Physics 224 | **Quiz 6, Wed 12 Feb** | Name _____
Winter 2003 | Equipartition of energy

The equipartition theorem says that the mean energy in thermal equilibrium is $(q/2)kT$, where $q$ is the number of 'quadratic degrees of freedom'. A quadratic degree of freedom is one on which the energy depends quadratically. For instance there may be energy terms $\frac{1}{2} mv_x^2$, $\frac{1}{2} kx^2$, $\frac{1}{2} ky^2$, $\frac{1}{2} I\omega_x^2$ associated with degrees of freedom $v_x$, $x$, $y$, and $\omega_x$. (You can prove the equipartition theorem if you assume a Boltzmann distribution of energies and finding the mean energy).

For a gas of free noninteracting atoms, there are three quadratic degrees of freedom for each atom, $v_x$, $v_y$ and $v_z$, so $q = 3$, leading to $pV = (3/2)kT$. (Although there are three more degrees of freedom associated with position, they don't count, because the energy doesn't depend on position). What is $q$ for each of the following have? In each case, try to describe, or sketch, the degrees of freedom. $E = \frac{1}{2}mv_x^2 + \ldots + \frac{1}{2}kx^2 + \ldots + \frac{1}{2}I_1\omega_1^2 + \ldots \frac{1}{2}k_r r^2 + \ldots + \frac{1}{2}m_r\dot{r}^2 + \ldots$

(a) An atom in a 3D parabolic potential well (eg, in an 'atom trap' in the Phys-Astr basement)

6 – why? $v_x$ $v_y$ $v_z$ $x$ $y$ $z$ $\left[E = \frac{1}{2}m|\vec{v}|^2 + \frac{1}{2}k|\vec{r}|^2\right]$

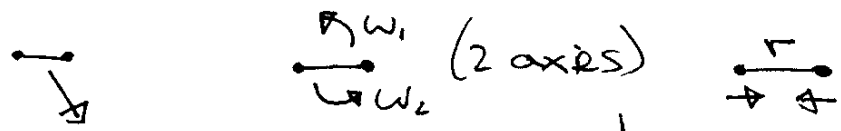
(b) An atom in a solid

6 – as for an atom in a trap. It is confined to a position by its neighbors.

(c) Two free O atoms

6 $v_{x1}$ $v_{y1}$ $v_{z1}$ $v_{x2}$ $v_{y2}$ $v_{z2}$

(d) A free diatomic molecule (eg $O_2$) – including vibrations and rotations! 7

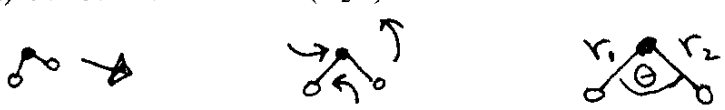



$v_x$ $v_y$ $v_z$ $\omega_1$ $\omega_2$ $r$ $\frac{dr}{dt}$

(e) Two free H atoms and one free O atom.

9 = 3 + 3 + 3 !

(f) A free water molecule ($H_2O$) 12




$v_x$ $v_y$ $v_z$ $\omega_1$ $\omega_2$ $\omega_3$ $r_1$ $r_2$ $\theta$ $\frac{dr_1}{dt}$ $\frac{dr_2}{dt}$ $\frac{d\theta}{dt}$

(f) A water molecule in liquid water (if you can do that, you will win a Nobel prize some day)

Lots.