

UW CSSS/POLS 512:
Time Series and Panel Data for the Social Sciences

Heteroskedasticity in Panel Data

Christopher Adolph

Department of Political Science
and

Center for Statistics and the Social Sciences
University of Washington, Seattle

Review of heteroskedasticity

Recall that in cross-sectional LS, heteroskedasticity

- is assumed away
- if present, biases our standard errors

We noted two approaches

- Model the heteroskedasticity directly with an appropriate ML model, *or*
- Less optimally, continue to use the wrong method (LS), but try to correct the se's; these are known as Huber-White, sandwich, or robust standard errors

How do these approaches transfer to the time series context? to panel data?

Dynamic heteroskedasticity

As with cross-sectional models, we can model heteroskedasticity directly

One possibility is to let heteroskedasticity evolve dynamically

Dynamic heteroskedasticity

As with cross-sectional models, we can model heteroskedasticity directly

One possibility is to let heteroskedasticity evolve dynamically

We can let heteroskedasticity be (sort-of) “ARMA”, under the name “GARCH”
Generalized Autoregressive Conditional Heteroskedasticity:

$$y_t = \mu_t + \varepsilon_t \quad \varepsilon_t \sim f_{\mathcal{N}}(0, \sigma_t^2)$$

Dynamic heteroskedasticity

As with cross-sectional models, we can model heteroskedasticity directly

One possibility is to let heteroskedasticity evolve dynamically

We can let heteroskedasticity be (sort-of) “ARMA”, under the name “GARCH”
Generalized Autoregressive Conditional Heteroskedasticity:

$$y_t = \mu_t + \varepsilon_t \quad \varepsilon_t \sim f_{\mathcal{N}}(0, \sigma_t^2)$$

where

$$\mu_t = \alpha + x_t\beta + \sum_{p=1}^P y_{t-p}\phi_p + \sum_{q=1}^Q \varepsilon_{t-q}\theta_q$$

Dynamic heteroskedasticity

As with cross-sectional models, we can model heteroskedasticity directly

One possibility is to let heteroskedasticity evolve dynamically

We can let heteroskedasticity be (sort-of) “ARMA”, under the name “GARCH”
Generalized Autoregressive Conditional Heteroskedasticity:

$$y_t = \mu_t + \varepsilon_t \quad \varepsilon_t \sim f_{\mathcal{N}}(0, \sigma_t^2)$$

where

$$\mu_t = \alpha + x_t\beta + \sum_{p=1}^P y_{t-p}\phi_p + \sum_{q=1}^Q \varepsilon_{t-q}\theta_q$$

$$\sigma_t^2 = \exp(\eta + z_t\gamma) + \sum_{c=1}^C \sigma_{t-c}^2\lambda_c + \sum_{d=1}^D \varepsilon_{t-d}^2\xi_d$$

Dynamic heteroskedasticity

As with cross-sectional models, we can model heteroskedasticity directly

One possibility is to let heteroskedasticity evolve dynamically

We can let heteroskedasticity be (sort-of) “ARMA”, under the name “GARCH”
Generalized Autoregressive Conditional Heteroskedasticity:

$$y_t = \mu_t + \varepsilon_t \quad \varepsilon_t \sim f_{\mathcal{N}}(0, \sigma_t^2)$$

where

$$\mu_t = \alpha + x_t\beta + \sum_{p=1}^P y_{t-p}\phi_p + \sum_{q=1}^Q \varepsilon_{t-q}\theta_q$$

$$\sigma_t^2 = \exp(\eta + z_t\gamma) + \sum_{c=1}^C \sigma_{t-c}^2\lambda_c + \sum_{d=1}^D \varepsilon_{t-d}^2\xi_d$$

In words, y_t is an $ARMA(P, Q)$ - $GARCH(C, D)$ distributed time-series

(Of course, we could leave out x and/or z if we wanted)

Dynamic heteroskedasticity

$$y_t = \mu_t + \varepsilon_t \quad \varepsilon_t \sim f_{\mathcal{N}}(0, \sigma_t^2)$$

where

$$\mu_t = \alpha + x_t\beta + \sum_{p=1}^P y_{t-p}\phi_p + \sum_{q=1}^Q \varepsilon_{t-q}\theta_q$$

$$\sigma_t^2 = \exp(\eta + z_t\gamma) + \sum_{c=1}^C \sigma_{t-c}^2\lambda_c + \sum_{d=1}^D \varepsilon_{t-d}^2\xi_d$$

Models like the above are workhorses of financial forecasting

Can estimated by ML as usual

In R, `garch()` in the `tseries` package does GARCH

May have to look around a bit for ARMA-GARCH

Dynamic and panel heteroskedasticity

Panel data allows for more complex forms of heteroskedasticity and serial correlation than cross-sectional data. For example. . .

- Serial correlation: $E(\varepsilon_{is}\varepsilon_{it}) = \sigma_{st} \neq 0$

(Reduced/eliminated by appropriate ARMA specification)

Dynamic and panel heteroskedasticity

Panel data allows for more complex forms of heteroskedasticity and serial correlation than cross-sectional data. For example. . .

- Serial correlation: $E(\varepsilon_{is}\varepsilon_{it}) = \sigma_{st} \neq 0$

(Reduced/eliminated by appropriate ARMA specification)

- Contemporaneous correlation: $E(\varepsilon_{it}\varepsilon_{jt}) = \sigma_{ij} \neq 0$

(Globally reduced by year fixed effects, but what about pairs of correlated units?)

Dynamic and panel heteroskedasticity

Panel data allows for more complex forms of heteroskedasticity and serial correlation than cross-sectional data. For example. . .

- Serial correlation: $E(\varepsilon_{is}\varepsilon_{it}) = \sigma_{st} \neq 0$

(Reduced/eliminated by appropriate ARMA specification)

- Contemporaneous correlation: $E(\varepsilon_{it}\varepsilon_{jt}) = \sigma_{ij} \neq 0$

(Globally reduced by year fixed effects, but what about pairs of correlated units?)

- Panel heteroskedasticity: $E(\varepsilon_{is}^2) = E(\varepsilon_{it}^2) = \sigma_i^2$, but $\sigma_i^2 \neq \sigma_j^2$

(How to fix?)

Dynamic and panel heteroskedasticity

Panel data allows for more complex forms of heteroskedasticity and serial correlation than cross-sectional data. For example. . .

- Serial correlation: $E(\varepsilon_{is}\varepsilon_{it}) = \sigma_{st} \neq 0$

(Reduced/eliminated by appropriate ARMA specification)

- Contemporaneous correlation: $E(\varepsilon_{it}\varepsilon_{jt}) = \sigma_{ij} \neq 0$

(Globally reduced by year fixed effects, but what about pairs of correlated units?)

- Panel heteroskedasticity: $E(\varepsilon_{is}^2) = E(\varepsilon_{it}^2) = \sigma_i^2$, but $\sigma_i^2 \neq \sigma_j^2$

(How to fix?)

- Dynamic heteroskedasticity: $\sigma_{it}^2 = f(\sigma_{i,t-k}^2)$

(How to fix?)

Dynamic and panel heteroskedasticity

Consider a Panel ARMA-GARCH model:

$$y_{it} = \mu_{it} + \varepsilon_{it} \quad \varepsilon_{it} \sim f_{\mathcal{N}}(0, \sigma_{it}^2)$$

Dynamic and panel heteroskedasticity

Consider a Panel ARMA-GARCH model:

$$y_{it} = \mu_{it} + \varepsilon_{it} \quad \varepsilon_{it} \sim f_{\mathcal{N}}(0, \sigma_{it}^2)$$

$$\mu_{it} = \alpha_i + \tau_t + x_{it}\beta + \sum_{p=1}^P y_{i,t-p}\phi_p + \sum_{q=1}^Q \varepsilon_{i,t-q}\theta_q$$

Dynamic and panel heteroskedasticity

Consider a Panel ARMA-GARCH model:

$$y_{it} = \mu_{it} + \varepsilon_{it} \quad \varepsilon_{it} \sim f_{\mathcal{N}}(0, \sigma_{it}^2)$$

$$\mu_{it} = \alpha_i + \tau_t + x_{it}\beta + \sum_{p=1}^P y_{i,t-p}\phi_p + \sum_{q=1}^Q \varepsilon_{i,t-q}\theta_q$$

$$\sigma_{it}^2 = \exp(\eta_i + \zeta_t + z_{it}\gamma) + \sum_{c=1}^C \sigma_{i,t-c}^2 \lambda_c + \sum_{d=1}^D \varepsilon_{i,t-d}^2 \xi_d$$

Dynamic and panel heteroskedasticity

Consider a Panel ARMA-GARCH model:

$$y_{it} = \mu_{it} + \varepsilon_{it} \quad \varepsilon_{it} \sim f_{\mathcal{N}}(0, \sigma_{it}^2)$$

$$\mu_{it} = \alpha_i + \tau_t + x_{it}\beta + \sum_{p=1}^P y_{i,t-p}\phi_p + \sum_{q=1}^Q \varepsilon_{i,t-q}\theta_q$$

$$\sigma_{it}^2 = \exp(\eta_i + \zeta_t + z_{it}\gamma) + \sum_{c=1}^C \sigma_{i,t-c}^2 \lambda_c + \sum_{d=1}^D \varepsilon_{i,t-d}^2 \xi_d$$

Suppose we estimated y_{it} as a function of μ_{it} ,
but ignored the structure of the error term

That is, we estimate a panel ARMA or GMM,
but assume ε_{it} is homoskedastic and serially uncorrelated,
conditional on the covariates and lags in μ_{it}

Dynamic and panel heteroskedasticity

If we ignore this:

$$\sigma_{it}^2 = \exp(\eta_i + \zeta_t + z_{it}\gamma) + \sum_{c=1}^C \sigma_{i,t-c}^2 \lambda_c + \sum_{d=1}^D \varepsilon_{i,t-d}^2 \xi_d$$

We would miss three non-standard features of the error variance-covariance:

- Panel heteroskedasticity, from η_i , the unit random effect in the variance function
- Contemporaneous correlation, from ζ_t , the time random effect in the variance function
- Conditional heteroskedasticity: λ_i and ξ_i make the variance time dependent

Thankfully, few reasonable models are this complex. . .

Dynamic and panel heteroskedasticity

Suppose we think this AR(1) with panel heteroskedasticity is appropriate:

$$y_{it} = \mu_{it} + \varepsilon_{it} \quad \varepsilon_{it} \sim f_{\mathcal{N}}(0, \sigma_i^2)$$

$$\mu_{it} = \alpha_i + x_{it}\beta + y_{i,t-p}\phi$$

$$\sigma_i^2 = \exp(\eta_i)$$

Only source of heteroskedasticity is now η_i :
panel heteroskedasticity, not dynamic heteroskedasticity

We could switch this to contemporaneous correlation, by swapping ζ_t for η_i

Roughly the model Beck & Katz advocate as a baseline for comparative politics

Suggest estimating by LS then correcting se's for omission of η_i & contemp. corr.

This procedure yields “panel-corrected standard errors”, PCSEs

What are they, and how do we compute them?

What would we do if we had a plain-vanilla cross-sectional regression and suspected or detected heteroskedasticity?

Recall the standard errors from LS are the square roots of the diagonal elements of

$$\hat{V}(\hat{\beta}) = \hat{\sigma}^2(\mathbf{X}'\mathbf{X})^{-1}$$

So if σ^2 varies by i , these will be badly estimated

What would we do if we had a plain-vanilla cross-sectional regression and suspected or detected heteroskedasticity?

Recall the standard errors from LS are the square roots of the diagonal elements of

$$\hat{V}(\hat{\beta}) = \hat{\sigma}^2(\mathbf{X}'\mathbf{X})^{-1}$$

So if σ^2 varies by i , these will be badly estimated

Instead, of the usual σ^2 estimator, we could use the residual from each observation as a robust estimate of its variance:

$$\hat{\sigma}_i^2 = \hat{\varepsilon}_i^2$$

What would we do if we had a plain-vanilla cross-sectional regression and suspected or detected heteroskedasticity?

Recall the standard errors from LS are the square roots of the diagonal elements of

$$\hat{V}(\hat{\beta}) = \hat{\sigma}^2(\mathbf{X}'\mathbf{X})^{-1}$$

So if σ^2 varies by i , these will be badly estimated

Instead, of the usual σ^2 estimator, we could use the residual from each observation as a robust estimate of its variance:

$$\hat{\sigma}_i^2 = \hat{\varepsilon}_i^2$$

A “heteroskedasticity robust” formula for the Var-Cov matrix follows:

$$\hat{V}(\hat{\beta}) = (\mathbf{X}'\mathbf{X})^{-1} \left(\sum_i \hat{\varepsilon}_i x_i' x_i \right) (\mathbf{X}'\mathbf{X})^{-1}$$

The standard errors of our parameters (β 's) are the square roots of the diagonal of this matrix

Review: Adjusting standard errors for heteroskedasticity

$$\hat{V}(\hat{\beta}) = (\mathbf{X}'\mathbf{X})^{-1} \left(\sum_i \hat{\varepsilon}_i x_i' x_i \right) (\mathbf{X}'\mathbf{X})^{-1}$$

SE's calculated from this equation are known by many names:

- Huber-White standard errors
- robust standard errors
- sandwich standard errors
- heteroskedasticity consistent standard errors

If you have a single time series, Newey-West standard errors generalize this concept to include robustness to serial correlation

For panel data there are many further options, leading to a vast literature exploring refinements to this basic concept

Panel-corrected standard errors

To calculate panel-corrected standard errors,
we need to estimate the correct variance-covariance matrix

Why not just use Huber-White? That would ignore panel structure,
which is inefficient if we know how that structure affects heteroskedasticity:

$$\hat{\sigma}_{it}^2 = \hat{\varepsilon}_{it}^2$$

Panel-corrected standard errors

To calculate panel-corrected standard errors,
we need to estimate the correct variance-covariance matrix

Why not just use Huber-White? That would ignore panel structure,
which is inefficient if we know how that structure affects heteroskedasticity:

$$\hat{\sigma}_{it}^2 = \hat{\varepsilon}_{it}^2$$

Beck and Katz's panel correction produces sharper estimates of $\hat{\sigma}_{it}^2$ by borrowing strength across the observations from a single unit:

$$\hat{\sigma}_{it}^2 = \hat{\sigma}_i^2 = \frac{1}{T}(\hat{\varepsilon}_{i,1}^2 + \hat{\varepsilon}_{i,2}^2 + \cdots + \hat{\varepsilon}_{i,T}^2)$$

Panel-corrected standard errors

To calculate panel-corrected standard errors, we need to estimate the correct variance-covariance matrix

Why not just use Huber-White? That would ignore panel structure, which is inefficient if we know how that structure affects heteroskedasticity:

$$\hat{\sigma}_{it}^2 = \hat{\varepsilon}_{it}^2$$

Beck and Katz's panel correction produces sharper estimates of $\hat{\sigma}_{it}^2$ by borrowing strength across the observations from a single unit:

$$\hat{\sigma}_{it}^2 = \hat{\sigma}_i^2 = \frac{1}{T}(\hat{\varepsilon}_{i,1}^2 + \hat{\varepsilon}_{i,2}^2 + \cdots + \hat{\varepsilon}_{i,T}^2)$$

Beck and Katz's panel correction also accounts for contemporaneous correlations across units:

$$\hat{\sigma}_{i,j} = \frac{1}{T}(\hat{\varepsilon}_{i,1}\hat{\varepsilon}_{j,1} + \hat{\varepsilon}_{i,2}\hat{\varepsilon}_{j,2} + \cdots + \hat{\varepsilon}_{i,T}\hat{\varepsilon}_{j,T})$$

Note the above will work better if T is large relative to N

Panel-corrected standard errors

Building this intuition out into a variance-covariance matrix involves a bit of algebra

To make PCSEs,

suppose the variance-covariance matrix Ω is $NT \times NT$ block-diagonal with an $N \times N$ matrix Σ of contemporaneous covariances on diagonal

Panel-corrected standard errors

Building this intuition out into a variance-covariance matrix involves a bit of algebra

To make PCSEs,
suppose the variance-covariance matrix Ω is $NT \times NT$ block-diagonal
with an $N \times N$ matrix Σ of contemporaneous covariances on diagonal

In other words, allow for unit or contemporaneous heteroskedasticity that stays the same over time

Visualizing this large matrix is tricky

Note that “ $NT \times NT$ block-diagonal” means we are ordering the observations first by time, then by unit (reverse of our usual practice)

Panel-corrected standard errors

Instead, suppose Ω is $NT \times NT$ block-diagonal
with an $N \times N$ matrix Σ of contemporaneous covariances on diagonal

Panel-corrected standard errors

Instead, suppose Ω is $NT \times NT$ block-diagonal with an $N \times N$ matrix Σ of contemporaneous covariances on diagonal

In other words, allow for unit or contemporaneous heteroskedasticity that stays the same over time

Beck and Katz (1995) estimate Σ using LS residuals $e_{i,t}$:

$$\hat{\Sigma}_{i,j} = \sum_{t=1}^T \frac{e_{i,t}e_{j,t}}{T}$$

And then use $\hat{\Sigma}$ to construct the covariance matrix

Panel-corrected standard errors

Monte Carlo experiments show panel-corrected standard errors are “correct” unless contemporaneous correlation is very high or T is small relative to N

(Note: alternative is to estimate random effects in variance by ML.)

Beck and Katz suggest using LS with PCSEs and lagged DVs as a baseline model

Most practitioners think fixed effects should also be used

Most important: getting the right lag structure & including FEs where appropriate

PCSEs (or other var-cov correction) is a second-order concern

In R, package `pcse` will calculate PCSEs for a linear regression

Even easier: In the `plm` package, `vcovBK()` will produce a panel corrected var-cov matrix from a `plm` object

If N is large relative to T , consider the Driscoll and Kraay alternative, `vcovSCC()`

Panel-corrected standard errors: Application

Let's apply Beck-Katz PCSEs to our panel ARIMA/plm example: we'll replace the usual variance-covariance matrix with the panel corrected variance covariance matrix

We must make this substitution *manually* after estimation to get corrected standard errors, confidence intervals, and var-cov matrices:

1. to print the `summary()` of a `plm` model

Example: `summary(plm.res, .vcov=vcovBK(plm.res))`

2. to use the `coeftest()` function on a `plm` model

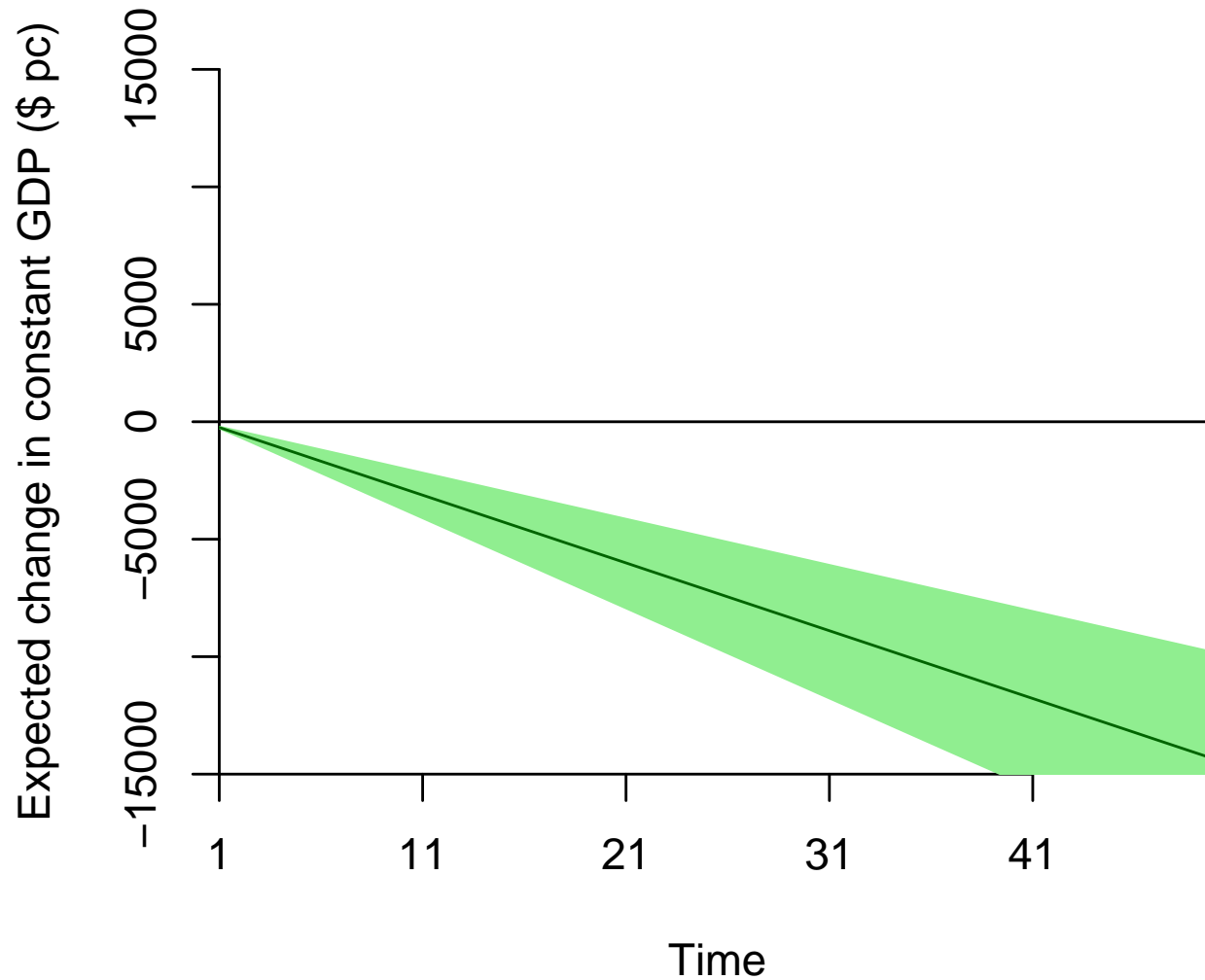
Example: `coeftest(plm.res, .vcov=vcovBK(plm.res))`

3. to simulate parameters with `mvrnorm()` for computing counterfactuals

Example: `mvrnorm(10000, coef(plm.res), vcovBK(plm.res))`

	Model				
	RE	FE	FE-pcse	FE	ME
Education _{it}	23.96	-75.56	-75.56	-86.68	-84.75
	5.59	12.16	13.48	12.45	14.56
Democracy _{it}	110.94	-12.90	-12.90	-26.15	-3.63
	34.63	47.69	50.58	47.97	55.22
Oil-Producer _{it}	-26.89	—	—	—	—
	44.84	—	—	—	—
GDP _{i,t-1}	0.23	0.15	0.15	0.17	0.20
		0.02	0.02	0.02	
GDP _{i,t-2}				-0.12	
				0.02	
σ_α	0.14	—	—	—	309.10
Fixed effects		x	x	x	x
Random effects	x				x
N	113	113	113	113	113
T	328	328	328	228	328
observed $N \times T$	2794	2794	2794	2741	2794
AIC	43376				42112
LM test p -value		<0.001	<0.001	0.131	

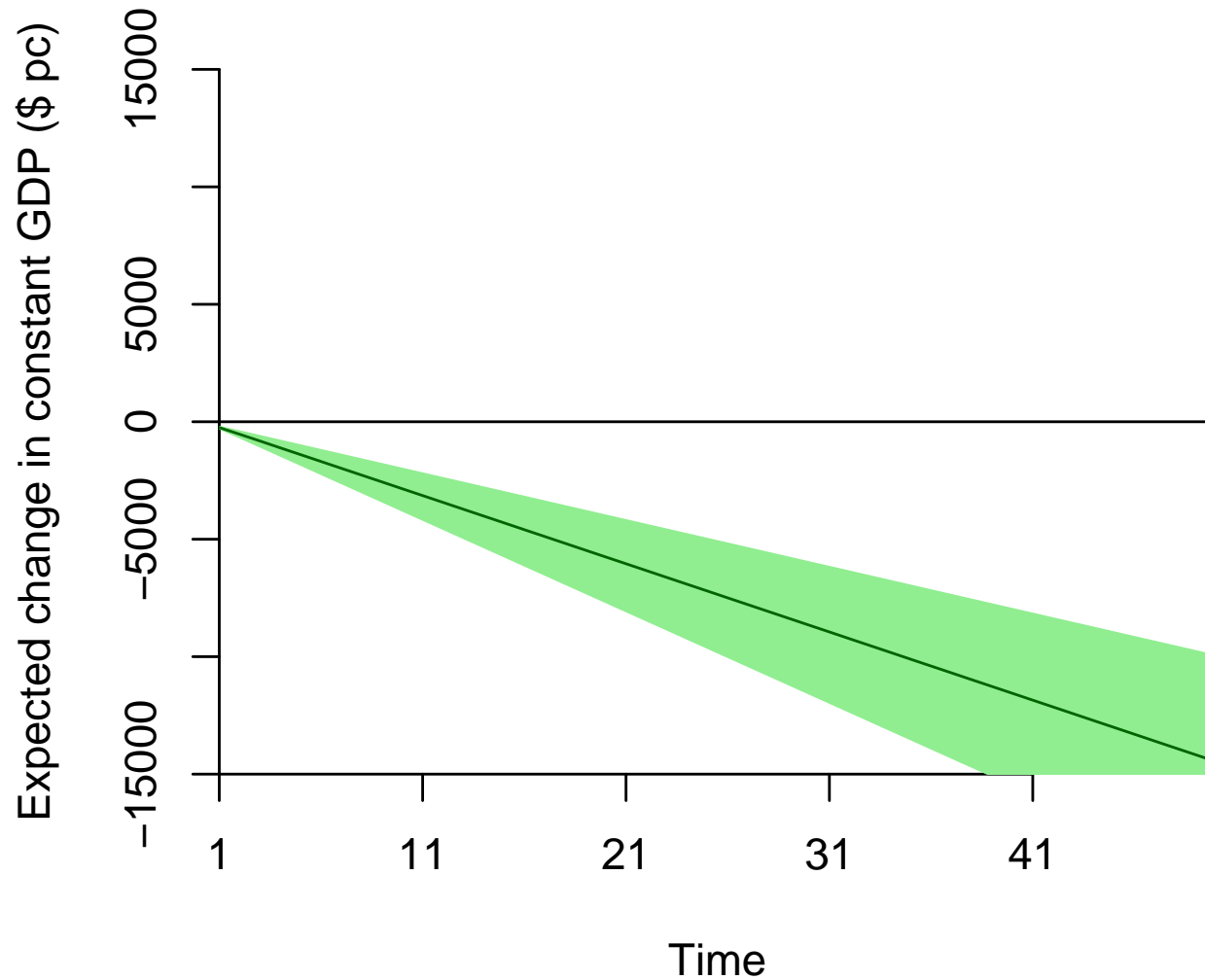
Fixed effects ARIMA(1,1,0)



Recall the fixed effects results. . .

These are uncorrected for panel heteroskedasticity or contemporaneous correlation

Fixed effects ARIMA(1,1,0), pcse



Panel correction usually makes little difference in long T small N contexts

But in short T , robust standard errors can be quite important. . .

Heteroskedastic and serial correlation consistent Var-Cov

In the PCSEs approach, the focus is on panel heteroskedasticity

It is assumed that serial correlation has been adequately modeled and purged

A reasonable check when we have a few dozen periods of data, though similar in most cases to either ordinary SEs or White SEs

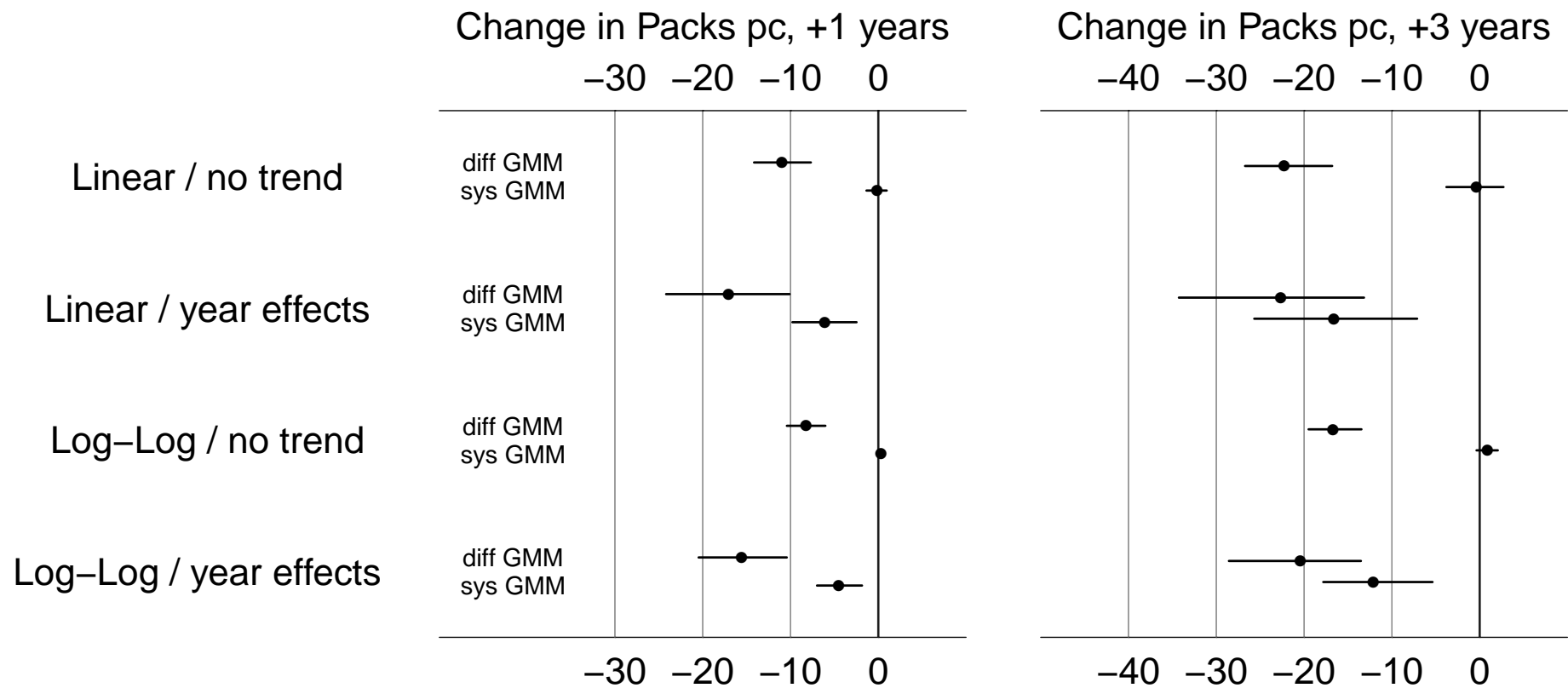
But what if we have a low T ? We might be more worried about residual serial correlation (and don't have practical access to ARMA diagnostics or fitting)

Now there is more need for a correction to the variance covariance that corrects for observed error correlation across units and across periods

Arellano (1987) provides a heteroskedastic and autocorrelation consistent variance-covariance matrix: in `p1m`, `vcovHC()`

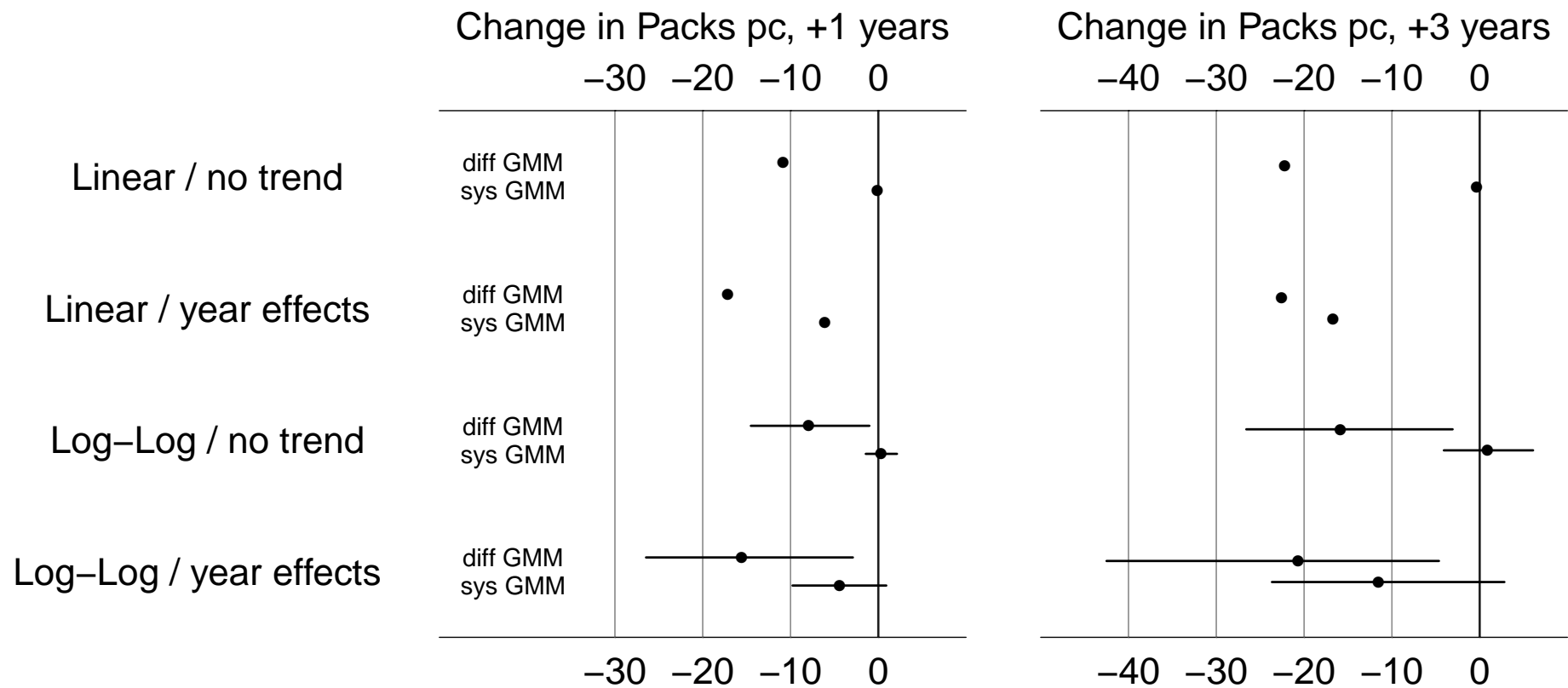
Use the same commands as above, but with `vcovHC()` instead of `vcovBK()`

Particularly important to correct with panel GMM estimators



Our prior results for cigarette taxes used the Arellano heteroskedastic and serial correlation consistent var-cov matrix

What would happen if we had used the ordinary, homoskedastic var-cov matrix to compute CIs?

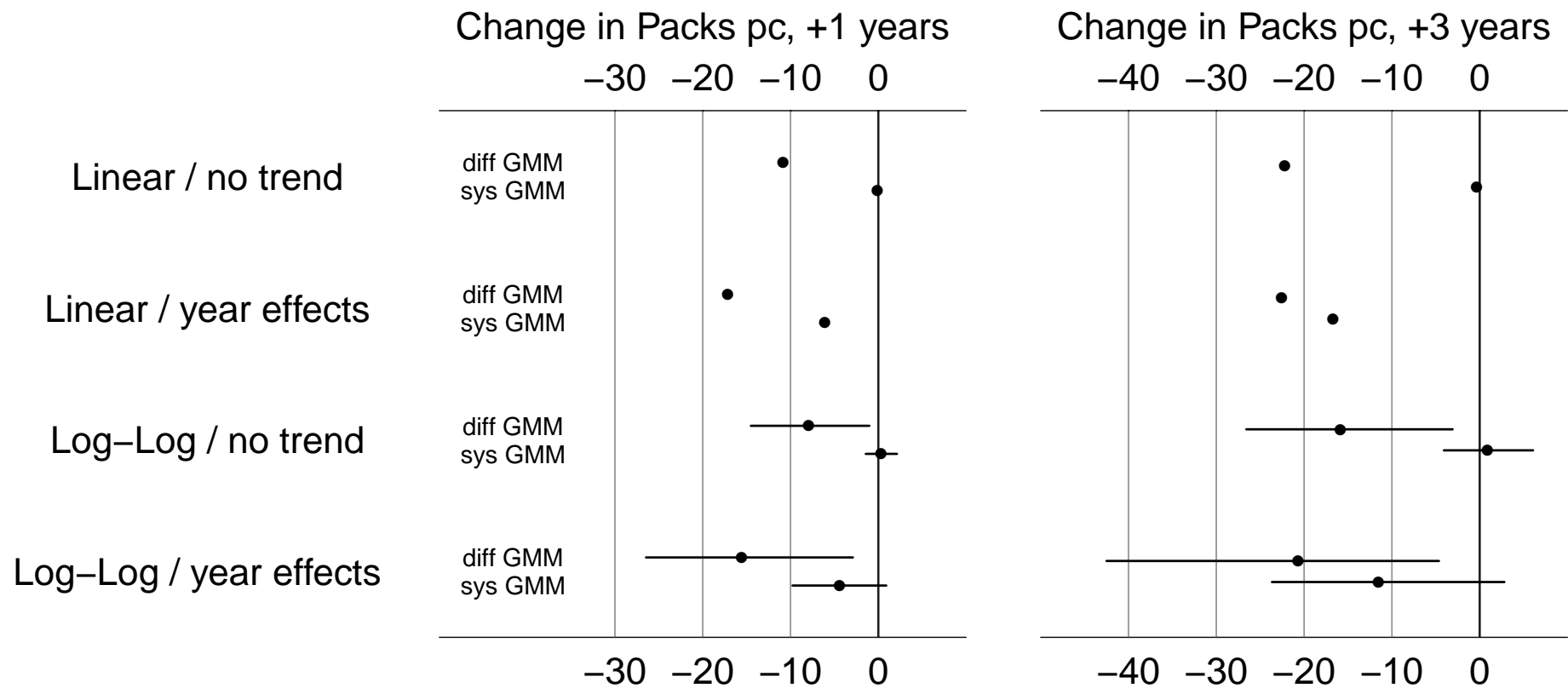


The effects sizes are mostly unchanged:
 adjustments to standard errors affect CIs, not point estimates

But the CIs are radically different under the traditional var-cov estimator

Far too small (invisible even!) for the misspecified linear models

And too large for the more correctly specified log-log models!



Just as panel GMM point estimate are sensitive to assumptions, so are the standard errors

Use caution, and prefer `vcovHC()` to `vcov()` in PGMM models

Be sure to check which var-cov matrix your functions are using: the default may be wrong!