Mesoscopic Oscillations of the Conductance of Disordered Metallic Samples as a Function of Temperature

B. Spivak,[1] A. Zyuzin,[2] and D. H. Cobden[1]
[1]*Physics Department, University of Washington, Seattle, Washington 98195, USA*
[2]*A. F. Ioffe Institute, 194021 St. Petersburg, Russia*
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We show theoretically and experimentally that the conductance of small disordered samples exhibits random oscillations as a function of temperature. The amplitude of the oscillations decays as a power law of temperature, and their characteristic period is of the order of the temperature itself.

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At low temperatures the conductance of small disordered metallic samples fluctuates from sample to sample. There are two contributions to the amplitude of the fluctuations. The first is associated with a classical effect: the Drude conductivity depends on the concentration of impurities, which fluctuates in space. The second effect is due to electron quantum interference. As a consequence of the latter, the conductance of an individual sample exhibits random oscillations as a function of the external magnetic field and chemical potential. The goal of this communication is to point out that the conductance $G(T)$ of an individual mesoscopic metallic sample also oscillates as a function of *temperature*.

The well-known picture of mesoscopic fluctuations of the conductance due to interference is as follows. When the sample conductance $G \gg e^2/\hbar$ is large and at zero temperature $T = 0$, the variance is universal,

$$\langle(\delta G)^2\rangle = \alpha \frac{e^4}{\hbar^2}, \tag{1}$$

and independent of sample size [1,2]. (For a review see Ref. [3]). Here $\delta G = G - \langle G\rangle$, the brackets $\langle\rangle$ denote averaging over a random scattering potential, and α is a coefficient of order unity which depends on the dimensionality of the sample and its geometry. One can get Eq. (1) by calculating the diagram shown in Fig. 1. (We use a standard diagram technique for averaging over random scattering potential [4].) The conductance of an individual sample, $G(\mathbf{H})$, exhibits random sample-specific oscillations as a function of external magnetic field $\mathbf{H}$ [1,5]. We will consider, for example, the sample geometry shown in the inset of Fig. 2, and assume that the sample size L is much larger than the elastic mean free path, $L \gg l$. If the magnetic length $L_H \ll L$ and at $T = 0$, the amplitude of the oscillations is given by Eq. (1), while their characteristic period is $H^* \sim \Phi_0/L^2$, where $\Phi_0 = h/ec$ is the flux quantum. This statement follows from the magnetic field dependence of the correlation function,

$$\langle(\delta G(\mathbf{H} + \Delta\mathbf{H})\delta G(\mathbf{H}))\rangle = \frac{e^4}{\hbar^2}\Gamma(\Delta\mathbf{H}). \tag{2}$$

At $\Delta H \gg H^*$ the correlation function has the asymptotic behavior $\Gamma(\Delta H) \sim L_{\Delta H}/L$ and approaches zero. This can be shown by calculating the diagram in Fig. 1, assuming that the inner solid lines correspond to electron Green functions at magnetic field $\mathbf{H}$ while the outer solid lines correspond to Green functions at $\mathbf{H} + \Delta\mathbf{H}$. The oscillations of the conductance as a function of H in the regime where $L_T \gg L_H \gg L$ were discussed in [6]. Here $L_T = \sqrt{D/T}$, where D is the diffusion coefficient of the metal. For example, in the three-dimensional (3D) case the amplitude of the oscillations decays as L_H^{-1} while their period is of order H. Thus in this regime the typical period of the oscillations decreases while the derivative dG/dH diverges as $H \to 0$. To get these results one has to assume that the electron diffusion coefficient in the leads is the same as in the sample.

The oscillations mentioned above are of a single-particle interference nature. Contributions to δG from electron wave functions with different energies, generally speaking, have different signs. As the temperature T increases, cancellation of contributions at different energies becomes more effective, leading to a decay of the amplitude of the mesoscopic oscillations, as has been long understood.

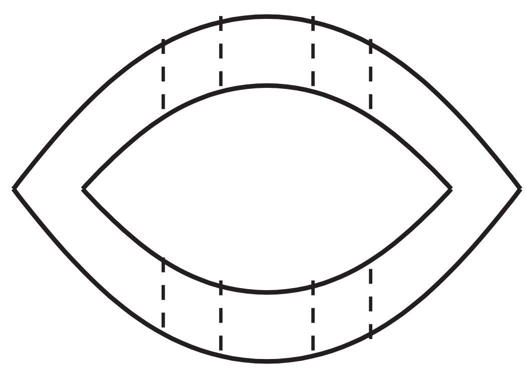

FIG. 1. Diagram describing the correlation function $\langle G(\mathbf{H}, T)G(\mathbf{H}', T')\rangle$. Solid lines correspond to electron Green functions, and thin dashed lines correspond to the correlation function of the random scattering potential.

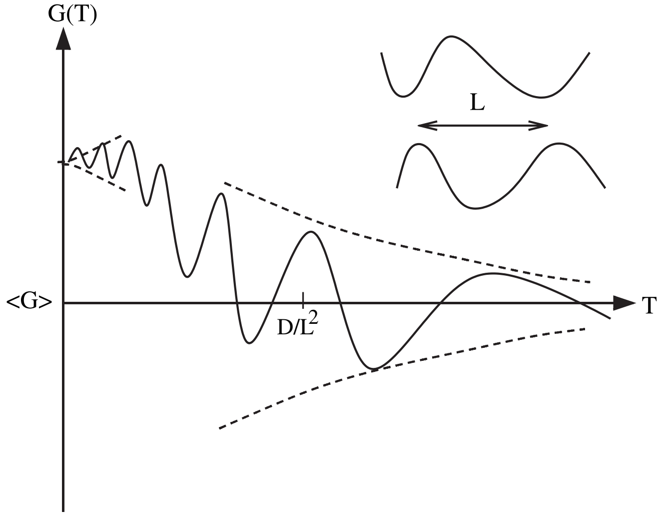

FIG. 2. Typical temperature dependence of the conductance $G(T)$. Inset: schematic diagram of the sample.

In this Letter we show that the temperature dependence of the conductance $G(T)$ of an individual sample is actually a *nonmonotonic* function of the temperature T and exhibits random sample-specific oscillations. The characteristic period of the oscillations T^* is of the order of the temperature itself, that is,

$$T^* \sim T. \tag{3}$$

To prove the existence of such oscillations we calculate the correlation function

$$\begin{aligned}\langle \delta G(T_1)\delta G(T_2)\rangle = \left(\frac{2e^2 D}{\pi h}\right)^2 \left(\frac{1}{L}\right)^{4-d} \\ \times \int_0^\infty d\mathbf{q}dt\left[\frac{2}{Dq^2+\tau_\phi^{-1}}+t\right] \\ \times \exp[-(Dq^2+\tau_\phi^{-1})t]B(tT_1)B(tT_2),\end{aligned} \tag{4}$$

where τ_ϕ is the electron phase breaking time, and

$$B(z) = \frac{\pi z}{\sinh \pi z}. \tag{5}$$

It follows from Eq. (4) that in the limit $T \gg T' \gg D/L^2$

$$\langle \delta G(T)^2\rangle = \alpha_1 \frac{e^4}{\hbar^2}\begin{cases}\frac{D^{1/2}}{T^{1/2}L} & \text{for } d=3\\ \frac{D}{TL^2}\ln(\tau_0 T) & \text{for } d=2\end{cases} \tag{6}$$

and

$$\langle \delta G(T')\delta G(T)\rangle = \alpha_1 \frac{e^4}{\hbar^2}\begin{cases}\frac{D^{1/2}}{T^{1/2}L}(1-0.4(\frac{T'}{T})^2) & \text{for } d=3\\ \frac{D}{TL^2}\ln(\tau_0 T)(1-0.8(\frac{T'}{T})^2) & \text{for } d=2\end{cases}. \tag{7}$$

Here $\tau_0 = \min\{\tau_\phi, D/L^2\}$, and α_1 is a coefficient of order unity which depends on the sample geometry. To get Eq. (6) one can calculate the diagram shown in Fig. 1 where the electron Green functions in the inner and the outer loops are taken at the same temperature T. To get Eq. (7), one has to take the Green functions in the inner loop at T' and in the outer loop at T.

The existence of oscillations of the conductance $G(T)$ as a function of T, [implying that $\delta G(T)$ changes sign with T], follow from the observation that for $T \gg T'$, Eqs. (6) and (7) have the same temperature dependence, together with the fact that the distribution function of $G(T)$ is Gaussian [7]. The latter means that the averages given by Eqs. (6) and (7) are dominated by typical realizations of $G(T)$. For example, a typical monotonic form $\delta G(T) \sim AT^{-\gamma}$ cannot satisfy both Eqs. (6) and (7), even if the coefficient A has a random sample-specific sign. As a result, a typical realization of the temperature dependence of the conductance has the form

$$\delta G(T) = \langle [\delta G(T)]^2\rangle^{1/2} F(T), \tag{8}$$

where the function $F(T)$ randomly oscillates about zero with a characteristic period of order T. The latter statement follows from comparing Eqs. (6) and (7).

In the opposite limit $T \ll D/L^2$, (i.e., $L_T \gg L$), the temperature dependence of $\delta G(T)$ depends on the properties of the leads. In the limit $D_L/D \to \infty$, where D_L is the diffusion coefficient in the leads, the function $\delta G(T)$ vanishes monotonically as $T \to 0$. However, if $D_L \sim D$ the T dependence of $G(T)$ has similar features to its H dependence in the limit $L_T \gg L_H \gg L$ [6]. That is, $\delta G(T)$ exhibits random oscillations whose amplitude decays as $T \to 0$. The period of the oscillations for $T \ll D/L^2$ is again of order $T^* \sim T$. The latter statement follows from the T dependence of the following correlation functions at $T' \ll T$,

$$\left\langle\left(\frac{d\delta G(T)}{dT}\right)^2\right\rangle \sim \frac{1}{T^{3-d/2}} \tag{9}$$

and

$$\left\langle\frac{d\delta G(T')}{dT'}\frac{d\delta G(T)}{dT}\right\rangle \sim -\frac{T'}{T^{4-d/2}}, \tag{10}$$

which can be obtained by calculation of the diagrams in Fig. 1 in a way similar to that in Ref. [6]. According to Eqs. (9) and (10) the value of the derivative dG/dT diverges as $T \to 0$. This is correct as long as $L_\phi \gg L_T$, where $L_\phi = \sqrt{D\tau_\phi}$ is the electron phase breaking length. The inequality holds if the value of L_ϕ is determined by electron-electron or electron-phonon scattering [8]. The overall qualitative temperature dependence of $\delta G(T)$ in the case $D_L \sim D$ is shown in Fig. 2.

At very low temperatures the value of L_ϕ is determined by the paramagnetic impurities in the sample and is temperature independent as long as the Kondo effect and exchange between paramagnetic spins are not significant. In the case $L_\phi \ll L_T$ the amplitude of the oscillations of dG/dT decays as $T \rightarrow 0$. Thus the typical amplitude of the oscillations of the derivative dG/dT has a maximum when $L_T \sim L_\phi$; and the total number of oscillations is of order $\ln(T\tau_s)$, where τ_s is the spin relaxation time.

In fact the oscillations as a function of temperature discussed above should be present in any thermodynamic or transport property of mesoscopic metallic samples. For instance, they should be present in the Ruderman-Kittel interaction [9] in disordered media.

To test the theory we study some measurements of conductance oscillations in a silicon metal-oxide-semiconductor field-effect transistor (MOSFET) as a joint function of gate voltage V_g and temperature T. The chosen device has a square channel of length and width $L \sim 1\ \mu$m. The oxide thickness is 25 nm, giving a gate capacitance per unit area of $8.6 \times 10^{11} e\ \mathrm{cm}^{-2}\,\mathrm{V}^{-1}$. The source and drain contacts are n++ doped silicon. The average conductance, measured by passing an ac current of 5 nA, varies approximately linearly from about $19e^2/h$ at $V_g = 4$ V to $26e^2/h$ at $V_g = 5$ V. For practical purposes, the device behaves as a square of disordered 2D electron gas with mobility $\mu \sim 2000\ \mathrm{cm}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$ and momentum scattering length $l \sim 30$ nm, having 3D metallic contacts. Measurements of conductance oscillations as a function of magnetic field [10] indicate that the channel is phase coherent at the base temperature of 35 mK achieved on the dilution refrigerator, which is approximately equal to the Thouless energy D/L^2.

The data presented in Fig. 3 are derived from sweeps of V_g at a series of temperatures between 35 mK and 1.2 K. (Note that a constant perpendicular magnetic field of 0.1 T was present in all measurements.) A smooth monotonic background variation of the mean conductance $\langle G \rangle$ with V_g and T has been subtracted from the raw data, so that the quantity plotted in the figure is the deviation from this background, $\delta G = G - \langle G \rangle$. The sweeps show reproducible oscillations as a function of V_g which decay and broaden as T increases, as illustrated in Fig. 3(a). The variance $\langle(\delta G)^2\rangle$ and correlation gate voltage V_g^* of these oscillations are plotted against T in Fig. 3(b). Figure 3(c) is a gray scale plot of δG vs V_g and T, where peaks are light and dips are dark. The appearance of this plot, where individual extrema can be seen to evolve steadily with T, gives us confidence that the data at different temperatures can be compared reliably.

Figure 3(d) shows the variation of δG with T on a logarithmic scale at a set of evenly spaced values of V_g. The curves here are smooth splines passing through the

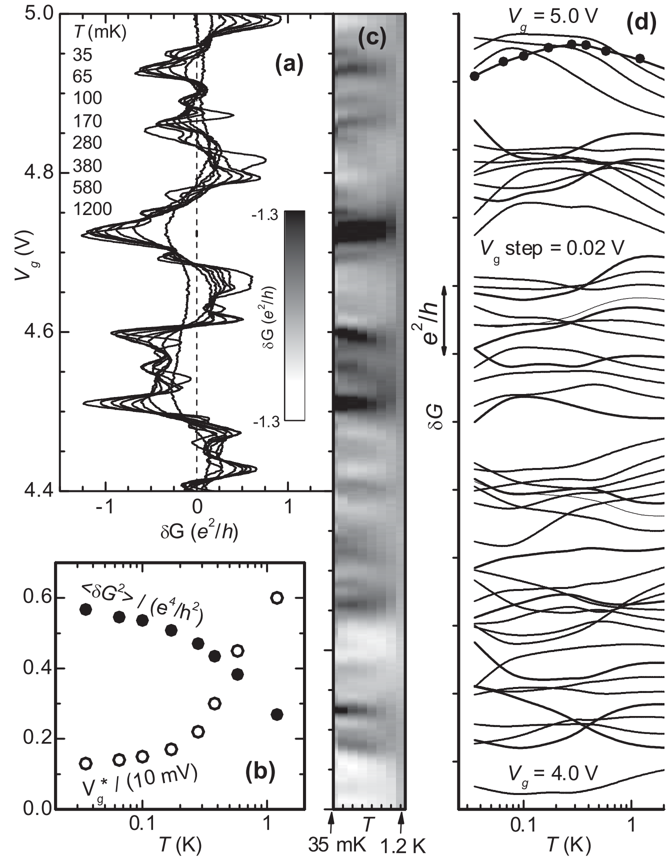

FIG. 3. Measurements of the conductance oscillations δG in a silicon MOSFET. (a) Gate voltage (V_g) sweeps at a series of temperatures (T) listed at the top left. (b) Variance $\langle(\delta G)^2\rangle$ and correlation gate voltage V_g^* (the half-width of the autocorrelation function) of the oscillations as a function of temperature, obtained by averaging over V_g. (c) Gray scale plot of δG vs T and V_g. (d) Temperature dependence of δG at $V_g = 4.0, 4.02, \ldots, 5.0$ V, with consecutive sweeps offset vertically by $0.2e^2/h$.

eight temperature points at each V_g and extrapolating towards $\delta G = 0$ at $T \gg 1.2$ K. For clarity, the actual data points are marked as solid circles on only one of the curves. The experimental data are in qualitative agreement with our predictions. It is apparent that δG oscillates randomly with T. The amplitude of the oscillations of the conductance is of order e^2/h, as expected. We typically see one period of oscillation over the factor-of-30 temperature range. This is not inconsistent with our theory, although the temperature range is insufficient to verify the prediction that at $T > D/L^2$ the period is of the order of the temperature itself. In addition, in our device $D_L \gg D$ so we would not expect oscillations at $T < D/L^2$.

It is quite surprising that these oscillations of the conductance as a function of temperature have never been pointed out in either the theoretical or the experimental literature.

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