Benacerraf on Mathematical Truth and Knowledge

Conor Mayo-Wilson
University of Washington

Phil 450: Epistemology
Lecture 13

1. Platonism
2. Tarski's Theory of Truth

Mathematical Platonism

Here are some intuitions you might have about numbers and mathematics more generally . . .

Mathematical objects are not sensible

Intuition 1: Mathematical objects (e.g., numbers, functions, etc.) are not sensible.

- Numbers seem unlike tables, chairs, etc. I can't see, touch, feel or smell numbers. Number don't seem to be **detectable via the senses**.
- And numbers don't seem like electrons or faraway galaxies either. That is, no matter how much we improve microscopes or telescopes, it doesn't seem like I will ever be able to "see", in any sense, the number three: I might see three pigs, or three blind mice, etc. But I don't see three.

Intuition 2: Mathematical facts are neither empirically confirmed nor empirically refuted.

Example: Suppose you found out that scientists conducted the following experiment. They place two coins in 100 empty cloth bags. Then they place two more coins in 50 of those bags. Finally, they emptied the bags and counted the total coins in each bag separately. The bags that didn't get additional coins all still had two coins in them. But in three bags that received the additional coins, there were only three coins, not four. The scientists conclude that the experiment confirms that $2 + 0 = 2$. However, it upends consensus that $2 + 2 = 4$ and instead provides evidence that the hypothesis that $2 + 2 = 3$ only 6% of the time (i.e., in $3/50$ cases) .

Mathematical Platonism

These intuitions are often used to support

Mathematical Platonism: Mathematical objects (like numbers, functions, etc.) are not physical objects or events.

- Note: Mathematical Platonism is often defined to be the conjunction of several theses, but this is good enough for us now.

Discussion:

- Benacerraf thinks that, if Mathematical Platonism is true, then we don't have mathematical knowledge. Your goal is to explain why. Review questions 9-11 from the reading assignment.
- Plato thought we had lots of mathematical knowledge. To understand why, discuss question 12 from the reading assignment.
- Discuss question 7 from the reading assignment.

Here's my rough understanding of **half** of Benacerraf's argument.

- **Premise 1** Causation is a relationship between physical objects and events.
- **Premise 2:** If mathematical Platonism is true, then mathematical objects (e.g., numbers, functions, etc.) are not physical objects.
- **Conclusion 1:** If mathematical Platonism is true, then mathematical objects (e.g., numbers, functions, etc.) do not stand in causal relationships to physical objects and events.
- **Premise 3:** The formation of a belief is a physical event.
- **Conclusion 2:** If mathematical Platonism is true, then mathematical objects do not stand in causal relationships to the formation of our beliefs.

- **Conclusion 2:** If mathematical Platonism is true, then mathematical objects do not stand in causal relationships to the formation of our beliefs.
- **Premise 4:** If the causal theory of knowledge is true, then we have mathematical knowledge only if mathematical objects stand in causal relationships to the formation of our beliefs.
- **Conclusion 3:** If mathematical Platonism and the causal theory of knowledge are true, we don't have mathematical knowledge.
- **Premise 5:** We do have mathematical knowledge.
- **Final Conclusion:** Either mathematical Platonism or the causal theory of knowledge is false.

Since Benacerraf endorses the causal theory of knowledge, he would like to reject mathematical Platonism.

Unfortunately, he runs into an issue.

Problem: According to Benacerraf, our best theory of truth (full stop) seems to imply mathematical Platonism.

Tarski's Theory of Truth

Question: When are declarative sentences **true**?

Example: Consider the sentence "Shaq is tall."

- That sentence seems true if the person called "Shaq" has the property that is denoted by the English word "tall".
- In general, the sentence "S has property P" seems to be true if the object denoted by "S" has the property denoted by "P".
  - This is more-or-less **Tarski's theory of truth**.

Question: What happens when we apply Tarski's theory to mathematical sentences?

Example: Consider the sentence "Two is prime."

- According to Tarski, that sentence is true if the number called "Two" has the property that is denoted by the English word "prIme".
- In other words, when applied to mathematics, Tarski's theory seems to imply that numbers exist and have properties, just like Shaq.
- But if numbers exist, they don't seem to exist in our physical world like Shaq.

Discussion: Review questions 3-6 from the reading assignment.

Looking Ahead: Mathematics and morality seem to face similar epistemological challenges.

- Consider the sentence "Generosity is good."
- I might be able to see generous **acts** (like charitable donations) just in the same way I can see three **physical objects** (like three pigs). But I don't seem to be able to sense generosity or goodness.
- So moral sentences likewise generate a Benacerraf-like dilemma: either we should be concerned we lack moral knowledge, or moral claims have entirely different truth conditions from other sentences.

Writing Prompt: Does Benacerraf's argument apply to theories of knowledge other than the causal theory? Explain.