FINANCIAL ECONOMICS (ECON 577)

Homework 3
Due April 28^{th}

1. Consider an economy extending over three dates $t = 0, 1, 2$ with uncertainty described by the set S of six states of the world $s_1, s_2, s_3, s_4, s_5, s_6$. The information structure is described by the partitions $\mathcal{F}_0 = \{\{s_1, s_2, s_3, s_4, s_5, s_6\}\}$, $\mathcal{F}_1 = \{\{s_1, s_2\}, \{s_3, s_4\}, \{s_5, s_6\}\}$, $\mathcal{F}_2 = \{\{s_1\}, \{s_2\}, \{s_3\}, \{s_4\}, \{s_5\}, \{s_6\}\}$. There are two securities with non-zero dividends only at date 2. Their date-2 dividends are given by: $x_{12} = (1, 1, 1, 1, 1, 1)$, $x_{22} = (1, 2, 4, 2, 2, 3)$. Date-1 prices of the securities are given by $p(\xi_a) = \left(1, \frac{3}{2}\right)$, $p(\xi_b) = (0.9, 2)$, $p(\xi_c) = (0.8, 2)$, where ξ_a, ξ_b, ξ_c are the sets of the partition $\mathcal{F}_1$. Date-0 prices are given by $p_0 = \left(0.9, \frac{7}{4}\right)$.

   (a) Find (the set of)event prices. What can you conclude from your findings about the market completeness or the existence of arbitrage opportunities?

   (b) Show that an European call option on security 2 with exercise price 2 and exercise date 2 is not in the asset span. Find the upper and lower bounds on the value of the option at date 0.

   (c) Suppose that the option of question (b) is traded at dates 0 and 1 along with the two primitive securities. Give an example of the prices of the option at dates 0 and 1 so that there is no arbitrage.

   (d) Suppose that the option is an American option, i.e., it can be exercised prior to expiration. Answer question (c) for the American option.

2. Consider an infinite horizon economy in which J securities, with *positive* dividends $(x_{jt})_{\substack{j=1...J \\ t=1...\infty}}$ are traded. Suppose that there is no finite-time arbitrage opportunity so that there exists a system of positive event prices $\{q(\xi_t)\}_t$. Further, suppose that the markets are complete in the sense that in every event there exist as many securities with linearly independent one-period payoffs as there are immediate successor events. Consider a borrowing constraint of the form:

$$p(\xi_t) h(\xi_t) \geq -\frac{1}{q(\xi_t)} \sum_{\tau=t+1}^{\infty} \sum_{\xi_\tau \subset \xi_t} q(\xi_\tau) w(\xi_\tau),$$

for every ξ_t, $t = 0, 1, ...,$ where w is an agent's endowment. Assuming that the expression on the right hand side is finite, show that there is no Ponzi scheme at any event satisfying this constraint.