Physics 224
Winter 2003

Quiz 4, Monday 27 Jan
Distribution functions

Name ______________________
Student number ______________________

Consider a one-dimensional kinetic theory gas (to avoid confusions with vectors): a long, thin box contains a gas of N molecules which have random positions and velocities v along the box. The distribution function of the velocity v is $f(v)$. By definition, $f(v)dv$ is the average number of molecules with velocities in the range v to $v+dv$.

1. What is the probability that the velocity of a molecule is between v_1 and v_2?

$$P = \int_{v_1}^{v_2} f(v)\,dv \Big/ N \qquad N = \int_{-\infty}^{\infty} f(v)\,dv$$

2. Write down the expression for the mean velocity, $\bar{v}$:

$$\bar{v} = \int_{-\infty}^{\infty} v f(v)\,dv \Big/ N$$

3. Write down the expression for the mean square velocity $\overline{v^2}$:

$$\overline{v^2} = \int_{-\infty}^{\infty} v^2 f(v)\,dv \Big/ N$$

4. Is $\overline{v^2}$ greater than, equal to, or less than $\bar{v}^2$, and why?

$$\overline{(v-\bar{v})^2} = \sigma^2 = \overline{v^2 - 2v\bar{v} + \bar{v}^2}$$
$$= \overline{v^2} - 2\bar{v}^2 + \bar{v}^2$$
$$= \overline{v^2} - \bar{v}^2$$
$$\text{so } \overline{v^2} = \bar{v}^2 + \sigma^2$$

$\overline{v^2} \geq \bar{v}^2$ always

5. Under what conditions will $f(v)$ not change with time?

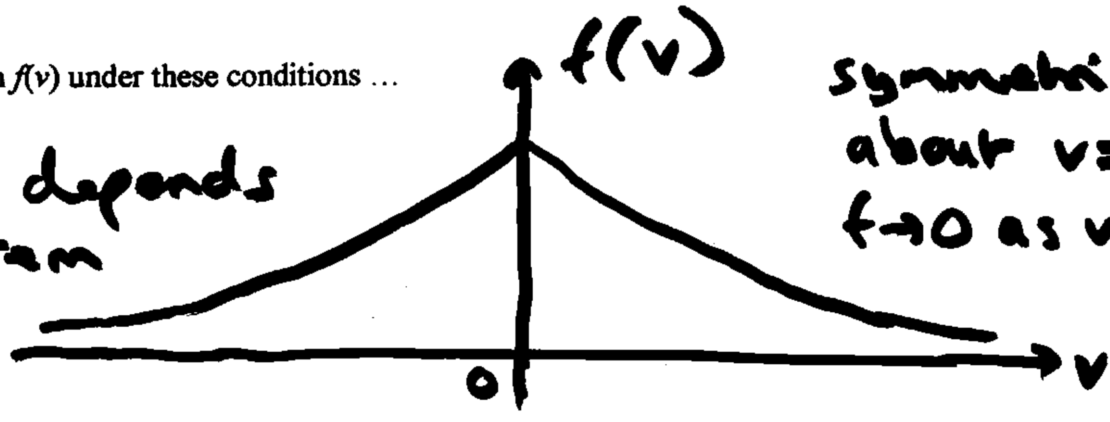
In equilibrium $\Rightarrow f(v)$ is stationary

6. Under what conditions is $\bar{v} = 0$? Why?

$\bar{v}$ = average motion

No flow (equilibrium)

7. Sketch $f(v)$ under these conditions …



Symmetric about $v=0$
$f \to 0$ as $v \to \infty$

shape depends on system