

FINANCIAL ECONOMICS (ECON 577)

Homework 2 Due March 22nd

1. Consider two-date security markets with I strictly risk averse agents and one risk neutral agent. Agents' endowments and a risk-free payoff lie in the asset span. The consumption set of each risk averse agent is the positive orthant R_+^S (there is no consumption at date 0). The consumption set of the risk neutral agent consists of all consumption plans c^1 such that $E(c^1) \geq 0$.
 - i. Show that security markets are effectively complete.
 - ii. Verify whether all equilibrium allocations in security markets are Pareto optimal.
2. Consider the CAPM model with mean-variance preferences. Suppose that the agents' endowments are in the asset span and that their preferences are strictly increasing in expectation and strictly decreasing in variance.

Suppose that agent A is more variance averse than agent B in the following sense:

$$\begin{aligned} \text{if } f_B(\mu, \sigma^2) &> f_B(\mu_1, \sigma_1^2) \text{ and } \mu \leq \mu_1, \\ \text{then } f_A(\mu, \sigma^2) &> f_A(\mu_1, \sigma_1^2) \end{aligned}$$

for all μ, σ .

Assuming that a risk-free security is traded, show that a more variance-averse agent invests a larger fraction of wealth in the risk-free security than a less variance-averse one, provided that both have the same wealth. You may assume that the optimal portfolios are unique.

3. Consider a portfolio choice problem of a strictly risk-averse individual with one risky and one risk-free security. Suppose that the risk-free return is 1 and that there is a tax levied on the risky investment income (i.e., the payoff on investment a is $a + (r - 1)(1 - t)a$). Show that the optimal investment \bar{a} in the risky security satisfies $\frac{\partial \bar{a}}{\partial t} = \frac{\bar{a}}{1-t}$ and that the expected utility of optimal consumption is independent of the tax rate. Do these results continue to hold if the risk-free return exceeds 1?