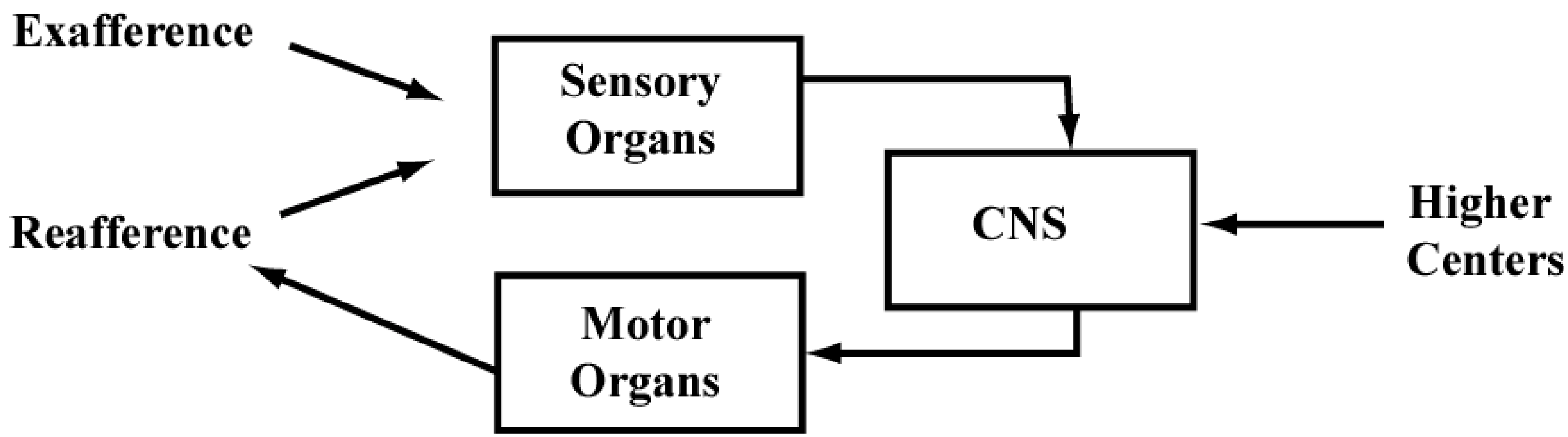
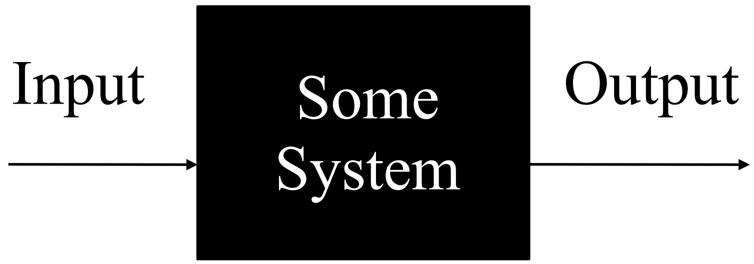
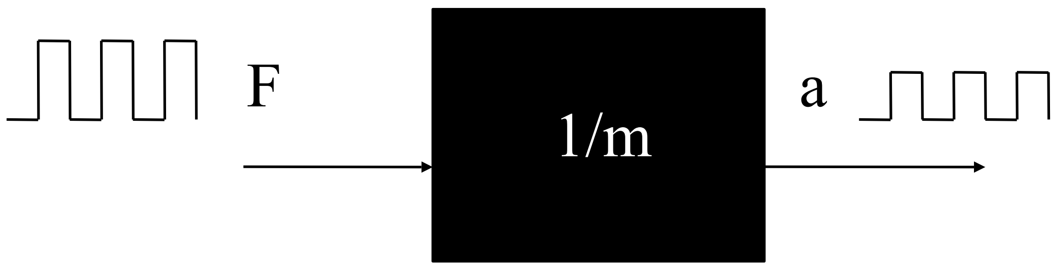

Exafference
Reafference
Sensory Organs
Motor Organs
CNS
Higher Centers

Transfer Functions



Simple Example: $F = ma$

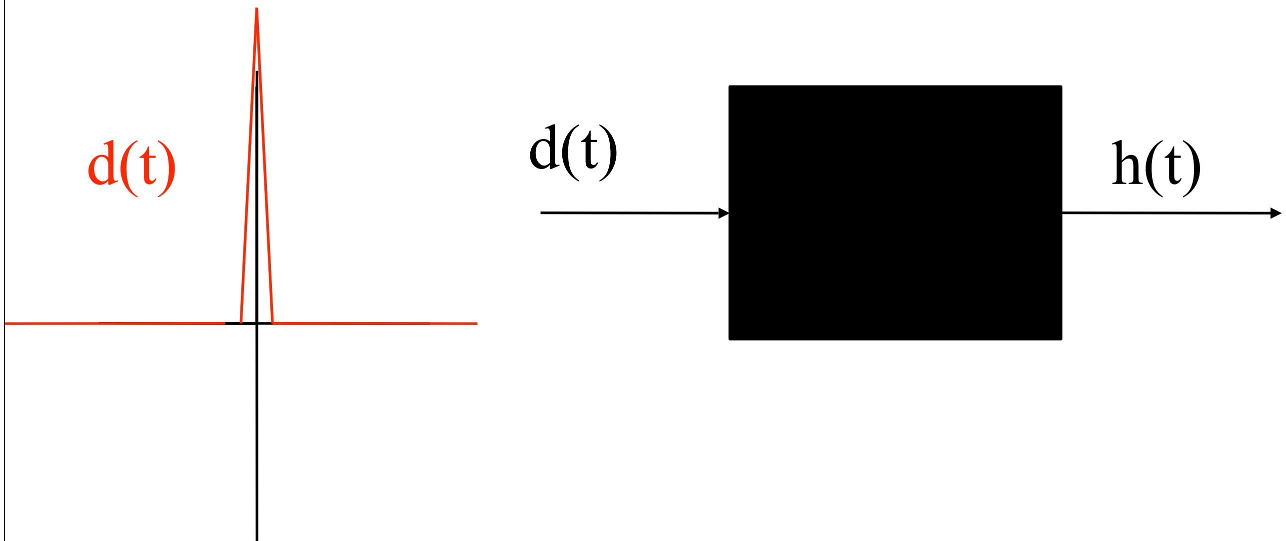


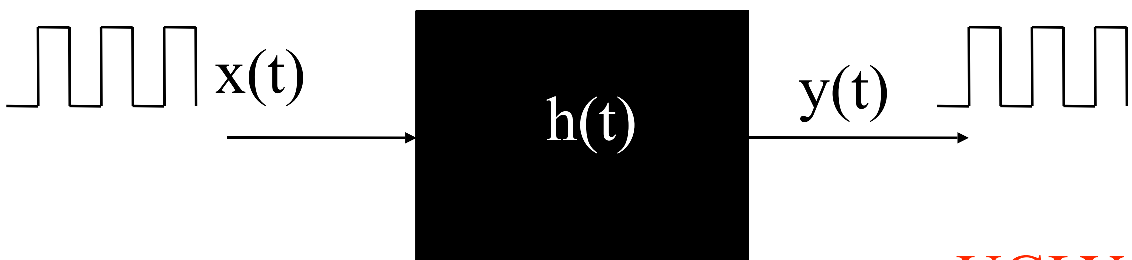
H=a/F=1/m

More Complex Example: $F = my'' + by' + ky$

How do you define the transfer function of this more complex system?

Transfer Function in Time Domain





UGLY!

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\delta\tau$$

Laplace Transform

$$F(s) = \int_0^{\infty} f(t)e^{-st}\,dt$$

Example

$$f(t) = e^{-at}$$

$$F(s) = \int_0^{\infty} e^{-at}e^{-st}\,dt = \int_0^{\infty} e^{-(s+a)t}\,dt$$

$$= \left.\frac{-e^{-(s+a)t}}{s+a}\right|_0^{\infty} = \frac{1}{s+a}$$

- Examples:

$f(t)$	$F(s)$
$\delta(t-t_0)$, $t_0 > 0$	$e^{-t_0 s}$
1	$\frac{1}{s}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
t	$1/s^2$
t^n	$n!/s^{n+1}$
$t\,e^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$

Laplace Transform

- In the time domain

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\delta\tau$$

- Laplace transform has the property that

$$x(t) * h(t) \Rightarrow X(s)H(s)$$

- Laplace domain

$$Y(s) = X(s)H(s)$$

Example

$$f(t) = my''(t) + by'(t) + ky(t) \Rightarrow F(s) = ms^2Y(s) + bsY(s) + kY(s)$$

$$H(s) = \frac{Y(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

Let mass(m) = 1, friction(b) = 2.5, and what spring stiffness(k)=1. What is the response of the system to a unit step?

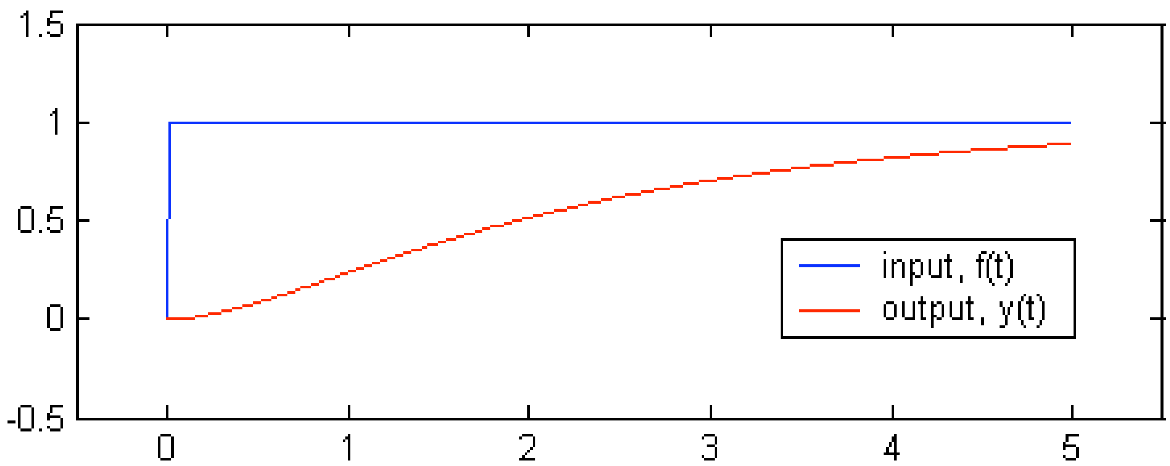
so

Example Continued

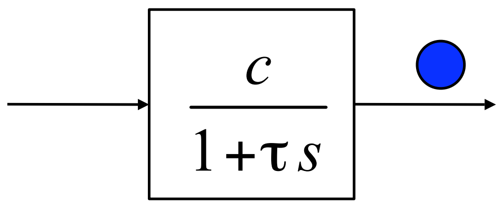
$$x(t) = u(t) \Rightarrow \frac{1}{s}$$

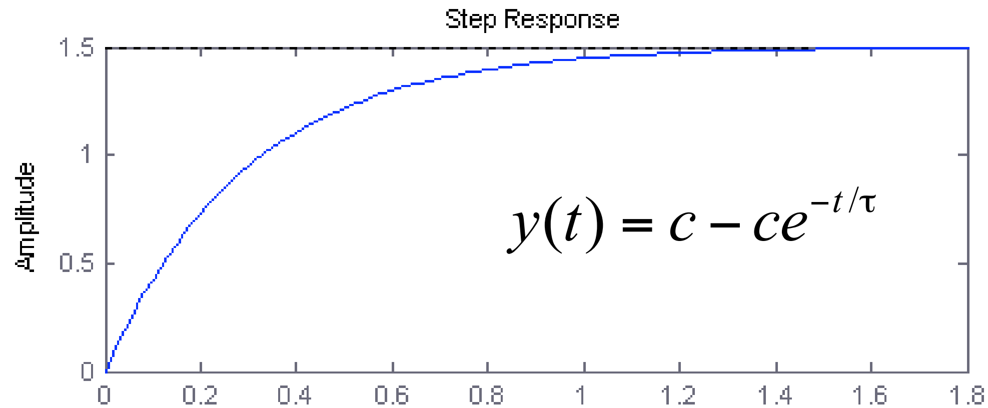
$$Y(s) = F(s)H(s) \Rightarrow \frac{1}{s} \cdot \frac{1}{s^2 + 2.5s + 1} = \frac{1}{s(s+2)(s+.5)}$$

$$Y(s) = \frac{1}{s(s+2)(s+.5)} = \frac{1}{s} + \frac{2/3}{s+2} - \frac{4/3}{s+.5} \Leftrightarrow y(t) = 1 + 2/3e^{-2t} - 4/3e^{-.5t}$$



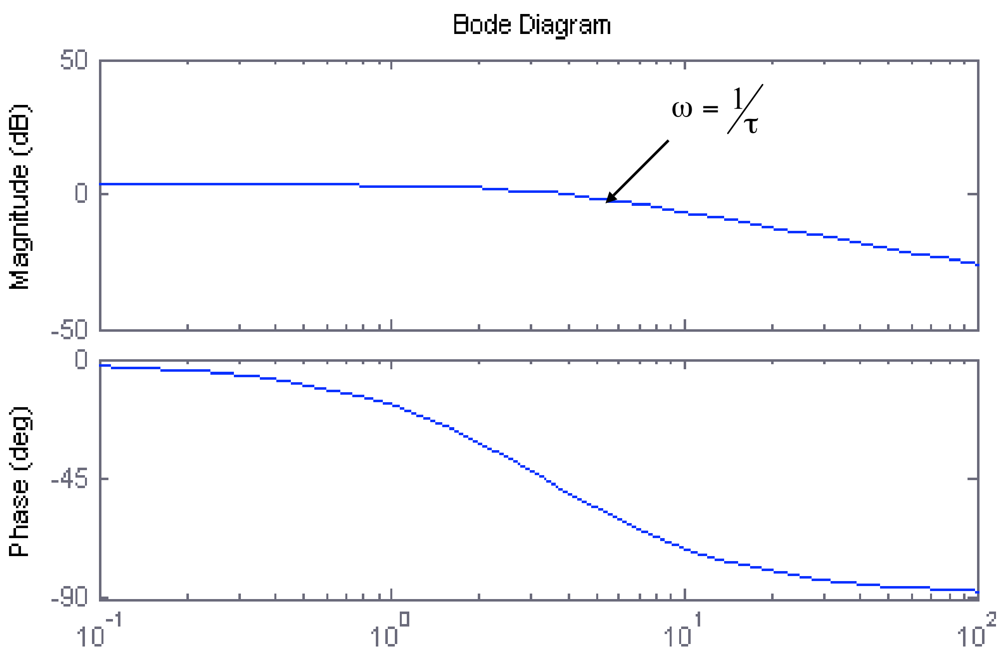
Lowpass Filter





$$y(t) = c - ce^{-t/\tau}$$

$j\omega$



$$G(j\omega) = \frac{c}{1 + j\omega\tau}$$

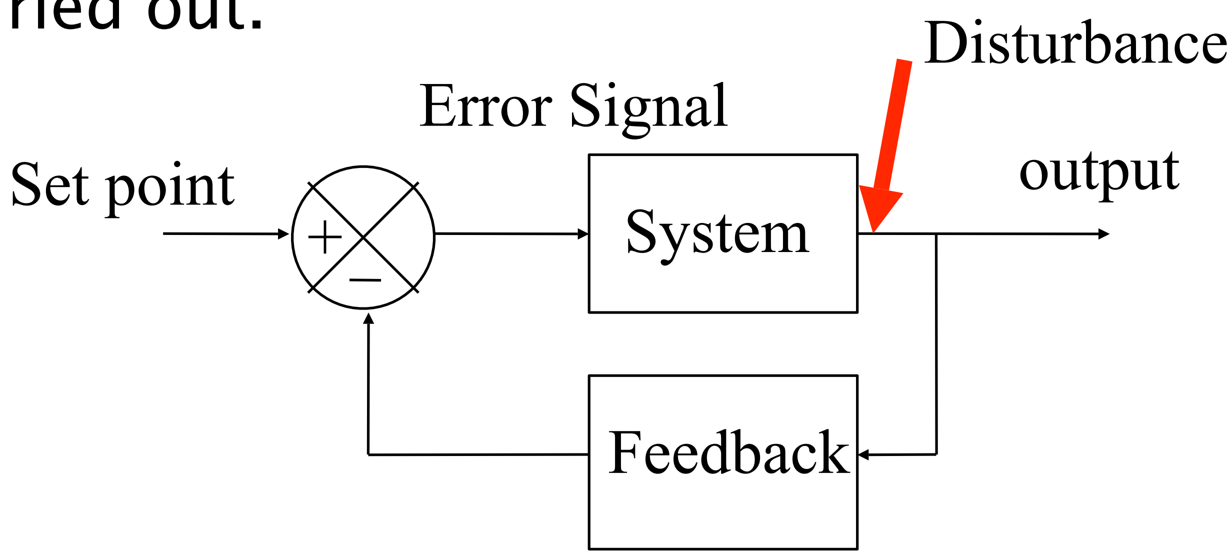
$$|G| = \frac{c}{\sqrt{1+\omega^2}}$$

$$\angle G = \angle c - \angle(1 + j\omega\tau) = -\arctan(\omega\tau)$$

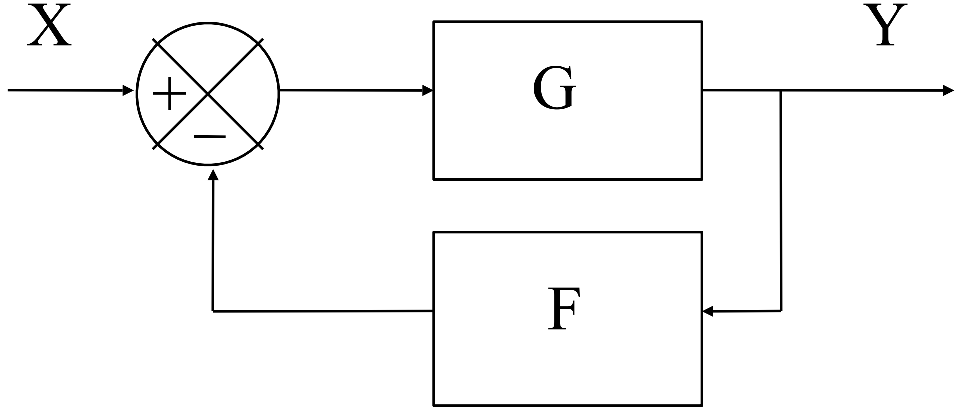
Feedback

- If everything were perfectly predictable, and every system perfectly understood, feedback would not be necessary.

- However, in real systems where parameters change and there are nonlinearities as well as outside disturbances, feedback is necessary to ensure that intended behavior is carried out.



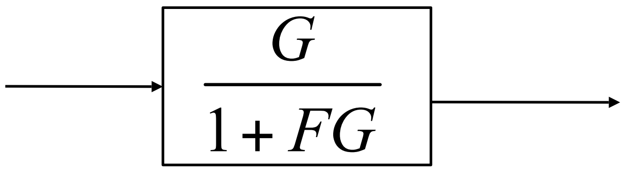
Feedback



$$Y = GX - FGY$$

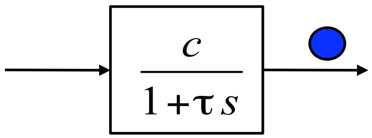
$$\frac{Y}{X} = \frac{G}{1+FG}$$

equivalent

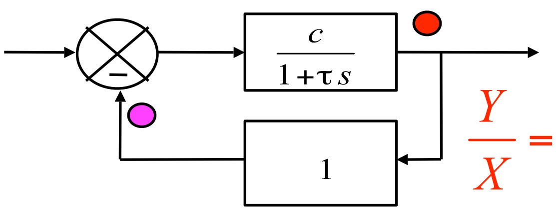


$$E = \frac{1}{1+FG}$$

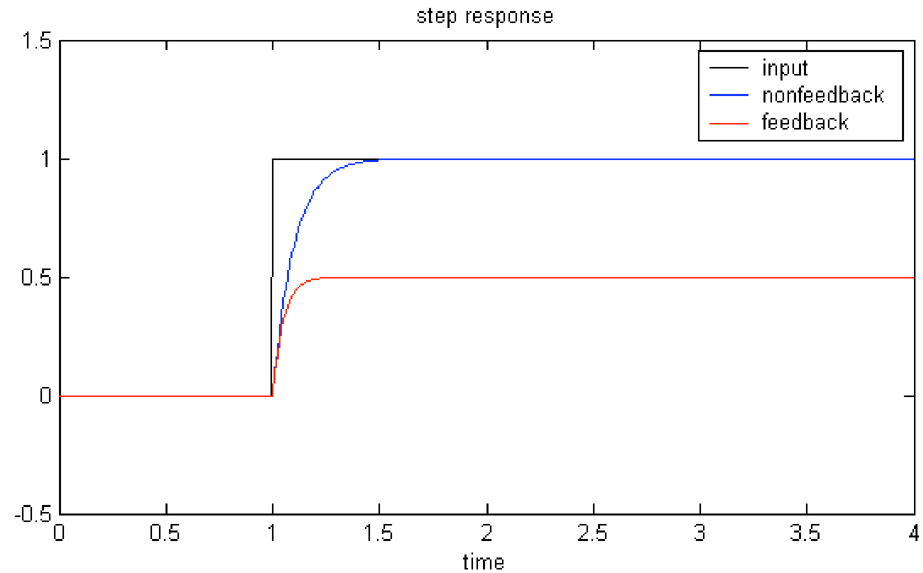
Feedback. c=1

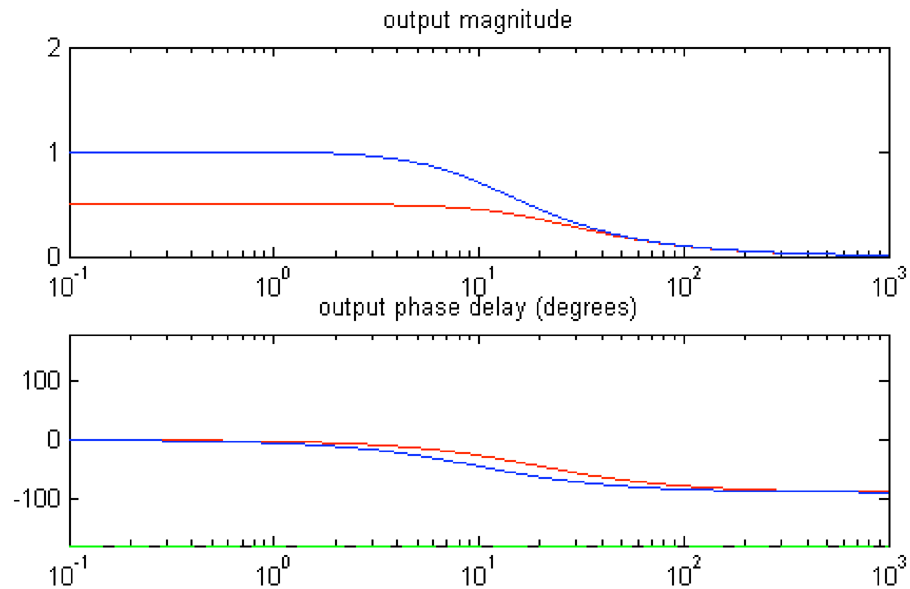


$$\frac{Y}{X} = \frac{c}{1+\tau s}$$



$$\frac{Y}{X} = \frac{c}{1+c+\tau s}$$

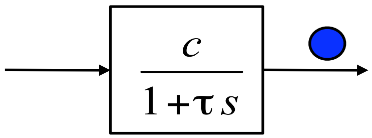




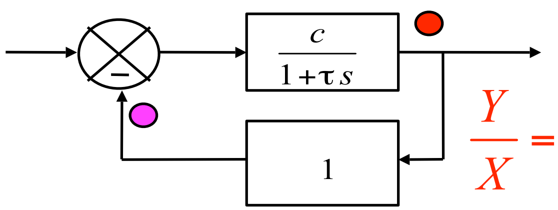
Sim parmtrs:

- t=.1
- C=1

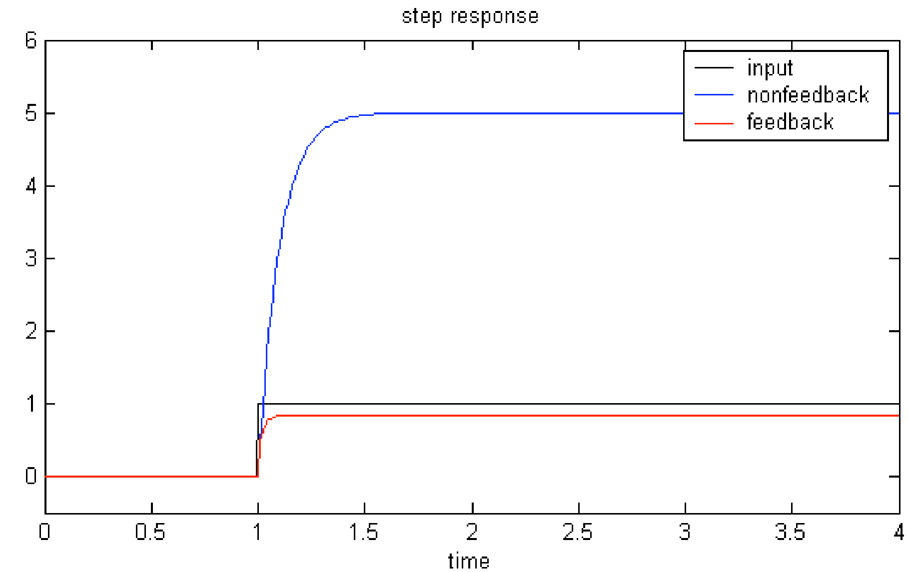
Feedback. c=5

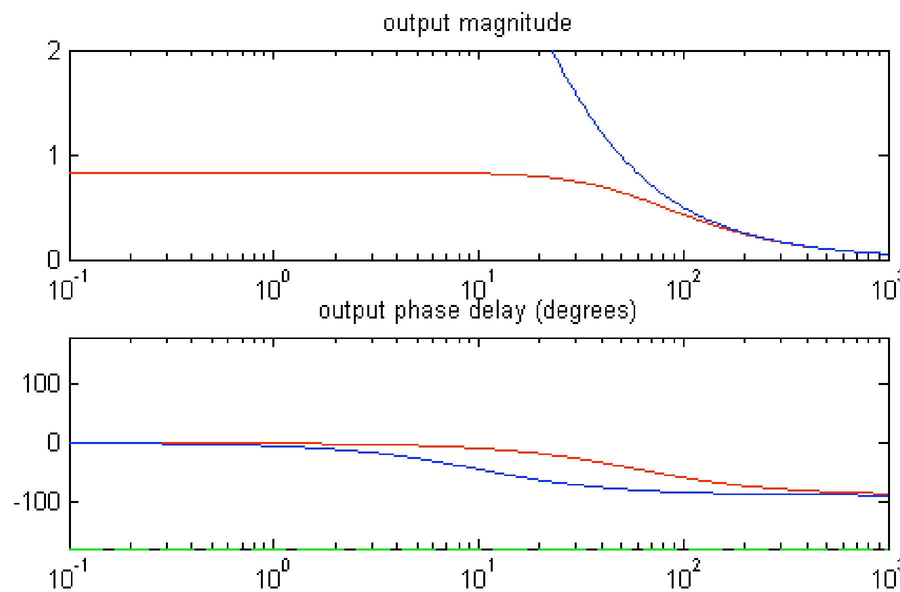


$$\frac{Y}{X} = \frac{c}{1+\tau s}$$



$$\frac{Y}{X} = \frac{c}{1+c+\tau s}$$





Sim parmtrs:

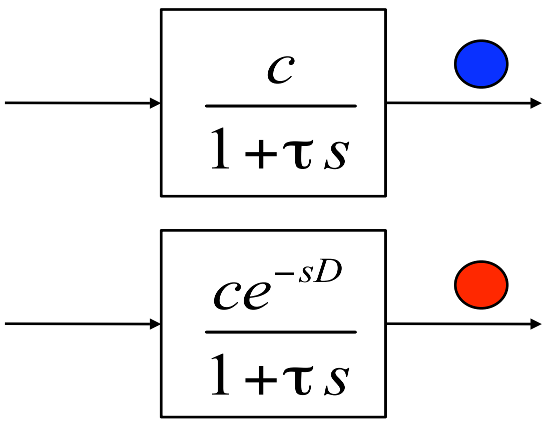
- t=.1
- C=5

Rx
Prescription Drug Costs Soar
Oil Money and Gas Price Hike
299
309
279
Republican Pharmaceutical Industry
Westar
Westar
Casino Scandal Senate Investigation Grand Jury Investigation
Medicare Vote Ethics Admonishment
Mike Scanlon
Jack Abramoff
Rep. Bob Ney
Enron
Enron
Junket Scandal Prospective Ethics Investigations
Ralph Reed
Warren RoBold
GOP Leader Tom DeLay
Grover Norquist
John Colyandro
"K Street Project" Ethics Admonishment
Republican National Committee
Jim Ellis
Abuse of FAA Ethics Admonishment
Texas Redistricting TRMPAC Indictments
House GOP Conference
Westar Ethics Admonishment

Time Delay

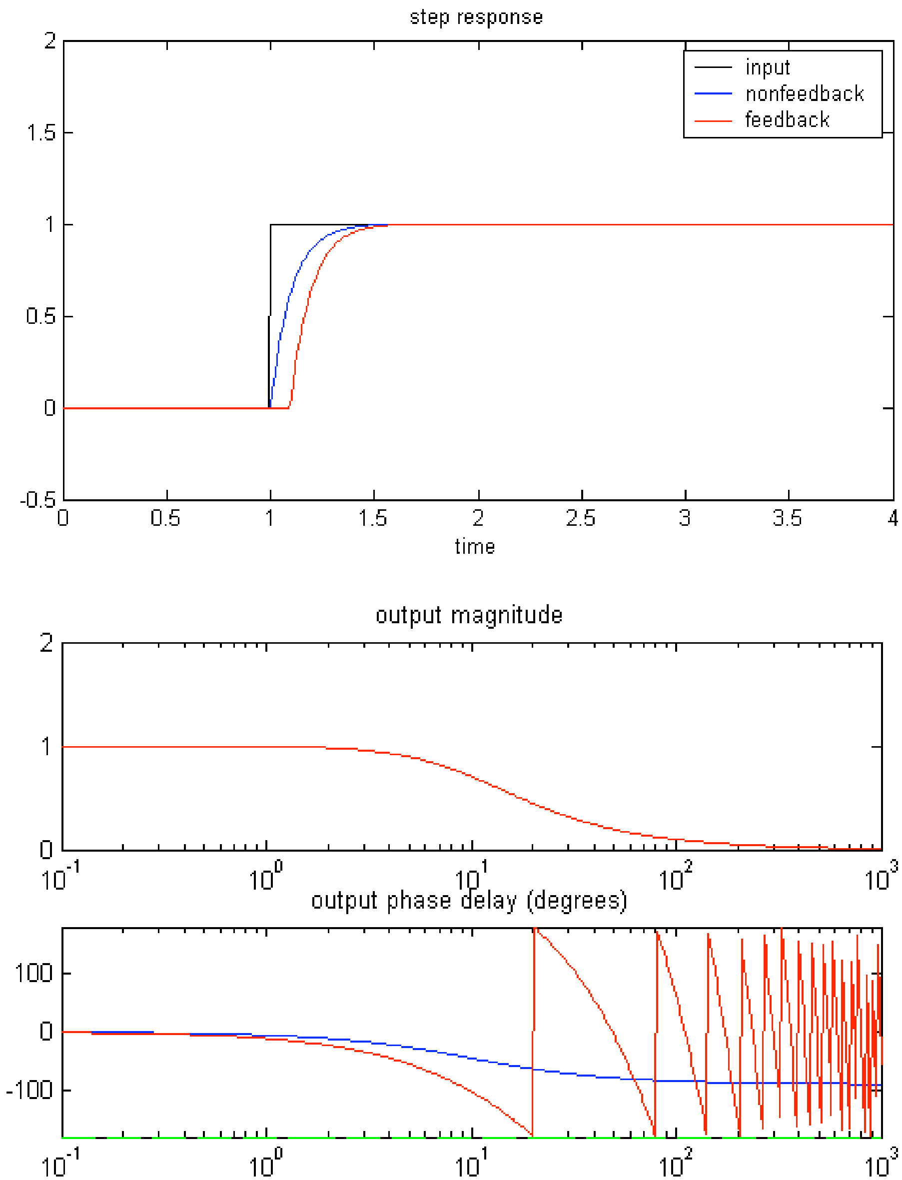
$$f(t) \Rightarrow F(s)$$

$$f(t-D) \Rightarrow F(s)e^{-sD}$$

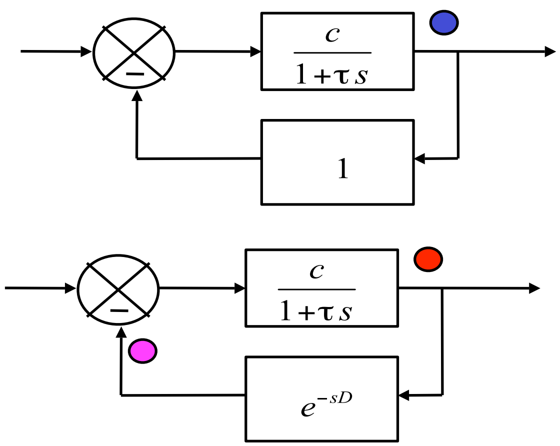


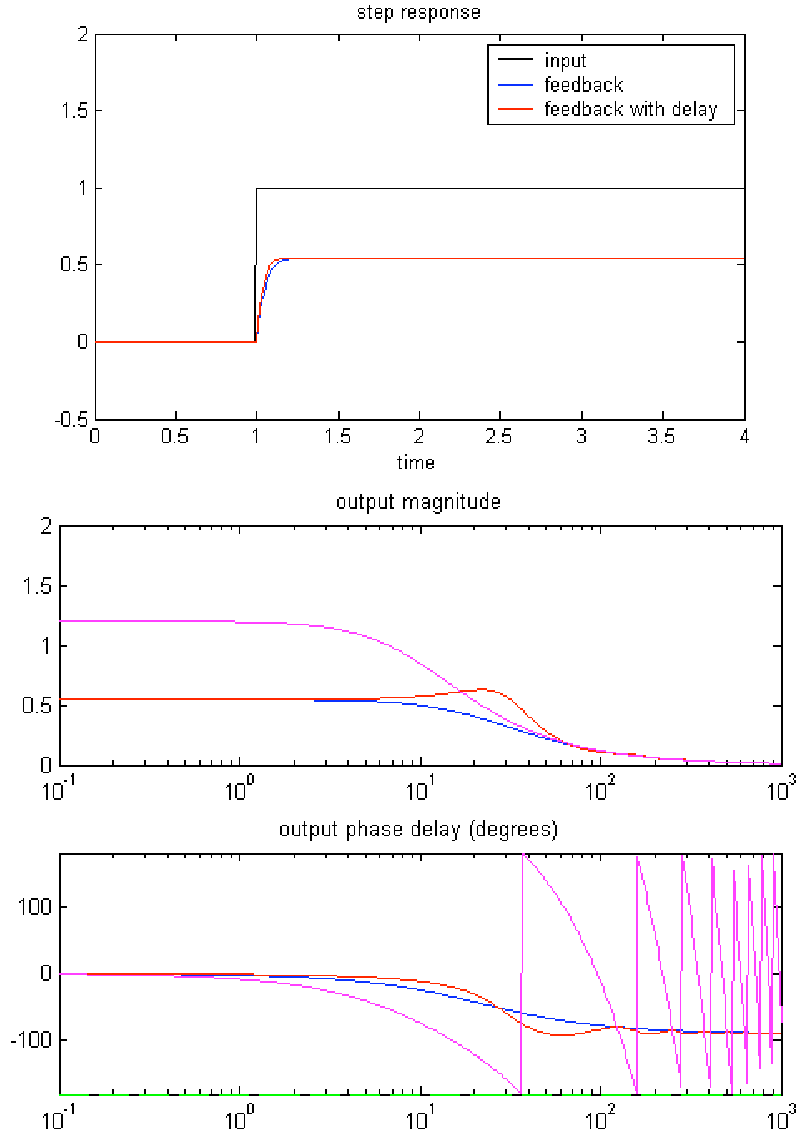
Sim parmtrs:

- t=.1
- C=1
- D=.1



Feedback with Time Delay (D=.02)

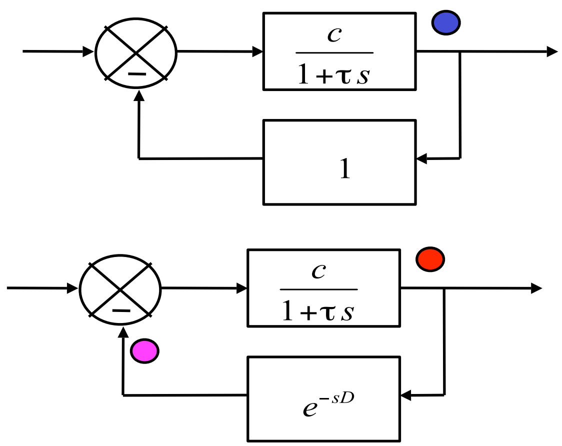


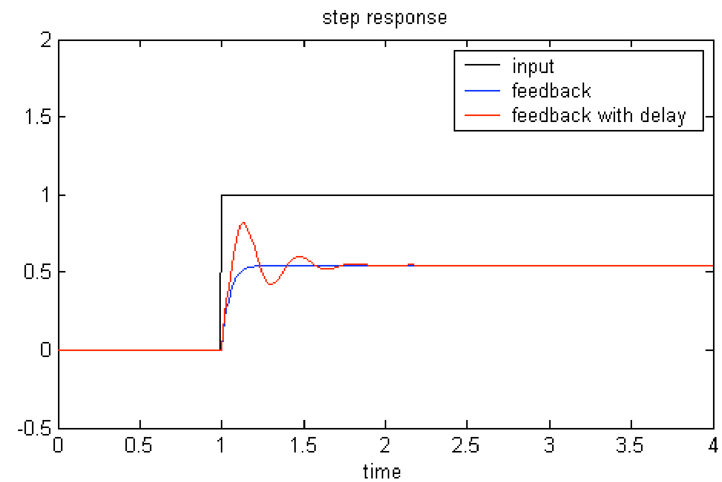


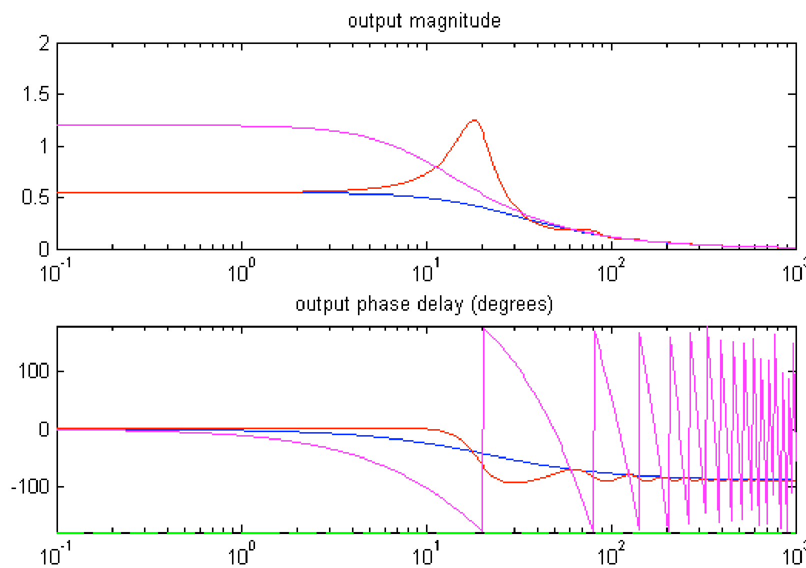
Sim parmtrs:

- t=.1
- C=1.2
- D=.02

Feedback with Time Delay (D=.1)



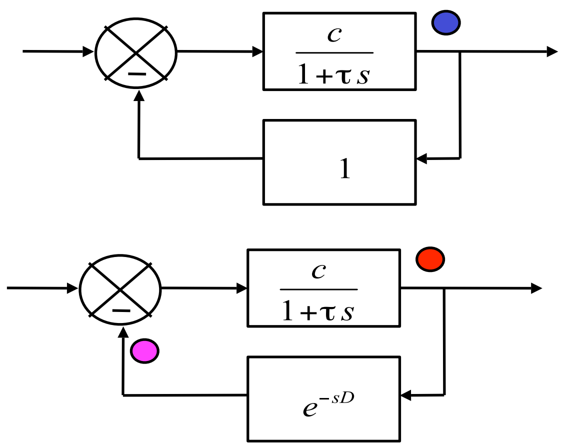


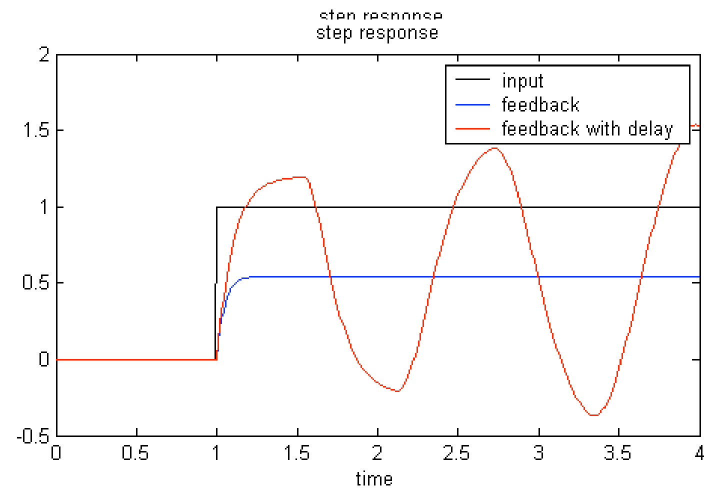


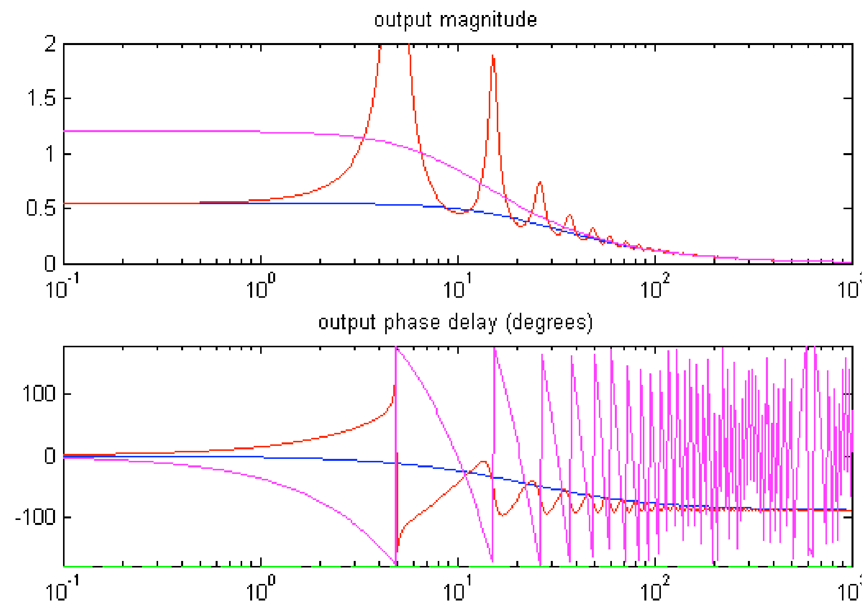
Sim parmtrs:

- t=.1
- C=1.2
- D=.1

Feedback with Time Delay(D=.55)







Sim parmtrs:

- t=.1
- C=1.2
- D=.55