Real-time Modelling of Tsunami Data

Applied Physics Laboratory
Department of Statistics
University of Washington
Seattle, Washington, USA

http://faculty.washington.edu/dbp

collaborative effort with NOAA Center for Tsunami Research

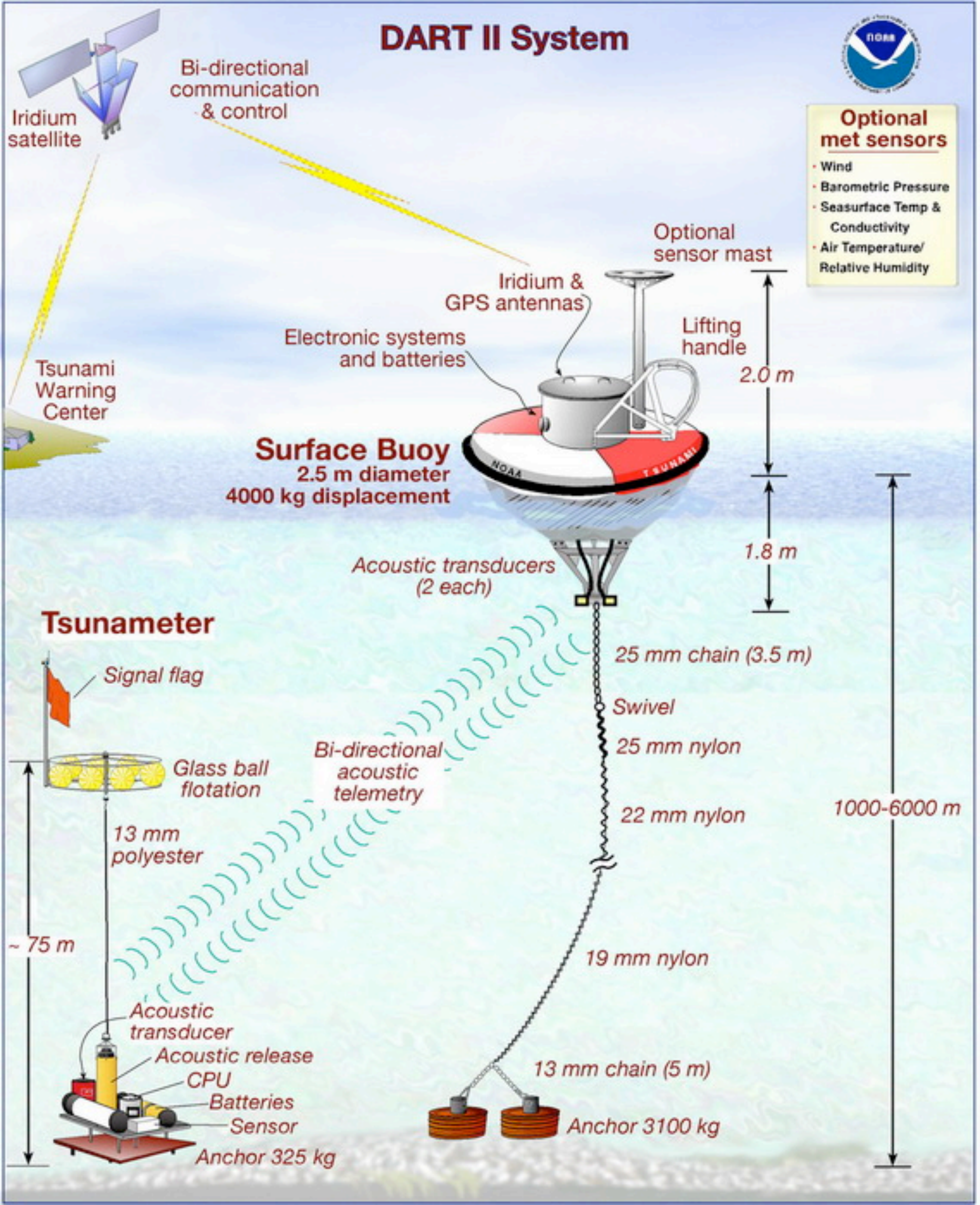
Background - I

- even before disasterous Sumatra tsunami in December 2004, destructive potential of earthquake-generated tsunamis was well-known
- due to rate at which a tsunami advances across the ocean, possible to lessen its effect on some coastal communities through advance warnings
- in USA, warning centers in Alaska and Hawaii are responsible for issuing timely bulletins about impending tsunamis
- seismometers give first indication of a potential tsunami event
  - starting time of earthquake event
  - location of epicenter
  - initial estimate of magnitude

Background - II

- seismic information alone is not enough to accurately forecast potential impact of tsnumai
- led to development of Deep-ocean Assessment and Reporting of Tsunamis (DART®) buoys

RONALD H BROWN
TSUNAMI


DART II System
Iridium satellite
Bi-directional communication & control
Optional met sensors
Wind
Barometric Pressure
Seasurface Temp & Conductivity
Air Temperature/ Relative Humidity
Optional sensor mast
Iridium & GPS antennas
Lifting handle
Electronic systems and batteries
2.0 m
Tsunami Warning Center
Surface Buoy
2.5 m diameter
4000 kg displacement
NOAA
TSUNAMI
1.8 m
Acoustic transducers (2 each)
Tsunameter
25 mm chain (3.5 m)
Signal flag
Swivel
25 mm nylon
Bi-directional acoustic telemetry
Glass ball flotation
22 mm nylon
1000-6000 m
13 mm polyester
~ 75 m
19 mm nylon
Acoustic transducer
Acoustic release
CPU
Batteries
Sensor
Anchor 325 kg
13 mm chain (5 m)
Anchor 3100 kg

Background - III

- prior to December 2004, there were six DART® buoys deployed in Pacific
- since then, many more have been added
- mindful of Percival's First Law

‘Real-time computer demonstrations are doomed to fail!’

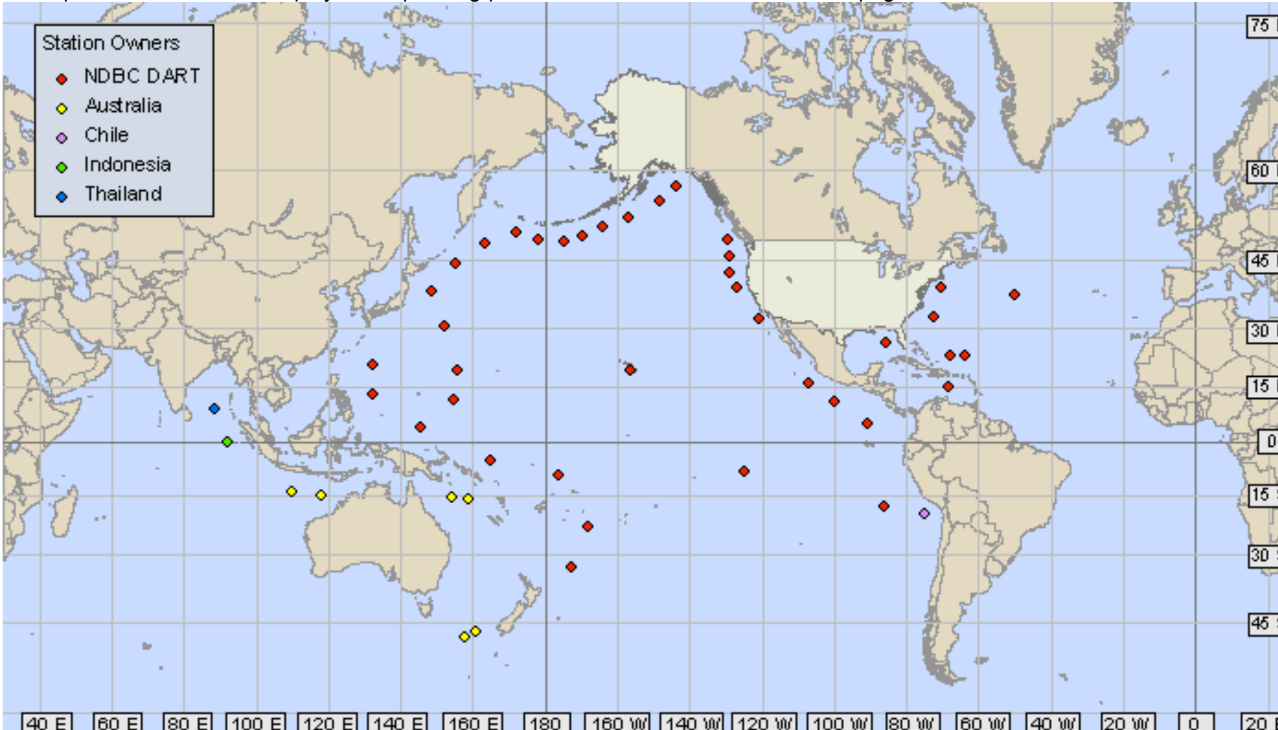
let's look at network of buoys and recent data by going to

`http://www.ndbc.noaa.gov/dart.shtml`

and movie of Nov. 2006 Kuril Island event downloadable from

`http://nctr.pmel.noaa.gov/kuril20061115.html`

Place pointer on station to display corresponding plot or click on station to view station page.



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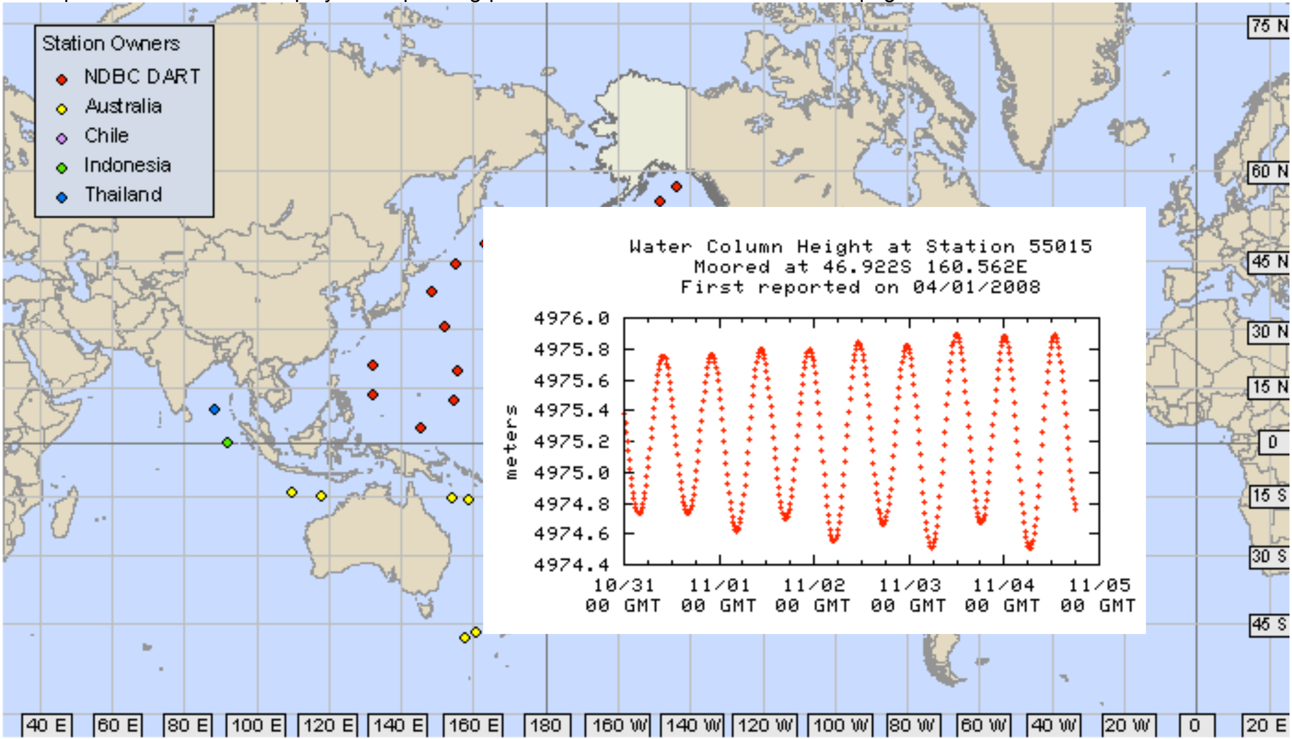


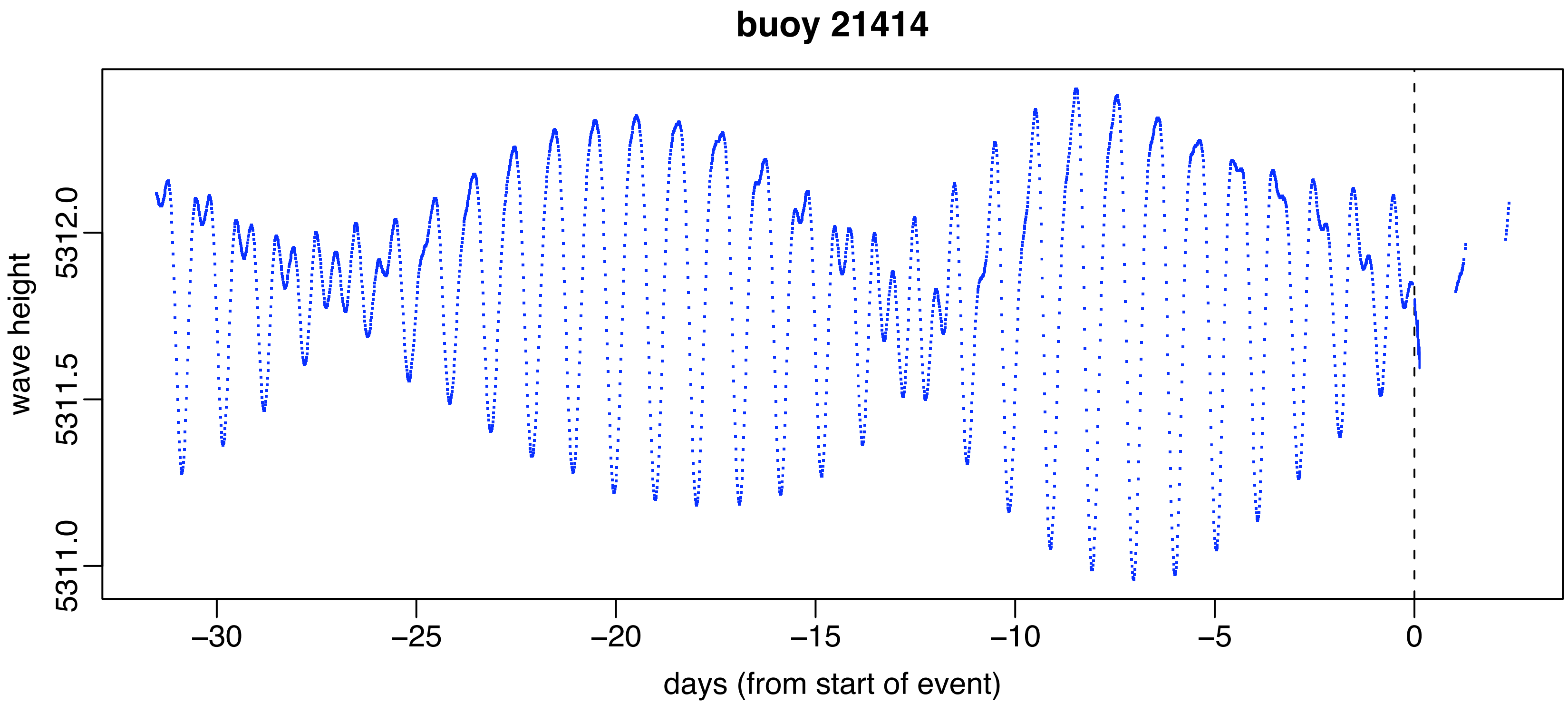
Format of Data

- buoy records one pressure measurement every 15 sec (represents an average over 15 sec)
- 15-min stream: when nothing is going on, buoy transmits one 15-sec measurement every 15 min
- 15-sec stream: when triggered by a seismic event, transmits all recorded measurements
- 1-min stream: during most of tsunami event, transmits averages of four consecutive 15-sec measurements

Short-term Inundation Forecast for Tsunamis (SIFT)

- SIFT is a computer application that uses DART® data and precomputed geophysically-based predictions to assess magnitude of tsunami while tsunami event is evolving
- SIFT currently used at warning centers in Alaska and Hawaii, but is still under development at NOAA Center for Tsunami Research
- statistical issues involved in processing data within SIFT
  - signal extraction: need to remove tidal component before precomputed predictions can be used
  - assessment of uncertainty: need error bars on estimates of magnitude of tsunami
  - variable selection: help operators select predictors

Buoy 21414 Data for Nov. 2006 Kuril Islands Event



Approaches for Signal Extraction (Detiding)

- three approaches, each with certain strengths and weaknesses
  1. empirical orthogonal functions (Tolkova, 2009)
  2. localized polynomial fits
  3. two-stage approach, the second of which involves Kalman filter/smoother
- will concentrate on third approach

Model for Data with Tides

- assume measured data at time t can be expressed as

$$x(t) = \mu(t) + \sum_{l=1}^{L} c_l(t) + \epsilon(t)$$

  where

  - $\mu(t) + \sum_l c_l(t)$ represents tides;
  - $\mu(t)$ is 'unpredictable' tidal component (has slowly varying mean level);
  - $c_l(t) = C_l \cos(2\pi f_l t + \phi_l)$ is lth part of 'predictable' tidal component (sinusoid with amplitude A_l, frequency f_l and phase ϕ_l); and
  - $\epsilon(t)$ is detided data

Model for 15-Min Stream Prior to Tsunami: I

- let $t = 0$ denote starting time (in days) of tsunami events
- can write $C_l \cos(2\pi f_l t + \phi_l) = A_l \cos(2\pi f_l t) + B_l \sin(2\pi f_l t)$, so assume data $x(t)$ at times $t < 0$ can be expressed as

$$x(t) = \mu + \sum_{l=1}^{L} A_l \cos(2\pi f_l t) + B_l \sin(2\pi f_l t) + \epsilon_t$$

  where

  - μ is an unknown overall mean level;
  - f_l is a known tidal frequency;
  - A_l and B_l are unknown amplitudes; and
  - ϵ_t is a residual term

- use linear least squares to estimate μ, A_l and B_l – denote these estimates as $\hat{\mu}$, $\hat{A}_l$ and $\hat{B}_l$

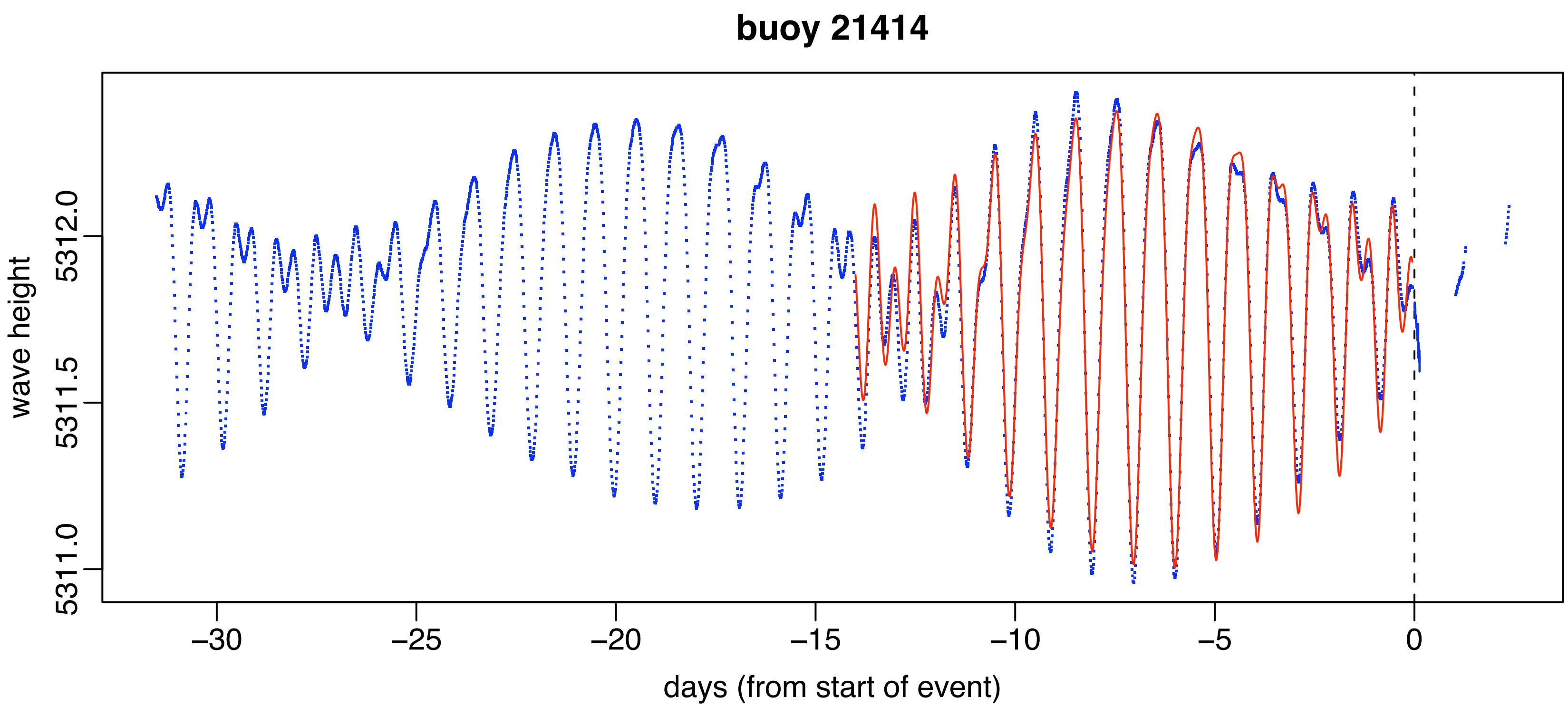
Model for 15-Min Stream Prior to Tsunami: II

- if using 14 days of prior data, let $L = 4$, and set f_1, f_2, f_3 and f_4 to so-called M2, S2, O1 and K1 frequencies
  (other rules apply if use, e.g., 1 day or 29 days of prior data)
- estimate tidal component at time t using

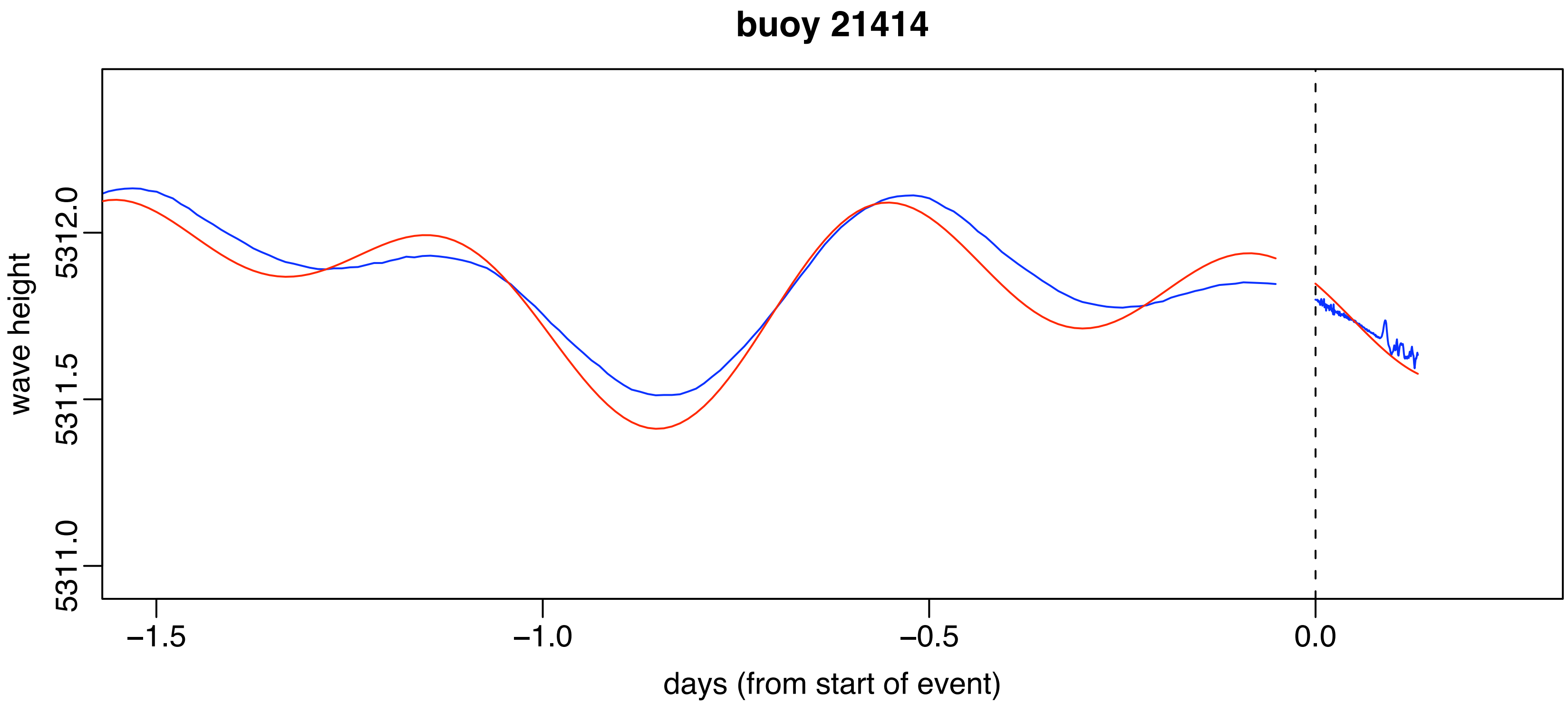
$$\hat{x}(t) = \hat{\mu} + \sum_{l=1}^{L} \hat{A}_l \cos(2\pi f_l t) + \hat{B}_l \sin(2\pi f_l t)$$

- for $t < 0$, can compare estimated tide to actual data
- for $t > 0$, can detide data from 1 min and/or 15 sec streams by subtracting off $\hat{x}(t)$

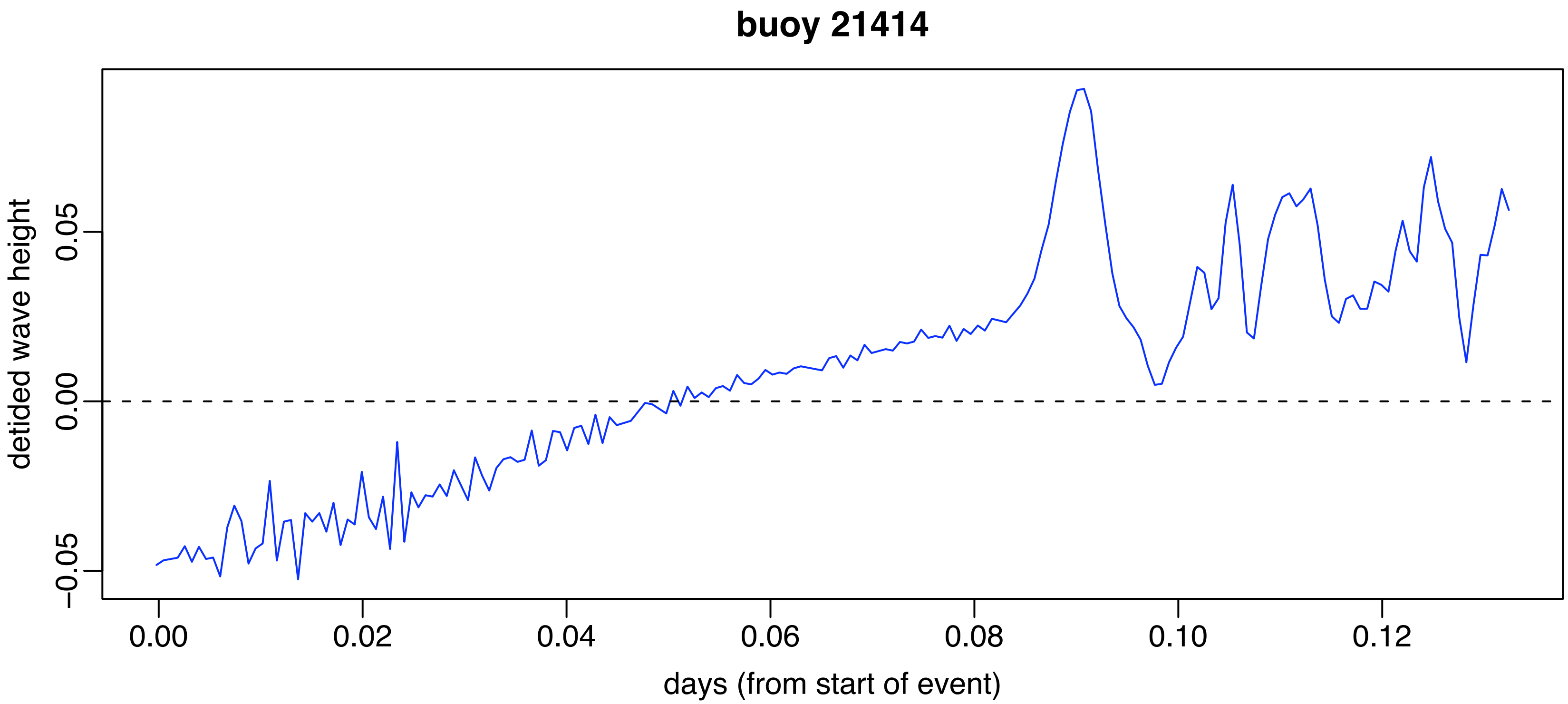
14 Day Model (M2, S2, O1 and K1)



14 Day Model and Extrapolation



Data Detided using 14 Day Model



Second-Stage Detiding via Kalman Filter/Smoother

- after first-stage detiding, can write

$$x(t) - \hat{x}(t) = \mu(t) + \epsilon(t),$$

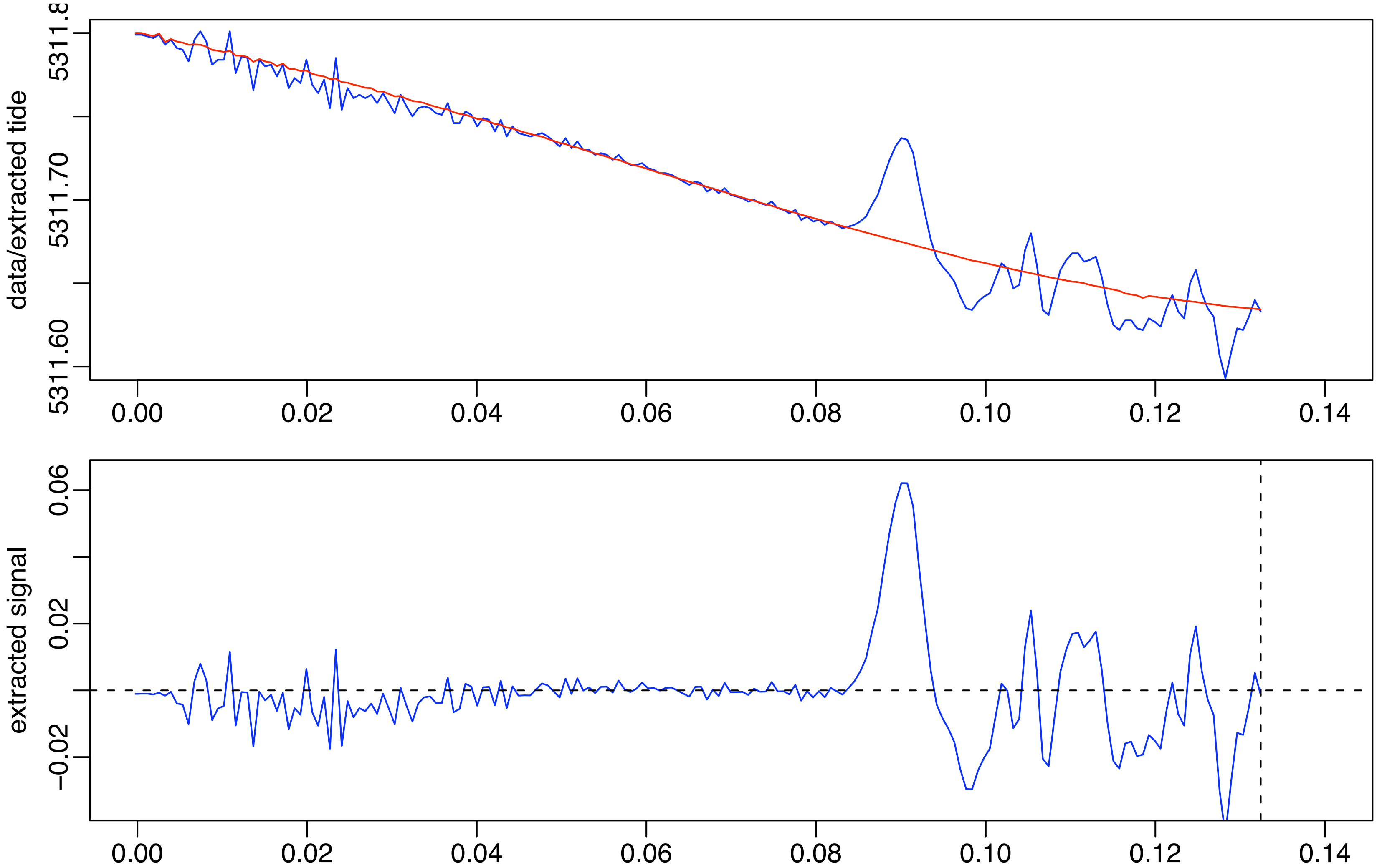
where $\mu(t)$ is 'unpredictable' tidal component

- extract $\mu(t)$ by formulating a state-space model, which allows use of Kalman filter/smoother:

$$\begin{aligned} \mu(t) &= \mu(t-1) + v(t) \\ v(t) &= v(t-1) + \zeta(t) \end{aligned}$$

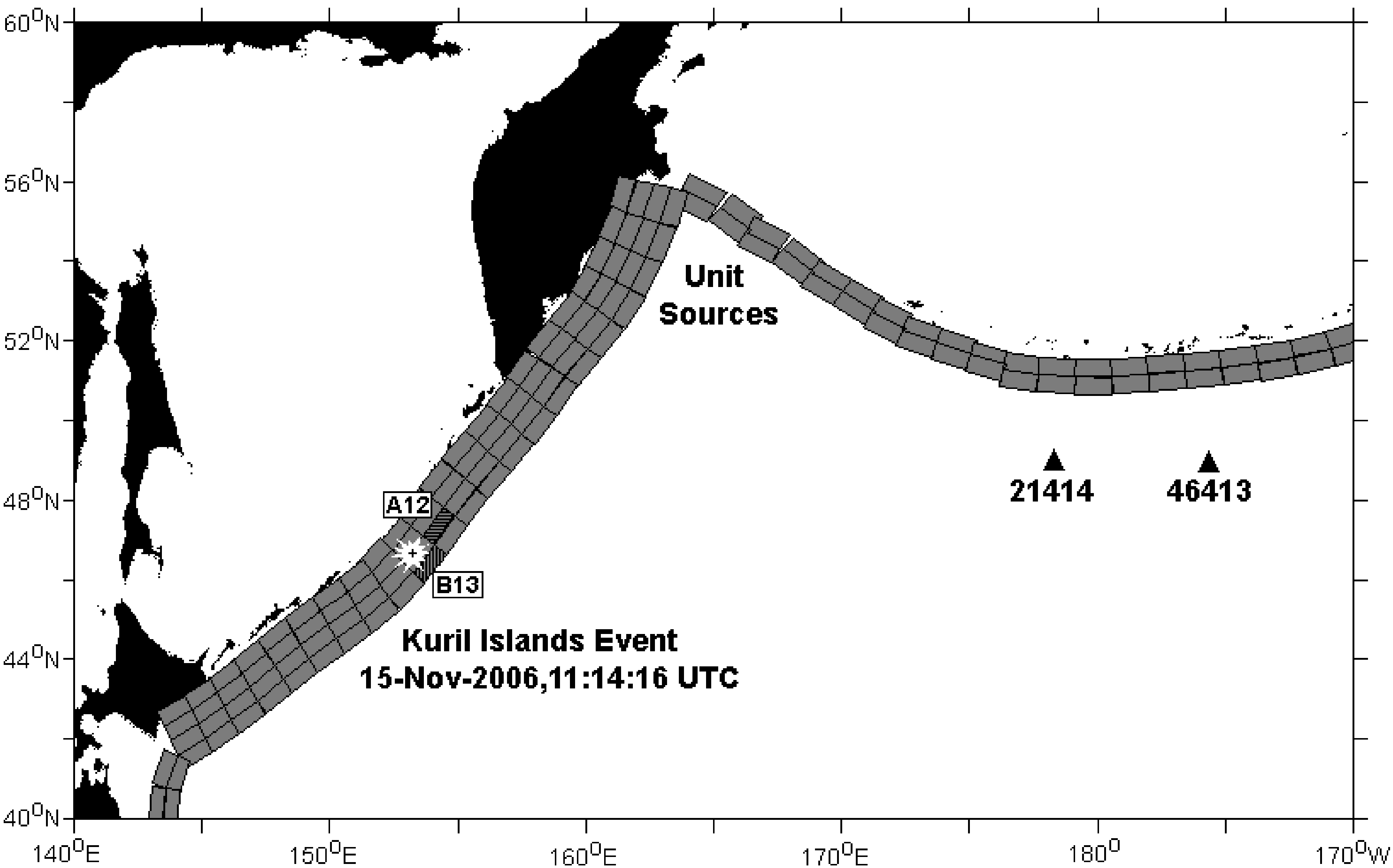
where $\zeta(t)$ is Gaussian white noise with mean zero and variance σ_ζ^2 (can be set using historical data)

Kalman Filtering/Smoothing – 14 Day Model



Modelling of Detided DART® Buoy Data: I

- geophysically-based propagation predictions are based on a simplification provided by so-called 'unit sources'
- unit sources are rectangles of area 100×50 km^2 covering portions of globe from which tsunami-generating earthquakes can occur

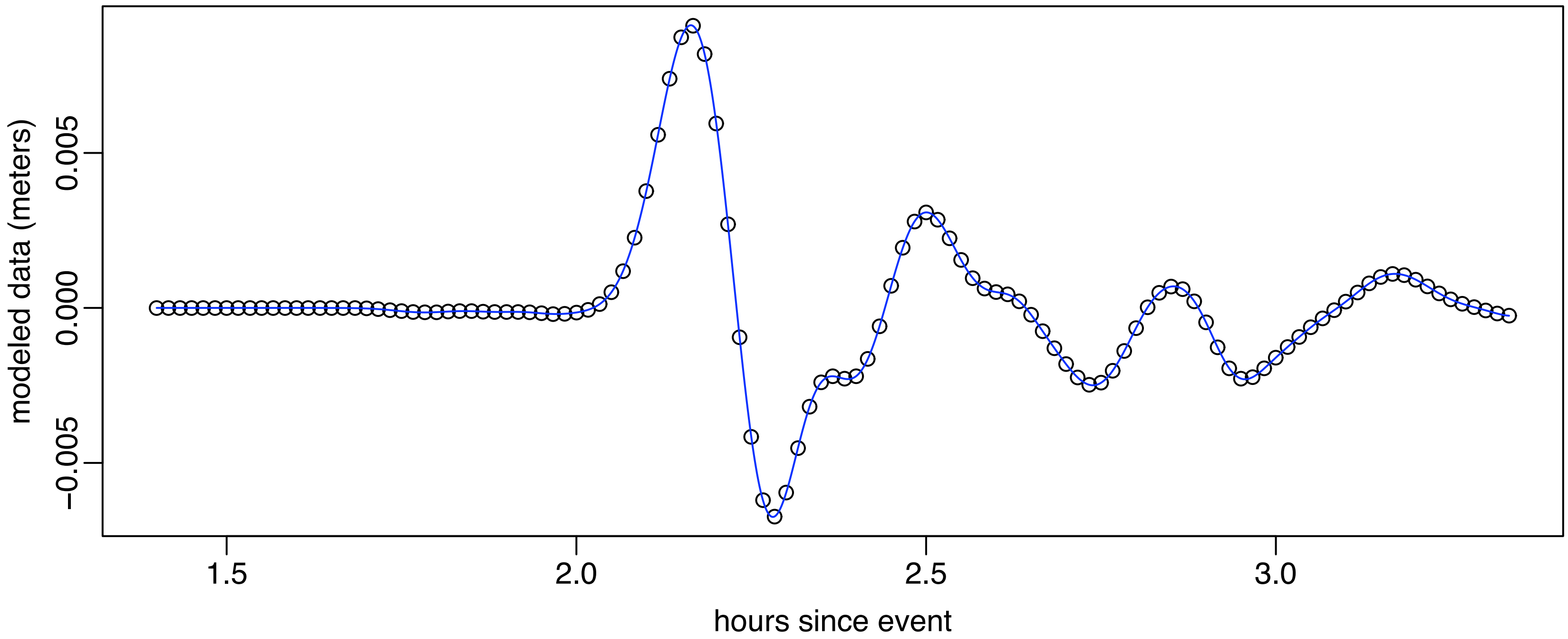


60°N
56°N
52°N
48°N
44°N
40°N
140°E
150°E
160°E
170°E
180°
170°W
Unit Sources
21414
46413
A12
B13
Kuril Islands Event
15-Nov-2006,11:14:16 UTC

Modelling of Detided DART® Buoy Data: II

- propagation predictions must be precomputed (not enough time to compute them once a tsunami is in progress)
- database has been established with precomputed predictions for each pairing of particular unit source and particular buoy
- predictions assume earthquake is located in center of unit source and is of magnitude 7.5
- entry in database predicts what will observed over time at a particular buoy given earthquake from a particular unit source

Model for Buoy 21414 and Unit Source a12



Modelling of Detided DART® Buoy Data: III

- will use buoy data to adjust predictions to handle earthquakes greater or less than 7.5 in magnitude
- consider case of 2 buoys (1, 2) and three unit sources (a, b, c)
- let $\mathbf{x}_1$ and $\mathbf{x}_2$ be relevant detided data from buoys 1 and 2
- let $\mathbf{g}_{1,a}$ etc. be prediction of what buoy 1 should see from earthquake at unit source a
- leads to following model for buoy data:

$$\mathbf{x} \equiv \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{1,a} & \mathbf{g}_{1,b} & \mathbf{g}_{1,c} \\ \mathbf{g}_{2,a} & \mathbf{g}_{2,b} & \mathbf{g}_{2,c} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\epsilon}_1 \\ \boldsymbol{\epsilon}_2 \end{bmatrix} \equiv G\boldsymbol{\alpha} + \boldsymbol{\epsilon}$$

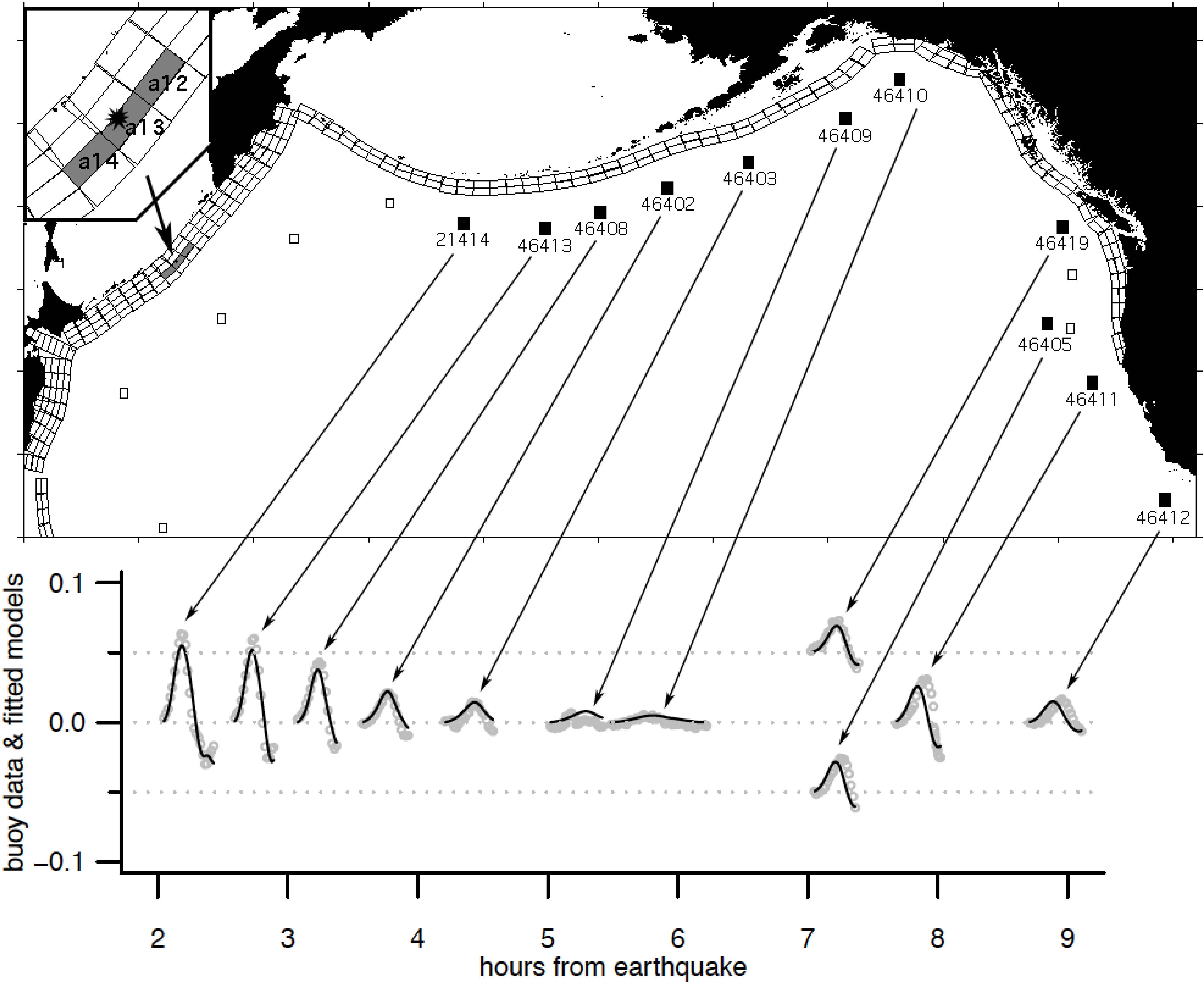
- error terms $\boldsymbol{\epsilon}_j$ assumed to obey first-order autoregressive model

Modelling of Detided DART® Buoy Data: IV

- solve for $\boldsymbol{\alpha}$ using constrained least squares:

$$\text{minimize } \|\boldsymbol{\epsilon}\|^2 = \|\mathbf{x} - G\boldsymbol{\alpha}\|^2 \text{ subject to } \boldsymbol{\alpha} \geq \mathbf{0}$$

- constraint needed to get physically reasonable solution $\hat{\boldsymbol{\alpha}}$
- note: if kth element of $\hat{\boldsymbol{\alpha}}$ is set to zero, kth unit source effectively eliminated from model
- sum of elements of $\hat{\boldsymbol{\alpha}}$ can be used to estimate tsunami magnitude T_M
- uncertainty in T_M assessable using covariance matrix for $\hat{\boldsymbol{\alpha}}$
- following example involves 11 buoys and 3 units sources



a12
a13
a14
21414
46413
46408
46402
46403
46409
46410
46419
46405
46411
46412
buoy data & fitted models
0.1
0.0
−0.1
2
3
4
5
6
7
8
9
hours from earthquake

Concluding Comments

- SIFT application currently relies on experienced operators to select appropriate unit sources
- desirable for future versions of SIFT to include statistically-oriented selection of unit sources (related to variable selection in linear regression)
- selection of data from buoy also currently done manually by operators – need for statistical guidance here also

Thanks to . . .

- Ross Darnell for invitation to speak
- colleagues at NOAA Center for Tsunami Research in Seattle
  - Don Denbo
  - Marie Eblé
  - Edison Gica
  - Hal Mofjeld
  - Mick Spillane
  - Elena Tolkova
  - Vasily Titov
- Crime Scene for 2nd place finish (got $13 in $2 sweep!!!)