 Name ____________________ Student ID ______________

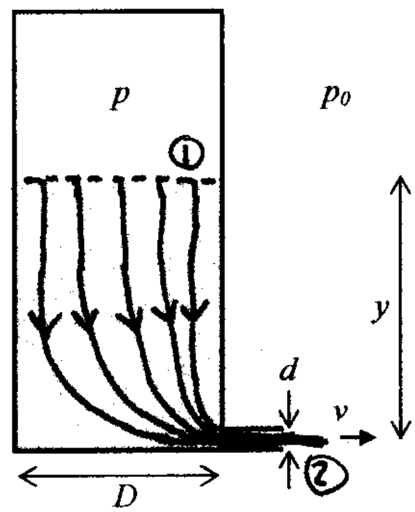
Thermal Physics, Physics 224 **Midterm exam 1** **January 31, 2003**
Winter 2003
Instructor: David Cobden **4 pages (printed on both sides)**

You have 50 minutes. Begin and end on the buzzer. Be sure to write your name on every page. Answer all questions. Write all your working on these question sheets. Watch the blackboard for corrections during the exam. This is a closed book exam. You are allowed one sheet (two sides) of notes. You are allowed a simple calculator. Throughout this exam, you will get credit for giving algebraic answers wherever possible as well as numerical answers.

Question I. Fluids [50 points total]

Part A.

The vessel sketched on the right is a vertical cylinder with diameter D containing water (grey, density ρ) to a depth y. A pressure p is maintained in the air above the water, which is allowed to squirt out at into the open air (pressure p_0) through a short, open-ended pipe of diameter d at the bottom.



1. [4] Sketch streamlines of the water flow. →

2. [8] Use Bernoulli's principle to ~~find~~ estimate the squirting speed v.

At ① at ②

$$\rho g y + p = \frac{1}{2}\rho v^2 + p_0$$

$$\therefore \frac{1}{2}\rho v^2 = \rho g y + p - p_0 \qquad \therefore v = \sqrt{2gy + \frac{2(p-p_0)}{\rho}}$$

3. [4] What approximations or assumptions did you make in applying this principle?

- Viscosity $\eta = 0$
- Laminar flow (no turbulence)
- $v = 0$ at ①
- water is incompressible so ρ is indep. of y.

4. [6] At what rate does the depth of the water y decrease?

Continuity: $\frac{dy}{dt} \times \pi\left(\frac{D}{2}\right)^2 = -v \times \pi\left(\frac{d}{2}\right)^2 \quad \therefore \frac{dy}{dt} = -\left(\frac{d}{D}\right)^2 v$

5. [6] What is the criterion that determines whether the flow in the pipe is turbulent?

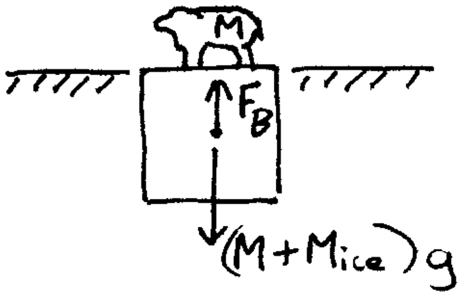
~~For~~ If Reynolds number $\frac{\rho v d}{\eta} > 2300$ it's turbulent.

Part B.

6. [6] State Archimede's principle for finding the buoyant force on an object. What is the center of buoyancy, and how do you find it?

The buoyant force is equal to the weight of the water displaced. It is directed through the center of buoyancy, which is at the center of mass of the displaced water.

7. [10] A cube of ice (density $\rho_i = 900$ kg/m^{-3}), of side L, is floating in the sea (density $\rho_w = 925$ kg/m^{-3}) with one face upwards. A polar bear of mass (M=) 200 kg stands on top of it. What is the minimum size L for his feet to be just above the water, assuming the ice cube does not tip over?



$F_B = (M + M_{ice})g$ in equilibrium

Just at this point,

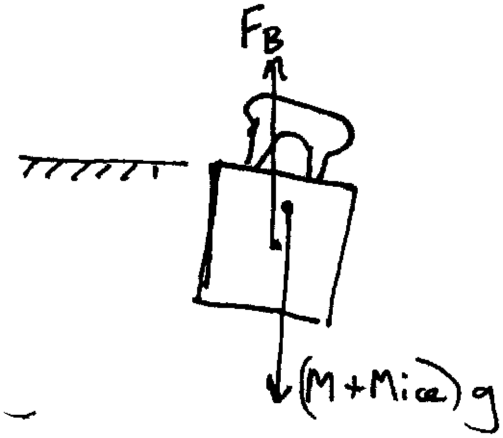
$F_B = \rho_w L^3 g$

$\therefore \rho_w L^3 g = (M + M_{ice})g \qquad M_{ice} = \rho_i L^3$

$\therefore L^3(\rho_w - \rho_i) = M$

$\therefore L = \left(\frac{M}{\rho_w - \rho_i}\right)^{1/3} = \left(\frac{200\text{ kg}}{25\text{ kgm}^{-3}}\right)^{1/3} = 2\text{ m}$

8. [6] Why is it virtually impossible for the bear to maintain his position on this mini-iceberg without it rolling over and dumping him in the water?



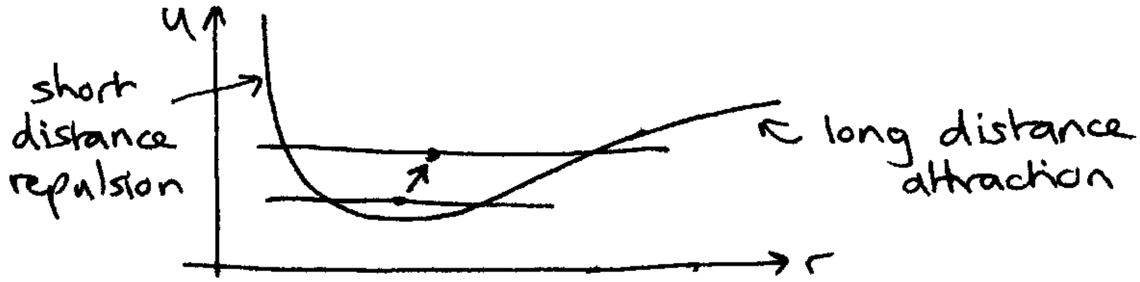
Center of buoyancy is in center of cube. Center of gravity is above center of buoyancy → unstable situation.

After the slightest tilt, there will be a torque tending to tip the bear/cube combination over further.

Question 2. Temperature and ideal gases [50 points total]

Part C.

9. [8] Sketch and annotate the interaction potential $U(r)$ between two adjacent atoms spaced by distance r in a solid. Explain why most solids expand as the temperature is increased.



As T increases, average vibrational energy increases → average r increases because the potential function is "softer" at larger r.

10. [8] A concrete highway is built of slabs which are 12 m long at 20° C. The coefficient of linear expansion is $\alpha = 12\times10^{-6}$ (°C)$^{-1}$. How wide should the expansion cracks between the slabs be at 20° C to prevent buckling if the range of temperatures throughout the year is -30° C to +50° C?

$L = 12\ m$ at $T = 20°C$

At 50°C, $\Delta L = \alpha L \Delta T = 12\times10^{-6}/°C \times 12 m \times (50°C - 20°C)$

$\approx 4.3\times10^{-3} m$

∴ each slab expands by up to 0.43 cm

∴ need gaps of say 0.5 cm between them

11. [6] The modulus of concrete under compression is $E = 2.0\times10^{10}$ N/m^{-2}. If the slabs are mistakenly laid end-to-end with no expansion cracks at 20° C, what pressure will develop between them (equal to the stress within them) at 30° C?

If free the slabs would expand by $\frac{\Delta L}{L} = \alpha \Delta T$.

The pressure (or stress) p must be such as to provide an equal and opposite contraction, $p = E\frac{\Delta L}{L}$

$\therefore p = E\alpha\Delta T$

$= (2.0\times10^{10} Nm^{-2}) \times (12\times10^{-6}/°C) \times 10°C$

$= 2.4\times10^{6} Nm^{-2}$

Part D.

The electrons moving in some solid behave as a low-density gas of free particles in thermal equilibrium at room temperature $T = 20°$ C. Their number density is $n = 10^{27}$ m^{-3}, and each has a mass $m = 10^{-4}$ a.u. (1 a.u. = mass of one hydrogen atom; $k_B = 1.38\times10^{-23}$ J/K; $N_A = 6.02\times10^{23}$).

12. [8] Estimate the pressure exerted by the electron gas on a surface of the solid.

Kinetic theory of ideal gas: 20°C = 293 K

$$p = \frac{N}{V} k_B T = n k_B T$$
$$= 10^{27} m^{-3} \times 1.38\times10^{-23} J/K \times 293 K$$
$$\doteq 4\times10^{6} Nm^{-2}$$

13. [8] Calculate the rms velocity, v_{rms}, of one of these electrons.

Equipartition: $\frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_B T$

$$\therefore v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

$$m = 10^{-4} \times \frac{1g}{N_A} = \frac{10^{-7} kg}{6.02\times10^{23}} \doteq 1.7\times10^{-31} kg$$

$$\therefore v_{rms} = \left(\frac{3\times1.38\times10^{-23} J/K \times 293 K}{1.7\times10^{-31} kg}\right)^{1/2} \doteq 2.7\times10^{5} ms^{-1}$$

14. [8] Is the pressure exerted by a real gas greater than, equal to, or less than that which would be exerted by a gas with the same short range interactions but no long range interactions? Why, in words? Show how this is accounted for in the van der Waals equation.

Less than, because atoms and molecules always attract at long range. The ones getting near a wall are slowed down as they are attracted back into the bulk.

van der Waals: $\left(p + \frac{a}{(V/n)^2}\right)(V - nb) = nRT$

ie $p = \frac{nRT}{V-nb} - \frac{a}{(V/n)^2}$

↑ pressure exerted if no long-range interactions

↖ reduction as above