



## Quantifier Shifts and Hume's Argument

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Phil 450: Epistemology

Lecture 12

1 Quantifier Shifts

2 Formality

## Quantifier Shifts

### Discussion:

1. Review your answers to the following question from the reading assignment.
  - ▶ Give your own example of a quantifier shift fallacy (i.e., an invalid inference from a statement of the form  $\forall \exists$  to a statement of the form  $\exists \forall$ ). Okasha does not explain what this fallacy is, and so if you haven't encountered it before, do a bit of research. Next, explain why, according to Sober, Hume is guilty of a quantifier-shift fallacy.
2. What premise might Sober advocate adding to this argument to strengthen it?

All reptiles today are cold-blooded.

Dinosaurs were cold-blooded.

3. Identify which premise of Hume's (generalized) argument that Okasha, Sober, and Norton reject.

## A generalization of Hume's argument

- P1. One is justified in placing confidence in or calling matter of fact about an unobserved object or event "probable" only if there is a good argument for it.
- P2. A good argument is non-circular and either deductively valid or inductively strong.
- C1. One is justified in placing confidence in or calling a calling matter of fact about an unobserved object or event "probable" only if there is a non-circular, deductively valid or inductively strong argument for it. [By P1 and P2]
- P3. One is justified in placing confidence in a matter of fact about an unobserved event or object only if it is the conclusion of an instance of Argument Schema  $S$ .
- C2. One is justified in placing confidence in a matter of fact about an unobserved event or object only if it is the conclusion of a non-circular, deductively valid or inductively strong instance of Argument Schema  $S$  [From C1 and P3].
- P4. No instance of Argument Schema  $S$  is deductively valid.
- P5. Any inductively strong instance of Argument Schema  $S$  is circular.
- C3. One is never justified placing confidence or calling a matter of fact about an unobserved event or object [From C2, P4, and P5].

## The Birthday Fallacy

Sober refers to the a quantifier-shift fallacy as the "birthday fallacy."

Why? Here's an example indicating the fallacy.

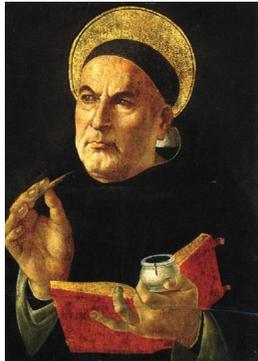
Everyone has a birthday.

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There is one day on which everyone was born.

This fallacy is more common than you might think ...

## Quantifier Order Fallacies



In Aquinas' second "way" of proving God's existence, he argues:  
*If something can fail to exist, there must have been a time at which it has failed to exist. Therefore, if everything could fail to exist, there would have been a time at which nothing existed.*

This is the same quantifier-order fallacy we saw above.

## Quantifier Order Fallacies



In the first sentence of *Nicomachean Ethics*, Aristotle claims:

*Every art and every inquiry, and similarly every action and pursuit, is thought to aim at some good; and for this reason the good has rightly been declared to be that at which all things aim.*

Aristotle defends this thesis (he was rather good at logic), but on first glance it appears to contain the same fallacy.

## Quantifier Order Fallacies



In 1821, the famous mathematician Augustin-Louis Cauchy published a fallacious proof (of a false “theorem”) that contained essentially the same error.

## Quantifier Order Fallacies



**Question:** According to Okasha, why is Hume guilty of the fallacy?

**Answer:** Hume claims that every “probable” argument is missing some premise that, if added, when yield a valid argument. Hume seems to infer that every “probable” argument is missing **the same** premise, namely, PUN.

### Small Group Discussion 10 minutes

Review your answers to questions 4 and 5 from the reading assignment.

## Formality

In logic courses, the rules you study are (purportedly) **formal**, i.e., the rules are applicable no matter what types of objects you're reasoning about.

### Example:

- ▶ Consider *modus ponens*, which recall is the rule that from  $P$  and “If  $P$ , then  $Q$ ”, you may infer  $Q$ .
- ▶ Here,  $P$  and  $Q$  could be propositions about microscopic objects, macroscopic physical objects, non-physical objects like numbers, fictional objects, etc.

## Lack of Formality

But we can also state rules of inference that might have more restricted scope, i.e., that apply only to certain types of objects.

**Example:** Consider the rule, “ $x$  is a bird. Therefore,  $x$  flies.” Here, the rule is applicable only when reasoning about birds.

## Formality, Validity, and Reliability

- ▶ Modus ponens and other typical logical rules are **both** (1) formal and (2) valid.
- ▶ Given a number  $P$  between 0 and 100, call a rule of inference  **$P\%$ -reliable** if the conclusion is true in  $P\%$  of the instances in which the premises of the rule are true.
  - ▶ For the moment, let's assume there are only finitely many instances of the rule so that percentages make sense.

**Question:** Are there *formal*,  $P\%$ -reliable rules for any number  $P > 50$ ?

## Example: Straight Rule

Consider the **straight rule**, which is the following rule:

$$\frac{x\% \text{ of observed/inspected } As \text{ have property } P.}{x\% \text{ of } \mathbf{all} \text{ } As \text{ have property } P.}$$

**Question:** Is any variant of the straight rule both formal and reliable?

- ▶ Most students note the straight rule as stated above is not reliable at all because the premises don't say anything about how many  $As$  have been observed, whether our sample of observations is random, how many total  $As$  there are, Etc. BUT ...
- ▶ Notice those considerations are all formal: we can ask questions about random sampling, numbers of  $As$ , etc. no matter what  $A$  is.
- ▶ So the question above asks, “Can we add appropriate premise to the above rule to yield a formal, reliable rule of inference?”

**Discuss In Small Groups:** Suppose three marbles have been drawn from the urn, and all three have been blue. Consider two different properties  $P_1$  and  $P_2$ . The property  $P_1$  is simply the property “is red.” What does induction by enumeration say we should predict about the proportion of the marbles in the urn that have property  $P_1$ , i.e., what proportion are red?

Next, we say a marble has property  $P_2$  if either (1) it is red and has yet to be drawn from the urn or (2) it is blue and has already been drawn from the urn. What proportion of the marbles drawn from the urn thusfar have property  $P_2$ ? What does induction enumeration say about the proportion of marbles in the urn that have property  $P_2$ ?

## Reading Assignment Review

Suppose three marbles have been drawn from the urn, and all three have been blue. Consider two different properties  $P_1$  and  $P_2$ . The property  $P_1$  is simply the property "is red." What does induction by enumeration say we should predict about the proportion of the marbles in the urn that have property  $P_1$ , i.e., what proportion are red?

**Answer:** 0% of the drawn/observed/inspected marbles have been red. So by induction by enumeration, we should predict 0% of the marbles in the urn are red.

## Reading Assignment Review

Next, we say a marble has property  $P_2$  if either (1) it is red and has yet to be drawn from the urn or (2) it is blue and has already been drawn from the urn. What proportion of the marbles drawn from the urn thusfar have property  $P_2$ ? What does induction enumeration say about the proportion of marbles in the urn that have property  $P_2$ ?

**Answer:** 100% of the drawn/observed/inspected marbles have been had property  $P_2$  because all have been blue. So by induction by enumeration, we should predict 100% of the marbles in the urn have property  $P_2$ . But since no such marbles have yet been drawn, that means we should predict 100% are RED!

## Upshot of the Reading Assignment

### Upshot:

- ▶ The example shows that, for every instance of the simple straight rule stated above that has a true conclusion, there is at least one instance of the straight rule (with some weird property like  $P_2$ ) that has a false conclusion.
- ▶ So the straight rule is not reliable.

That might not be surprising BUT ...

## Puzzle

**Question:** Consider adding purely **formal** premises to the straight rule (e.g., about sample size, randomness, etc.). Is there any such modification that yields a reliable rule?

### Sober and Norton's Answer:

1. No.
2. But this doesn't matter for justifying induction ...

**Sober and Norton's Thesis 1:** There are no reliable rules of inference that are also formal.

- ▶ Defense: Sober and Norton argue that particular formal rules aren't reliable, but they don't provide a mathematical proof that no formal, reliable rules exist.
- ▶ Nonetheless, notice that our criticism of the straight rule above would still apply **even if we added premises** stating our sample size was large and drawn at random. For example, suppose we had drawn 1 million blue marbles, at random without replacement, from an urn containing only 1 million and one marbles in total.

**Sober and Norton's Thesis 2:** Many inductive inferences are nonetheless justified. Conclusions of inductive inferences do not need to be justified by a **formal** rule.

Why?

## Return to Justification and Reliabilism

- ▶ According to the reliabilism, when we draw an inference from some premises  $\varphi_1, \varphi_2, \dots, \varphi_n$  to some conclusion  $\psi$ , our belief in  $\psi$  is justified if
  - ▶ We justifiably believe the conjunction of  $\varphi_1, \varphi_2, \dots, \varphi_n$ , and
  - ▶ The inference " $\varphi_1, \varphi_2, \dots, \varphi_n$ , Therefore,  $\psi$ " is an instance of a reliable rule  $R$ , and
  - ▶ We infer  $\psi$  because we're disposed to reason with rule  $R$ .
- ▶ **Sober and Norton's Point:** Every inference we draw is an instance of many different rules, many of which are unreliable because they're purely formal.
  - ▶ But the fact that our inference is an instance of *some* unreliable rule doesn't mean we're unjustified in drawing it, according to the reliabilist.

**Writing Prompt:** Produce what you consider to be a strong inductive argument that Hume would nonetheless judge to be circular. Identify whether there is premise that you could add to make the argument deductively valid. Finally, argue that Sober and Norton would agree with your assessment that your original argument (i.e., without the additional premise) is strong.