FINANCIAL ECONOMICS (ECON 577)

Homework 1
Due February 3rd

1. Consider the model of security markets discussed in class. Assume the following:

   A1. Utility functions u is strictly increasing, continuous and strictly quasi-concave on the consumption set $\mathbb{R}_+^{S+1}$.

   A2. Security payoffs are strictly positive and there are no redundant securities.

   A3. Initial endowments are strictly positive in every component.

   Show that the consumption-portfolio demand is a continuous function on the set of arbitrage-free prices. Describe the "boundary behavior" of the consumption-portfolio demand.

2. Consider two securities: security 1 is risk-free with return r^*, security 2 is risky with return (d, r^*, u) in the three possible states of nature, with $d < r^* < u$.

   i. Find the set of state prices and risk-neutral probabilities.

   ii. Give an example of an option on security 2 that is not in the asset span, and find the upper and lower bounds, q_u, q_l on the value of the option.

3. Consider a two-period economy with $S = 3$ states of nature at date 1. Suppose there are $J = 2$ securities with date 1 payoffs

$$x_1 = (10 - 20, 60)\,; \ x_2 = (20, 30, 10)$$

   i. Find the set $\mathcal{P}$ of all prices $p = (p_1, p_2)$ which are arbitrage free. Show that $\mathcal{P}$ is an open cone.

   ii. For each $p \in \mathcal{P}$ find all state price vectors $q = (q_1, q_2, q_3)$ such that $p = Xq$.

   iii. Consider a vector of security prices which offers arbitrage opportunities and construct a corresponding arbitrage portfolio.