Wavelet Methods for Time Series Analysis

Part IV: Wavelet-Based Decorrelation of Time Series

- DWT well-suited for decorrelating certain time series, including ones generated from a fractionally differenced (FD) process
- on synthesis side, leads to
  - DWT-based simulation of FD processes
  - wavelet-based bootstrapping
- on analysis side, leads to
  - wavelet-based estimators for FD parameters
  - test for homogeneity of variance
  - test for trends (won’t discuss – see Craigmiles et al., 2004, for details)

Wavelets and FD Processes: I

- wavelet filters are approximate band-pass filters, with nominal pass-bands \([1/2^j+1, 1/2^j]\) (called \(j\)th ‘octave band’)
- suppose \(\{X_t\}\) has \(S_X(\cdot)\) as its spectral density function (SDF)
- statistical properties of \(\{W_j,t\}\) are simple if \(S_X(\cdot)\) has simple structure within \(j\)th octave band
- example: FD process with SDF
  \[
  S_X(f) = \frac{\sigma_X^2}{[4 \sin^2(\pi f)]^\delta}
  \]

Wavelets and FD Processes: II

- FD process controlled by two parameters: \(\delta\) and \(\sigma_X^2\)
- for small \(f\), have \(S_X(f) \approx C |f|^{-2\delta}\); i.e., a power law
- \(\log(S_X(f)) \approx \log(f)\) is approximately linear with slope \(-2\delta\)
- for large \(\tau_j\), wavelet variance at scale \(\tau_j\), namely \(\nu_X^2(\tau_j)\), satisfies \(\nu_X^2(\tau_j) \approx C' \tau_j^{2\delta-1}\)
- \(\log(\nu_X^2(\tau_j)) \approx \log(\tau_j)\) is approximately linear, slope \(2\delta - 1\)
- approximately ‘self-similar’ (or ‘fractal’)
- FD process is stationary with ‘long memory’ if \(0 < \delta < 1/2\): correlation between \(X_t \& X_{t+\tau}\) dies down slowly as \(\tau\) increases

Wavelets and FD Processes: III

- power law model ubiquitous in physical sciences
  - voltage fluctuations across cell membranes
  - density fluctuations in hour glass
  - traffic fluctuations on Japanese expressway
  - impedance fluctuations in geophysical borehole
  - fluctuations in the rotation of the earth
  - X-ray time variability of galaxies
- DWT well-suited to study FD and related processes
  - ‘self-similar’ filters used on ‘self-similar’ processes
  - key idea: DWT approximately decorrelates LMPs
DWT of an FD Process: I

- realization of an FD(0.4) time series \( X \) along with its sample autocorrelation sequence (ACS): for \( \tau \geq 0 \),

\[
\hat{\rho}_{X,\tau} = \frac{\sum_{t=0}^{N-1-\tau} X_t X_{t+\tau}}{\sum_{t=0}^{N-1} X_t^2}
\]

- note that ACS dies down slowly

WMTSA: 341–342

IV–5

DWT of an FD Process: II

- LA(8) DWT of FD(0.4) series and sample ACSs for each \( W_j \) & \( V_7 \), along with 95% confidence intervals for white noise

WMTSA: 341–342

IV–6

MODWT of an FD Process

- in contrast to \( X \), ACSs for \( W_j \) consistent with white noise

- variance of \( W_j \) increases with \( j \) – can argue that

\[
\text{var} \{ W_j, t \} \approx \frac{1}{2^j - 2^{j+1} + 1} \int_{1/2^{j+1}}^{1/2^{j+1}} S_X(f) df \equiv C_j,
\]

where \( C_j \) is average value of \( S_X(\cdot) \) over \([1/2^{j+1}, 1/2^j]\)

- for FD process, have \( C_j \approx S_X(1/2^{j+1}) \), where \( 1/2^{j+1} \) is midpoint of interval \([1/2^{j+1}, 1/2^j]\)

WMTSA: 341–344

IV–7

DWT of an FD Process: III

- LA(8) MODWT of FD(0.4) series & sample ACSs for MODWT coefficients, none of which are approximately uncorrelated

WMTSA: 341–344

IV–8
DWT of an FD Process: IV

- plot shows \( \text{var} \{W_{j,t}\} \) (circles) & \( S_X(1/2^{j+1/2}) \) (curve) versus \( 1/2^{j+1/2} \), along with 95% confidence intervals for \( \text{var} \{W_{j,t}\} \)
- observed \( \text{var} \{W_{j,t}\} \) agrees well with theoretical \( \text{var} \{W_{j,t}\} \)

Correlations Within a Scale and Between Two Scales

- let \( \{s_{X,\tau}\} \) denote autocovariance sequence (ACVS) for \( \{X_t\} \); i.e., \( s_{X,\tau} = \text{cov} \{X_t, X_{t+\tau}\} \)
- let \( \{h_{j,l}\} \) denote equivalent wavelet filter for \( j \)th level
- to quantify decorrelation, can write

\[
\text{cov} \{W_{j,t}, W_{j',t'}\} = \sum_{l=0}^{L_j-1} \sum_{l'=0}^{L_{j'}-1} h_{j,l} h_{j',l'} s_{X,2^j(t+1) - l - 2^{j'+1} + l'}
\]

from which we can get ACVS (and hence within-scale correlations) for \( \{W_{j,t}\} \):

\[
\text{cov} \{W_{j,t}, W_{j,t+\tau}\} = \sum_{m=-(L_j-1)}^{L_j-|m|-1} s_{X,2^j \tau + m} \sum_{l=0}^{L_j-|m|-1} h_{j,l} h_{j,l+|m|}
\]

Correlations Within a Scale

- correlations between \( W_{j,t} \) and \( W_{j,t+\tau} \) for an FD(0.4) process
- correlations within scale are slightly smaller for Haar
- maximum magnitude of correlation is less than 0.2

Correlations Between Two Scales: I

- correlation between Haar wavelet coefficients \( W_{j,t} \) and \( W_{j',t'} \) from FD(0.4) process and for levels satisfying \( 1 \leq j < j' \leq 4 \)
Correlations Between Two Scales: II

\[
\begin{array}{cccccc}
  j' = 2 & j' = 3 & j' = 4 \\
  j = 1 & j = 2 & j = 3 \\
  \tau & \tau & \tau \\
-8 & 0 & 8 & -8 & 0 & 8 & -8 & 0 & 8 \\
\end{array}
\]

- $\rho_{j,j'}$ are correlations
- $\tau$ is lag
- $\rho_{j,j'}$ is given by

- same as before, but now for LA(8) wavelet coefficients
- correlations between scales decrease as $L$ increases

Wavelet Domain Description of FD Process

- DWT acts as a decorrelating transform for FD process (also true for fractional Gaussian noise, pure power law etc.)
- wavelet domain description is simple
  - wavelet coefficients within a given scale approximately uncorrelated (refinement: assume 1st order autoregressive model)
  - wavelet coefficients have scale-dependent variance controlled by the two FD parameters ($\delta$ and $\sigma_\varepsilon^2$)
  - wavelet coefficients between scales also approximately uncorrelated (approximation improves as filter width $L$ increases)

DWT-Based Simulation

- properties of DWT of FD processes lead to schemes for simulating time series $X \equiv [X_0, \ldots, X_{N-1}]^T$ with zero mean and with a multivariate Gaussian distribution
- with $N = 2^J$, recall that $X = W^T W$, where

\[
W = \begin{bmatrix}
  W_1 \\
  W_2 \\
  \vdots \\
  W_j \\
  \vdots \\
  W_J \\
  V_J
\end{bmatrix}
\]

- assume $W$ to contain $N$ uncorrelated Gaussian (normal) random variables (RVs) with zero mean
- assume $W_j$ to have variance $C_j \approx S_X(1/2^{j+1/2})$
- assume single RV in $V_J$ to have variance $C_{j+1}$ (see Percival and Walden, 2000, for details on how to set $C_{j+1}$)
- approximate FD time series $X$ via $Y \equiv W^T \Lambda^{1/2} Z$, where
  - $\Lambda^{1/2}$ is $N \times N$ diagonal matrix with diagonal elements
    - $C_1^{1/2}, \ldots, C_{1/2}, \ldots, C_{1/2}, \ldots, C_{J-1}^{1/2}, C_{J-1}^{1/2}, \ldots, C_{J+1}^{1/2}$
  - $Z$ is vector of deviations drawn from a Gaussian distribution with zero mean and unit variance

Basic DWT-Based Simulation Scheme
Refinements to Basic Scheme: I

- covariance matrix for approximation $Y$ does not correspond to that of a stationary process
- recall $W$ treats $X$ as if it were circular
- let $T$ be $N \times N$ ‘circular shift’ matrix:
  $$T = \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \\ Y_0 \end{bmatrix}, \quad T^2 = \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \\ Y_0 \end{bmatrix} = \begin{bmatrix} Y_2 \\ Y_3 \\ Y_0 \\ Y_1 \\ Y_2 \end{bmatrix}; \text{ etc.}$$

- let $\kappa$ be uniformly distributed over $0, \ldots, N - 1$
- define $\tilde{Y} \equiv T^\kappa Y$
- $\tilde{Y}$ is stationary with ACVS given by, say, $s_{\tilde{Y}, \tau}$

Refinements to Basic Scheme: II

- Q: how well does $\{s_{\tilde{Y}, \tau}\}$ match $\{s_{X, \tau}\}$?
- due to circularity, find that $s_{\tilde{Y}, N-\tau} = s_{\tilde{Y}, \tau}$ for $\tau = 1, \ldots, N/2$
- implies $s_{\tilde{Y}, \tau}$ cannot approximate $s_{X, \tau}$ well for $\tau$ close to $N$
- can patch up by simulating $\tilde{Y}$ with $M > N$ elements and then extracting first $N$ deviates ($M = 4N$ works well)

Example and Some Notes

- plot shows true ACVS $\{s_{X, \tau}\}$ (thick curves) for FD(0.4) process and wavelet-based approximate ACVSs $\{s_{\tilde{Y}, \tau}\}$ (thin curves) based on an LA(8) DWT in which an $N = 64$ series is extracted from $M = N$, $M = 2N$ and $M = 4N$ series

- simulated FD(0.4) series (LA(8), $N = 1024$ and $M = 4N$)
- notes:
  - can form realizations faster than best exact method
  - can efficiently simulate extremely long time series in ‘real-time’ (e.g. $N = 2^{30} = 1,073,741,824$ or even longer!)
  - effect of random circular shifting is to render time series slightly non-Gaussian (a Gaussian mixture model)
Wavelet-Domain Bootstrapping

- for many (but not all!) time series, DWT acts as a decorrelating transform: to a good approximation, each $W_j$ is a sample of a white noise process, and coefficients from different sub-vectors $W_j$ and $W_{j'}$ are also pairwise uncorrelated
- variance of coefficients in $W_j$ depends on $j$
- scaling coefficients $V_{J_0}$ are still autocorrelated, but there will be just a few of them if $J_0$ is selected to be large
- decorrelating property holds particularly well for FD and other processes with long-range dependence
- above suggests the following recipe for wavelet-domain bootstrapping of a statistic of interest, e.g., sample autocorrelation sequence $\hat{\rho}_{X,\tau}$ at unit lag $\tau = 1$

Illustration of Wavelet-Domain Bootstrapping

Recipe for Wavelet-Domain Bootstrapping

1. given $X$ of length $N = 2^J$, compute level $J_0$ DWT (the choice $J_0 = J - 3$ yields 8 coefficients in $W_{J_0}$ and $V_{J_0}$)
2. randomly sample with replacement from $W_j$ to create bootstrapped vector $W_j^{(b)}$, $j = 1, \ldots, J_0$
3. create $V_{J_0}^{(b)}$ using 1st-order autoregressive parametric bootstrap
4. apply $W_j^T$ to $W_1^{(b)}$, $\ldots$, $W_{J_0}^{(b)}$ and $V_{J_0}^{(b)}$ to obtain bootstrapped time series $X^{(b)}$ and then form $\hat{\rho}_{X,1}^{(b)}$
- repeat above many times to build up sample distribution of bootstrapped autocorrelations

Wavelet-Domain Bootstrapping of FD Series

- approximations to true PDF using (a) Haar & (b) LA(8) wavelets
- using 50 FD time series and the Haar DWT yields:
  - average of 50 sample means $\hat{\rho}_1^{(m)} = 0.35$  (truth $= 0.53$)
  - average of 50 sample SDs $\hat{\rho}_1^{(m)} = 0.096$  (truth $= 0.107$)
- using 50 FD time series and the LA(8) DWT yields:
  - average of 50 sample means $\hat{\rho}_1^{(m)} = 0.43$  (truth $= 0.53$)
  - average of 50 sample SDs $\hat{\rho}_1^{(m)} = 0.098$  (truth $= 0.107$)
MLEs of FD Parameters: I

- FD process depends on 2 parameters, namely, $\delta$ and $\sigma_\varepsilon^2$:
  $$S_X(f) = \frac{\sigma_\varepsilon^2}{[4 \sin^2(\pi f)]^{\delta}}$$
- given $X = [X_0, X_1, \ldots, X_{N-1}]^T$ with $N = 2^J$, suppose we want to estimate $\delta$ and $\sigma_\varepsilon^2$
- if $X$ is stationary (i.e., $\delta < 1/2$) and multivariate Gaussian, can use the maximum likelihood (ML) method

MLEs of FD Parameters: II

- definition of Gaussian likelihood function:
  $$L(\delta, \sigma_\varepsilon^2 | X) \equiv \frac{1}{(2\pi)^{N/2}|\Sigma_X|^{1/2}}e^{-X^T\Sigma_X^{-1}X/2}$$
  where $\Sigma_X$ is covariance matrix for $X$, with $(s, t)$th element given by $s_X, s-t$, and $|\Sigma_X|$ & $\Sigma_X^{-1}$ denote determinant & inverse
- ML estimators of $\delta$ and $\sigma_\varepsilon^2$ maximize $L(\delta, \sigma_\varepsilon^2 | X)$ or, equivalently, minimize
  $$-2 \log (L(\delta, \sigma_\varepsilon^2 | X)) = N \log (2\pi) + \log (|\Sigma_X|) + X^T\Sigma_X^{-1}X$$
- exact MLEs computationally intensive, mainly because of the need to deal with $|\Sigma_X|$ and $\Sigma_X^{-1}$
- good approximate MLEs of considerable interest

MLEs of FD Parameters: III

- key ideas behind first wavelet-based approximate MLEs
  - have seen that we can approximate FD time series $X$ by $Y = \mathbf{W}^T\Lambda^{1/2}\mathbf{Z}$, where $\Lambda^{1/2}$ is a diagonal matrix, all of whose diagonal elements are positive
  - since covariance matrix for $Z$ is $I_N$, the one for $Y$ is $\mathbf{W}^T\Lambda^{1/2}I_N(\mathbf{W}^T\Lambda^{1/2})^T = \mathbf{W}^T\Lambda^{1/2}\Lambda^{1/2}\mathbf{W} = \mathbf{W}^T\Lambda\mathbf{W} \equiv \widehat{\Sigma}_X$, where $\Lambda \equiv \Lambda^{1/2}\Lambda^{1/2}$ is also diagonal
  - can consider $\widehat{\Sigma}_X$ to be an approximation to $\Sigma_X$
- leads to approximation of log likelihood:
  $$-2 \log (L(\delta, \sigma_\varepsilon^2 | X)) \approx N \log (2\pi) + \log (|\widehat{\Sigma}_X|) + X^T\widehat{\Sigma}_X^{-1}X$$

MLEs of FD Parameters: IV

- Q: so how does this help us?
  - easy to invert $\widehat{\Sigma}_X$:
    $$\widehat{\Sigma}_X^{-1} = (\mathbf{W}^T\Lambda\mathbf{W})^{-1} = (\mathbf{W})^{-1}\Lambda^{-1}(\mathbf{W}^T)^{-1} = \mathbf{W}^T\Lambda^{-1}\mathbf{W},$$
    where $\Lambda^{-1}$ is another diagonal matrix, leading to $X^T\widehat{\Sigma}_X^{-1}X = X^T\mathbf{W}^T\Lambda^{-1}\mathbf{W}X = \mathbf{W}^T\Lambda^{-1}\mathbf{W}$
  - easy to compute the determinant of $\widehat{\Sigma}_X$:
    $$|\widehat{\Sigma}_X| = |\mathbf{W}^T\Lambda\mathbf{W}| = |\Lambda\mathbf{W}\mathbf{W}^T| = |\Lambda I_N| = |\Lambda|,$$
    and the determinant of a diagonal matrix is just the product of its diagonal elements
**MLEs of FD Parameters: V**

- define the following three functions of $\delta$:
  \[ C'_j(\delta) \equiv \int_{1/2+1}^{1/2j+1+1} \frac{2^{j+1}}{4 \sin^2(\pi f)} \cdot \delta^{2j} \cdot \delta^{2} \cdot \rangle \]
  \[ C'_{j+1}(\delta) \equiv \frac{N^j(1 - 2\delta)}{1^2(1 - \delta)} - \sum_{j=1}^{N} \frac{N}{2j} C'_j(\delta) \]
  \[ \sigma^2_{\varepsilon}(\delta) \equiv \frac{1}{N} \left( \frac{V_{j,0}^2}{C'_{j+1}(\delta)} + \sum_{j=1}^{N} \frac{1}{C'_j(\delta)} \sum_{t=0}^{j-1} W^2_{j,t} \right) \]

**Other Wavelet-Based Estimators of FD Parameters**

- second MLE approach: formulate likelihood directly in terms of nonboundary wavelet coefficients
  - handles stationary or nonstationary FD processes (i.e., need not assume $\delta < 1/2$)
  - handles certain deterministic trends
- alternative to MLEs are least square estimators (LSEs)
  - recall that, for large $\tau$ and for $\beta = 2\delta - 1$, have
    \[ \log(\nu^2_X(\tau)) \approx \zeta + \beta \log(\tau) \]
  - suggests determining $\delta$ by regressing $\log(\nu^2_X(\tau))$ on $\log(\tau)$
    over range of $\tau$
  - weighted LSE takes into account fact that variance of $\log(\nu^2_X(\tau))$
    depends on scale $\tau$ (increases as $\tau$ increases)

**MLEs of FD Parameters: VI**

- wavelet-based approximate MLE $\hat{\delta}$ for $\delta$ is the value that minimizes the following function of $\delta$:
  \[ \tilde{l}(\delta | X) \equiv N \log(\sigma^2_{\varepsilon}(\delta)) + \log(C'_{j+1}(\delta)) + \sum_{j=1}^{N} \frac{N}{2j} \log(C'_j(\delta)) \]
- once $\hat{\delta}$ has been determined, MLE $\sigma^2_{\varepsilon}(\hat{\delta})$
- computer experiments indicate scheme works quite well

**Homogeneity of Variance: I**

- because DWT decorrelates LMPs, nonboundary coefficients in $W_j$ should resemble white noise; i.e., $\text{cov} \{W_{j,t}, W_{j,t'}\} \approx 0$ when $t \neq t'$, and $\text{var} \{W_{j,t}\}$ should not depend upon $t$
- can test for homogeneity of variance in $X$ using $W_j$ over a range of levels $j$
- suppose $U_0, \ldots, U_{N-1}$ are independent normal RVs with $E\{U_t\} = 0$ and $\text{var} \{U_t\} = \sigma^2_t$
- want to test null hypothesis $H_0 : \sigma^2_0 = \sigma^2_1 = \cdots = \sigma^2_{N-1}$
- can test $H_0$ versus a variety of alternatives, e.g., $H_1 : \sigma^2_0 = \cdots = \sigma^2_k \neq \sigma^2_{k+1} = \cdots = \sigma^2_{N-1}$
  using normalized cumulative sum of squares
Homogeneity of Variance: II

- to define test statistic $D$, start with
\[ P_k = \frac{\sum_{j=0}^{k} U_j^2}{\sum_{j=1}^{N-1} U_j^2} \quad k = 0, \ldots, N - 2 \]
and then compute $D \equiv \max (D^+, D^-)$, where
\[ D^+ \equiv \max_{0 \leq k \leq N-2} \left( \frac{k + 1}{N - 1} - P_k \right) \quad \text{and} \quad D^- \equiv \max_{0 \leq k \leq N-2} \left( P_k - \frac{k}{N - 1} \right) \]
- can reject $H_0$ if observed $D$ is ‘too large,’ where ‘too large’ is quantified by considering distribution of $D$ under $H_0$
- need to find critical value $x_\alpha$ such that $P[D \geq x_\alpha] = \alpha$ for, e.g., $\alpha = 0.01, 0.05$ or $0.1$

Homogeneity of Variance: III

- once determined, can perform $\alpha$ level test of $H_0$:
  - compute $D$ statistic from data $U_0, \ldots, U_{N-1}$
  - reject $H_0$ at level $\alpha$ if $D \geq x_\alpha$
  - fail to reject $H_0$ at level $\alpha$ if $D < x_\alpha$
- can determine critical values $x_\alpha$ in two ways
  - Monte Carlo simulations
  - large sample approximation to distribution of $D$
    \[ P((N/2)^{1/2}D \geq x) \approx 1 + 2 \sum_{l=1}^{\infty} (-1)^l e^{-2l^2 x^2} \]
    (reasonable approximation for $N \geq 128$)

Homogeneity of Variance: IV

- idea: given time series $\{X_t\}$, compute $D$ using nonboundary wavelet coefficients $W_{j,t}$ (there are $M'_j \equiv N_j - L'_j$ of these):
\[ P_k = \frac{\sum_{t=L'_j}^{k} W_{j',t}^2}{\sum_{t=L'_j}^{N_j-1} W_{j',t}^2} \quad k = L'_j, \ldots, N_j - 2 \]
- if null hypothesis rejected at level $j$, can use nonboundary MODWT coefficients to locate change point based on
\[ \tilde{P}_k = \frac{\sum_{t=L_j-1}^{k} \tilde{W}_{j',t}^2}{\sum_{t=L_j-1}^{N_j-1} \tilde{W}_{j',t}^2} \quad k = L_j - 1, \ldots, N - 2 \]
along with analogs $\tilde{D}_k^+$ and $\tilde{D}_k^-$ of $D_k^+$ and $D_k^-$

Example – Annual Minima of Nile River: I

- left-hand plot: annual minima of Nile River
- new measuring device introduced around year 715
- right: Haar $\chi^2(\tau_j)$ before (x’s) and after (0’s) year 715.5, with 95% confidence intervals based upon $\chi^2_{(n)}$ approximation
Example – Annual Minima of Nile River: II

- based upon last 512 values (years 773 to 1284), plot shows \( l(\delta \mid X) \) versus \( \delta \) for the first wavelet-based approximate MLE using the LA(8) wavelet (upper curve) and corresponding curve for exact MLE (lower)
  - wavelet-based approximate MLE is value minimizing upper curve: \( \hat{\delta} = 0.4532 \)
  - exact MLE is value minimizing lower curve: \( \hat{\delta} = 0.4452 \)

Example – Annual Minima of Nile River: III

- using last 512 values again, variance of wavelet coefficients computed via LA(8) MLEs \( \hat{\delta} \) and \( \sigma^2(\hat{\delta}) \) (solid curve) as compared to sample variances of LA(8) wavelet coefficients (circles)
- agreement is almost too good to be true!

Example – Annual Minima of Nile River: IV

- results of testing all Nile River minima for homogeneity of variance using the Haar wavelet filter with critical values determined by computer simulations

<table>
<thead>
<tr>
<th>( \tau_j )</th>
<th>( M_j' )</th>
<th>( D )</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
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<td>1 year</td>
<td>331</td>
<td>0.1559</td>
<td>0.0945</td>
<td>0.1051</td>
<td>0.1262</td>
</tr>
<tr>
<td>2 years</td>
<td>165</td>
<td>0.1754</td>
<td>0.1320</td>
<td>0.1469</td>
<td>0.1765</td>
</tr>
<tr>
<td>4 years</td>
<td>82</td>
<td>0.1000</td>
<td>0.1855</td>
<td>0.2068</td>
<td>0.2474</td>
</tr>
<tr>
<td>8 years</td>
<td>41</td>
<td>0.2313</td>
<td>0.2572</td>
<td>0.2864</td>
<td>0.3436</td>
</tr>
</tbody>
</table>

- can reject null hypothesis of homogeneity of variance at level of significance 0.05 for scales \( \tau_1 \) & \( \tau_2 \), but not at larger scales

Example – Annual Minima of Nile River: V

- Nile River minima (top plot) along with curves (constructed per Equation (382)) for scales \( \tau_1 \) & \( \tau_2 \) (middle & bottom) to identify change point via time of maximum deviation (vertical lines denote year 715)
Summary

- DWT approximately decorrelate certain time series, including ones coming from FD and related processes
- leads to schemes for simulating time series and bootstrapping
- also leads to schemes for estimating parameters of FD process
  - approximate maximum likelihood estimators (two varieties)
  - weighted least squares estimator
- can also devise wavelet-based tests for
  - homogeneity of variance
  - trends (see Craigmire et al., 2004, for details)

References: I

- bootstrapping

References: II

- decorrelation property of DWTs
- parameter estimation for FD processes
- testing for homogeneity of variance
- wavelet-based trend assessment