How to Solve Systems of Equations With a Calculator

J. B. Callis, Instructor
Chemistry 142

Let us begin with a system of two equations in two unknowns:

$$\begin{aligned} 2\,x + 3\,y &= 18 \\ 3\,x + 2\,y &= 17 \end{aligned} \qquad (1)$$

This system is solved with $x = 3$ and $y = 4$. Equations (1) may be expressed in matrix - vector form as:

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 17 \end{bmatrix} \qquad (2)$$

Where the first term in brackets is the matrix of coefficients, the second term is the column vector of unknowns and the third term is the column vector of constants.

Equation two is now converted to an augmented matrix as follows:

$$\begin{bmatrix} 2 & 3 & 18 \\ 3 & 2 & 17 \end{bmatrix} \qquad (3)$$

Where the matrix of Equation two has been augmented by appending the vector of constants. Expression 3 may now be solved by the operator **rref()** which yields the following result:

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \end{bmatrix} \qquad (4)$$

The solutions ($x = 3$, $y = 4$) are found in the last column of the matrix in expression 4.

Now we apply the foregoing procedure to solving a chemical equation, such as:

$$x\ H_2\ (g) + y\ O_2\ (g) \longrightarrow z\ H_2O(g) \qquad (5)$$

The two atom conservation equations implied by equation 5 are

$$\begin{aligned} &\text{H: } 2\,x = 2\,z, \text{ or } x = z \\ &\text{O: } 2\,y = z \end{aligned} \qquad (6)$$

This is a system of two equations in 3 unknowns. To obtain a unique solution we need a third equation. This can be obtained by setting one of the unknowns equal to one.

$$x = 1 \tag{7}$$

Combining equations 6 and 7 in a form suitable for expression as an augmented matrix we obtain:

$$\begin{aligned} 1\,x + 0\,y - 1\,z &= 0 \\ 0\,x + 2\,y - 1\,z &= 0 \\ 1\,x + 0\,y + 0\,z &= 1 \end{aligned} \tag{8}$$

We now state the augmented matrix for the system of equations in (8).

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Solving equation 9 using the **rref()** operator of the calculator we obtain $x = 1$, $y = 0.5$, $z = 1$. By multiplying x, y, and z by two we obtain the desired solution $x = 2$, $y = 1$, $z = 2$.