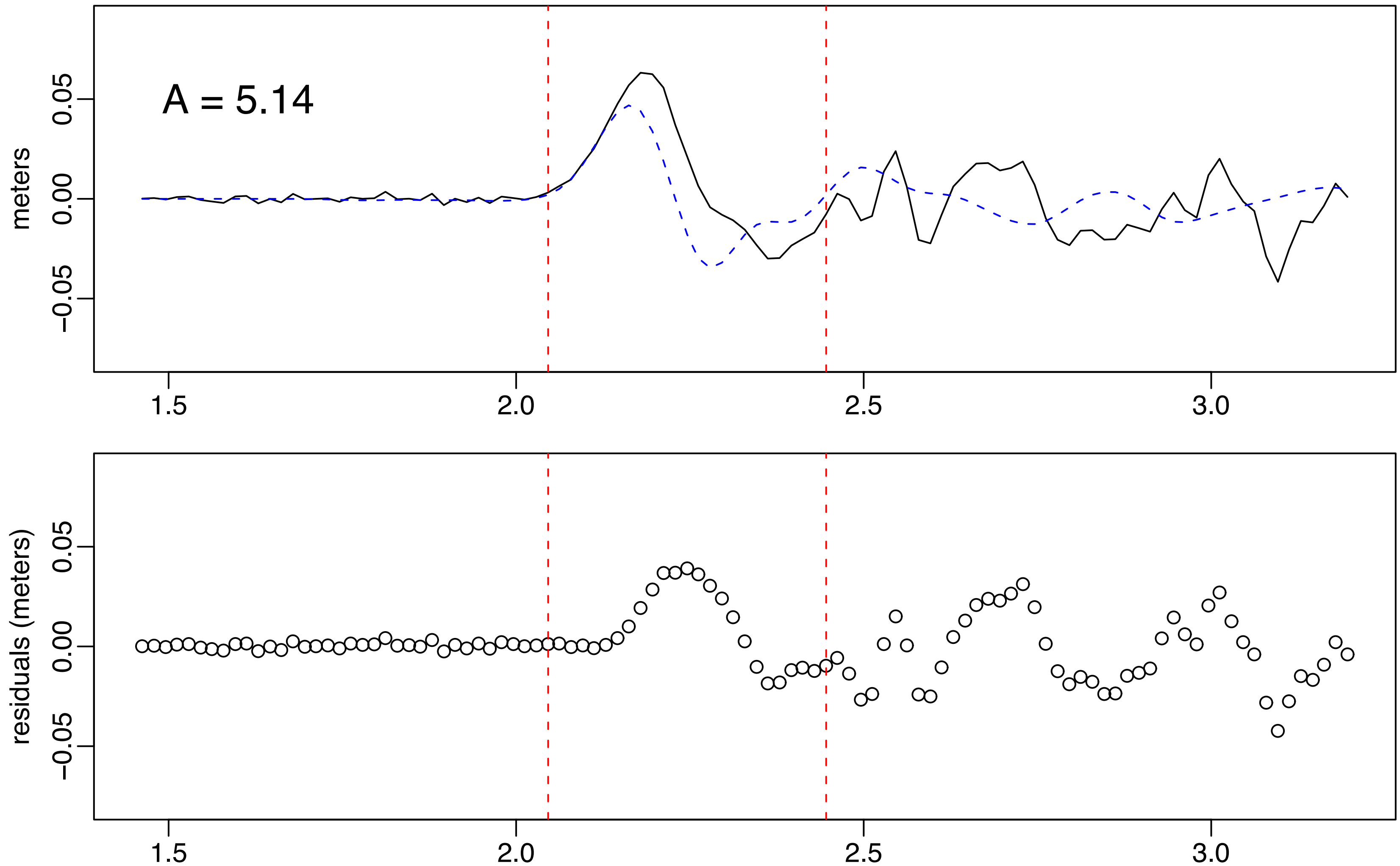

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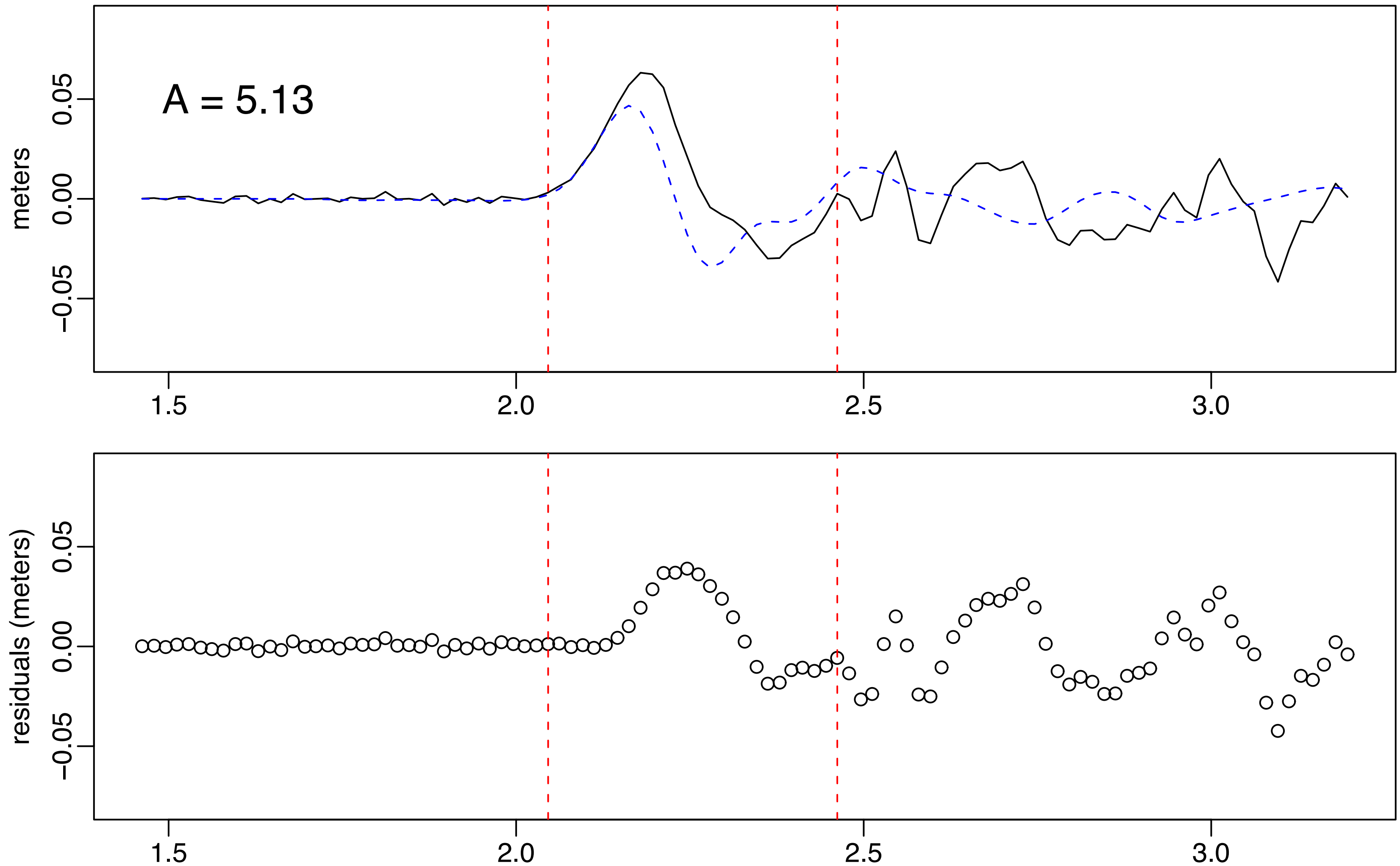

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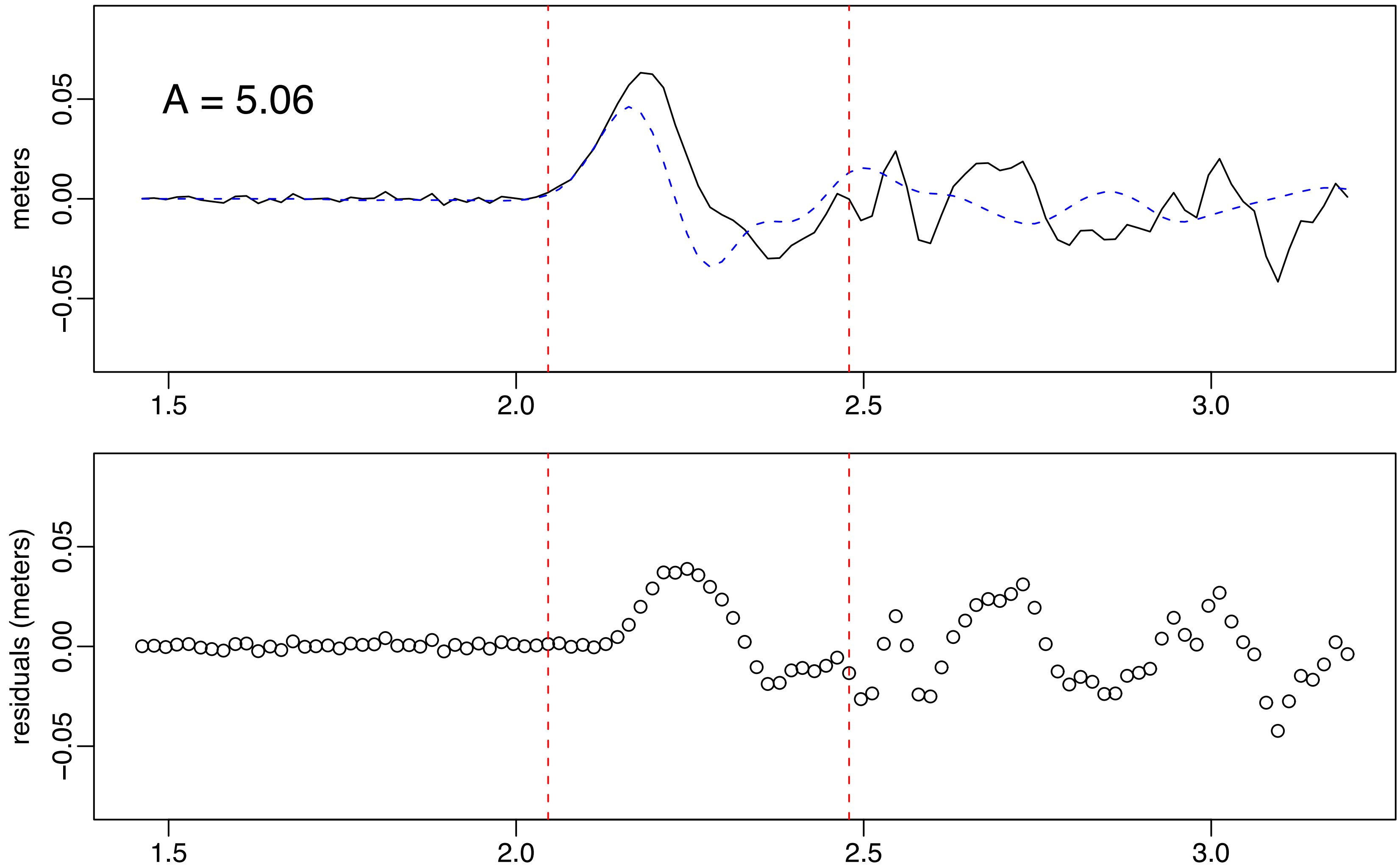

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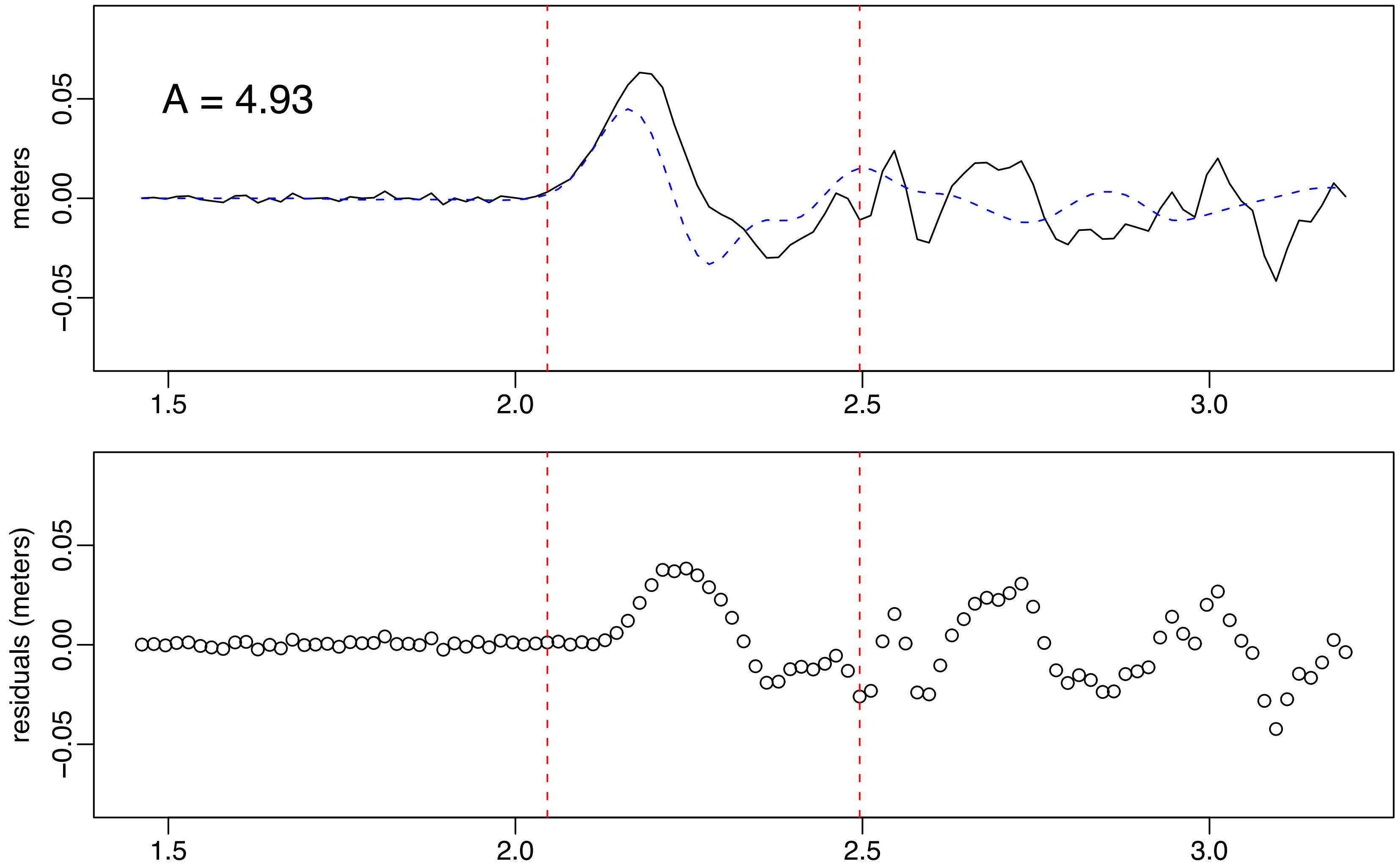

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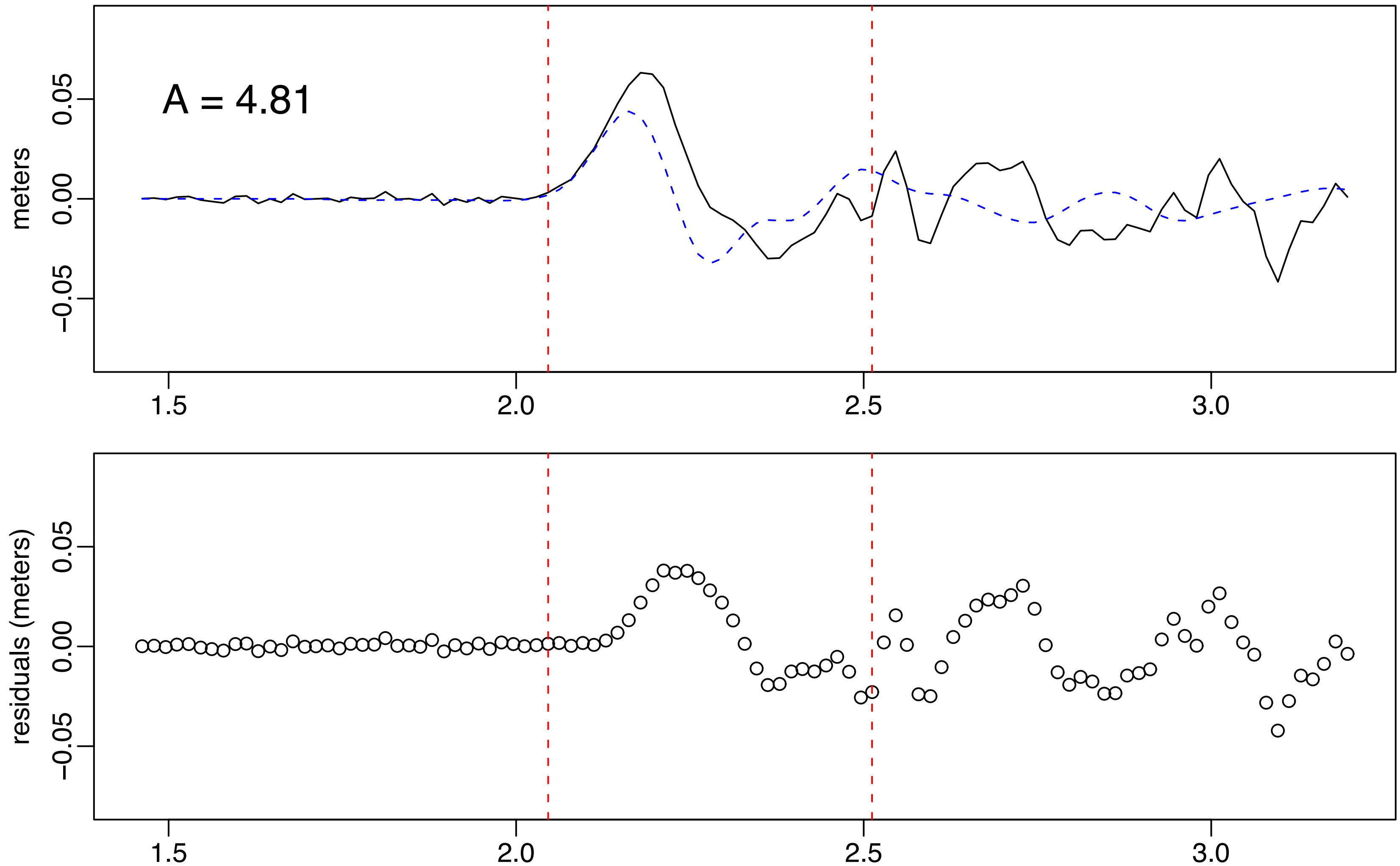

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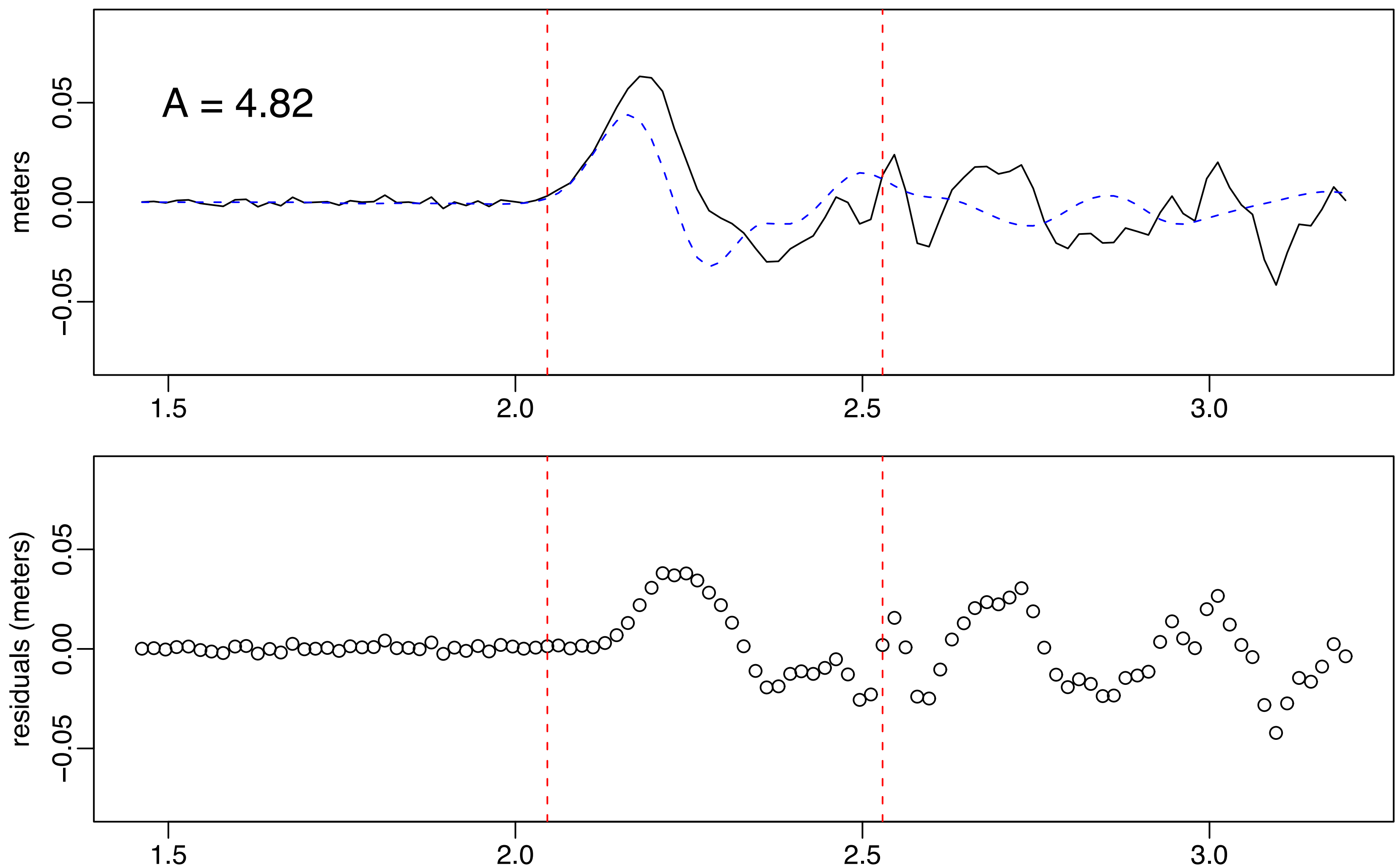

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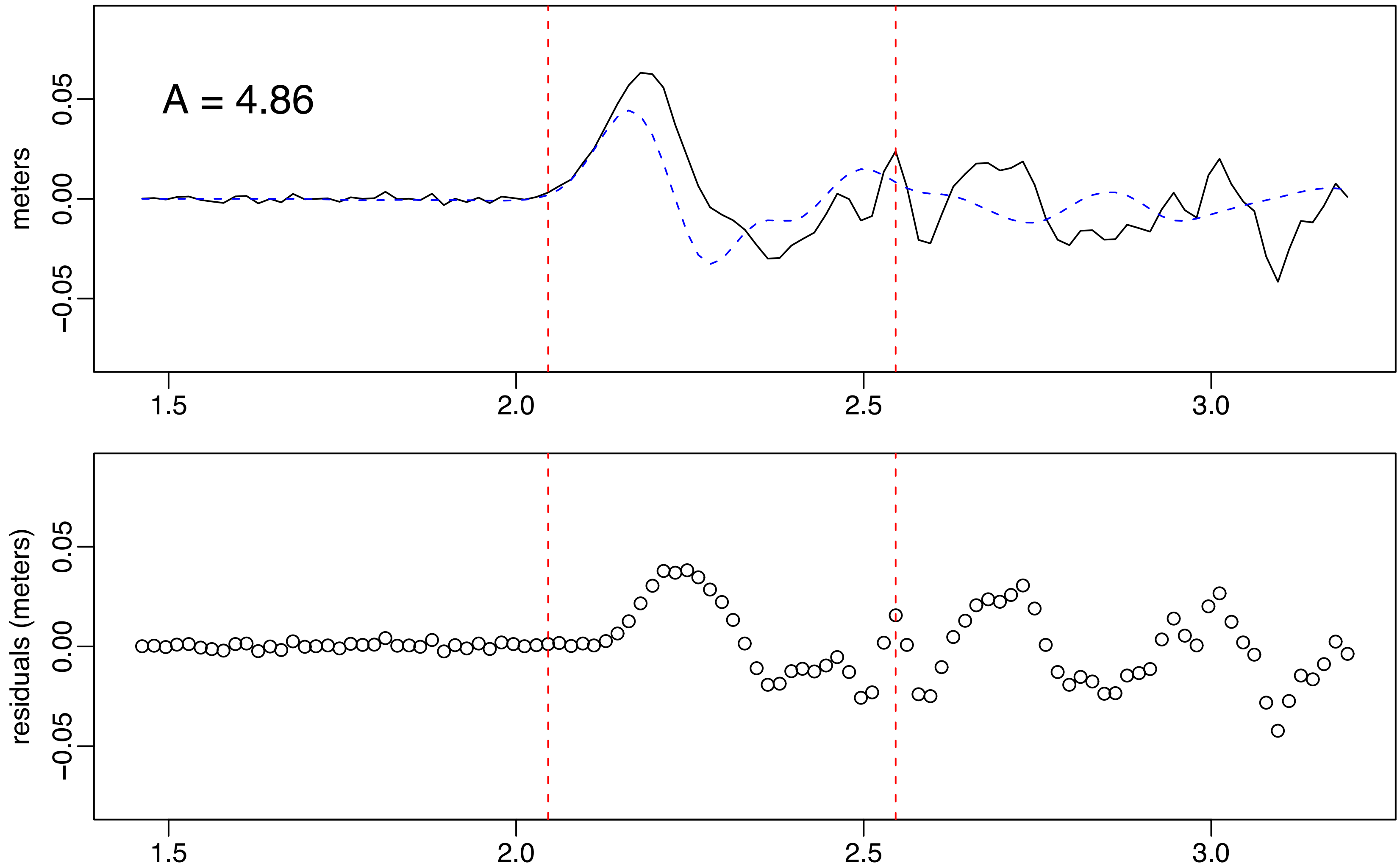

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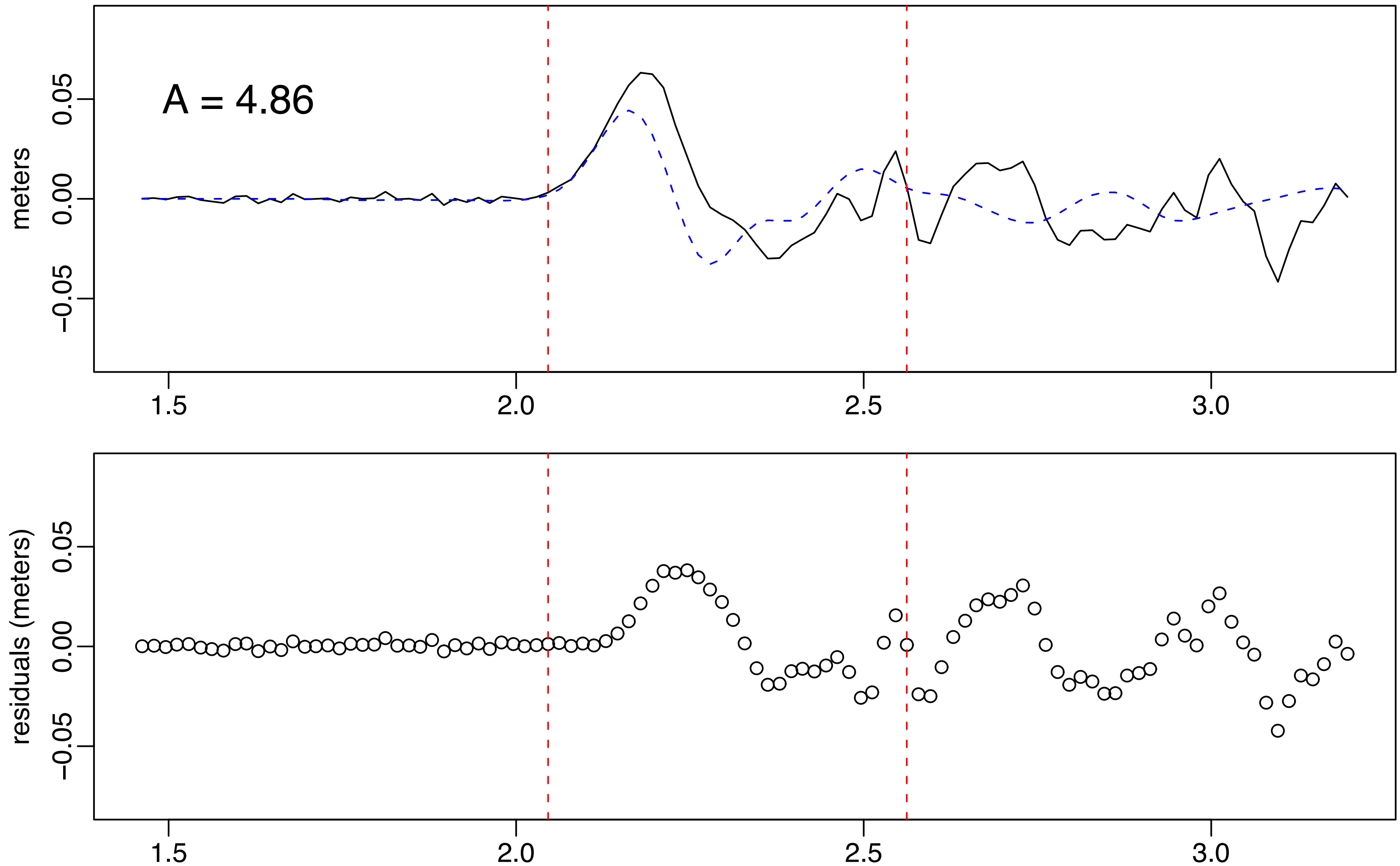

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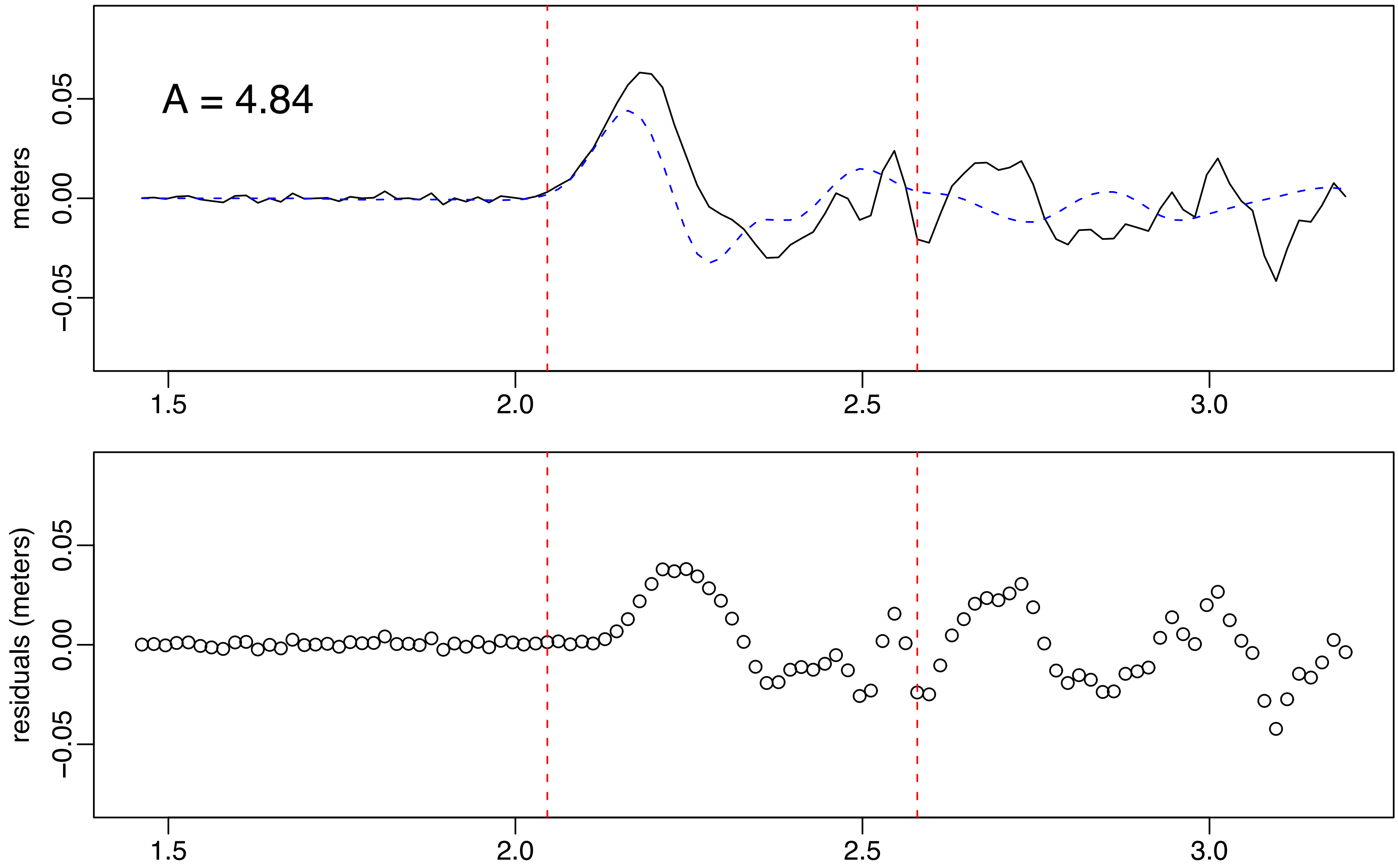

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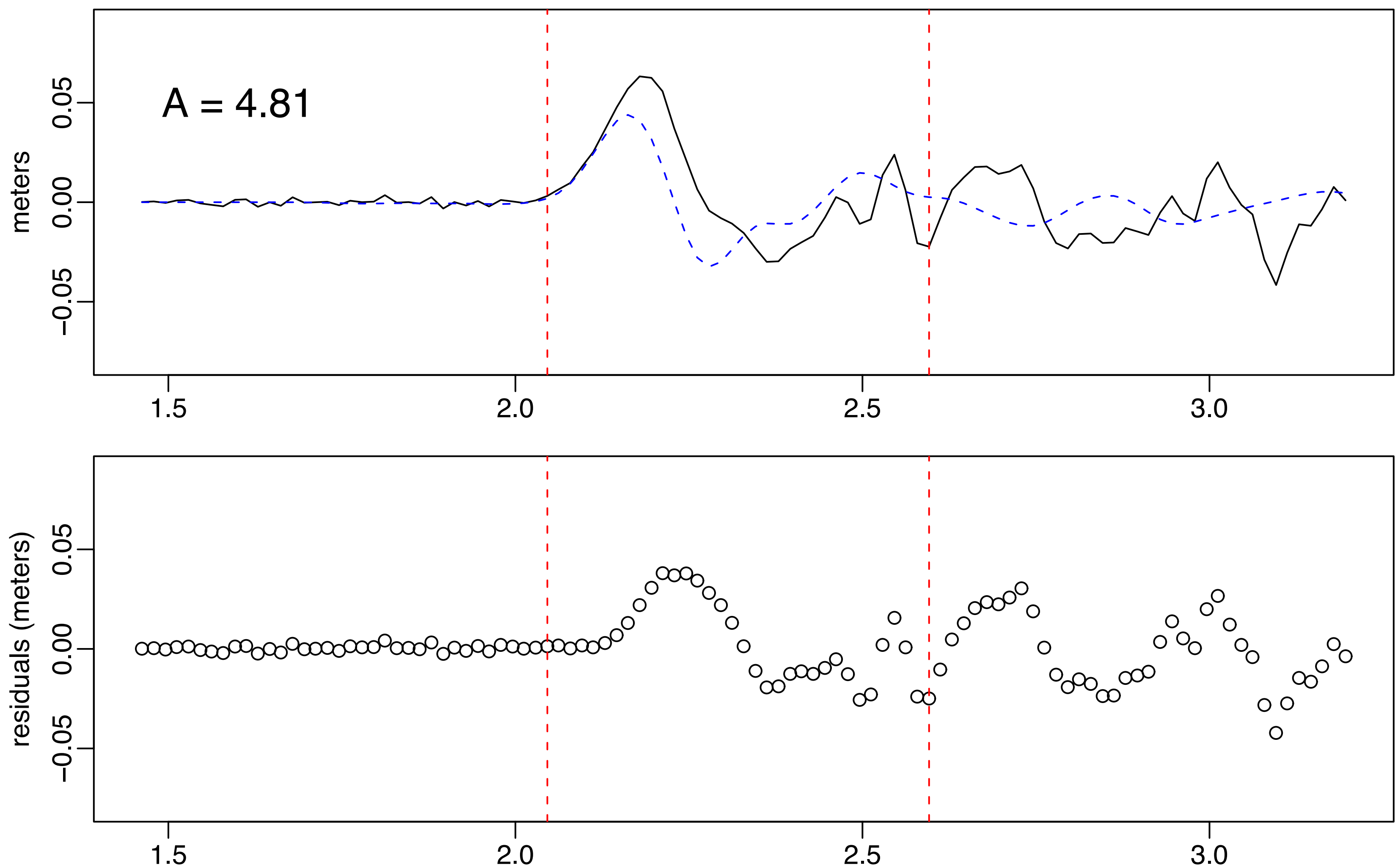

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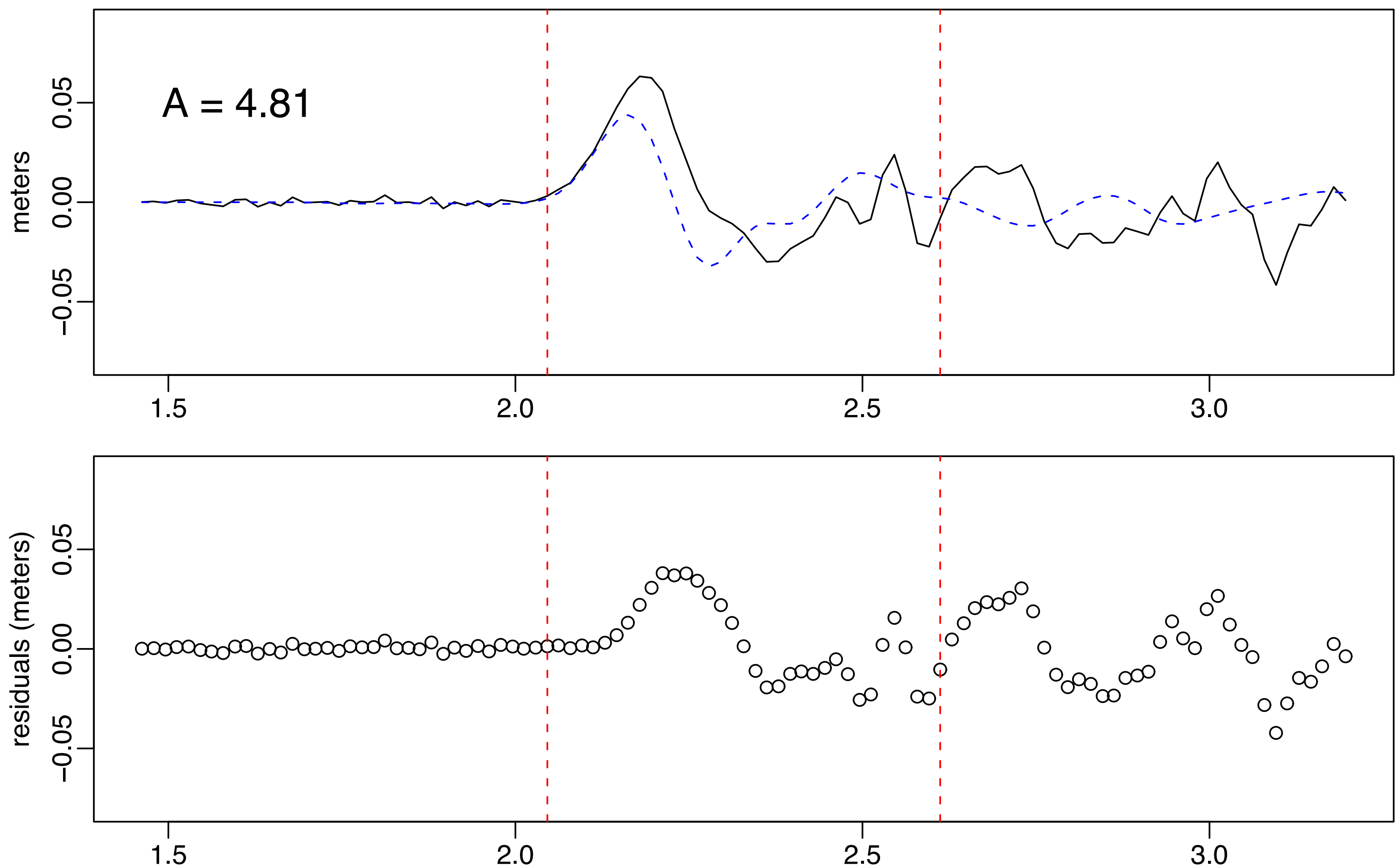

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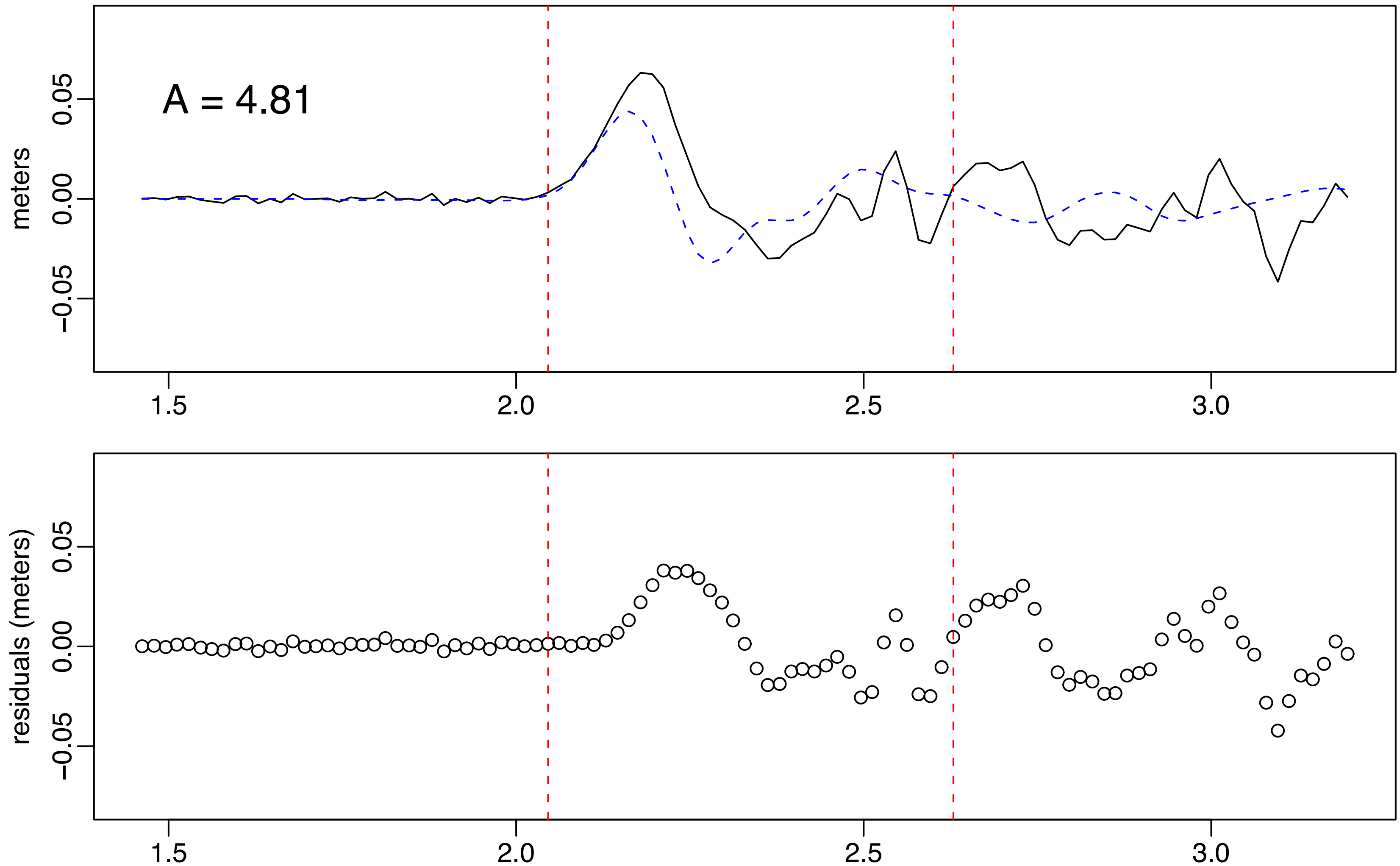

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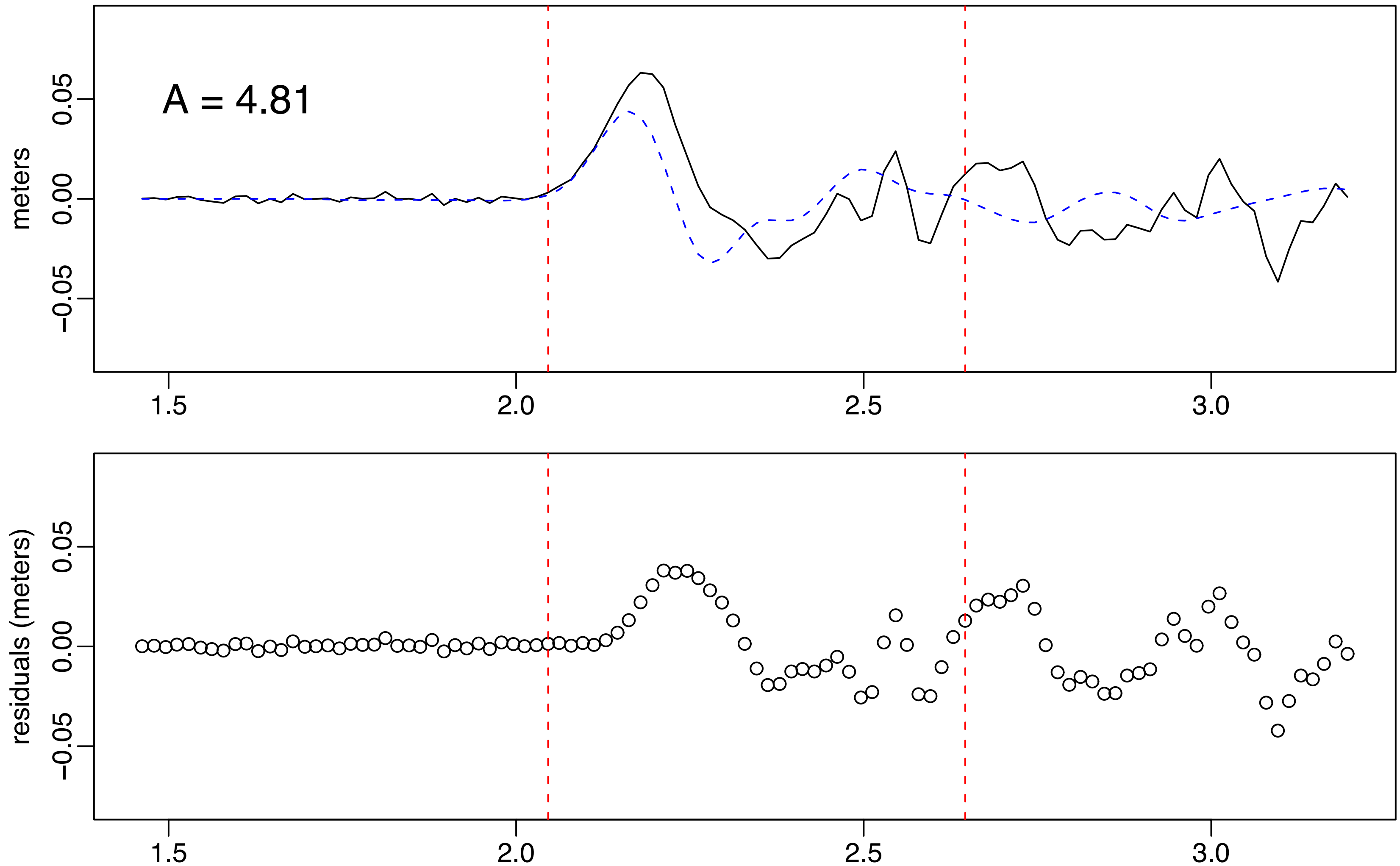

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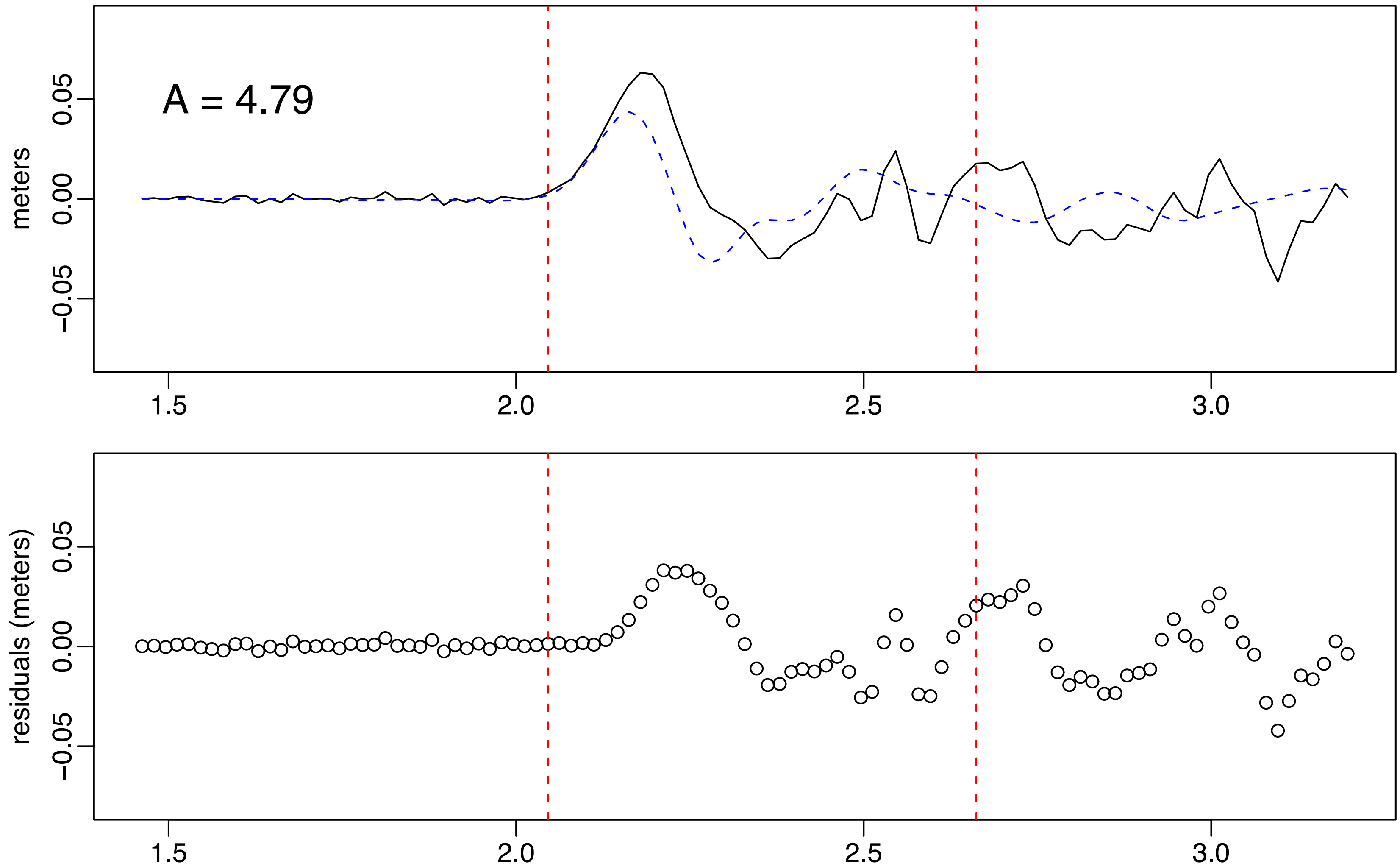

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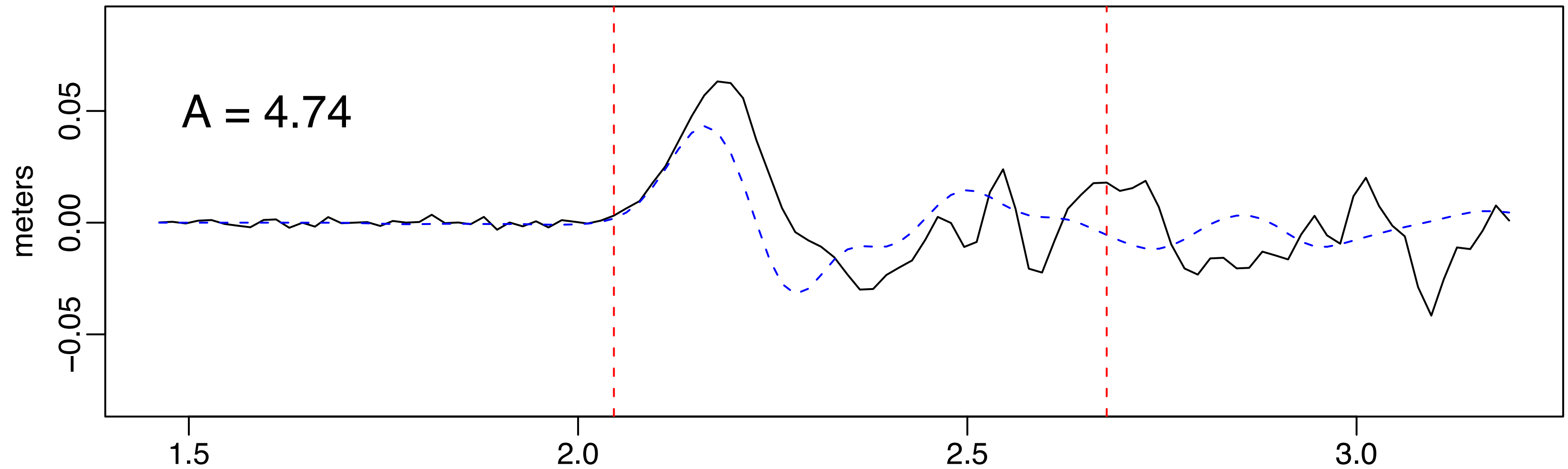

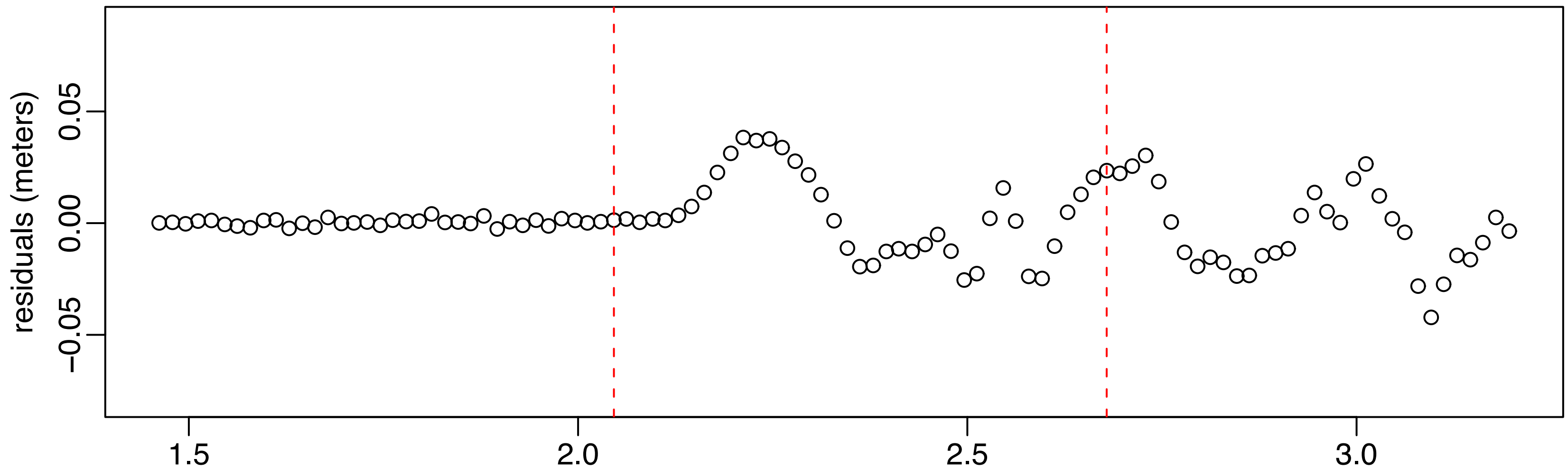

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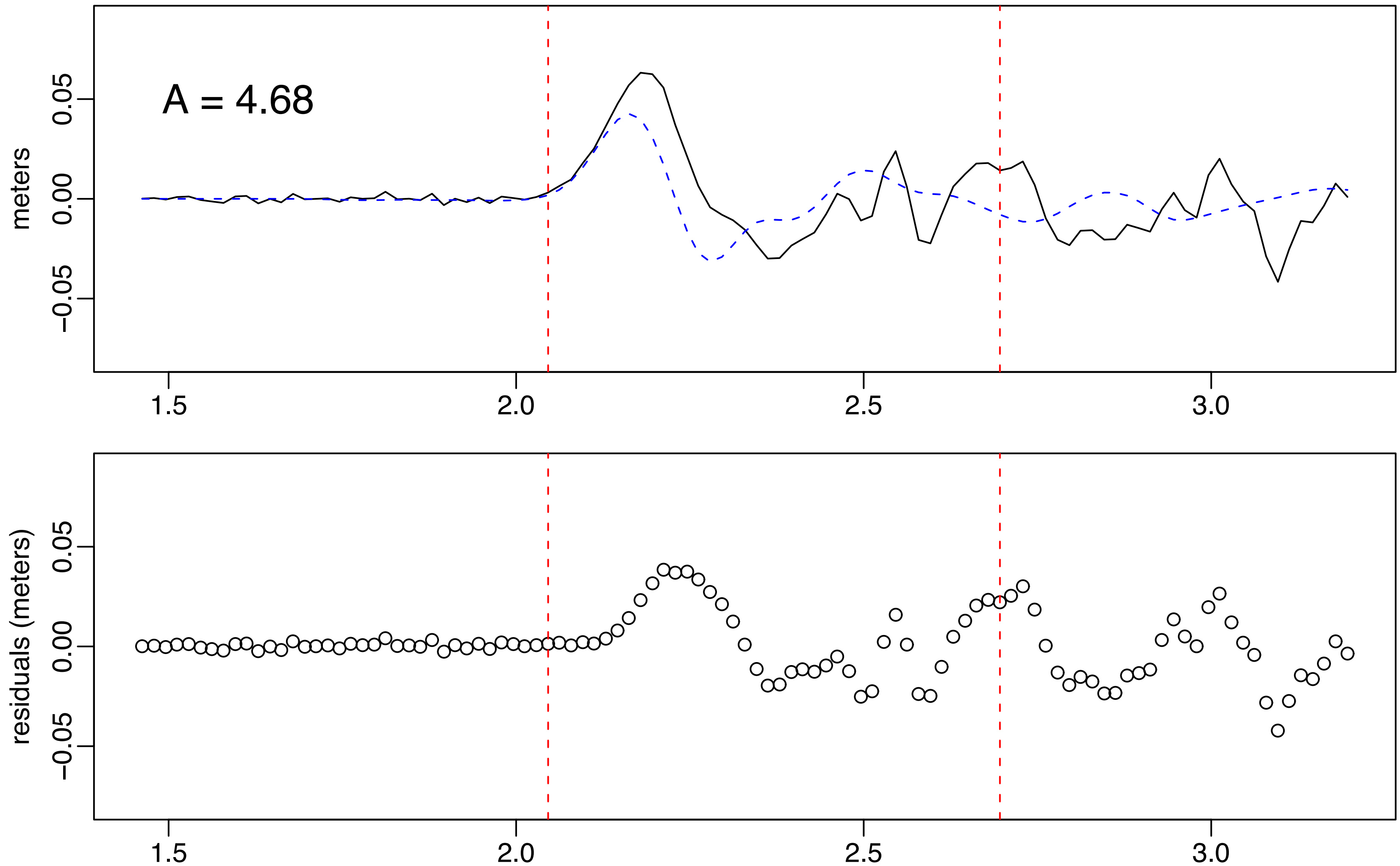

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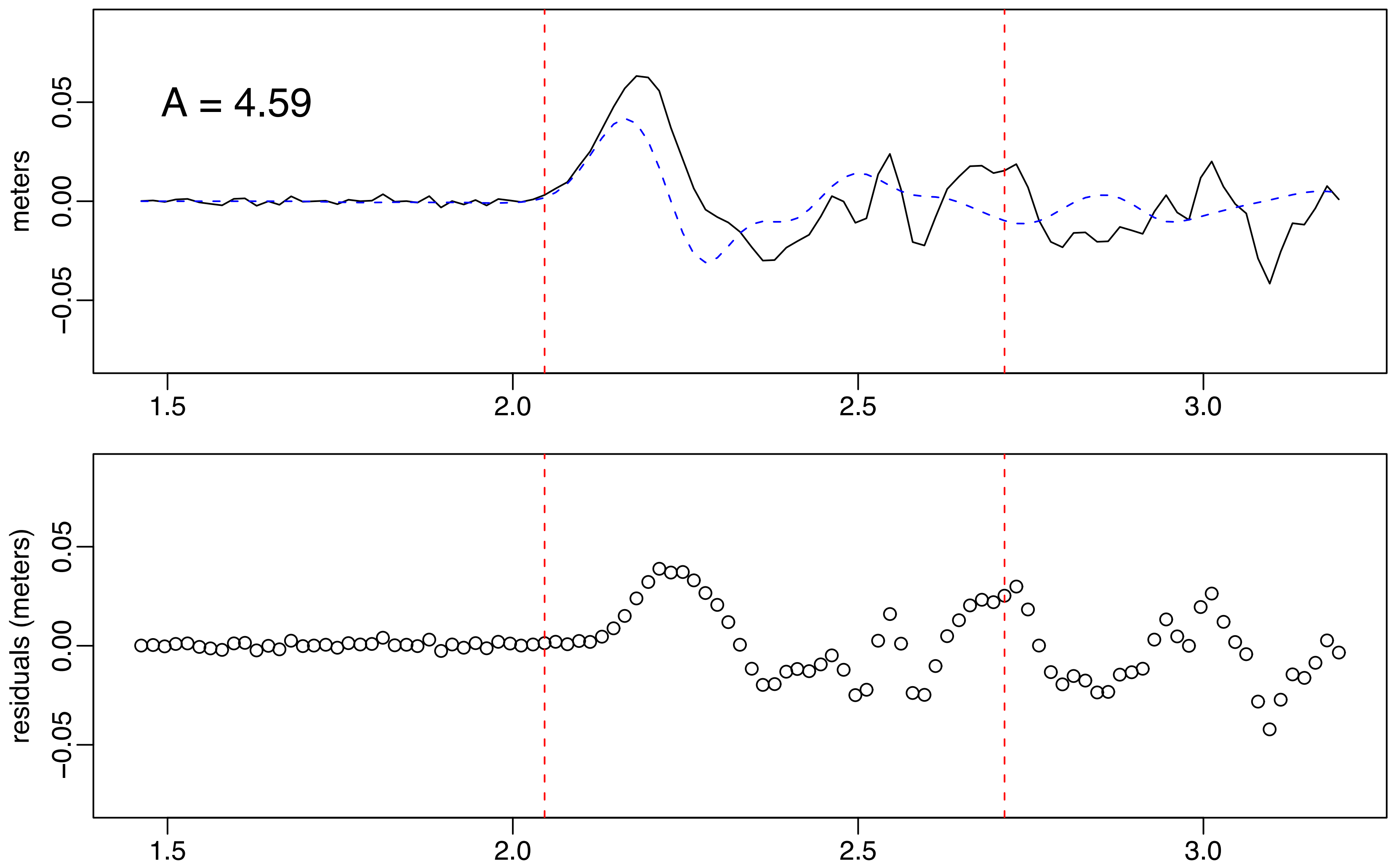

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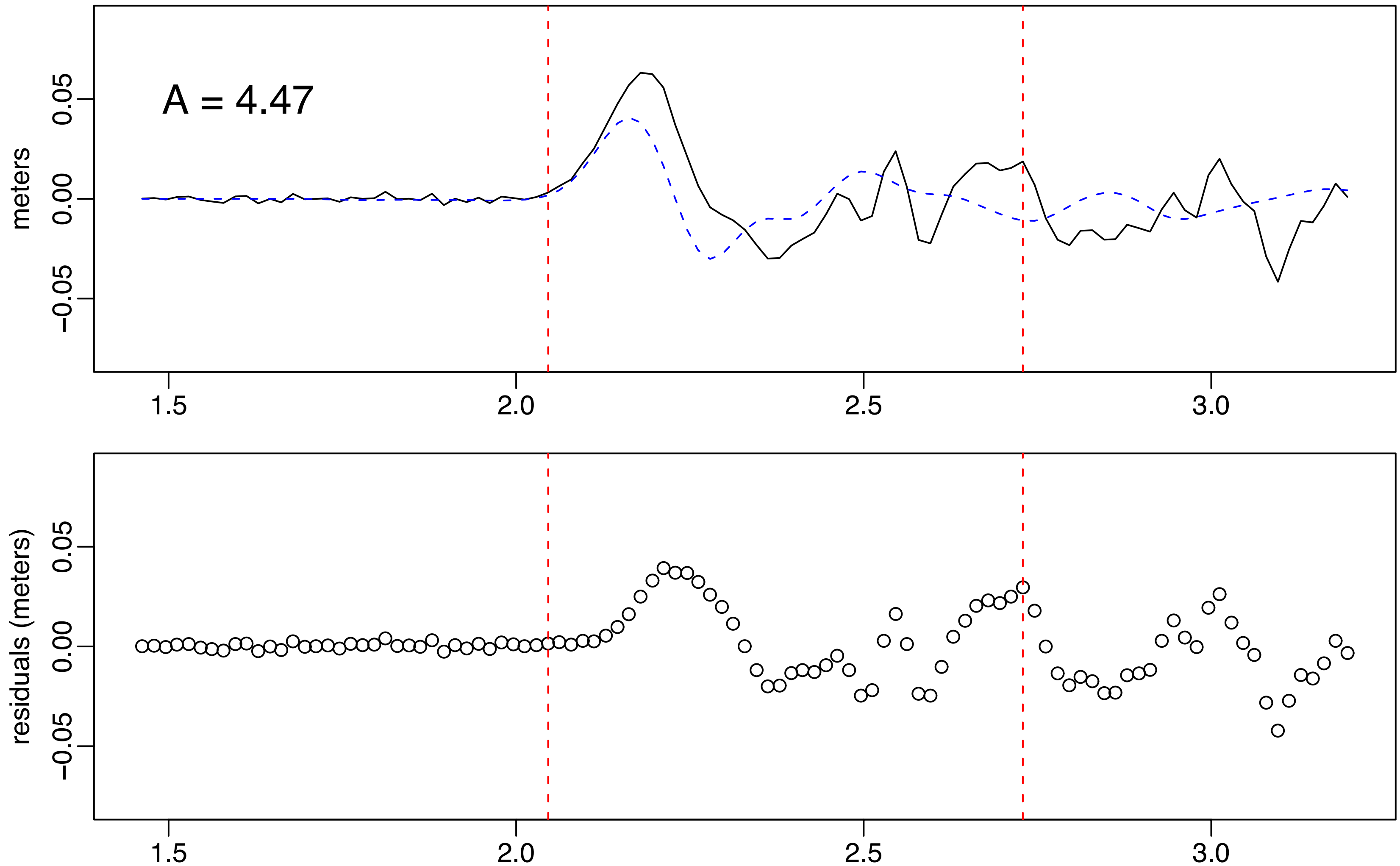

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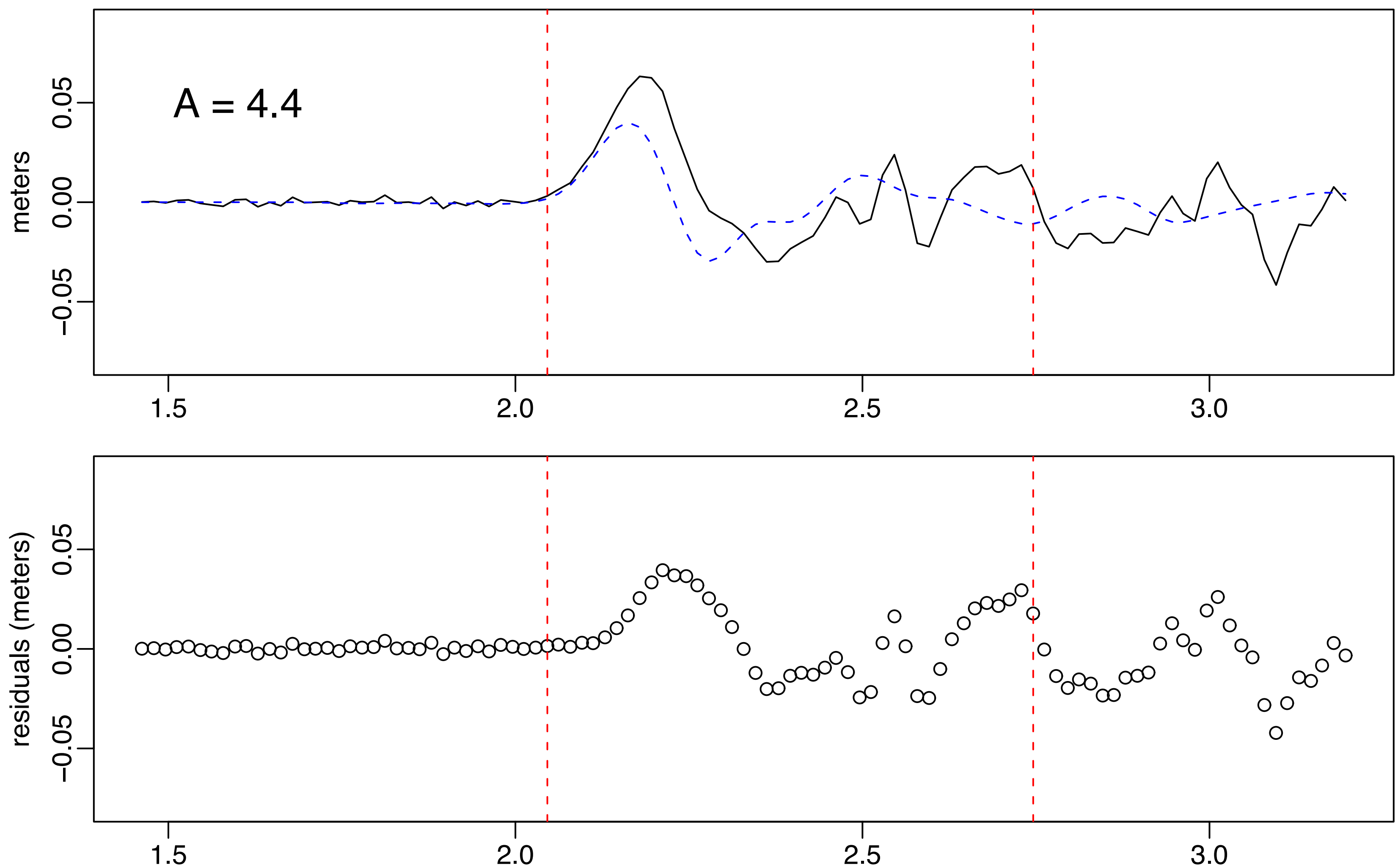

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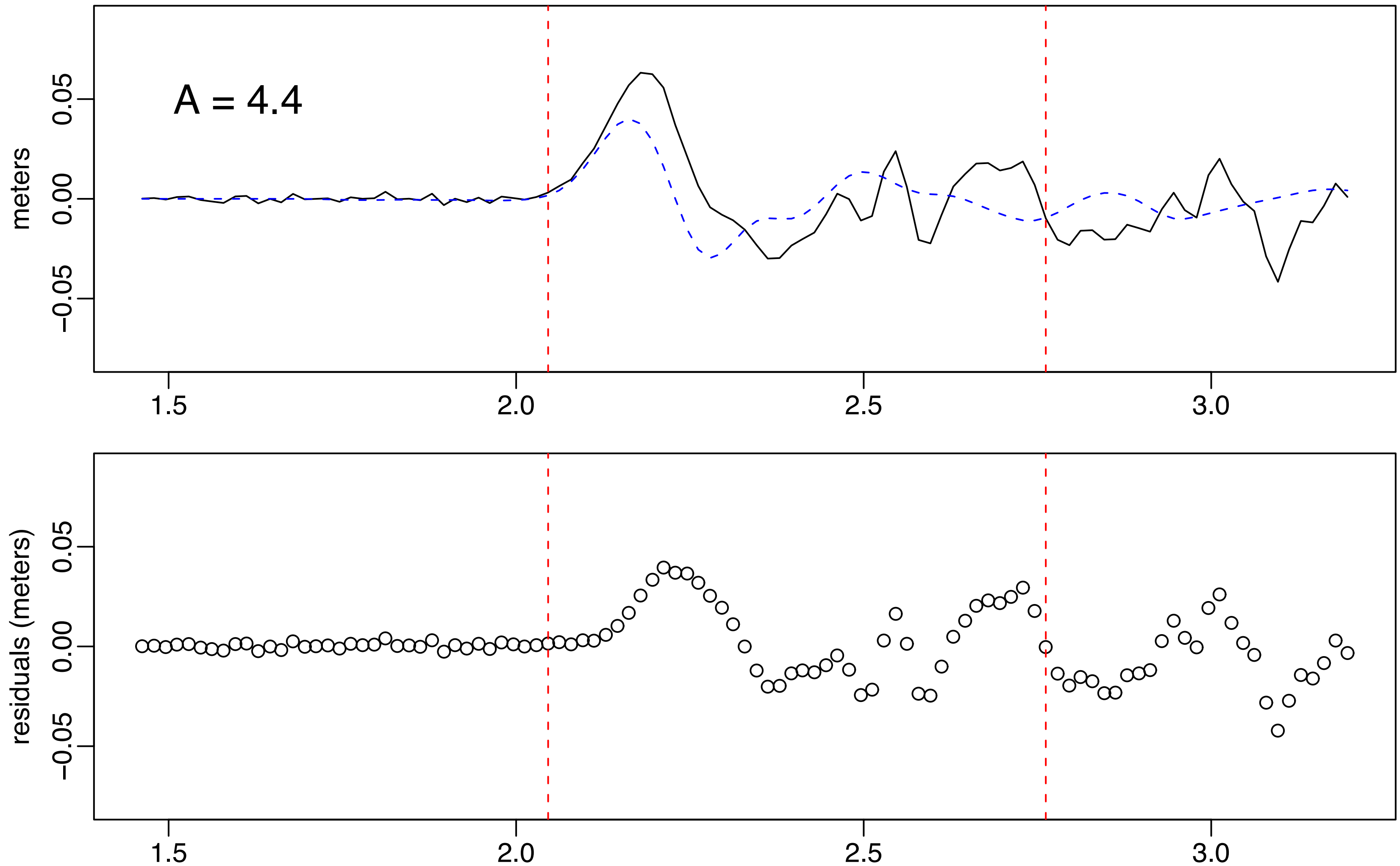
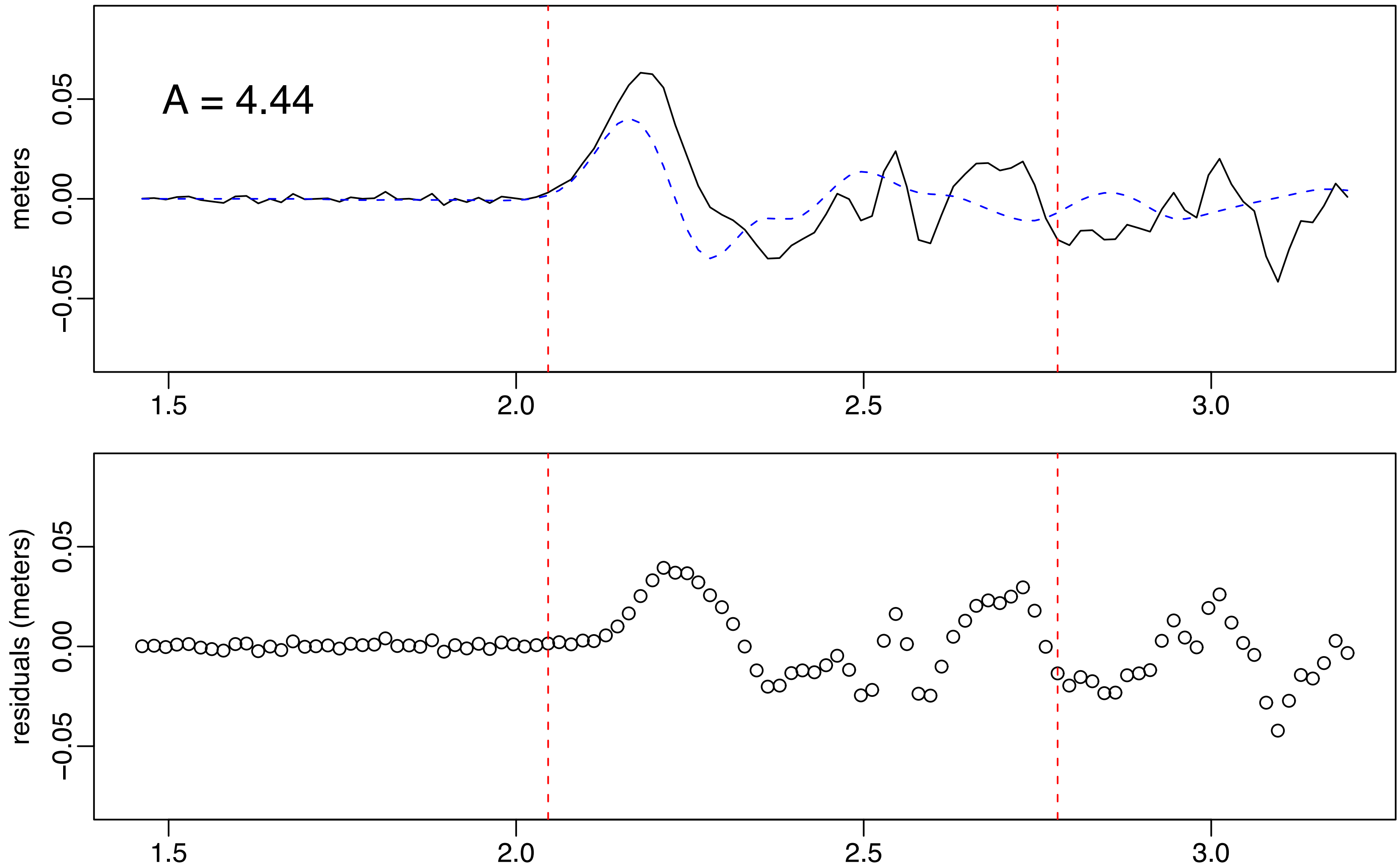
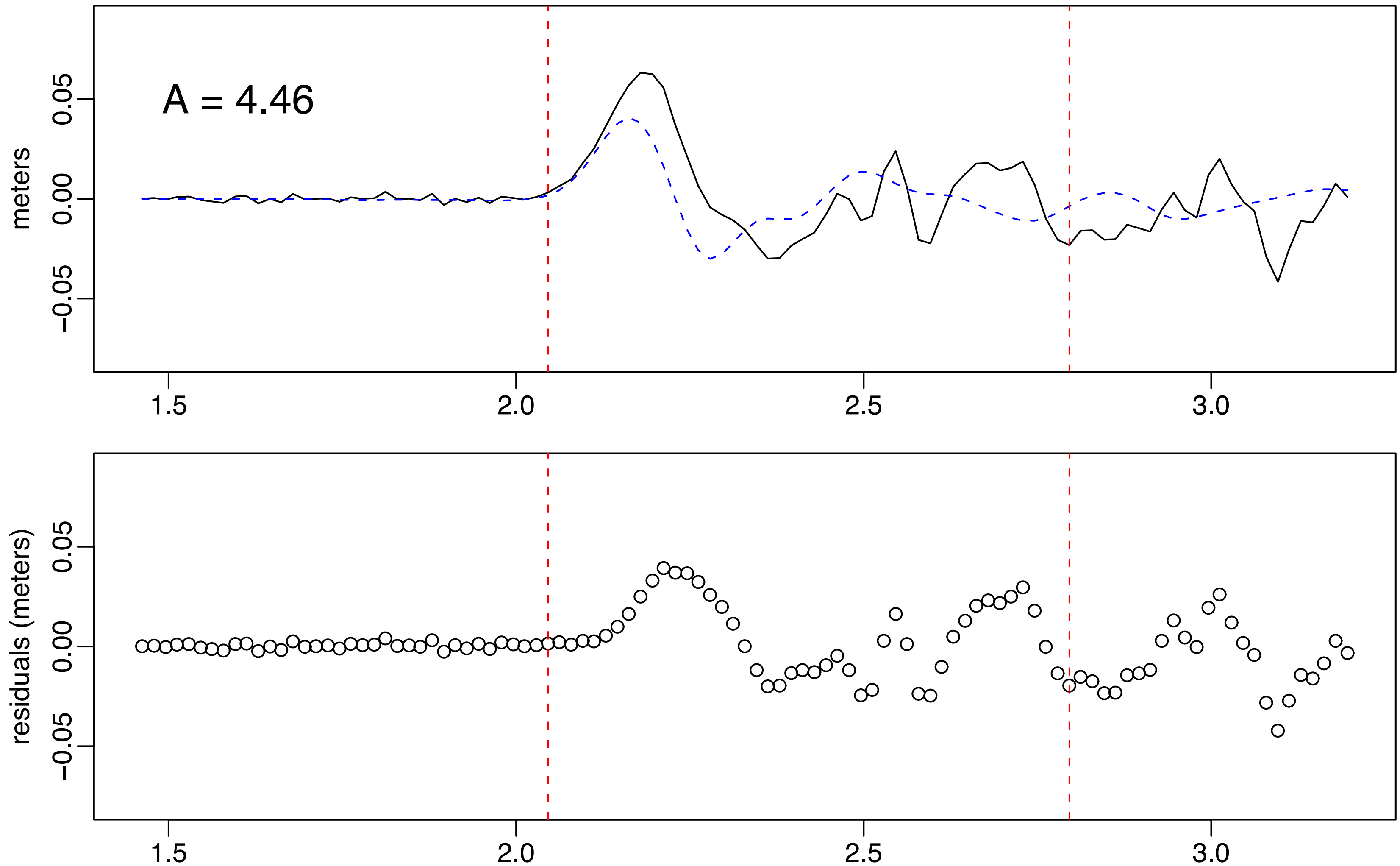


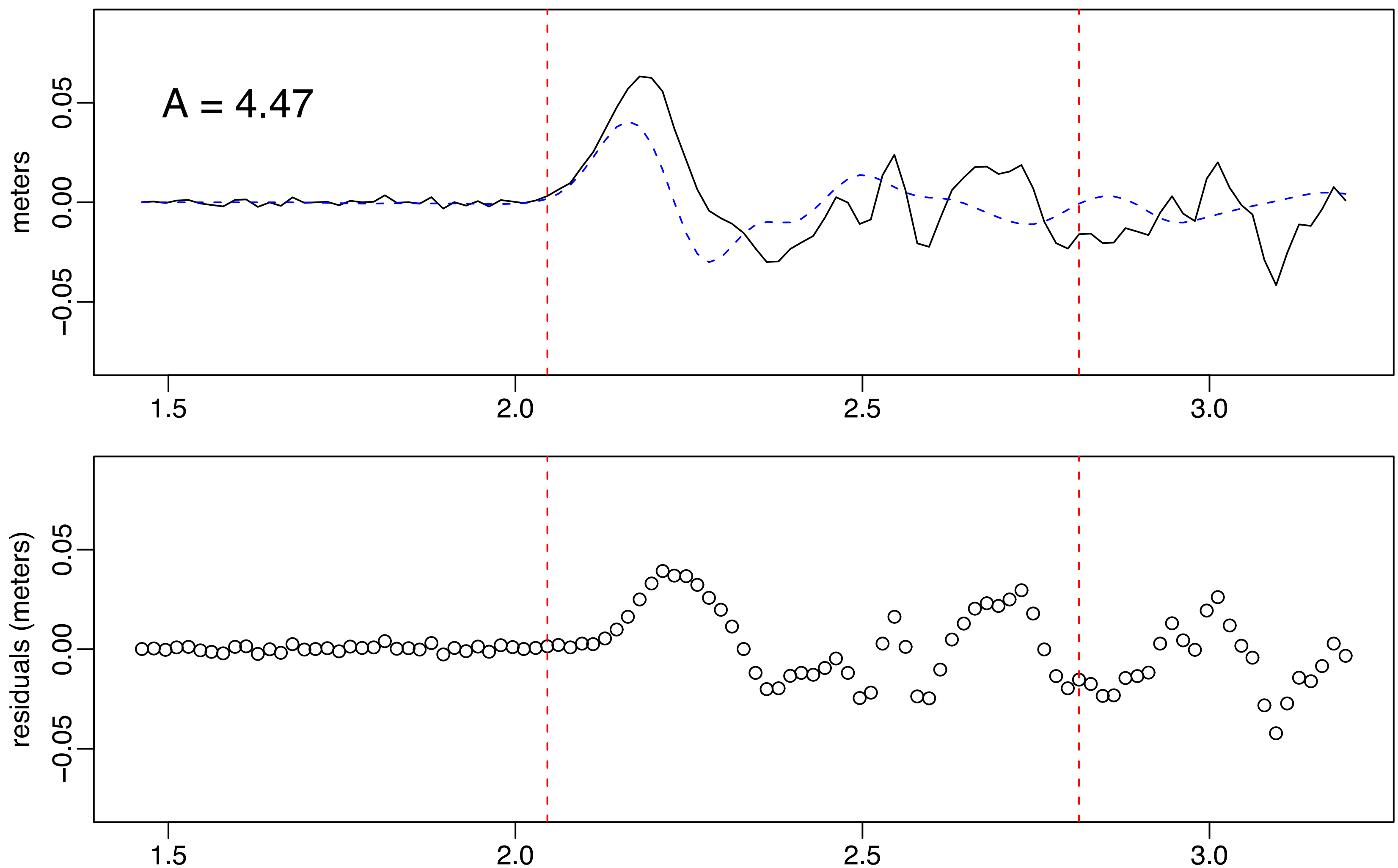
LS Fit of a12 Model to Selected 21414 Data



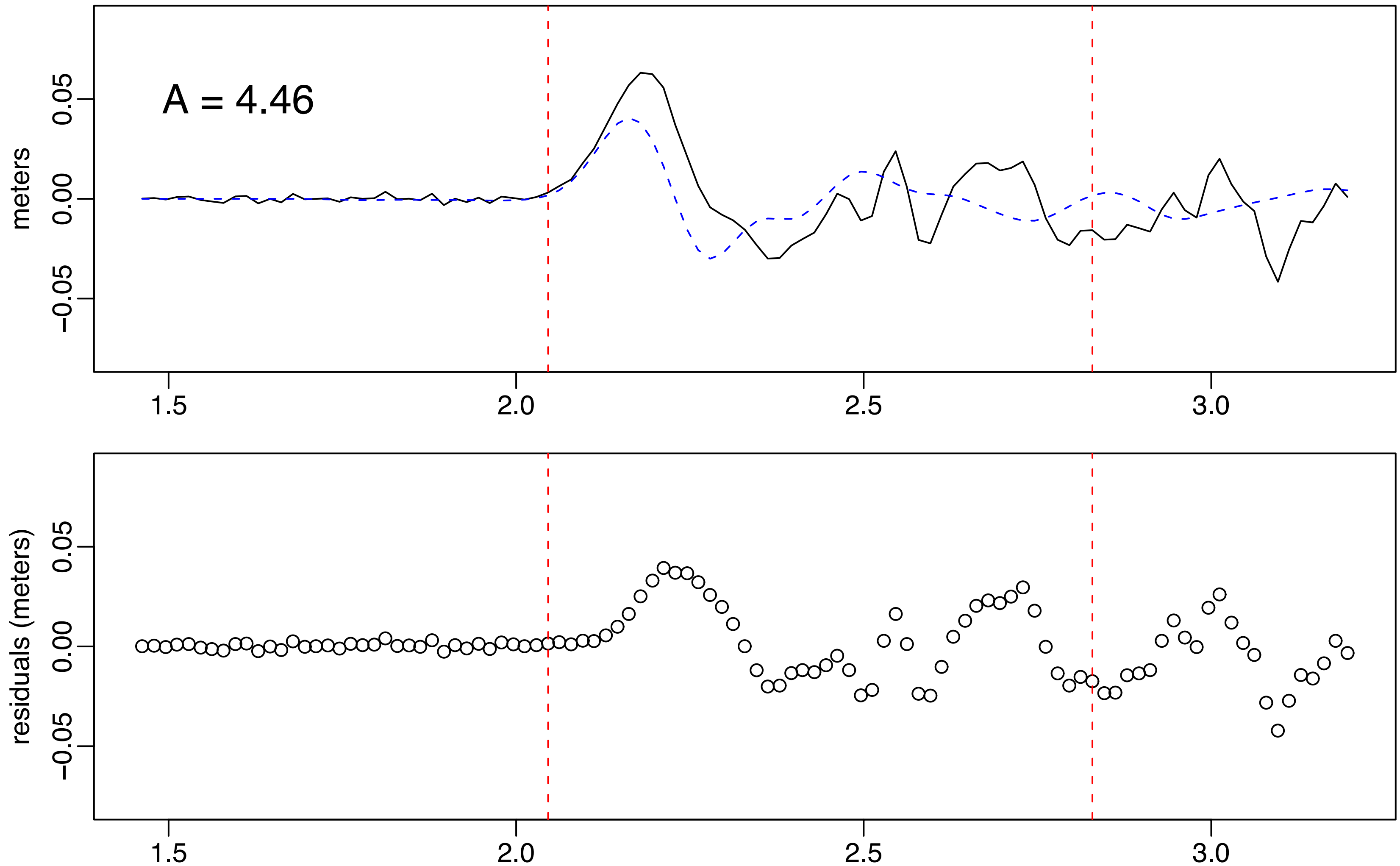
LS Fit of a12 Model to Selected 21414 Data



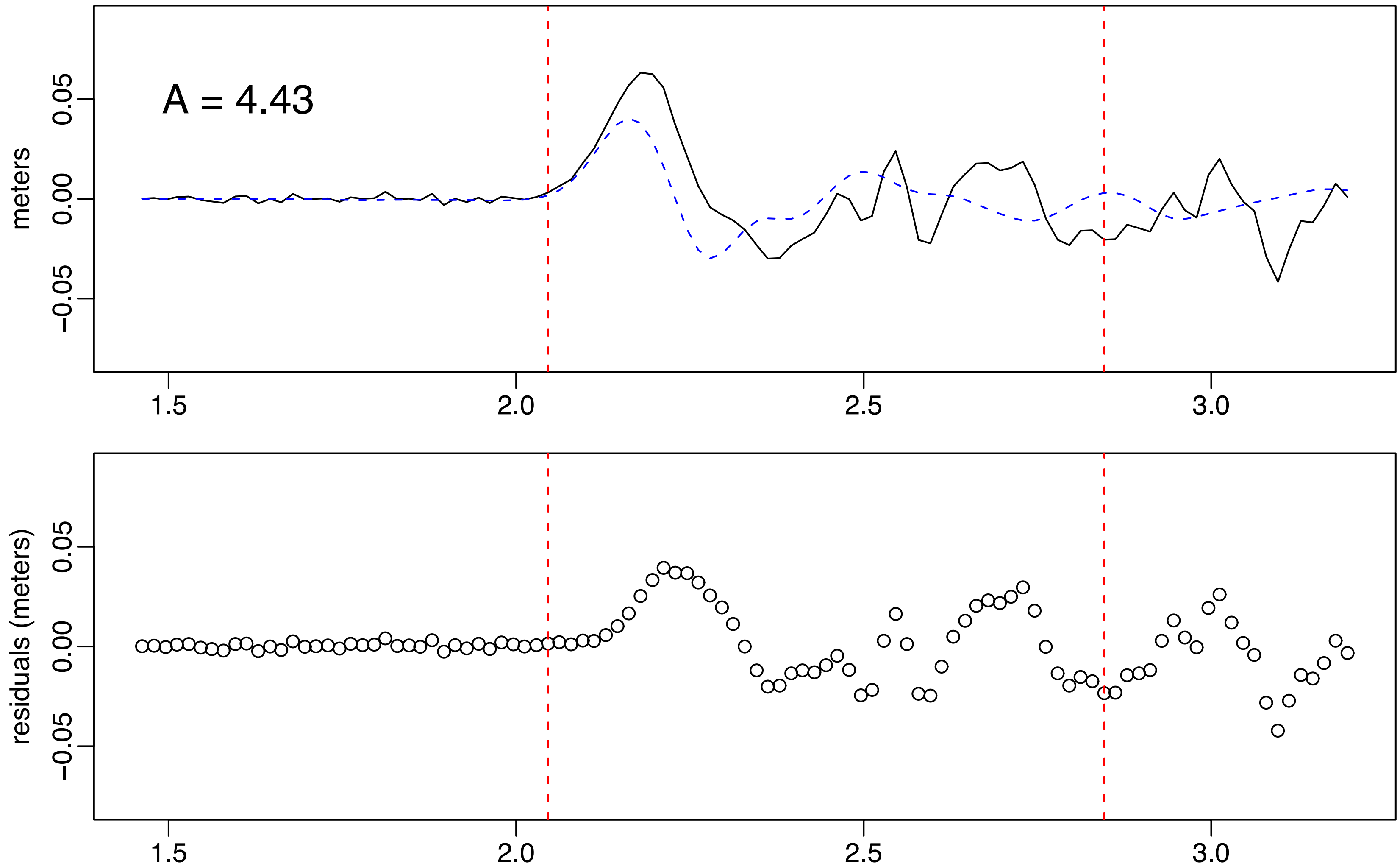
LS Fit of a12 Model to Selected 21414 Data



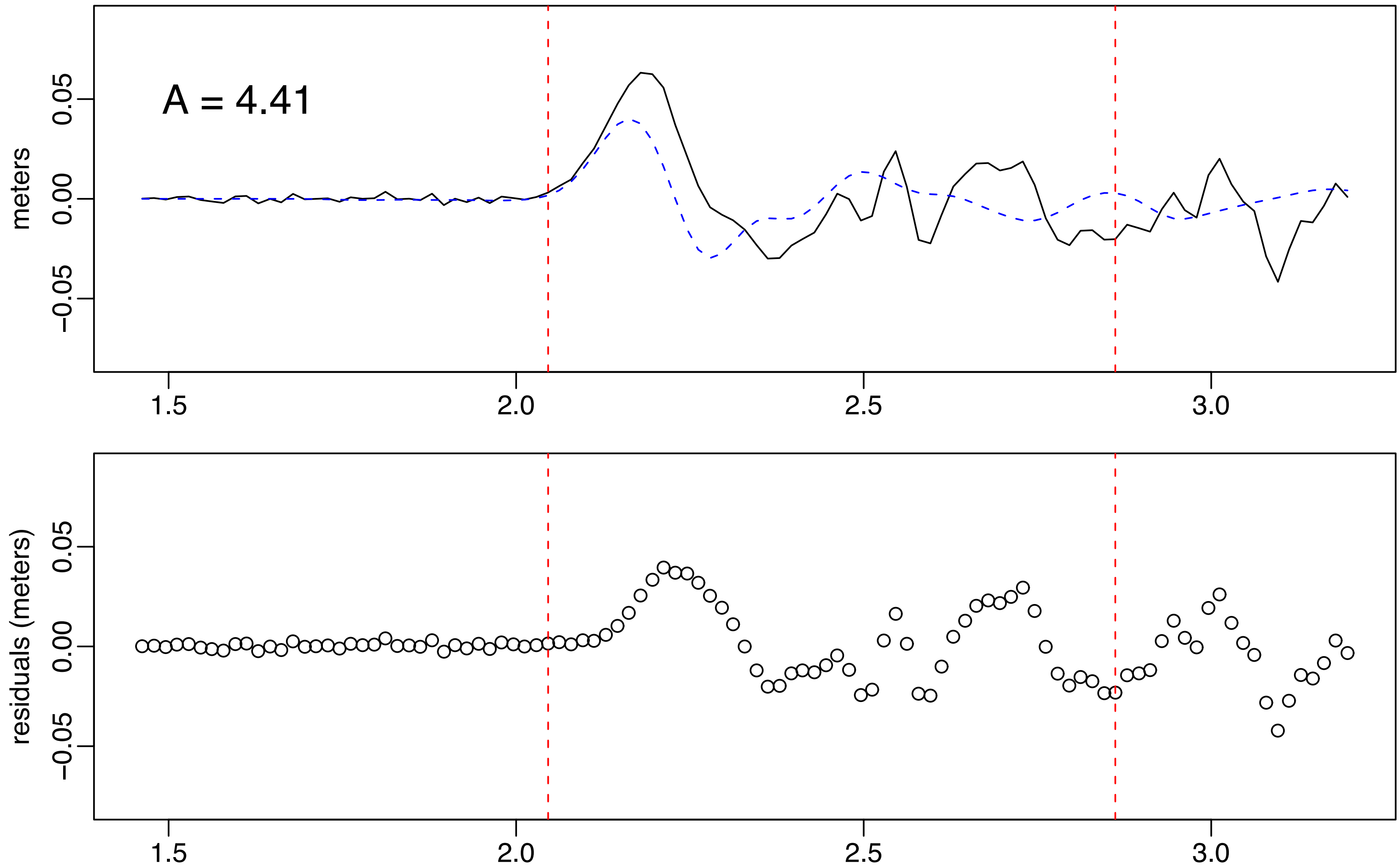
LS Fit of a12 Model to Selected 21414 Data



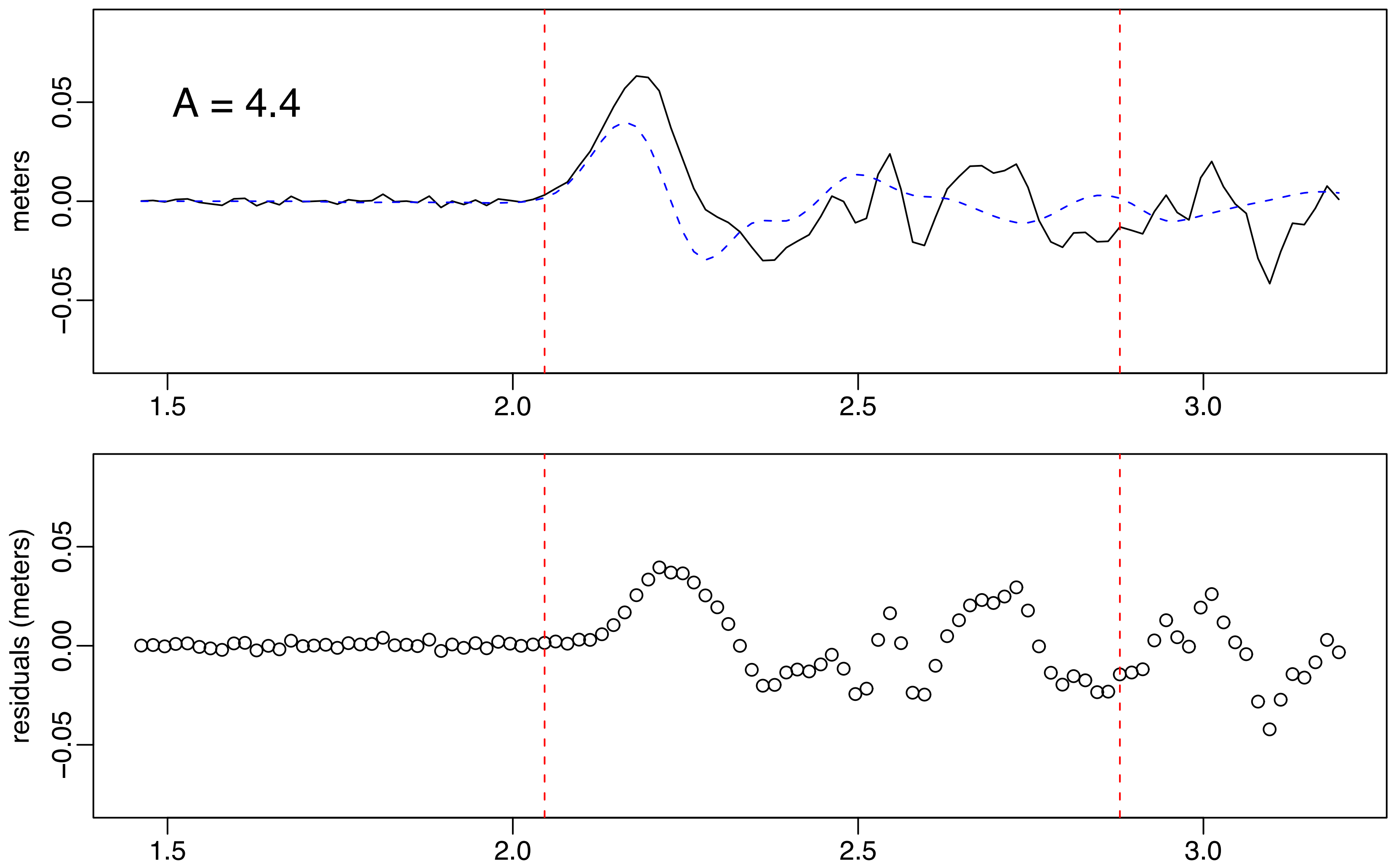
LS Fit of a12 Model to Selected 21414 Data



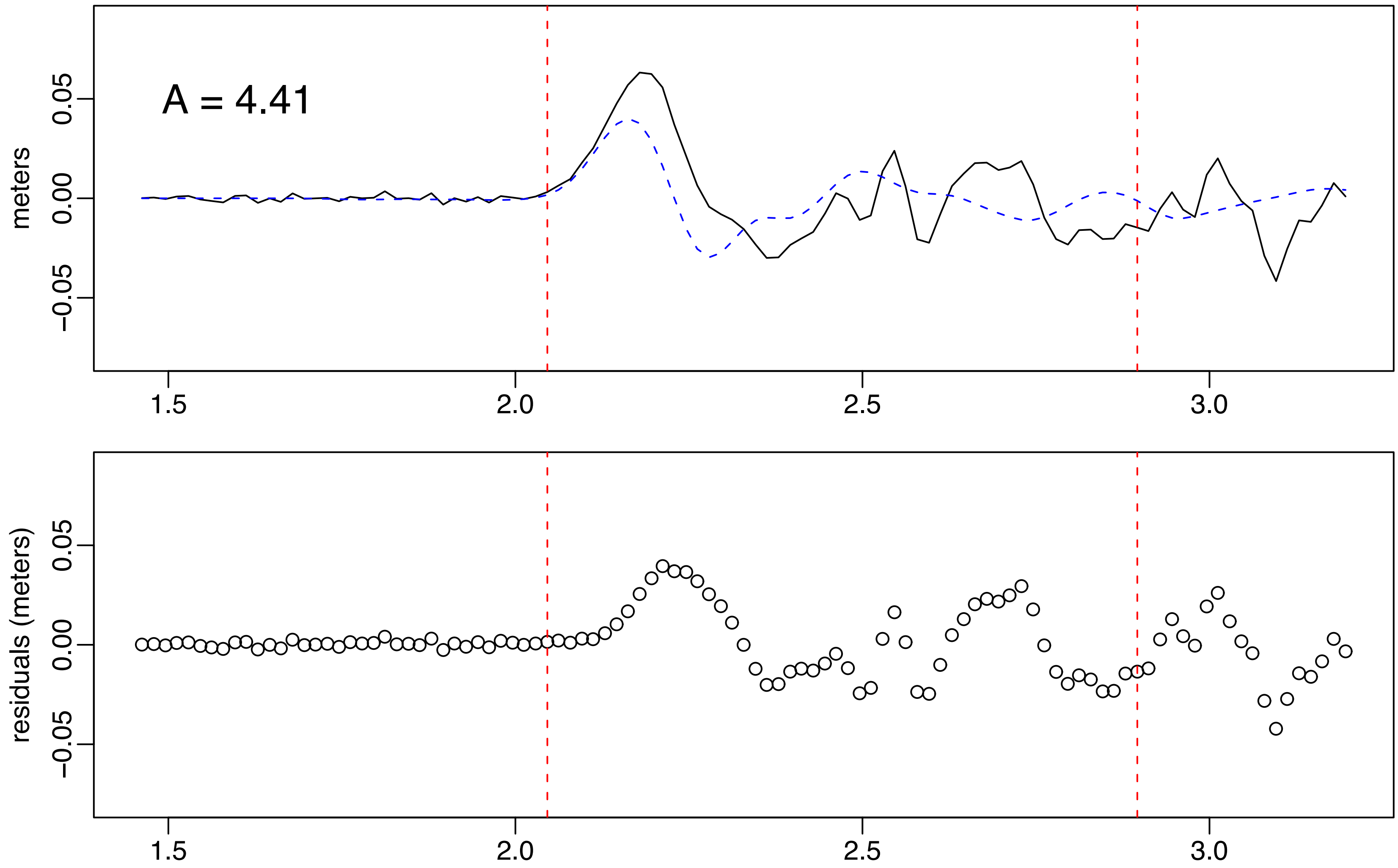
LS Fit of a12 Model to Selected 21414 Data



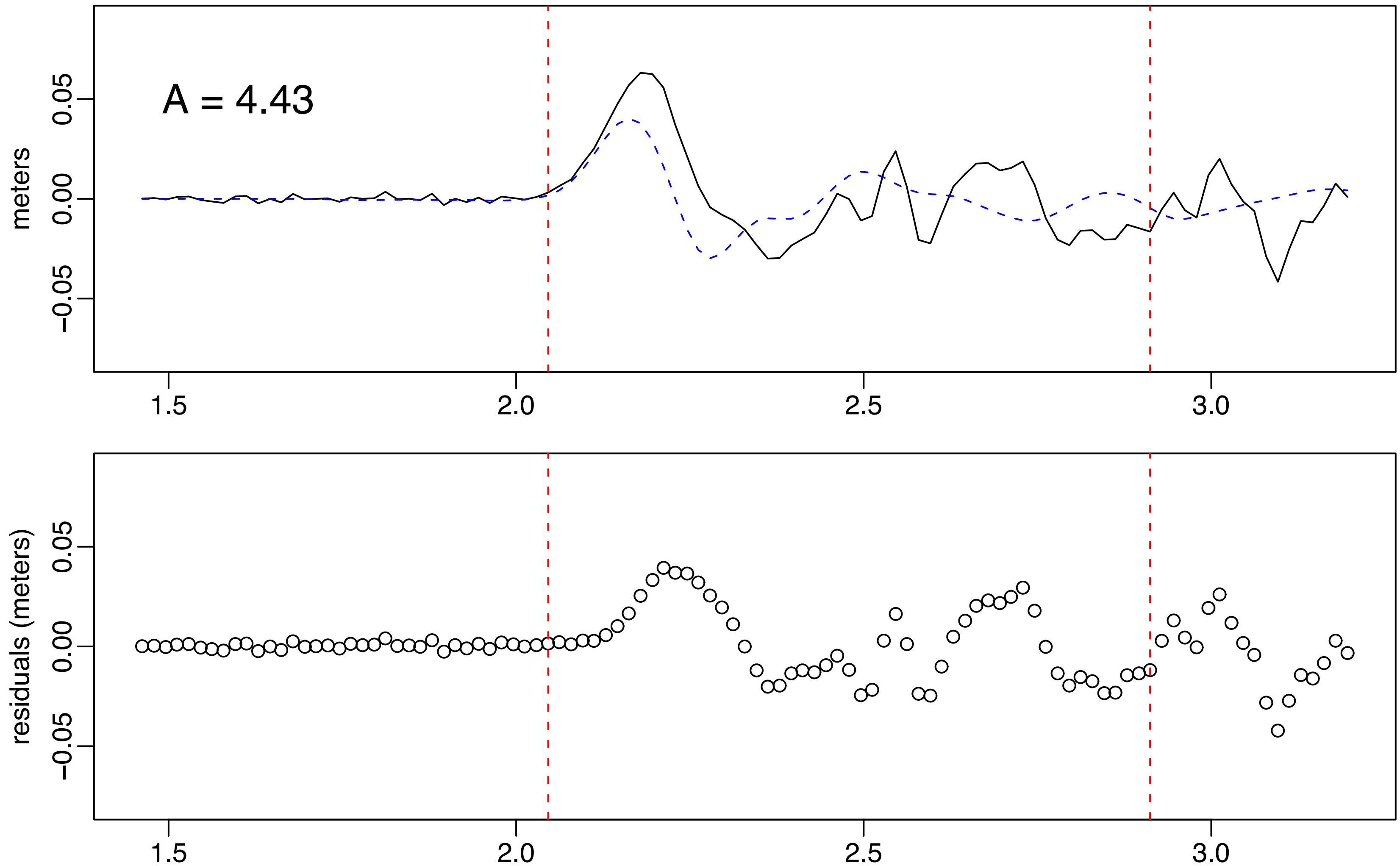
LS Fit of a12 Model to Selected 21414 Data



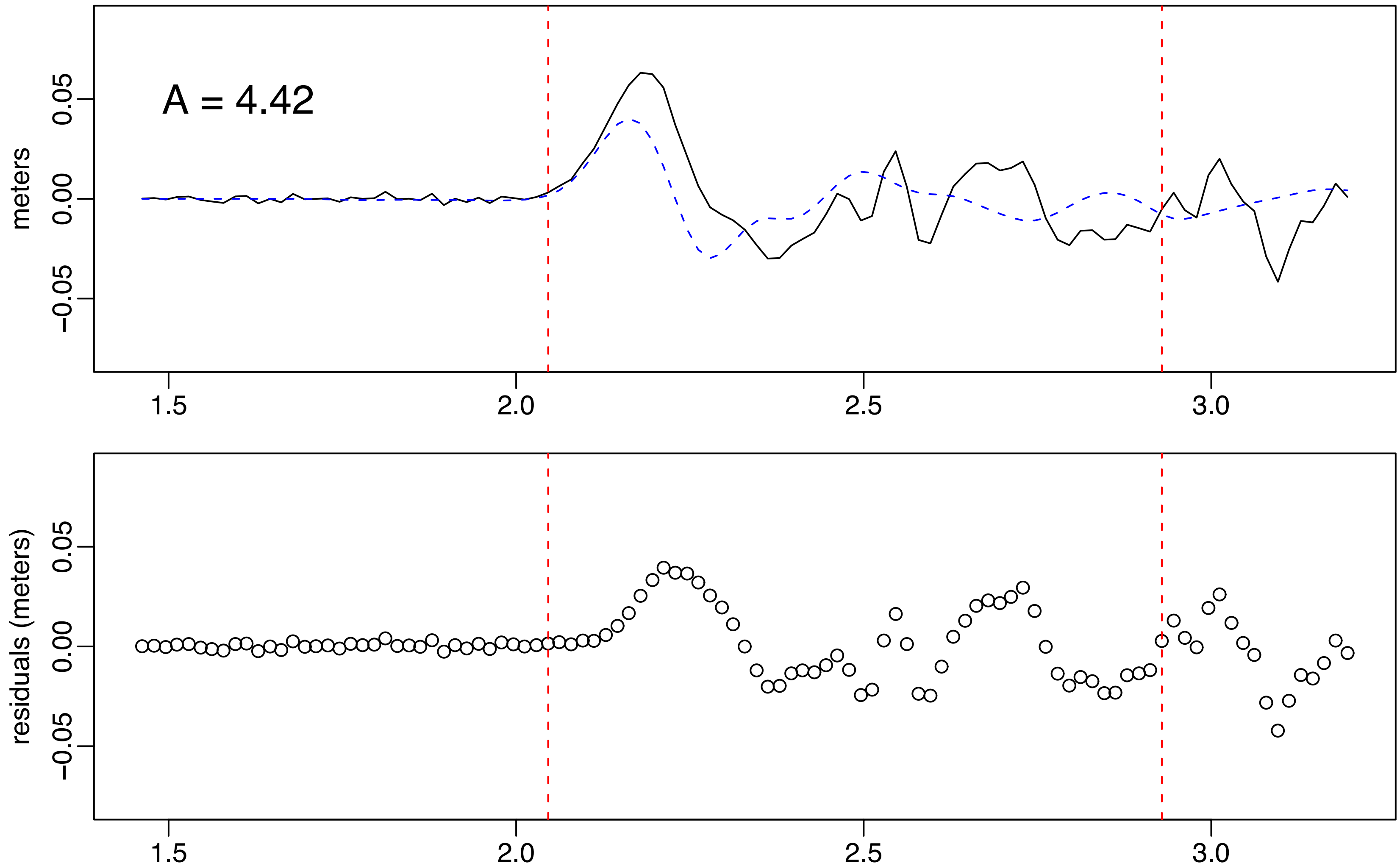
LS Fit of a12 Model to Selected 21414 Data



LS Fit of a12 Model to Selected 21414 Data



LS Fit of a12 Model to Selected 21414 Data



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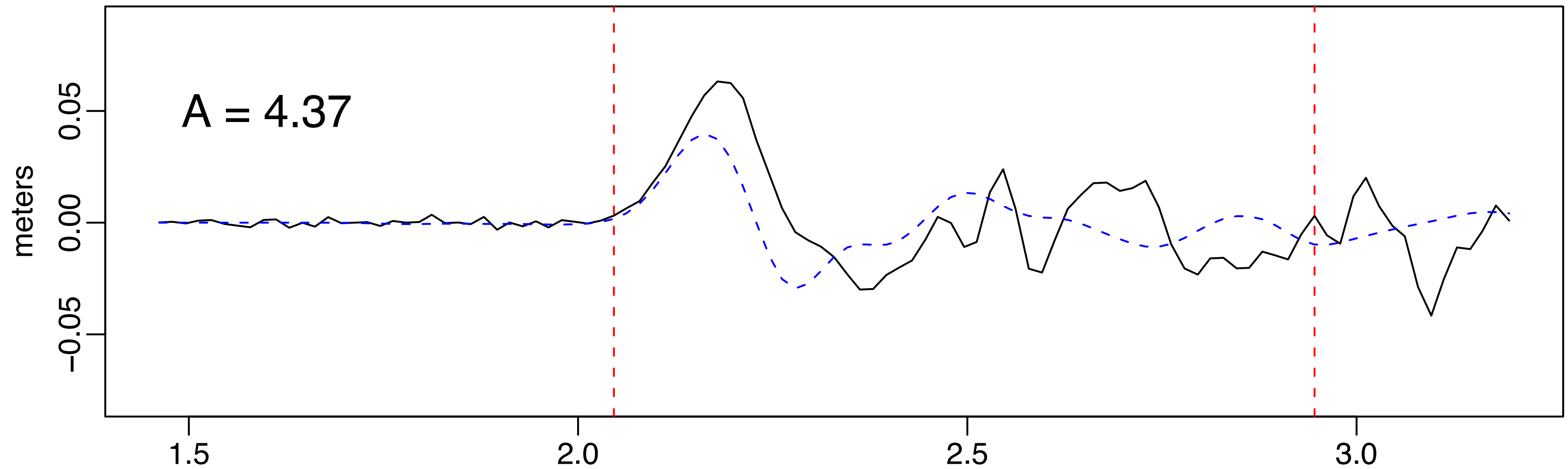

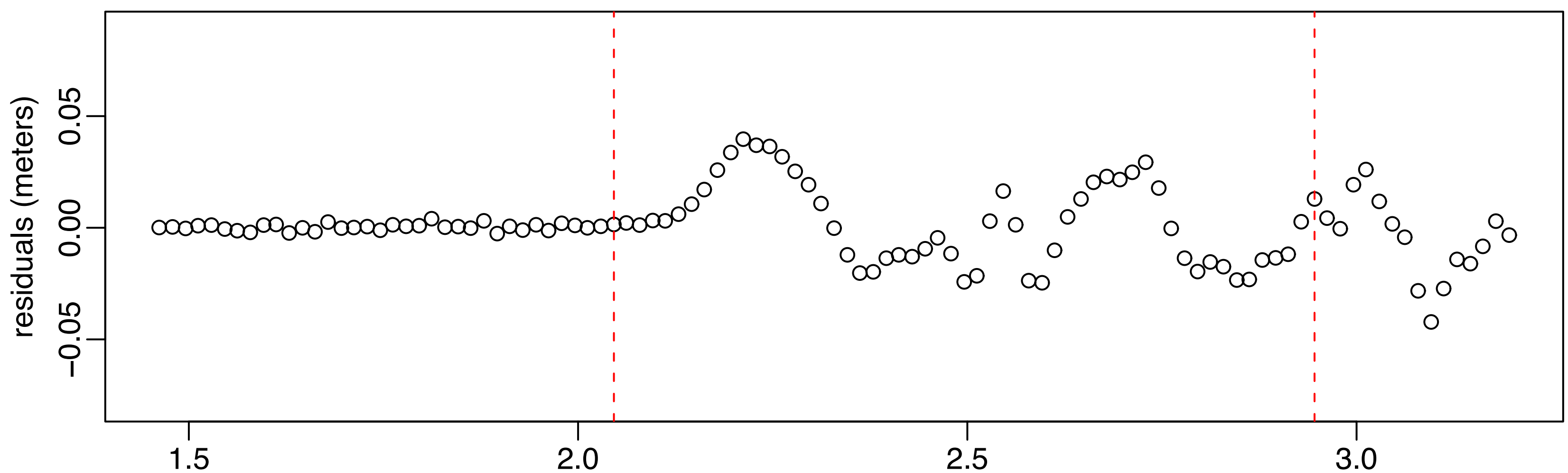

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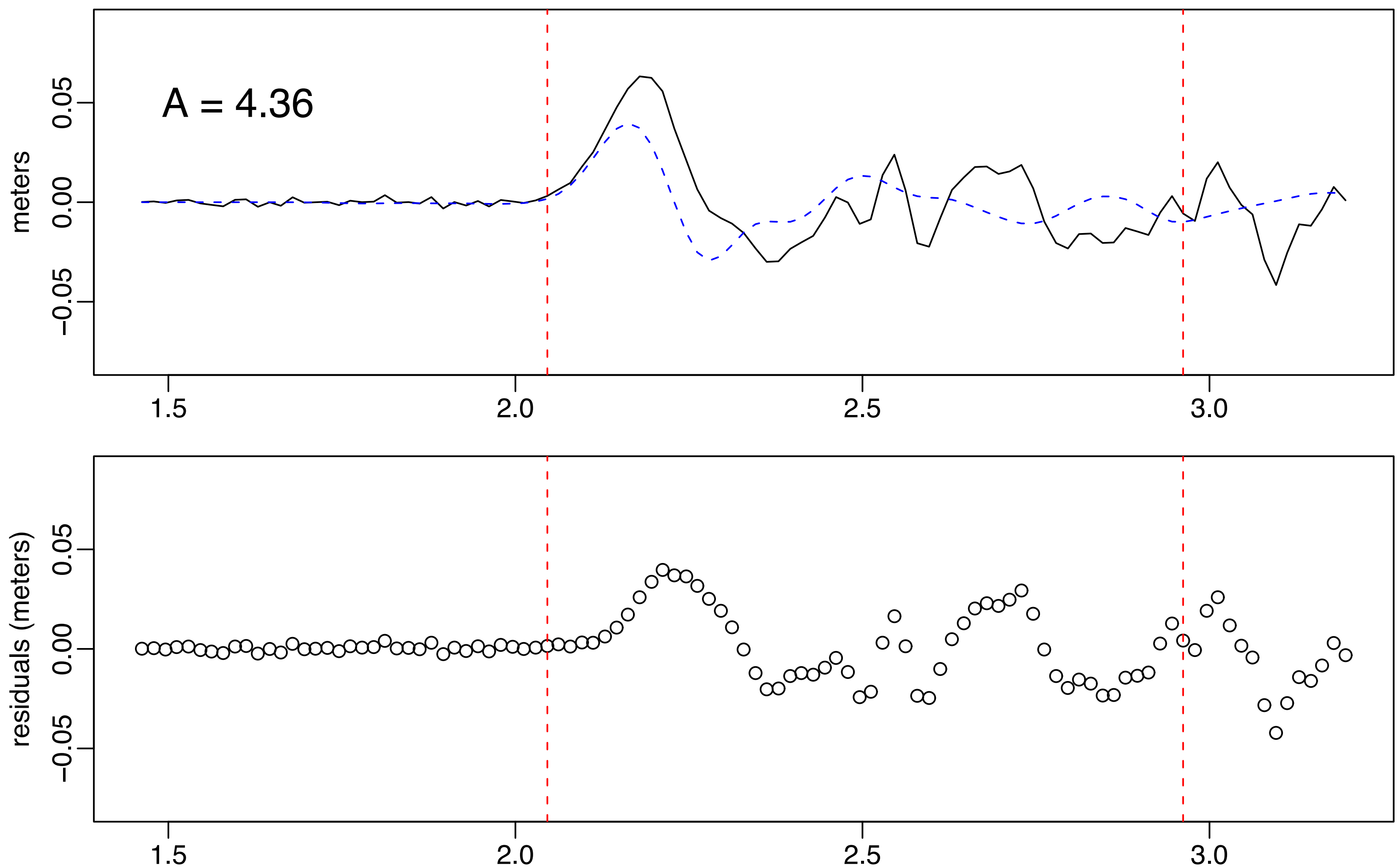

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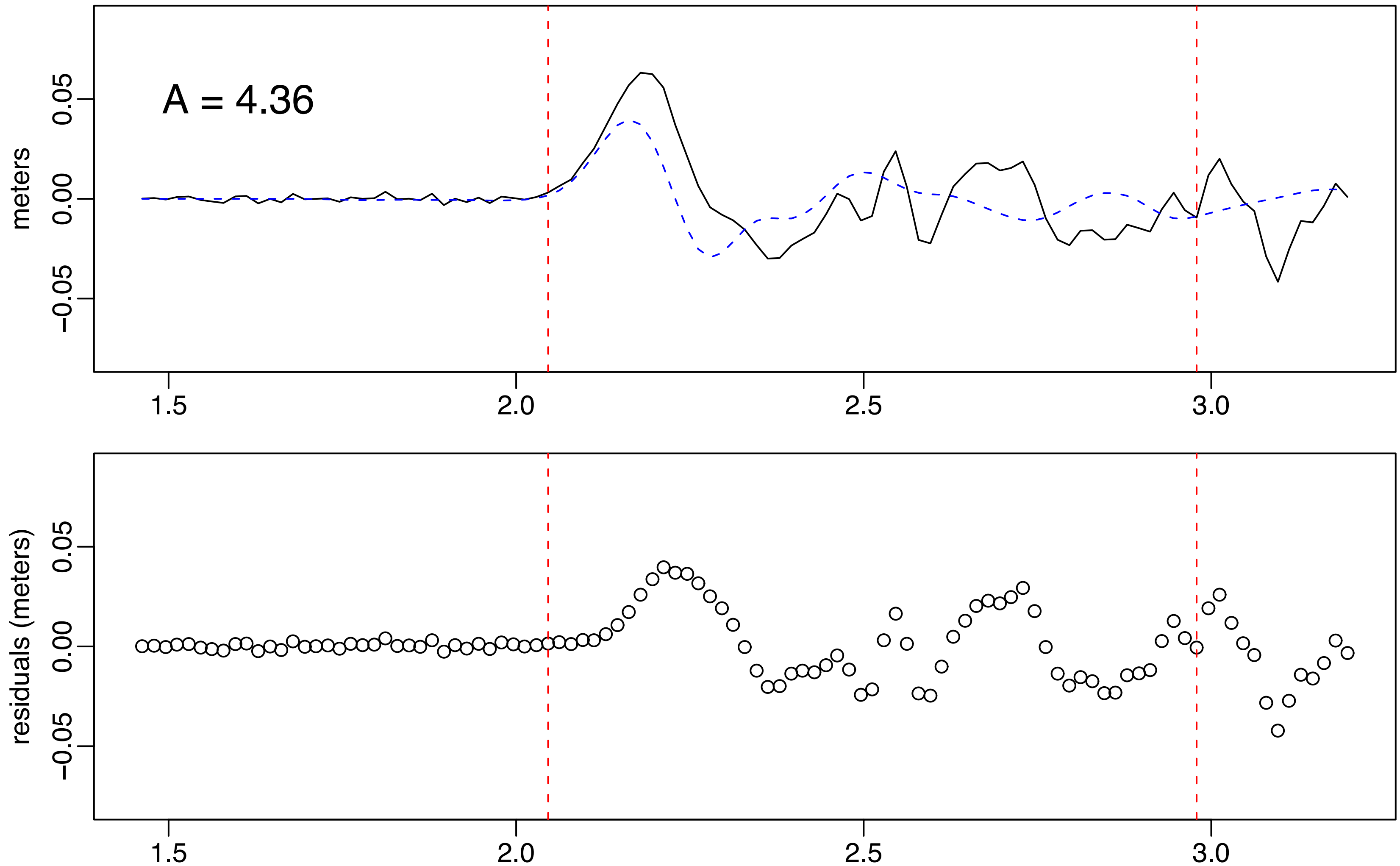

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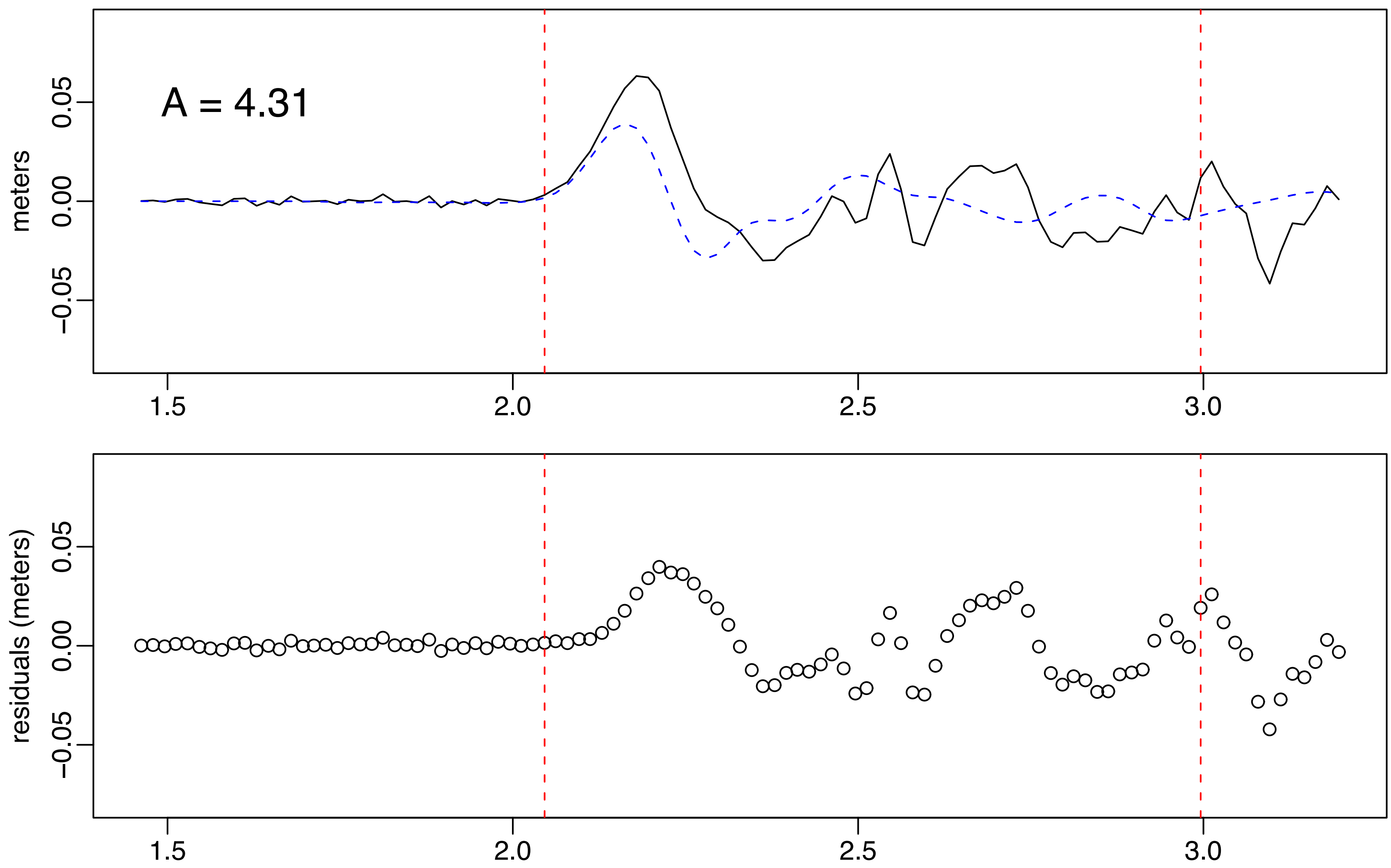
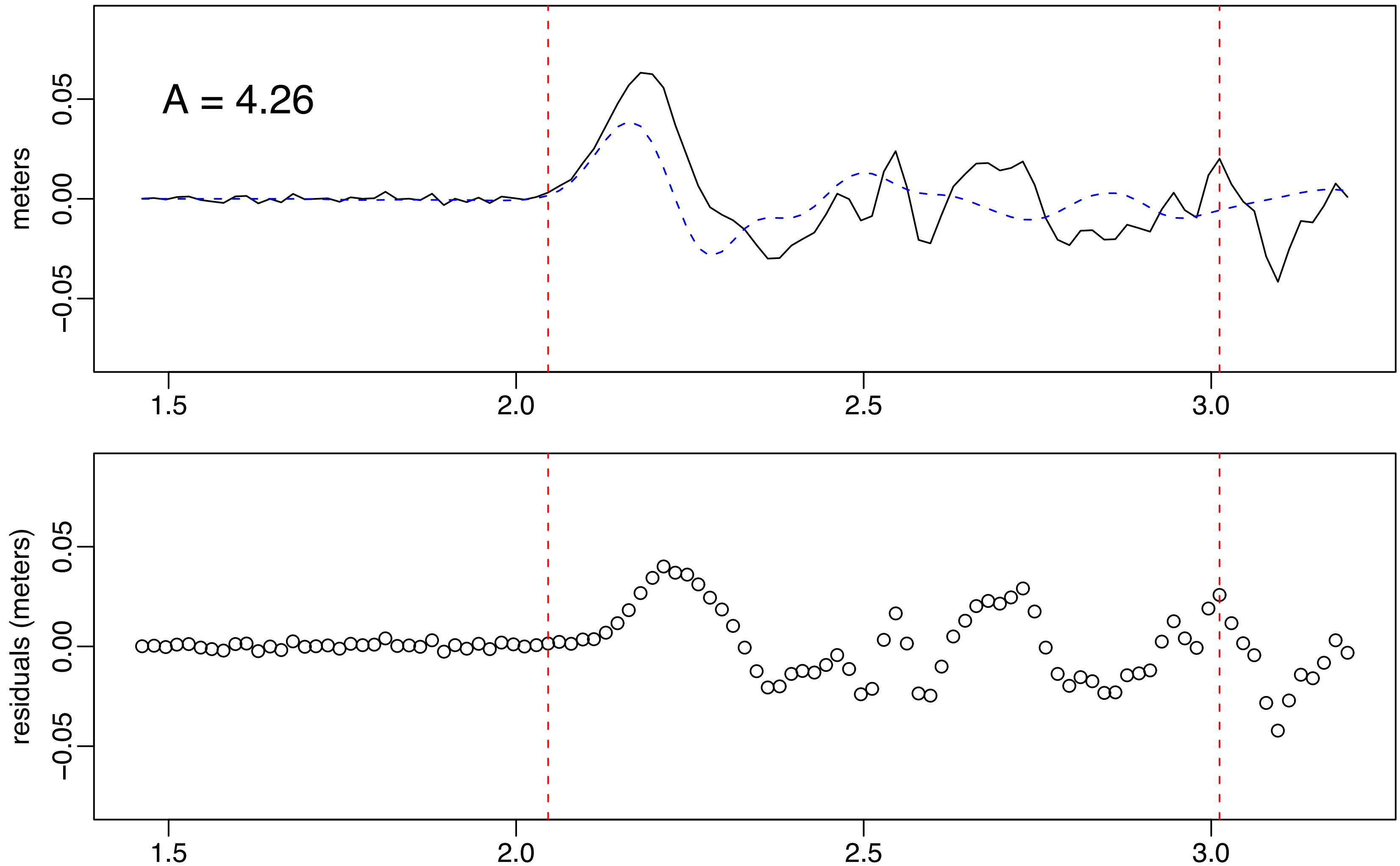


LS Fit of a12 Model to Selected 21414 Data



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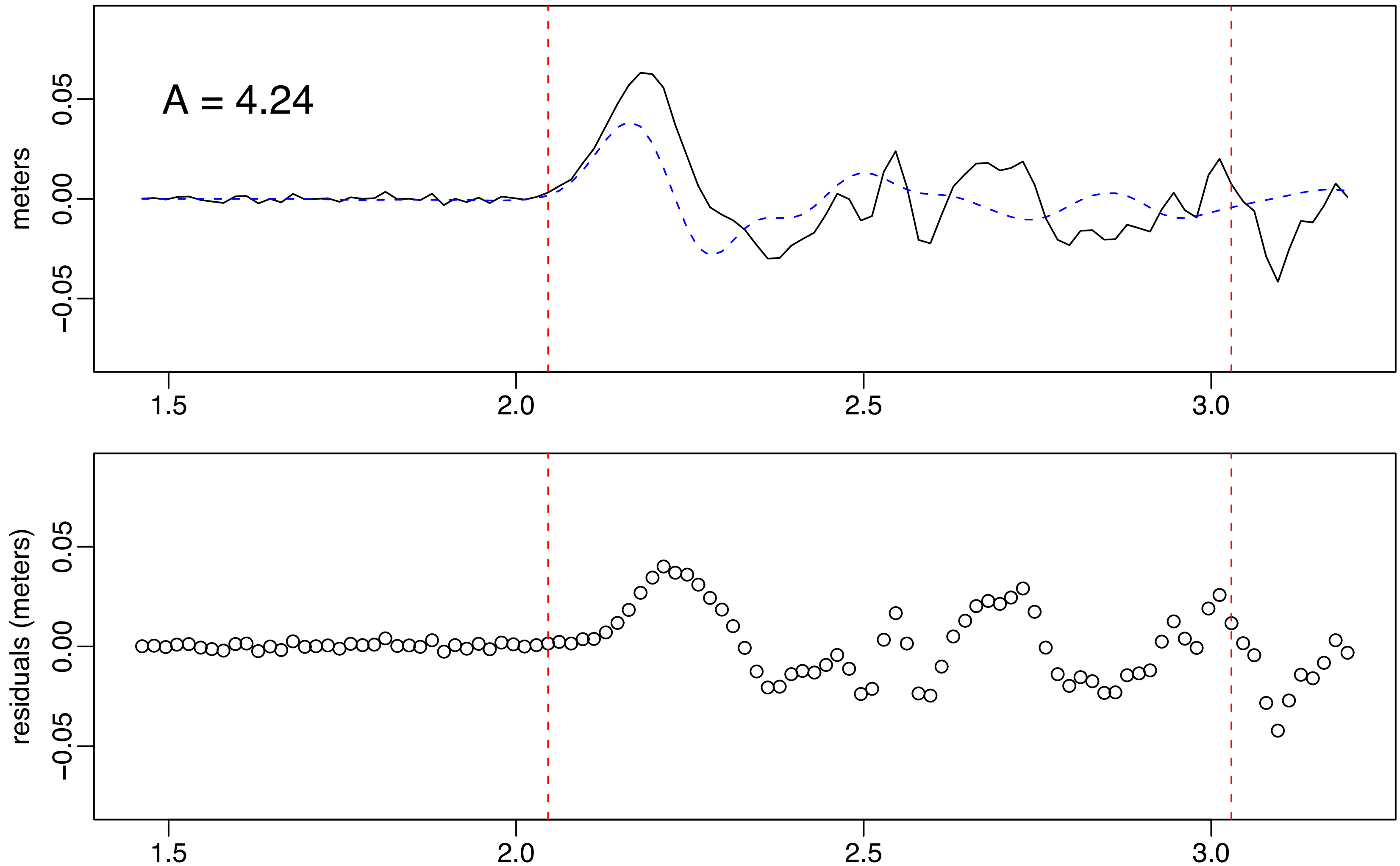
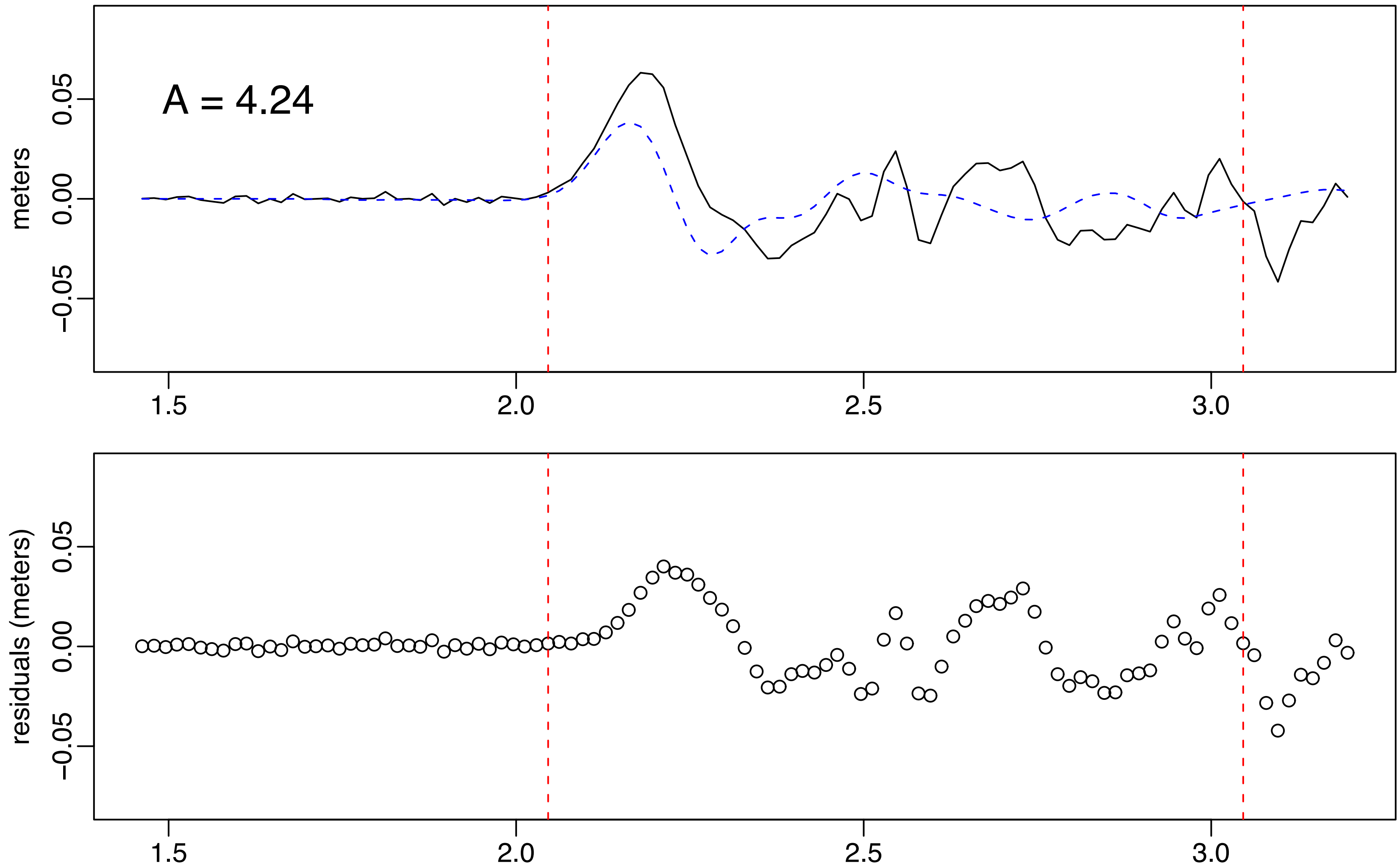


LS Fit of a12 Model to Selected 21414 Data



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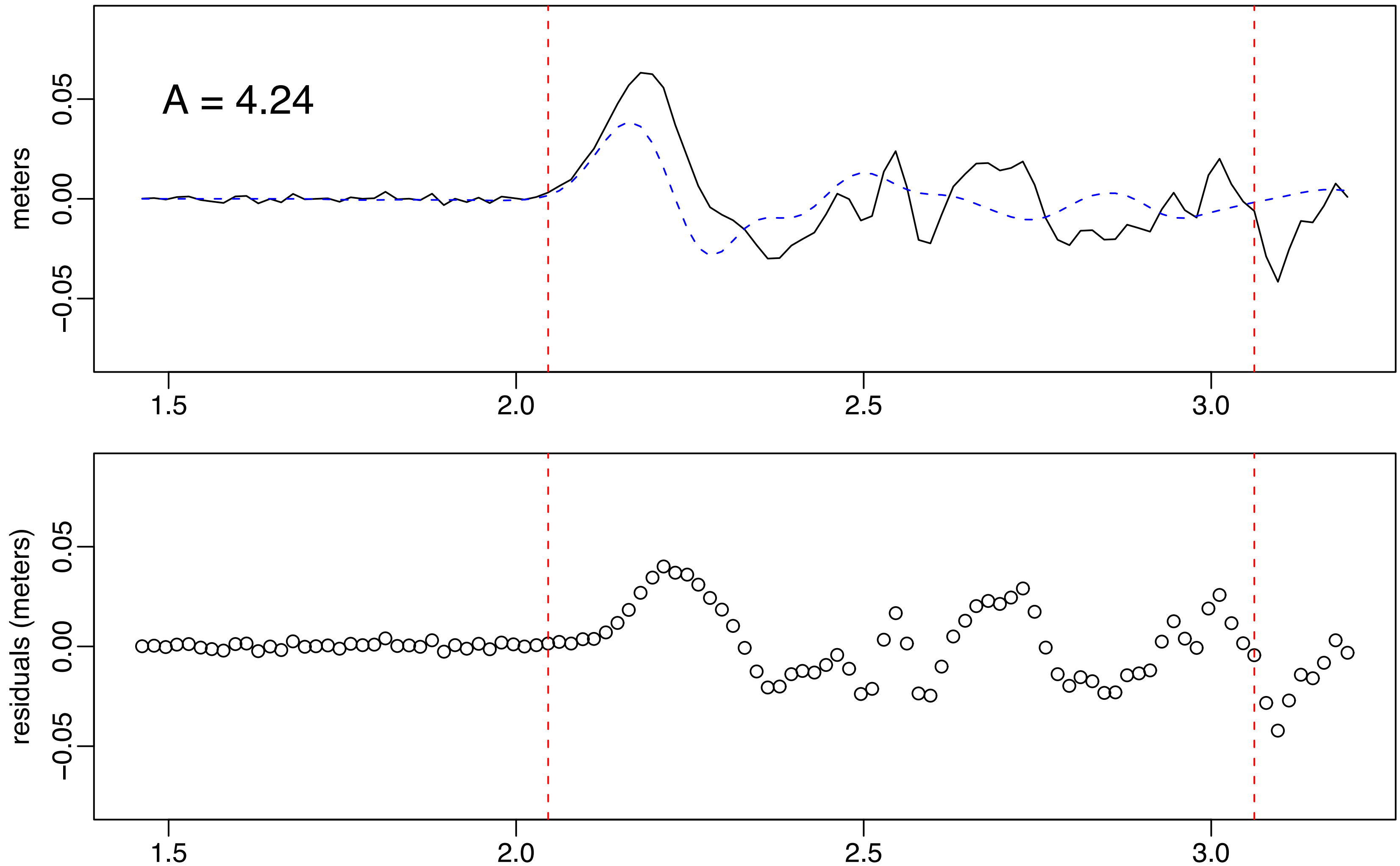

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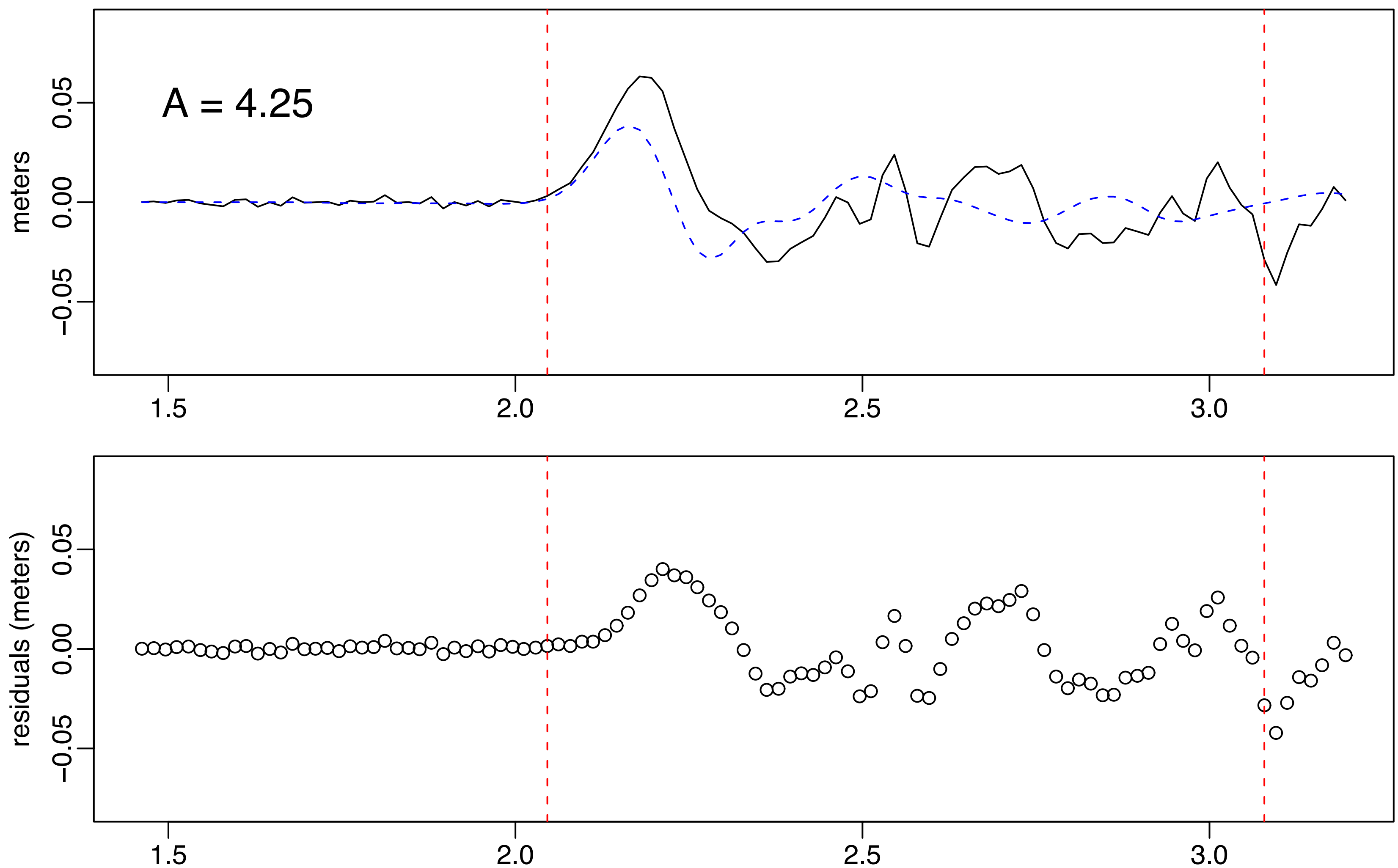

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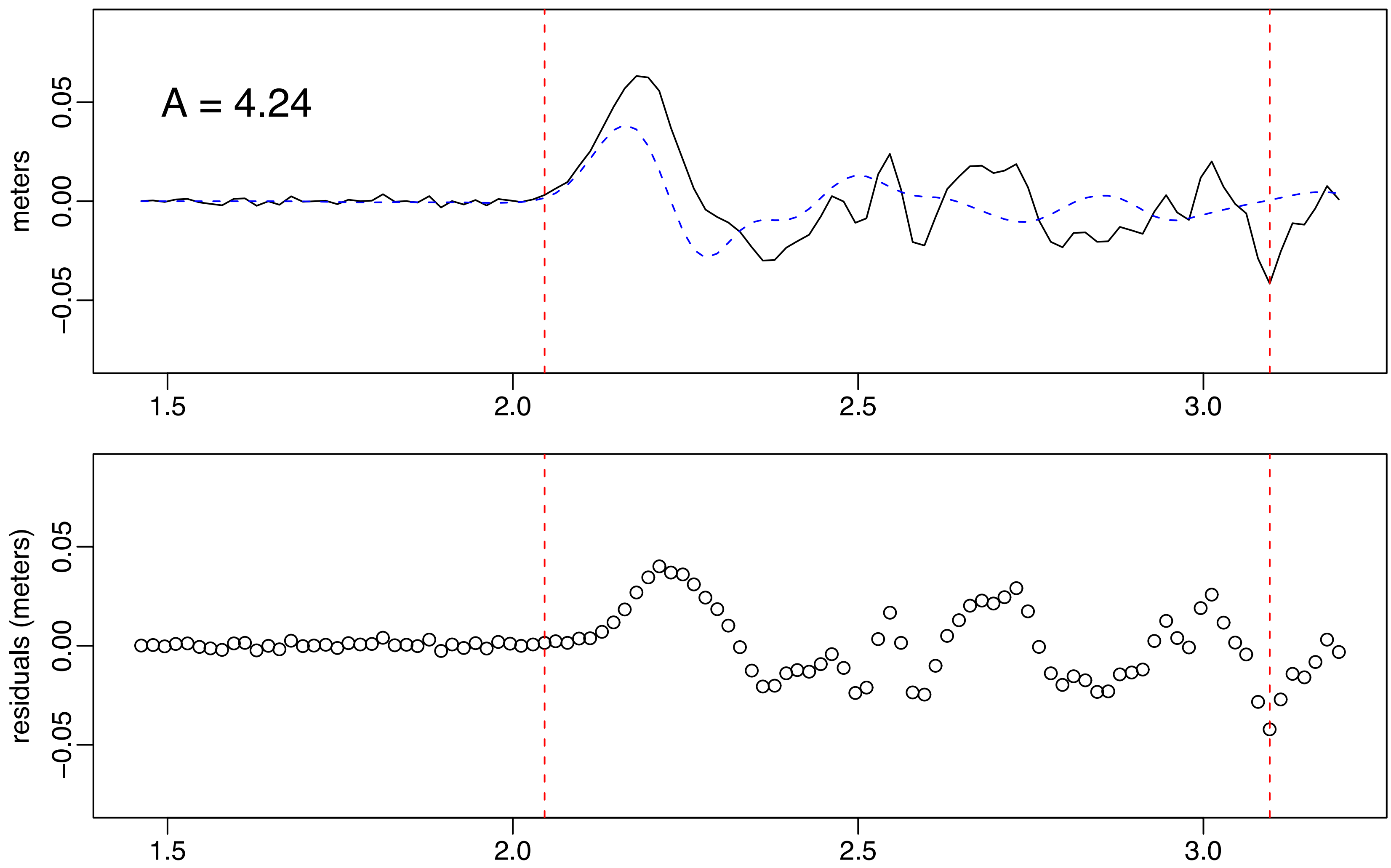

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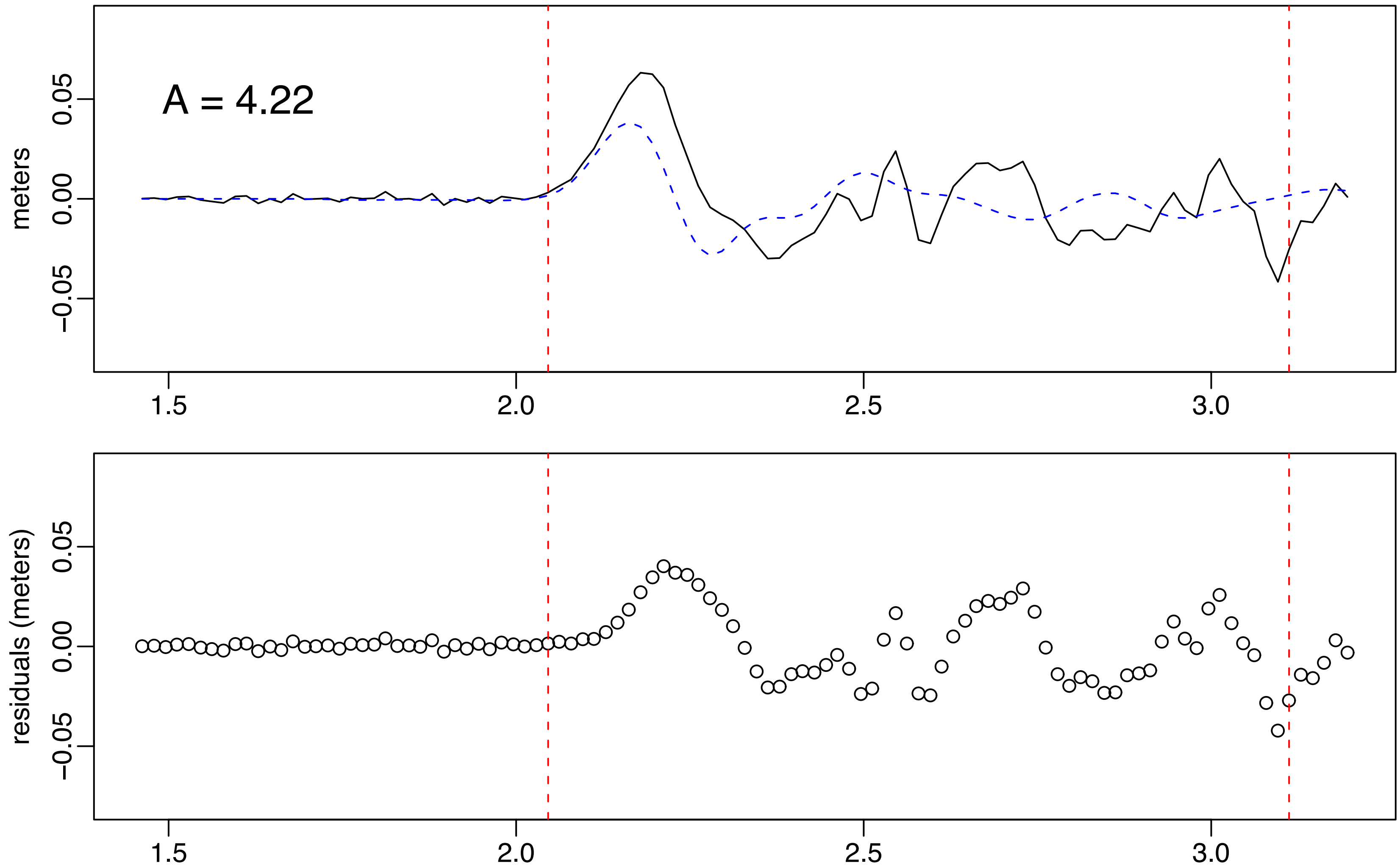
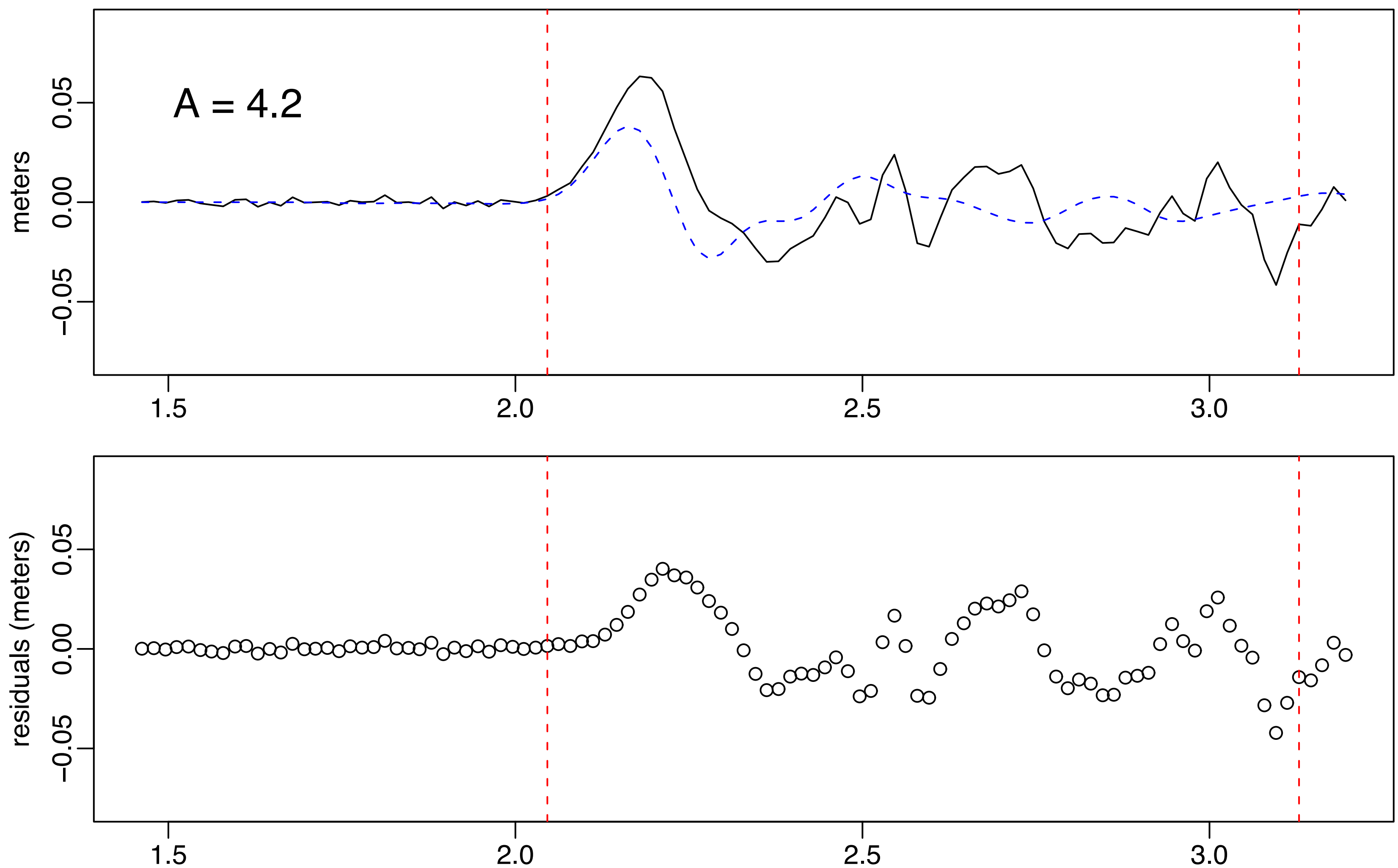
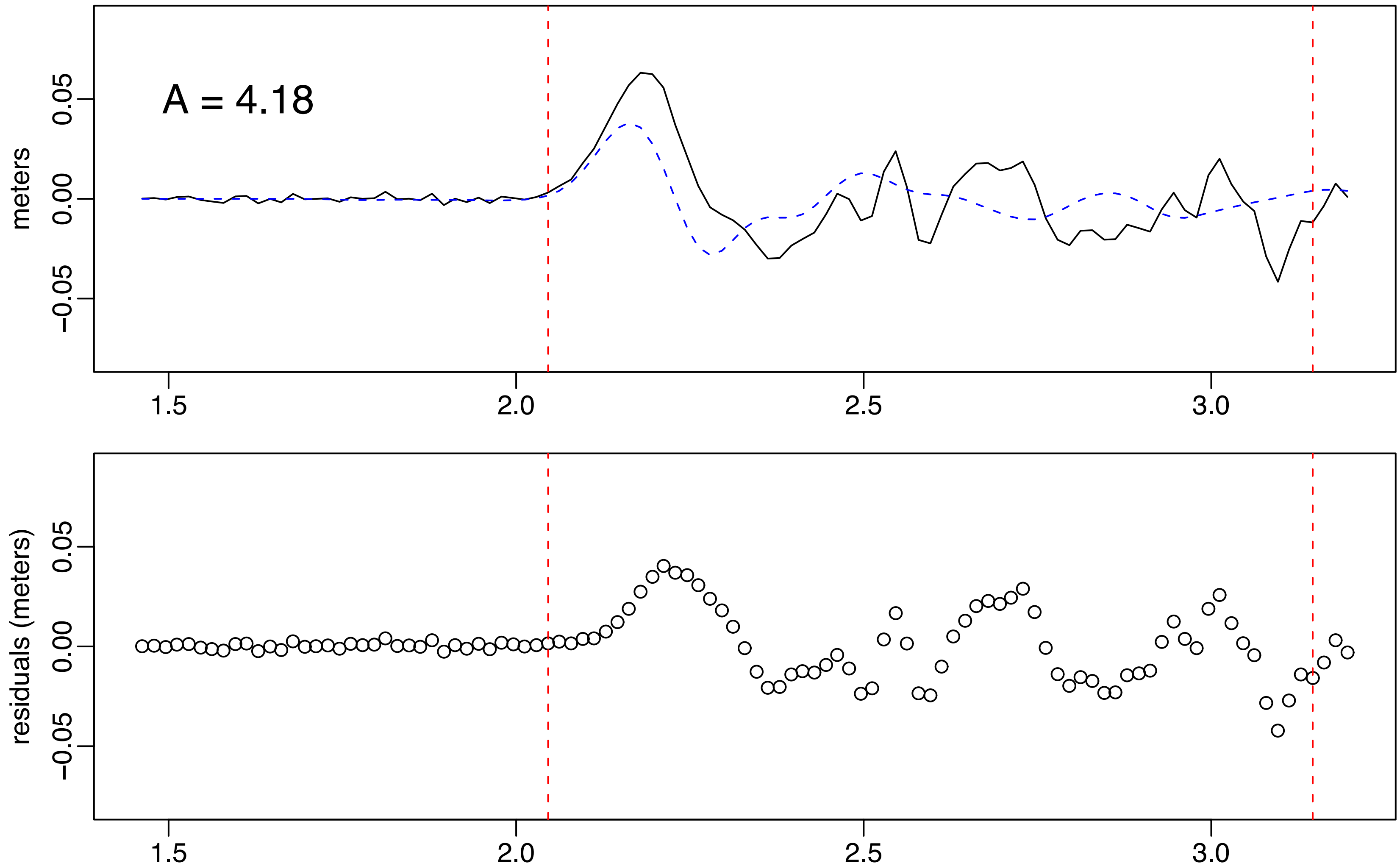


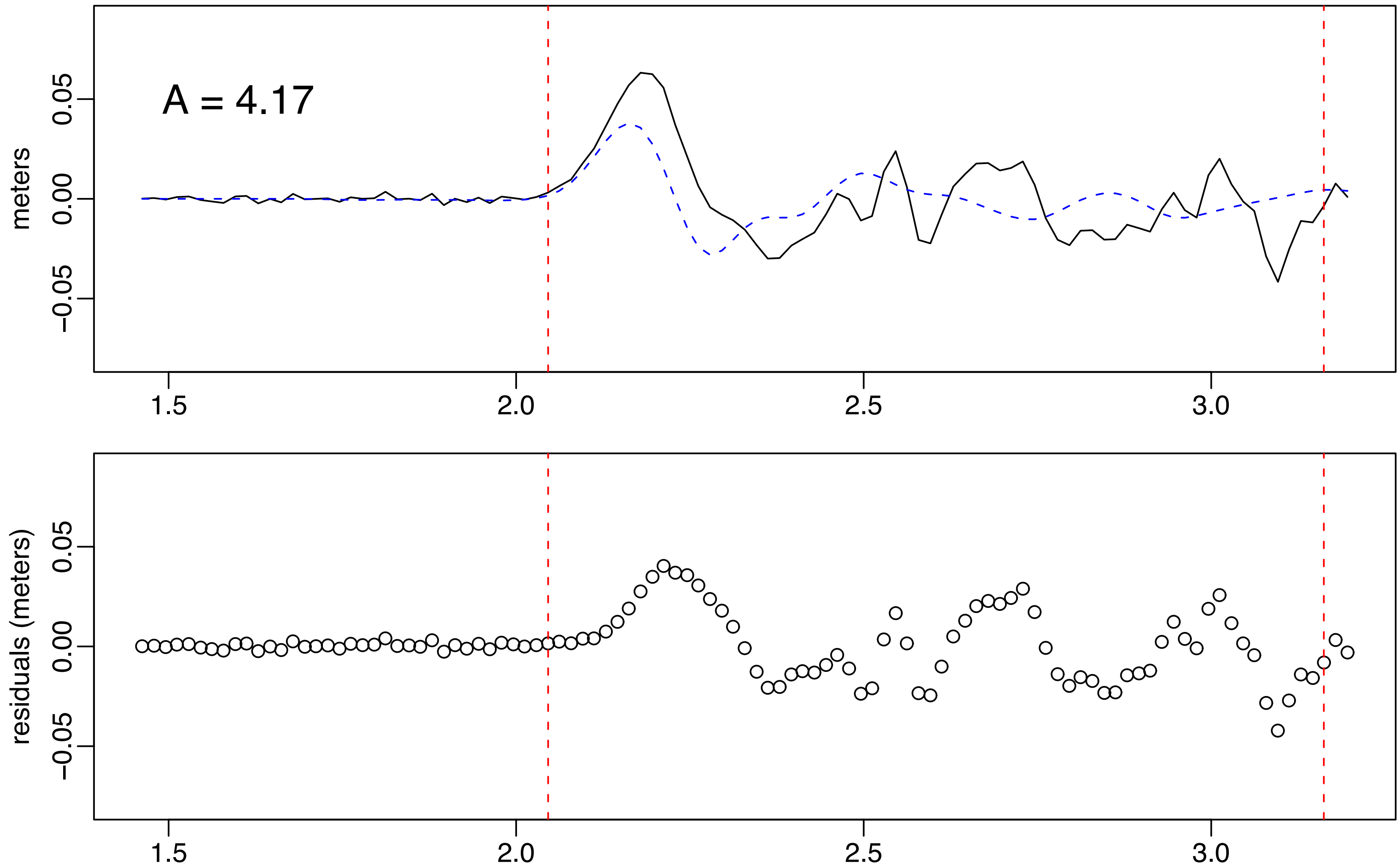
LS Fit of a12 Model to Selected 21414 Data



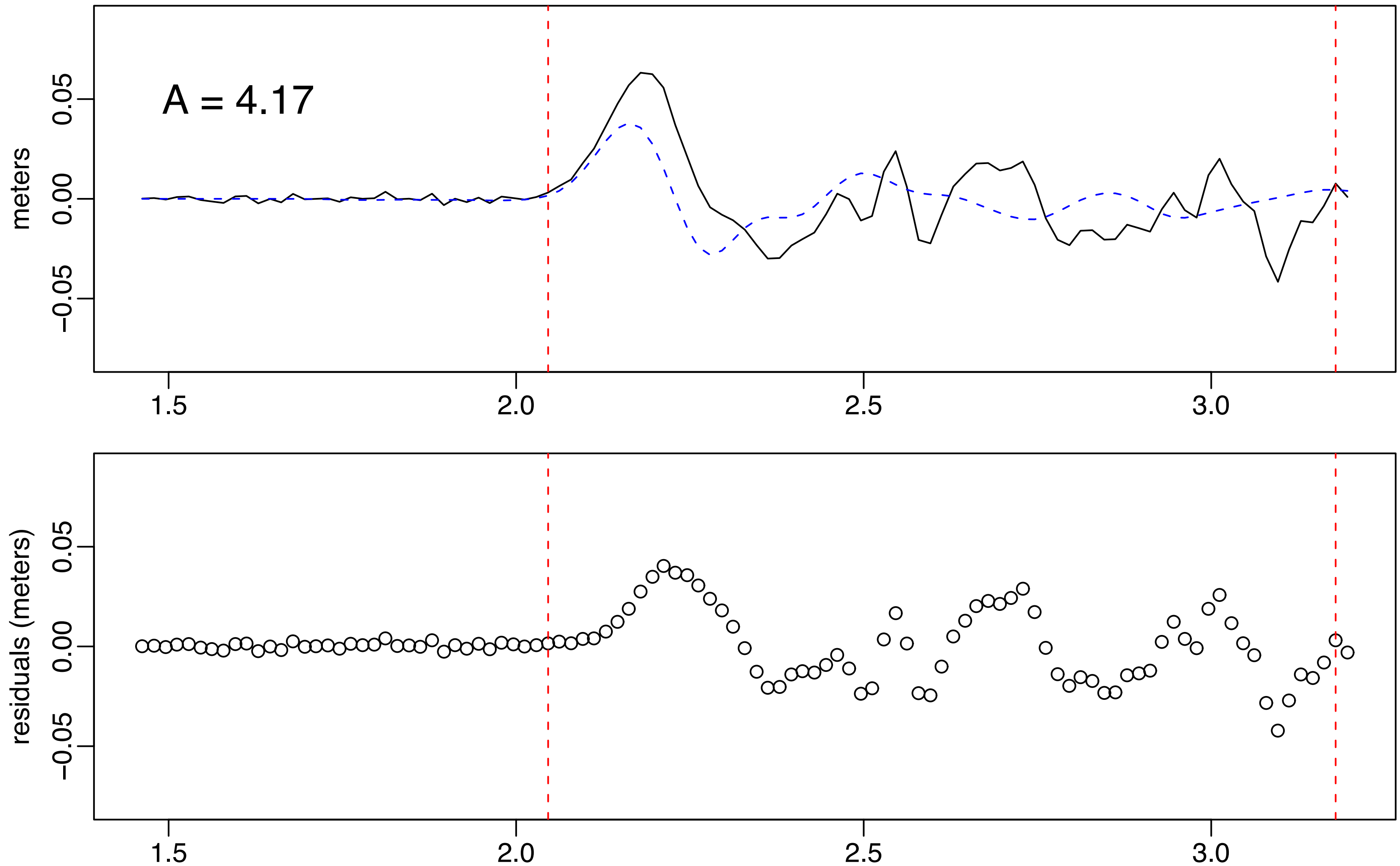
LS Fit of a12 Model to Selected 21414 Data



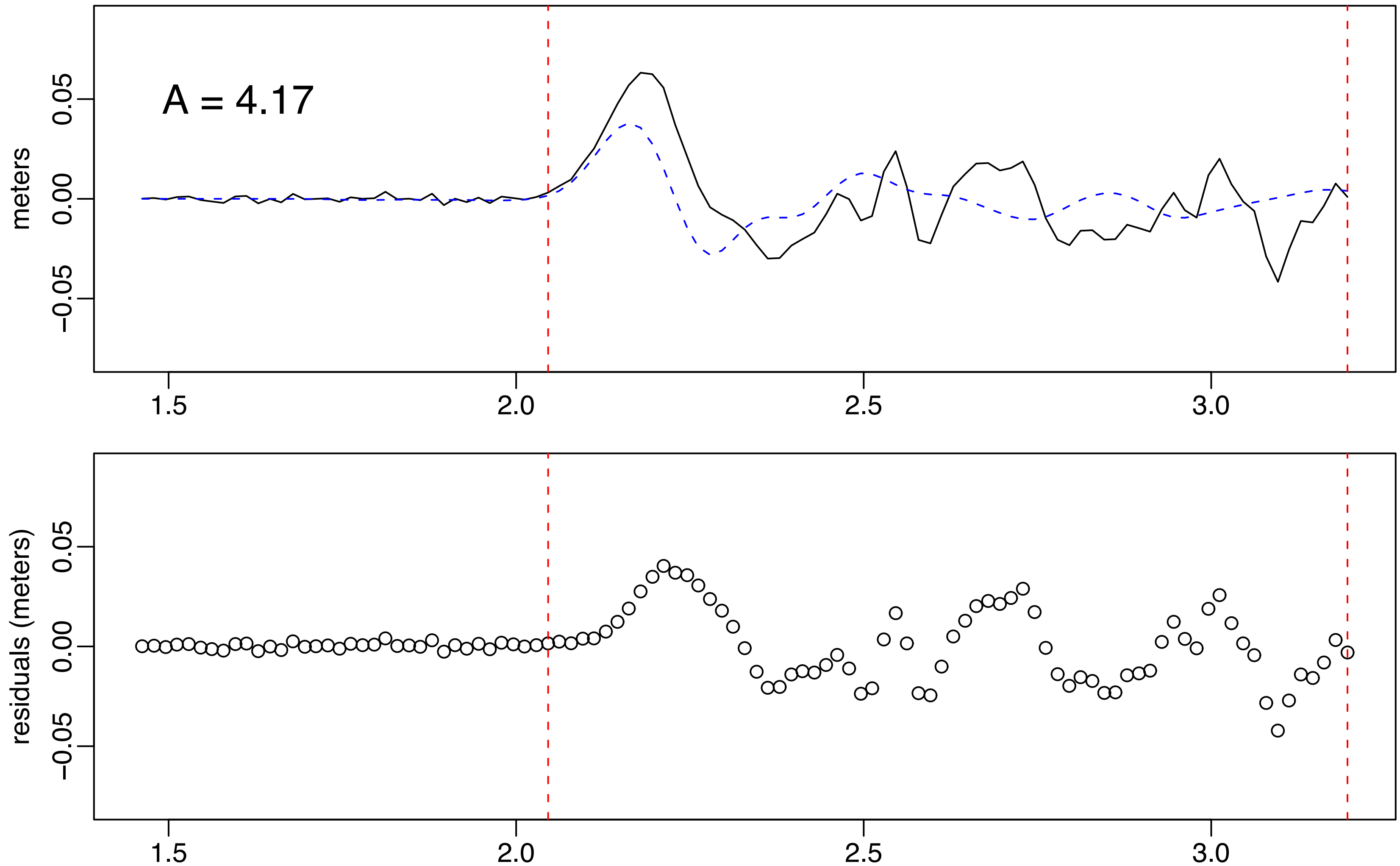
LS Fit of a12 Model to Selected 21414 Data



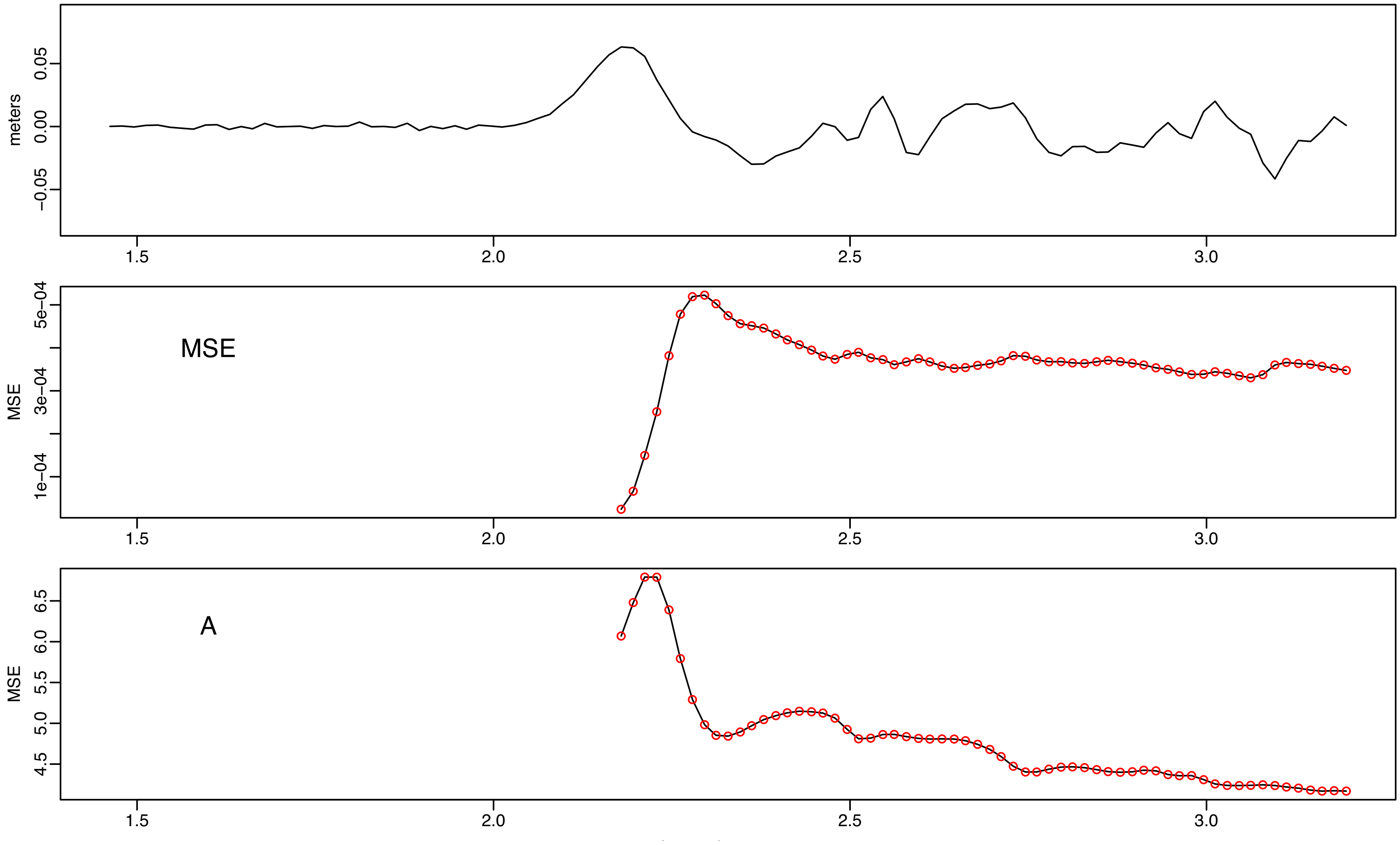
LS Fit of a12 Model to Selected 21414 Data



LS Fit of a12 Model to Selected 21414 Data



Mean Squared Errors and $\hat{A}$ for Selected 21414 Data



Incorporating Data from a Second Buoy: I

- so far, have modeled data from buoy 21414 in terms of an earthquake coming from source a12
- in vector notation, we have $\mathbf{x} = A\mathbf{g} + \mathbf{e}$
- in preparation for looking at additional buoys and additional sources, let's rewrite model as $\mathbf{x}_1 = A_{a12} \cdot \mathbf{g}_{1,a12} + \mathbf{e}_1$, where
  - $\mathbf{x}_1$ is a vector containing data from first buoy (here 21414)
  - A_{a12} is a scalar representing slip associated with source a12
  - $\mathbf{g}_{1,a12}$ is a vector containing unit slip model for what first buoy should see from earthquake originating at source a12
  - $\mathbf{e}_1$ is a vector of residuals (represents combination of measurements errors and model inaccuracies)

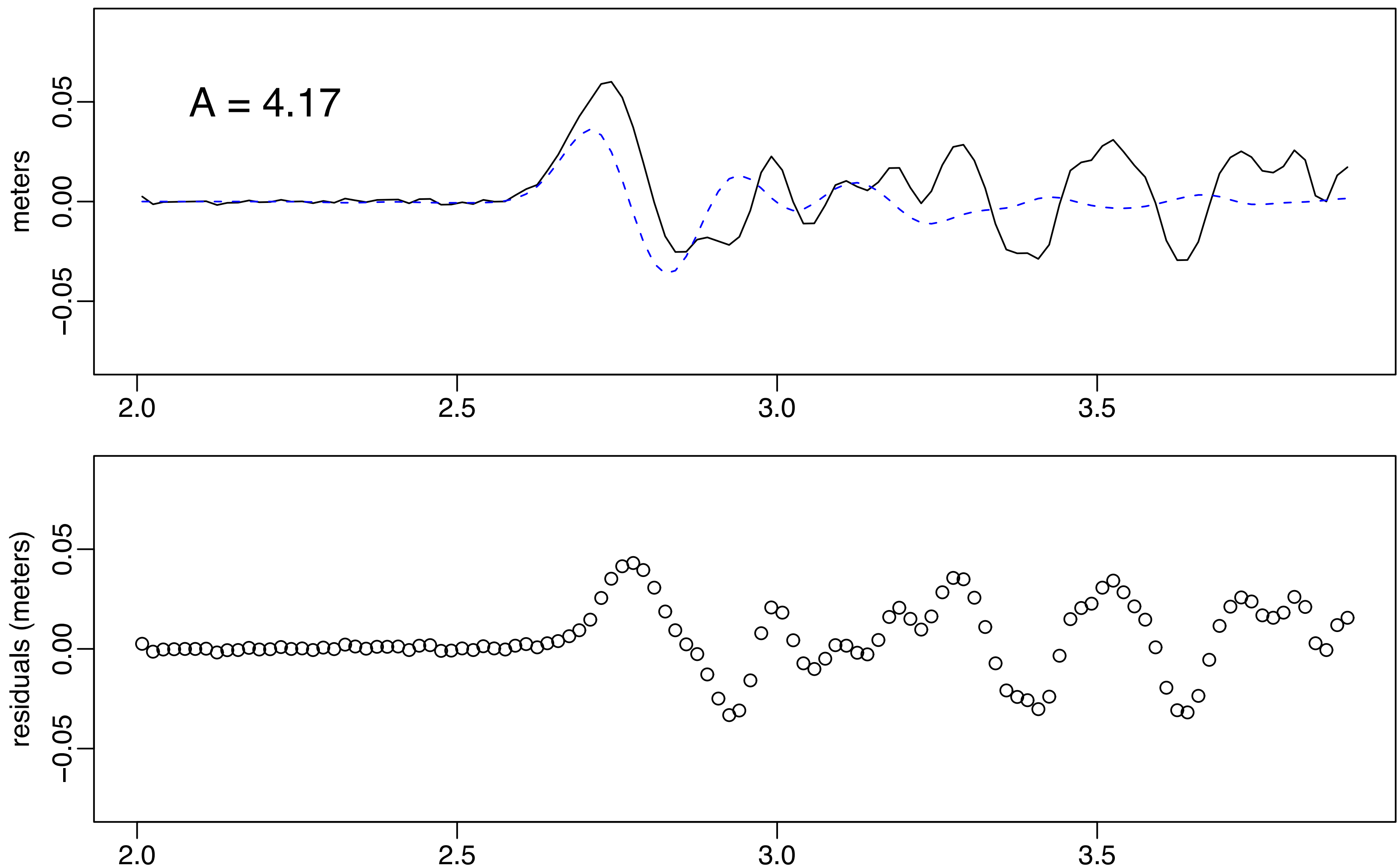
Incorporating Data from a Second Buoy: II

- in this notation, LS estimator of A_{a12} is given by

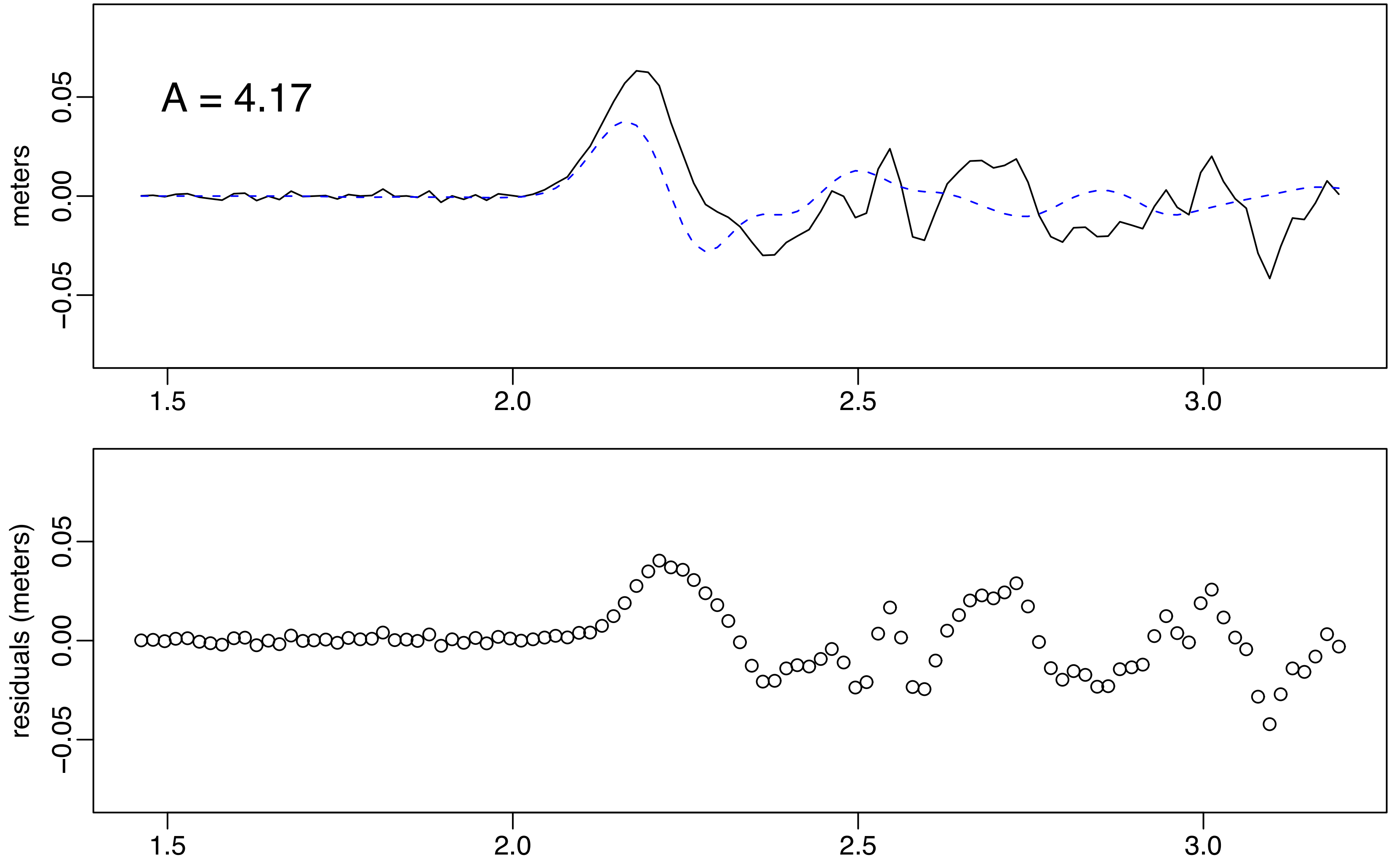
$$\hat{A}_{a12} = \mathbf{g}_{1,a12}^T \mathbf{x}_1 / \mathbf{g}_{1,a12}^T \mathbf{g}_{1,a12} = \left(\mathbf{g}_{1,a12}^T \mathbf{g}_{1,a12}\right)^{-1} \mathbf{g}_{1,a12}^T \mathbf{x}_1$$

  (last expression of interest for generalizations to come)
- now consider data $\mathbf{x}_2$ from a second buoy (46413)
- model this data as $\mathbf{x}_2 = A_{a12} \cdot \mathbf{g}_{2,a12} + \mathbf{e}_2$
- note that, while $\mathbf{g}_{2,a12}$ for buoy 46413 is different from $\mathbf{g}_{1,a12}$ for buoy 21414, both models have the same slip A_{a12}
- given our estimate $\hat{A}_{a12} \doteq 4.17$ based upon just $\mathbf{x}_1$, let's see how well $\mathbf{x}_2$ and $\hat{A}_{a12} \cdot \mathbf{g}_{2,a12}$ match up ('cross-validation')

a12 Slip from 21414 Data Applied to 46413 Data



a12 Slip from 21414 Data Applied to 21414 Data



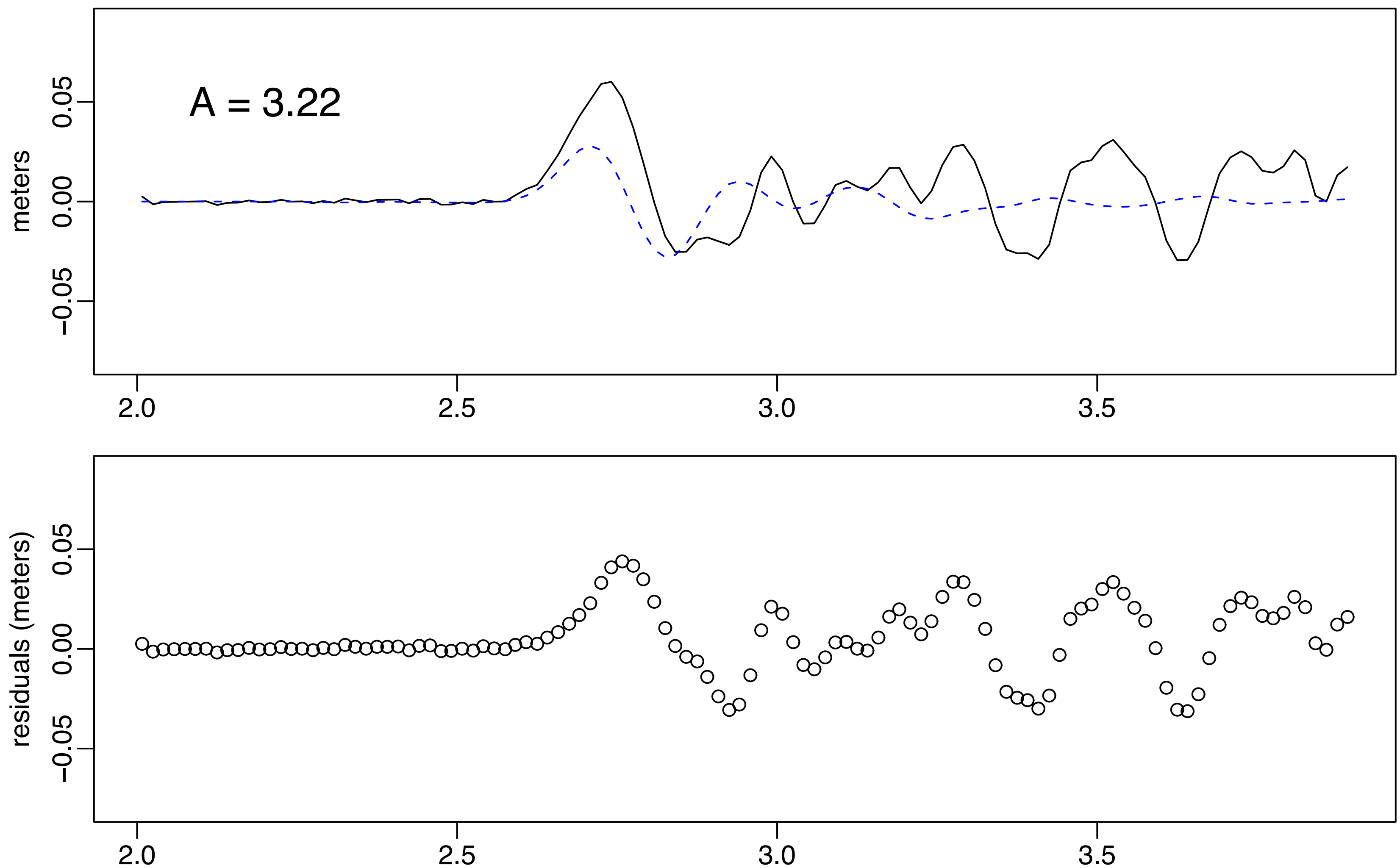
Incorporating Data from a Second Buoy: III

- can also estimate A_{a12} using just data from 46413:

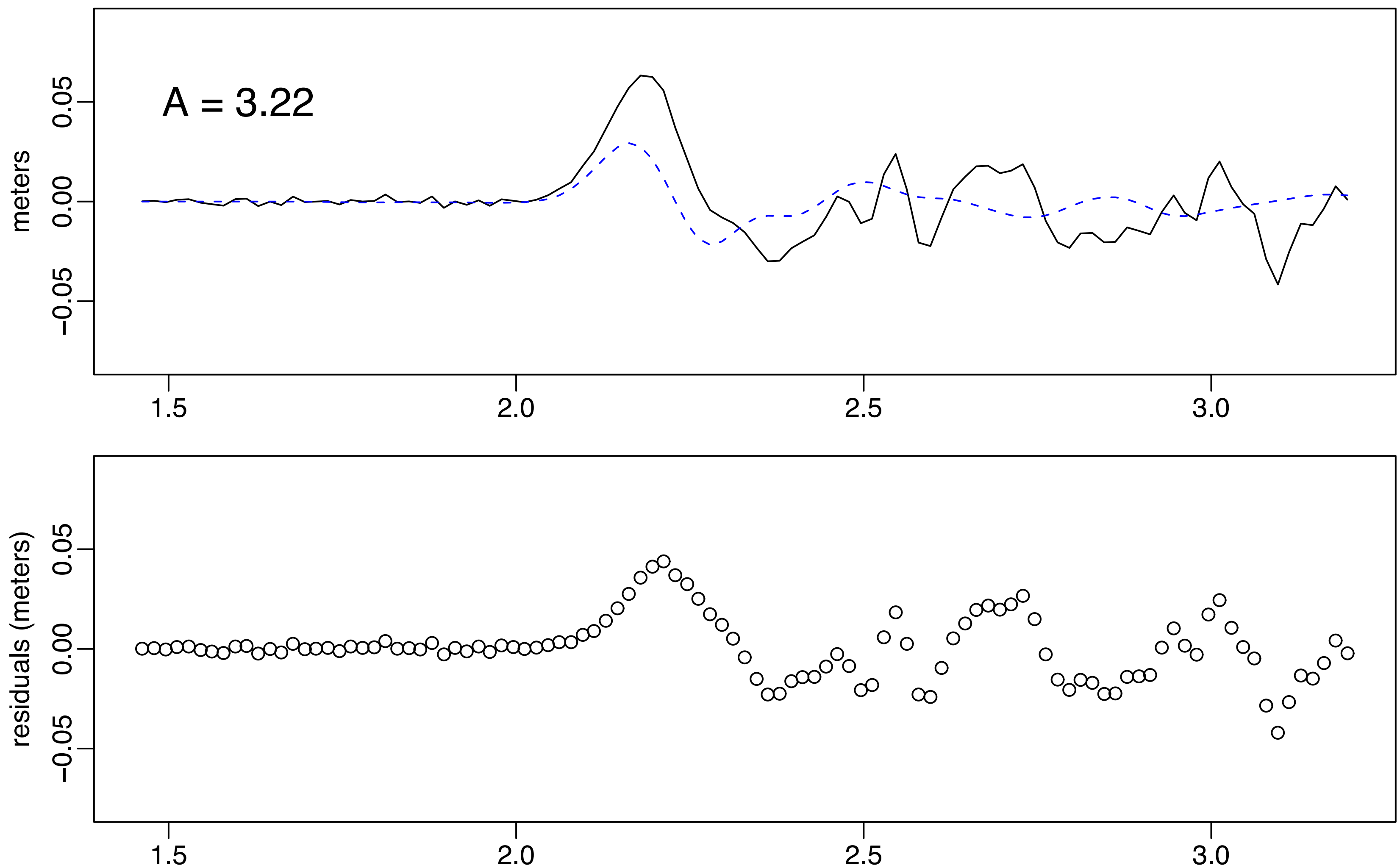
$$\hat{A}_{a12} = \left(\mathbf{g}_{2,a12}^T \mathbf{g}_{2,a12}\right)^{-1} \mathbf{g}_{2,a12}^T \mathbf{x}_2$$

- yields $\hat{A}_{a12} \doteq 3.22$, whereas we had $\hat{A}_{a12} \doteq 4.17$ from 21414
- can look at plots corresponding to the ones we had before

a12 Slip from 46413 Data Applied to 46413 Data



a12 Slip from 46413 Data Applied to 21414 Data



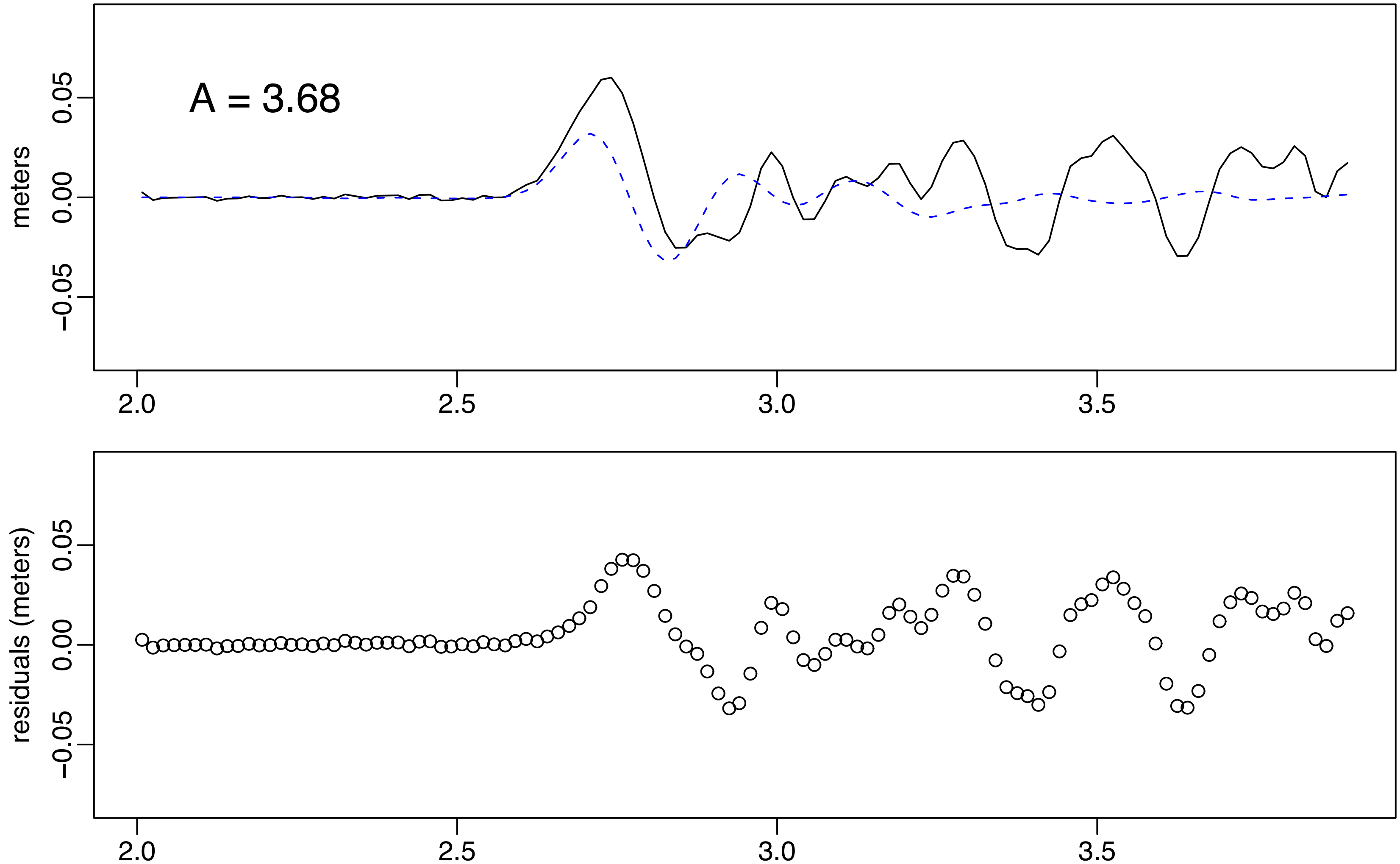
Incorporating Data from a Second Buoy: IV

- another approach is to use data from both buoys to get a joint estimate for A_{a12}
- joint model is $\mathbf{x}_{1:2} = A_{a12} \cdot \mathbf{g}_{1:2,a12} + \mathbf{e}_{1:2}$, where
  - $\mathbf{x}_{1:2}$ is a vector formed by stacking $\mathbf{x}_1$ on top of $\mathbf{x}_2$
  - A_{a12} is a scalar representing slip associated with source a12
  - $\mathbf{g}_{1:2,a12}$ is a vector formed by stacking $\mathbf{g}_{1,a12}$ on top of $\mathbf{g}_{2,a12}$
  - $\mathbf{e}_{1:2}$ is a vector of residuals
- LS estimator of A_{a12} now takes the form

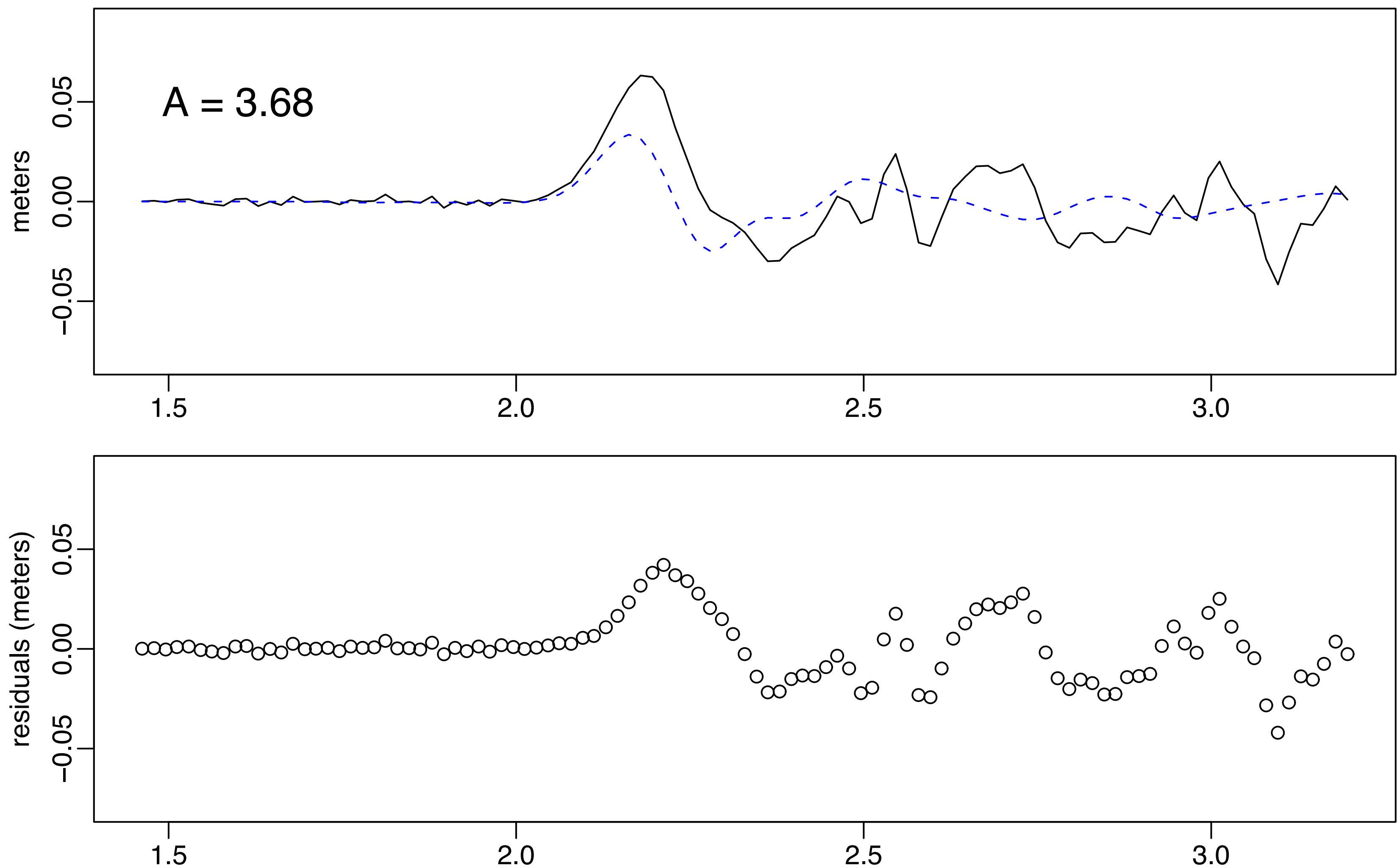
$$\hat{A}_{a12} = \left(\mathbf{g}_{1:2,a12}^T \mathbf{g}_{1:2,a12}\right)^{-1} \mathbf{g}_{1:2,a12}^T \mathbf{x}_{1:2}$$

- yields $\hat{A}_{a12} \doteq 3.68$ (cf. 4.17 from 21414 and 3.22 from 46413)
- can look at plots corresponding to the ones we had before

a12 Slip from Both Buoys Applied to 46413 Data



a12 Slip from Both Buoys Applied to 21414 Data



Using Linear Combinations of Sources: I

- so far, have modeled data in terms of a single source (a12)
- in vector notation, our model is $\mathbf{x}_{1:2} = A_{a12} \cdot \mathbf{g}_{1:2,a12} + \mathbf{e}_{1:2}$
- suppose earthquake is actually a linear combination of two sources, namely, a12 and b13
- our model is now $\mathbf{x}_{1:2} = A_{a12} \cdot \mathbf{g}_{1:2,a12} + A_{b13} \cdot \mathbf{g}_{1:2,b13} + \mathbf{e}_{1:2}$
- can reexpress this model as $\mathbf{x}_{1:2} = G\mathbf{A} + \mathbf{e}_{1:2}$, where
  - G is a matrix with two columns, namely, $\mathbf{g}_{1:2,a12}$ and $\mathbf{g}_{1:2,b13}$
  - $\mathbf{A}$ is a vector with two elements, namely, A_{a12} and A_{b13}
- LS estimator of $\mathbf{A}$ is given by $\hat{\mathbf{A}} = (G^T G)^{-1} G^T \mathbf{x}_{1:2}$, which is similar in form to

$$\hat{A}_{a12} = \left(\mathbf{g}_{1:2,a12}^T \mathbf{g}_{1:2,a12}\right)^{-1} \mathbf{g}_{1:2,a12}^T \mathbf{x}_{1:2}$$

Using Linear Combinations of Sources: II

- complication: models from two sources can be very similar!
- in worse case scenario, have $\mathbf{g}_{1:2,b13} = \alpha\mathbf{g}_{1:2,a12} \equiv \alpha\mathbf{g}$
- in this case, $G = [\mathbf{g}, \alpha\mathbf{g}]$ and

$$G^T G = \begin{bmatrix} \mathbf{g}^T\mathbf{g} & \alpha\mathbf{g}^T\mathbf{g} \\ \alpha\mathbf{g}^T\mathbf{g} & \alpha^2\mathbf{g}^T\mathbf{g} \end{bmatrix},$$

  which has a determinant of zero, so $(G^T G)^{-1}$ does not exist
- instead of using $\hat{\mathbf{A}} = (G^T G)^{-1} G^T \mathbf{x}_{1:2}$, can handle this case by solving equation $G^T G \hat{\mathbf{A}} = G^T \mathbf{x}_{1:2}$ with help of a singular value decomposition (SVD) of the matrix $G^T G$
- in general, use of SVD yields protection against problems of numerical stability in computing $\hat{\mathbf{A}}$

Using Linear Combinations of Sources: III

- using sources a12 and b13 to model data from buoys 21414 and 46413, LS estimates of slips are

$$\hat{\mathbf{A}} \equiv [\hat{A}_{a12}, \hat{A}_{b13}]^T \doteq [2.61, 3.81]^T$$

- fitted model and residuals for 21414 are given by

$$\mathbf{f}_1 \equiv \hat{A}_{a12} \cdot \mathbf{g}_{1,a12} + \hat{A}_{b13} \cdot \mathbf{g}_{1,b13} \text{ and } \mathbf{e}_1 = \mathbf{x}_1 - \mathbf{f}_1$$

likewise, fitted model and residuals for 46413 are given by

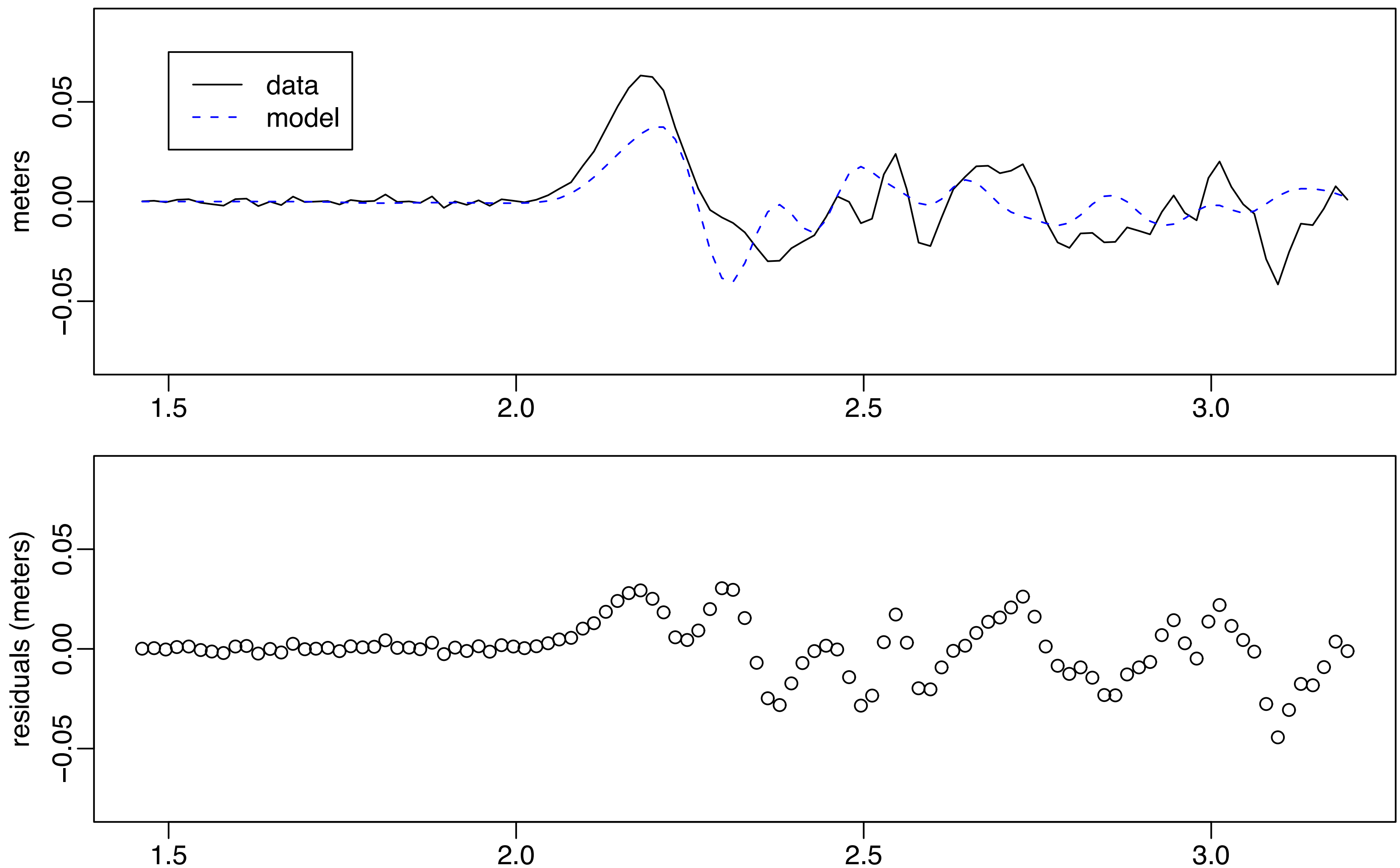
$$\mathbf{f}_2 \equiv \hat{A}_{a12} \cdot \mathbf{g}_{2,a12} + \hat{A}_{b13} \cdot \mathbf{g}_{2,b13} \text{ and } \mathbf{e}_2 = \mathbf{x}_2 - \mathbf{f}_2$$

- can use this model to predict what a third buoy should see:

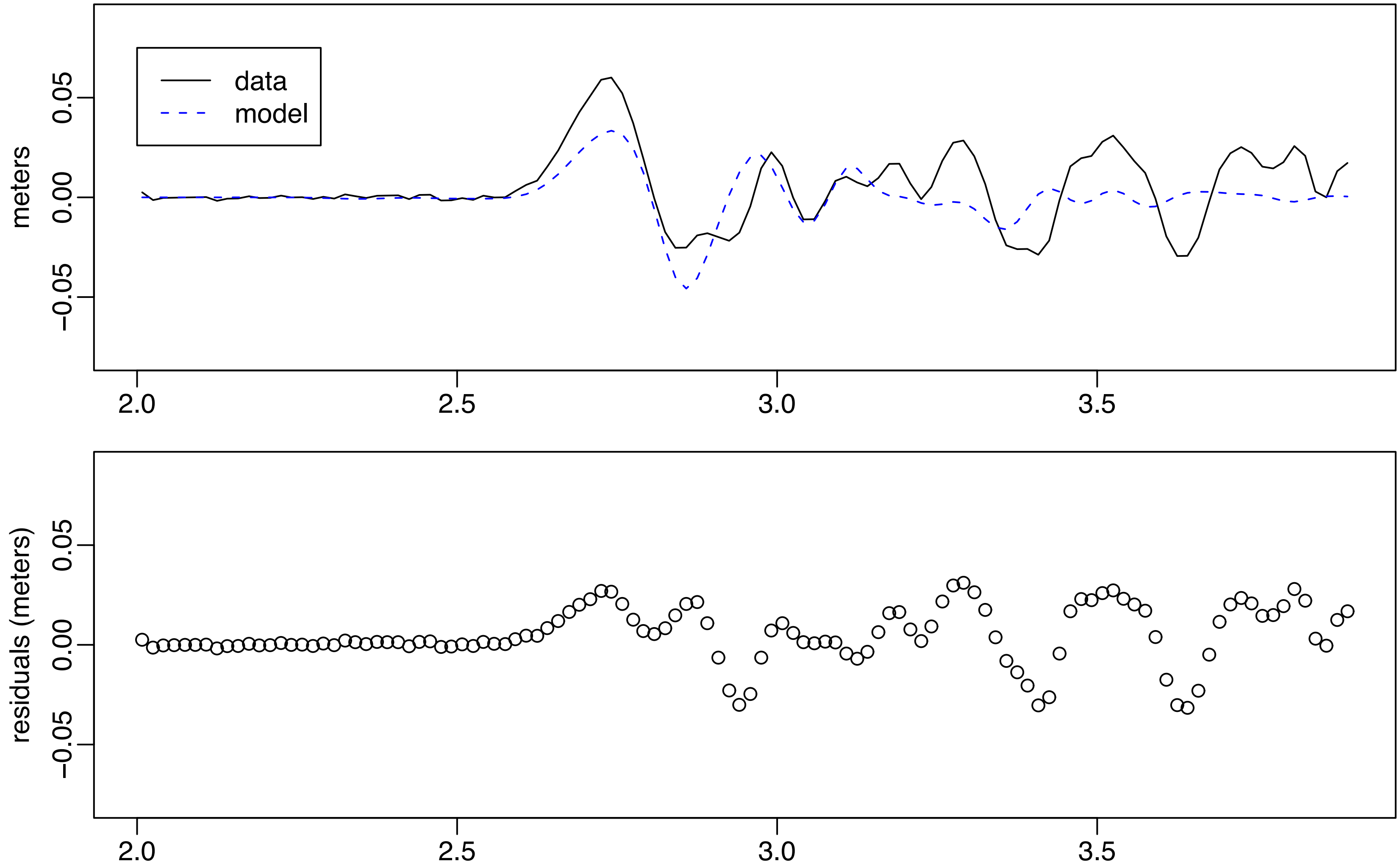
$$\mathbf{f}_3 \equiv \hat{A}_{a12} \cdot \mathbf{g}_{3,a12} + \hat{A}_{b13} \cdot \mathbf{g}_{3,b13}$$

('cross-validation')

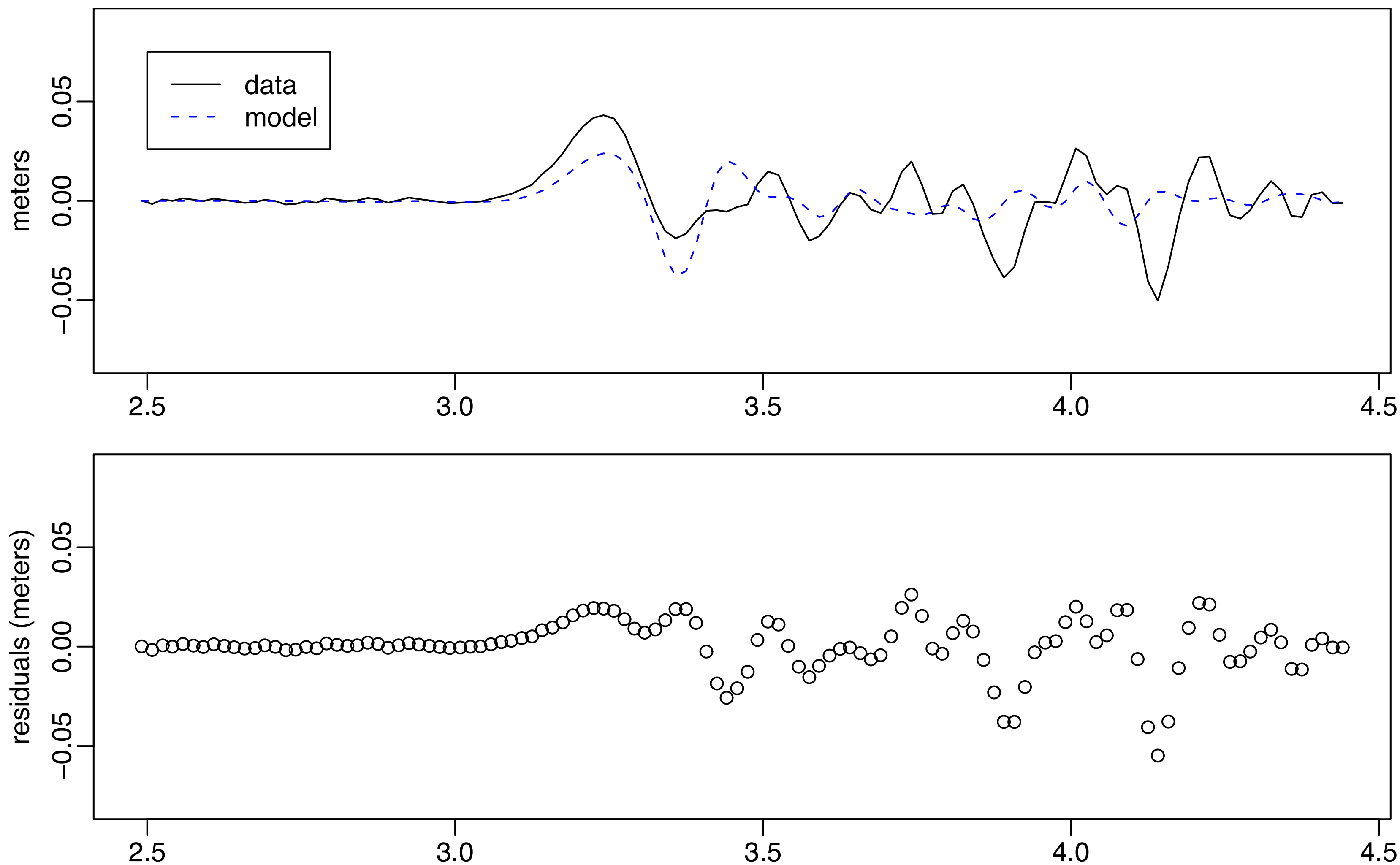
Data, Fitted Model and Residuals for 21414



Data, Fitted Model and Residuals for 46413



Data, Fitted Model and Residuals for 46408



Bells & Whistles

- current implementation of inversion algorithm allows for
  - constraints on slips (either $A \geq 0$ or $A \leq 0$)
  - shifting of source models, i.e., use of

    $$\tilde{g}(t) = g(t - a),$$

    where a is a shift that can be constrained to interval $[a_l, a_u]$
  - stretching/shrinking of source models, i.e., use of

    $$\tilde{g}(t) = g(t/b),$$

    where b is a stretch/shrink factor that can be constrained to interval $[b_l, b_u]$
  - shifting and stretching/shrinking together, i.e., use of

    $$\tilde{g}(t) = g([t - a]/b)$$

    with constraints on both a and b

Future Directions

- inversion algorithm requires choice of sources as part of input
- seismic information might suggest, say, eight sources
- currently user can do a joint fit and then manually select sources
- want to investigate use of statistical tests to select sources
- two approaches: step-up and step-down
- step-up approach starts with one source and uses statistical tests to add other sources one at a time
- step-down approach starts with, say, eight sources and uses statistical tests to remove sources one at a time
- idea is that these approaches might provide guidance on source selection for users

Demo of R Implementation

- R is an interpretive statistical language freely available from

  `http://www.r-project.org/`

  under the General Public License (GPL)
- R is popular in the statistical community for
  - testing out new ideas in statistics,
  - performing statistical analysis and
  - creating graphics
- inversion algorithm has been bread-boarded in R