

Figure 1. ACVS  $\{s_{X,\tau}\}$ , spectrum  $S_X(\cdot)$  and three realization of an FD process with  $\delta = 0.4$ .



Figure 2. Haar wavelet filters for scales  $\tau_j = 2^{j-1}, j = 1, 2, ..., 7$ .



**Figure 3.** D(4), C(6) and LA(8) wavelet filters for scales  $\tau_j = 2^{j-1}$ , j = 1, 2, ..., 7.



**Figure 4.** Squared gain functions  $\mathcal{H}_{j}^{(D)}(\cdot)$ , j = 1, 2 and 3 (left, middle and right columns, respectively), for Daubechies wavelet filters of widths  $L = 2, 4, \ldots, 14$  (top to bottom rows, respectively). The two thin vertical lines in each plot delineate the nominal pass-band for the filter. The vertical axis is in decibels (i.e., we plot  $10 \cdot \log_{10}(\mathcal{H}_{j}^{(D)}(f))$  versus f).



**Figure 5.** LA(8) DWT coefficients for simulated FD(0.4) time series and sample ACSs.



**Figure 6.** SDFs for an FD(0.4) process (left-hand plot) and for nonboundary LA(8) wavelet coefficients in  $\mathbf{W}_1$ ,  $\mathbf{W}_2$ ,  $\mathbf{W}_3$  and  $\mathbf{W}_4$  (right-hand). The vertical axis is in units of decibels (i.e., we plot  $\log_{10}(S_X(f))$  versus f). The vertical lines in the left-hand plot denote the nominal pass-bands for the four  $\mathbf{W}_j$ .



**Figure 7.** ACSs at  $\tau = 1, \ldots 4$  for Haar, D(4) and LA(8) wavelet coefficients  $W_{j,t}$ ,  $j = 1, \ldots 4$ , of an FD(0.4) process. The ACS values are plotted as deviations from zero (some are not visible because they are so close to zero).

$$j' = 2 j' = 3 j' = 4$$

$$[1] \qquad [1] \qquad [$$

**Figure 8a.** Correlation between the Haar wavelet coefficients  $W_{j,t}$  and  $W_{j',t'}$  formed from an FD(0.4) process and for levels satisfying  $1 \le j < j' \le 4$ .

$$j' = 2 j' = 3 j' = 4$$

$$[1^{j} - 1^{j} -$$

Figure 8b. As in Figure 8a, but now using the LA(8) DWT.



Figure 9. Diagonal elements  $\Sigma_{\mathbf{Y},m,m+\tau}$  and  $\Sigma_{\mathbf{X},m,m+\tau}$ ,  $m = 0, \ldots, N-1-\tau$ , of the covariance matrices  $\Sigma_{\mathbf{Y}}$  and  $\Sigma_{\mathbf{X}}$  (thick jagged curves and thin horizontal lines, respectively) for sample size N = 64 from an FD(0.4) process with  $\sigma^2 = 1$  and with  $\Sigma_{\mathbf{Y}}$  constructed using an LA(8) DWT. Three diagonals are plotted for each covariance matrix, namely, the main diagonal ( $\tau = 0$ ) and the first two off-diagonals ( $\tau = 1$  and 2). Whereas  $\Sigma_{\mathbf{X}}$  exhibits the Toeplitz structure required for a stationary process, its approximation  $\Sigma_{\mathbf{Y}}$  does not.



Figure 10. True ACVS (thin curves) and wavelet-based approximate ACVSs (thick) for an FD(0.4) process. The approximating ACVSs are based on an LA(8) DWT in which we generate a series of length M and then extract a series of length N = 64. As M goes from N to 4N, the approximate ACVS gets closer to the true ACVS.



Figure 11. LA(8) wavelet-based simulation of a series of length N = 1024 from an FD process with zero mean and with parameters  $\delta = 0.4$  and  $\sigma^2 = 1.0$ .



Figure 12. PDFs for  $\mathcal{N}(0, 1)$  and  $\mathcal{N}(0, 10)$  RVs (left-hand plot, thin and thick curves, respectively) and for an RV obeying a Gaussian mixture model (right-hand plot). The mixture PDF is non-Gaussian and is formed by adding the  $\mathcal{N}(0, 1)$  and  $\mathcal{N}(0, 10)$  PDFs, weighted by p = 0.75 and 1-p = 0.25, respectively (adapted from Figure 1 of Chipman *et al.*, 1997).



Figure 13. Three simulated time series generated using the wavelet-based scheme. The top series was constructed using homogeneous variances for each scale, while the bottom two series use inhomogeneous variances. The middle series has a quiesent period of about a hundred points, whereas the bottom has a noticeable burst of about the same duration. While all three series are realizations of stationary processes with spectra that are designed to approximate that of an FD process with  $\delta = 0.4$ , their marginal distributions obey quite different Gaussian mixture models (the distribution for the top series is in fact very close to Gaussian).



Figure 14. LA(8) wavelet-based simulation of a series of length N = 1024 from process with time varying statistical properties.



Figure 15. SDFs for AR(1) processes (top plot) with  $\phi = 0.9$  (thick curve) and -0.9 (thin) and for corresponding nonboundary LA(8) wavelet coefficients in  $\mathbf{W}_1$  to  $\mathbf{W}_4$  (bottom four plots). The vertical axes are in decibels, and the vertical lines in the top plot denote the nominal pass-bands for the four  $\mathbf{W}_j$ .



Figure 16. Flow diagram illustrating the analysis of **X** into  $\mathbf{W}_{3,0}, \ldots, \mathbf{W}_{3,7}$  (recall that  $N_j \equiv N/2^j$ ).



Figure 17. Flow diagram illustrating the analysis of X into  $\mathbf{W}_{3,0}$ ,  $\mathbf{W}_{3,1}$ ,  $\mathbf{W}_{2,1}$  and  $\mathbf{W}_{1,1}$ , which is identical to a partial DWT of level  $J_0 = 3$ .



Figure 18. Flow diagram illustrating the analysis of  $\mathbf{X}$  into  $\mathbf{W}_{2,0}$ ,  $\mathbf{W}_{3,2}$ ,  $\mathbf{W}_{3,3}$  and  $\mathbf{W}_{1,1}$ , an arbitrary disjoint dyadic decomposition.