Maximum Likelihood Methods
for the Social Sciences
POLS 510 · CSSS 510

Introduction to the Course, Probability, and R

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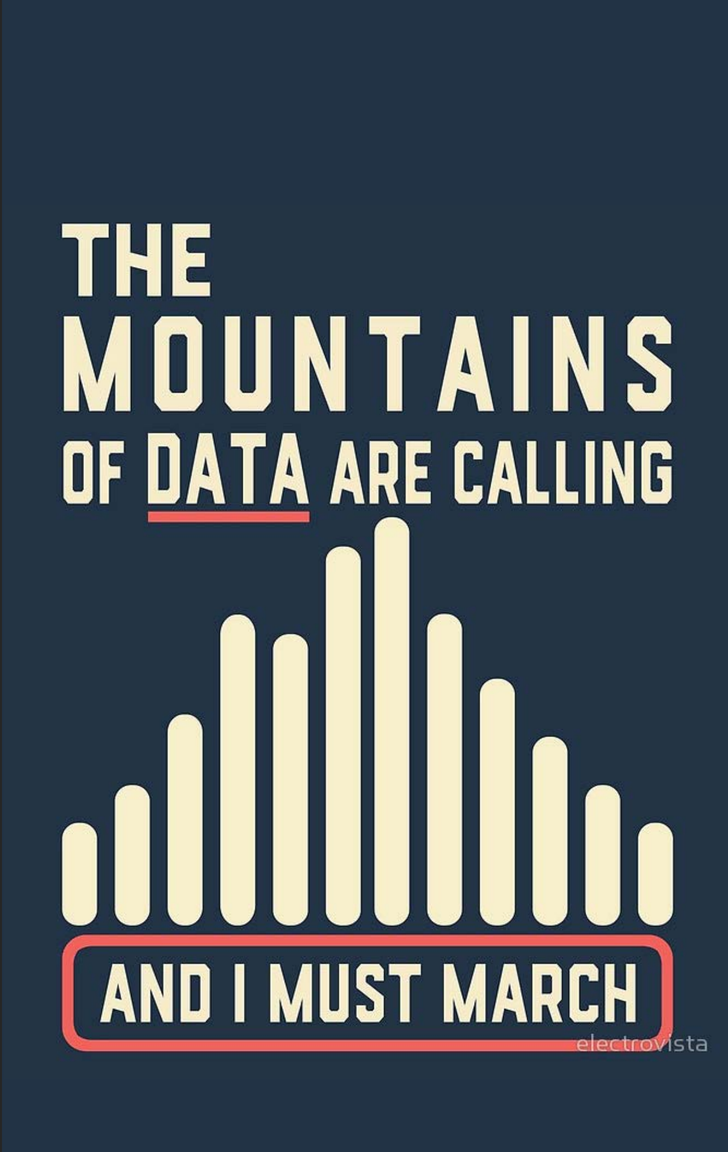

Electrovista

Welcome

Class goals

Go beyond the linear model to develop models for real-world data

messy data with substantively interesting quirks

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Gateway to CSSS and other classes

Bayesian inference
Hierarchical/multilevel modeling
Event history analysis
Panel data analysis
Social network analysis
. . .

Challenges

1. Hard new concepts

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2. A fair bit of math

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   - Empowering for any research involving data:
     you'll be surprised how many problems can be simplified by programming

MLE for Categorical & Count Data

First half of course focuses on inference about discrete data: categories & counts

Foundational quantitative methods classes focus on the linear regression model

- Assume data consist of a systematic component $\mathbf{x}_i\boldsymbol{\beta}$ and a continuous, Normally distributed disturbance ε_i
- Easy to implement, estimate, and interpret
- A reasonable starting place for many analyses, with some robust features

But do the assumptions of linear regression (aka least squares) always fit?

Do they fit with discrete data?

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- Whether a person rides the bus, drives, or walks to work?
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- The number of wars fought per decade?
- Whether someone taunted in a bar ignores it, argues back, or throws a punch

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No. All of these variables violate basic linear regression assumptions

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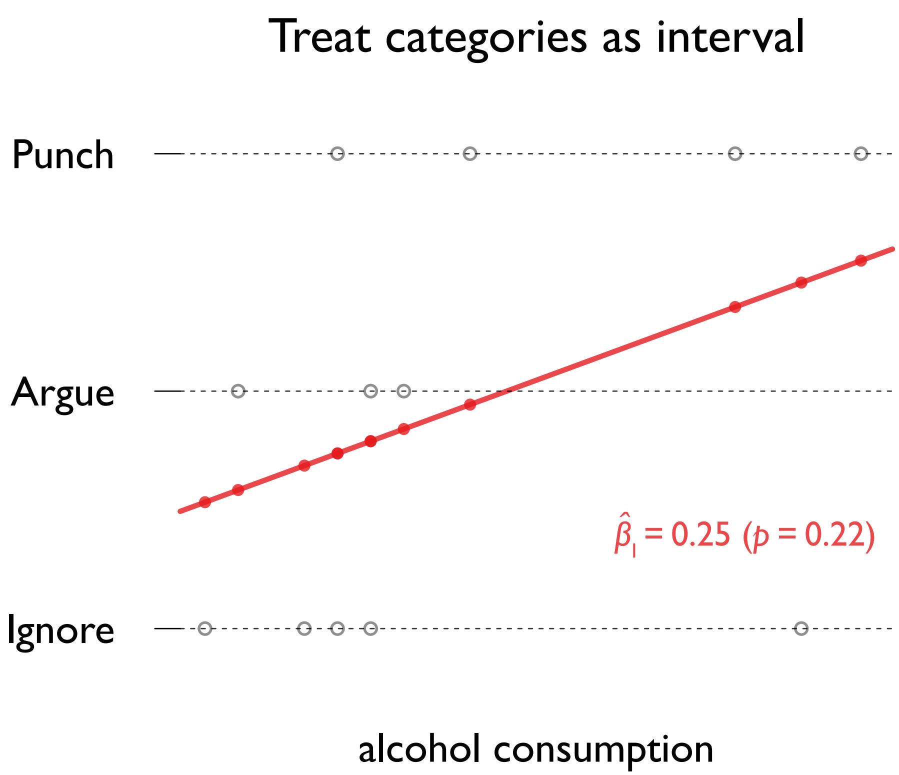
Let's take a closer look the last example . . .

To ignore, argue, or punch – does this escalation follow a uniform pattern?

Problems for linear regression in this case?



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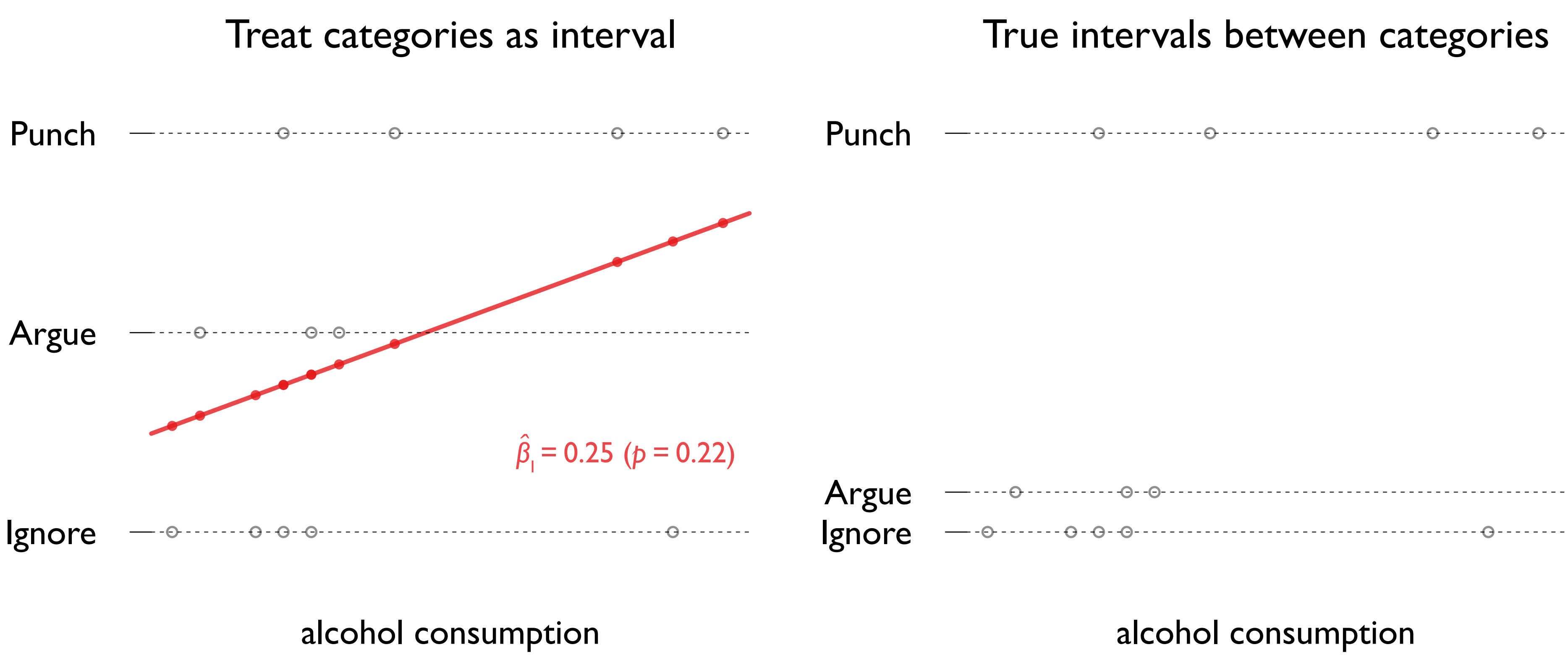

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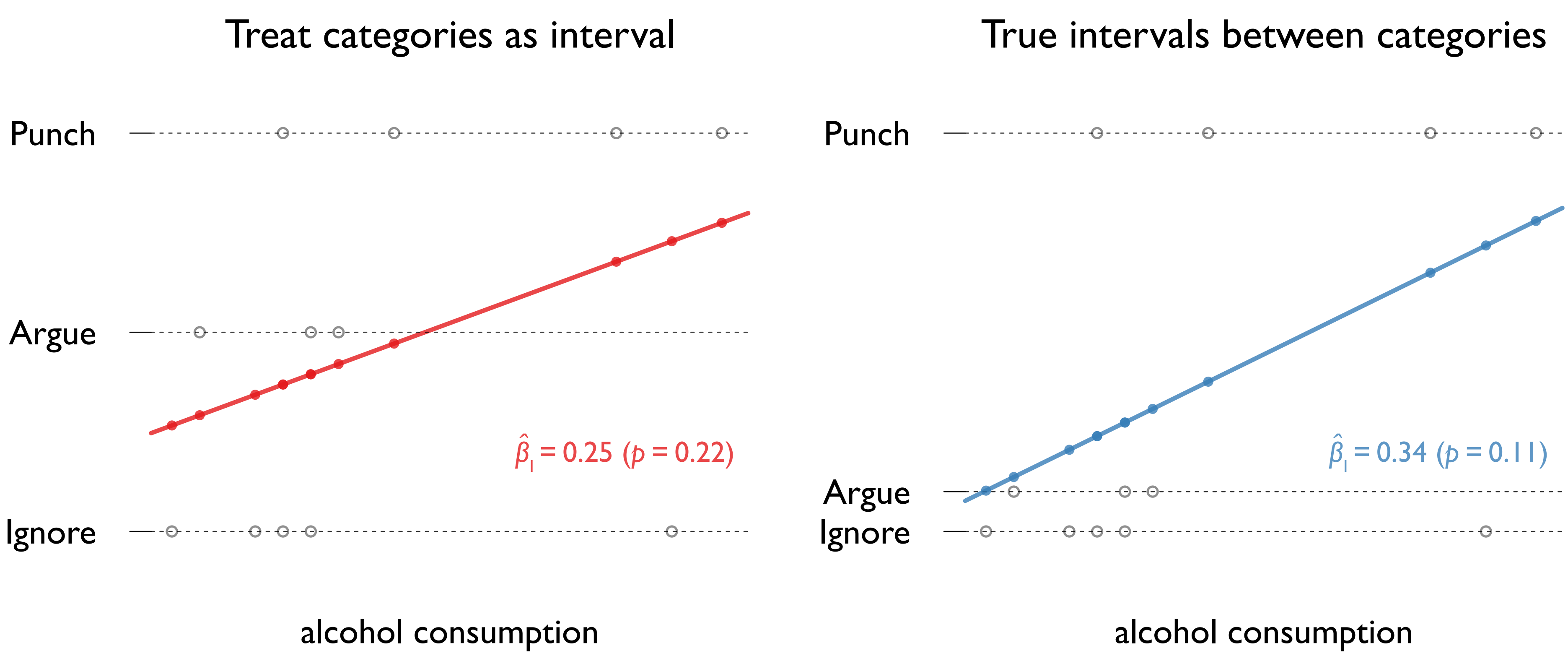

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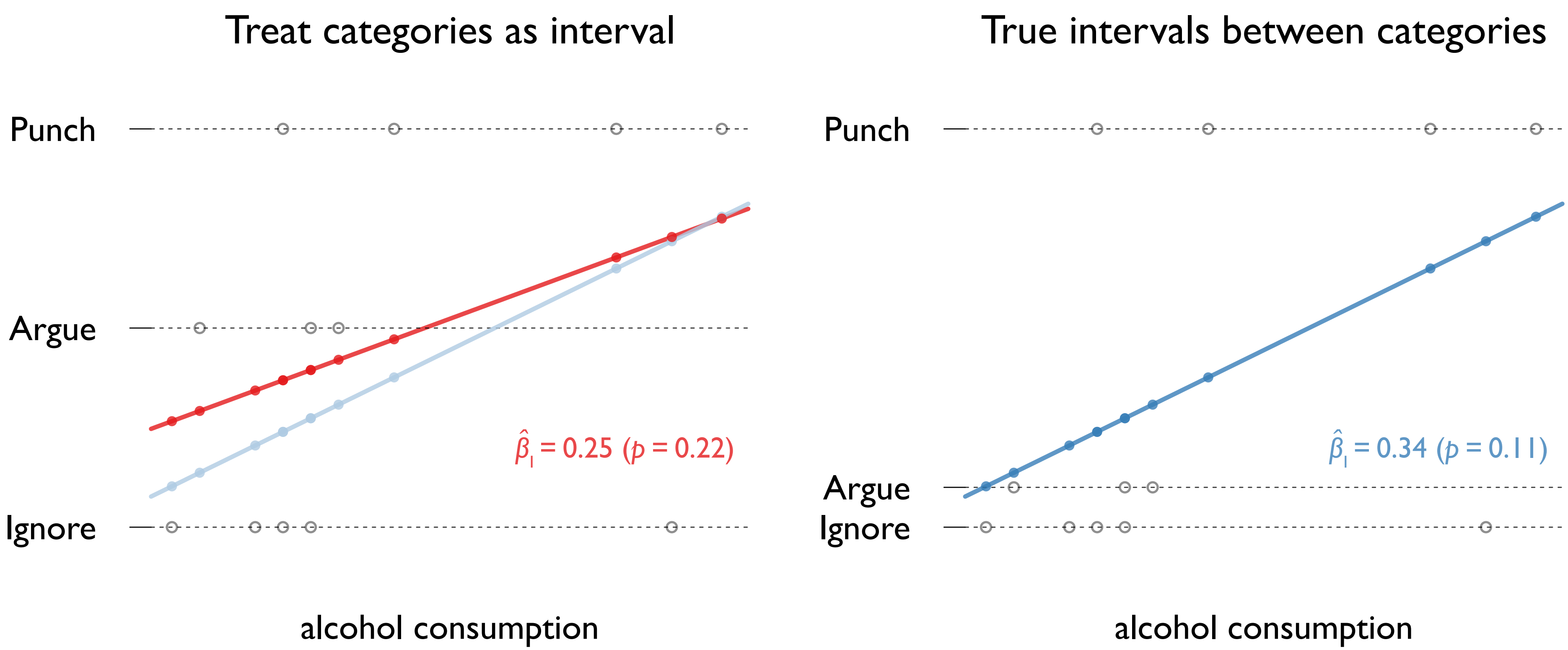

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Application: Selecting and presenting models to uncover substantive relationships

Practice: Programming skills to implement, fit, and interpret these models

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We'll start on Step 1 today . . .

Outline for today

1. Course administration

2. Review basic probability

3. Review some fundamental probability distributions

Course administration

1. Syllabus
2. Paper requirements
3. Survey
4. Introductions

Lightning course in basic probability: Sets

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A set may also be empty, e.g., $\mathcal{B} = \emptyset = \{\}$

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subset $\subset$ union $\cup$ intersection $\cap$

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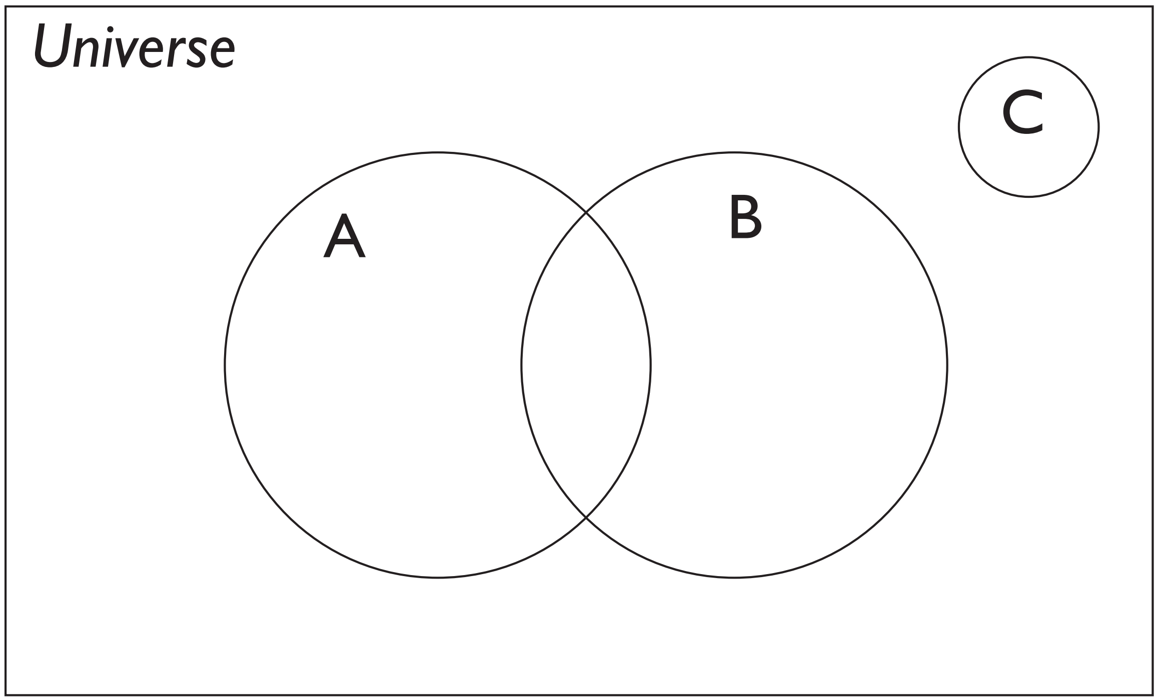
(Remember Venn Diagrams?)



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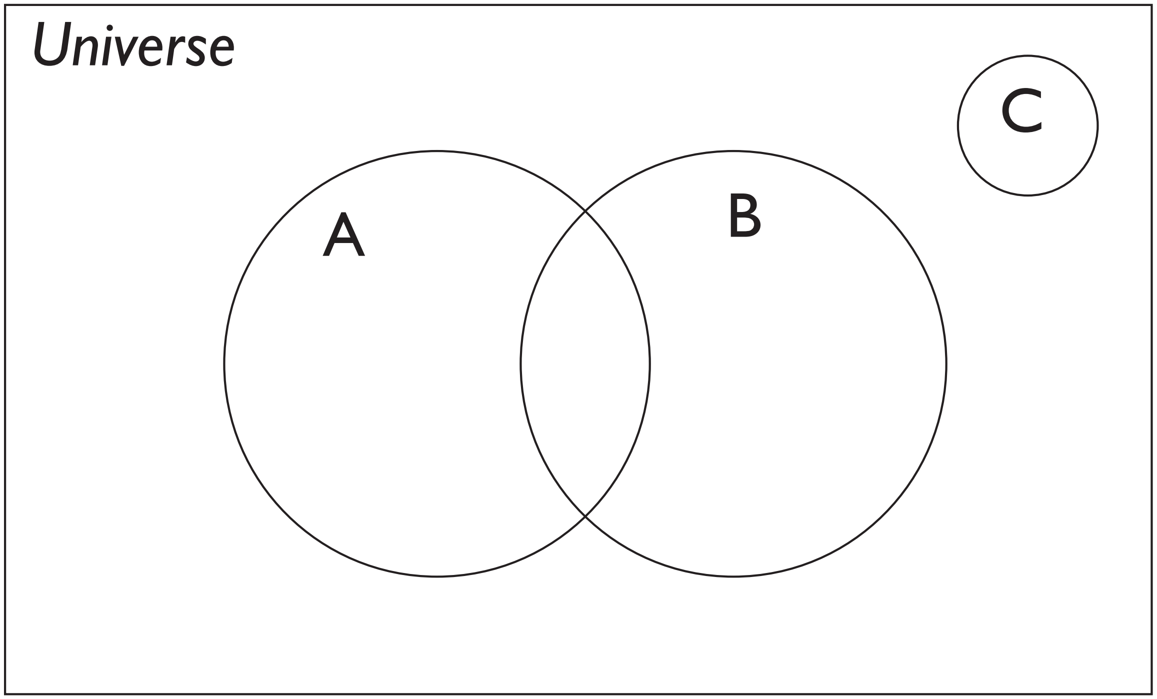

An important definition:

If $\mathcal{A} \cap \mathcal{C} = \emptyset$, then $\mathcal{A}$ and $\mathcal{C}$ are *disjoint*.

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Sets will help us define probability

Suppose we toss a coin twice and record the results.

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The universe of possible results is the *sample space*. It is a set of sets:

$$\Omega = \{\{H, H\}, \{H, T\}, \{T, H\}, \{T, T\}\}$$

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- $\Pr(A) \geq 0 \quad \forall A$
- $\Pr(\Omega) = 1$
- $A \cap B = \emptyset \quad \Longleftrightarrow \quad \Pr(\bigcup(A,B)) = \Pr(A) + \Pr(B)$

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We'll use these terms a lot:

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These concepts are linked by a simple identity:

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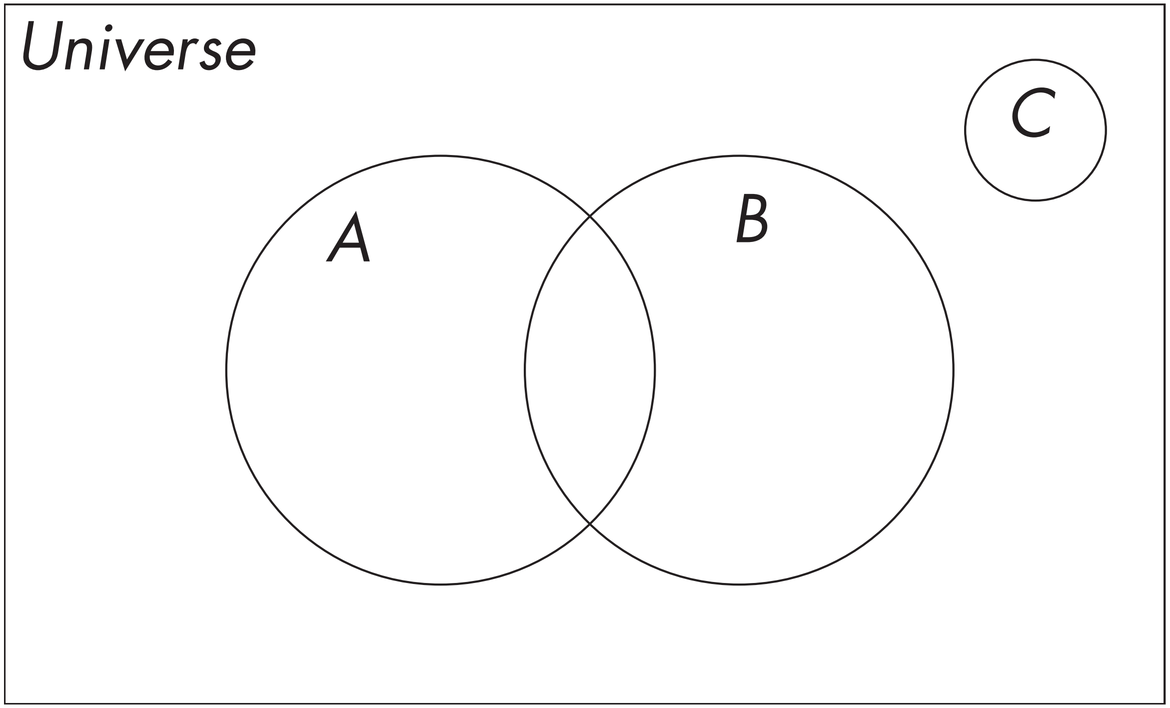

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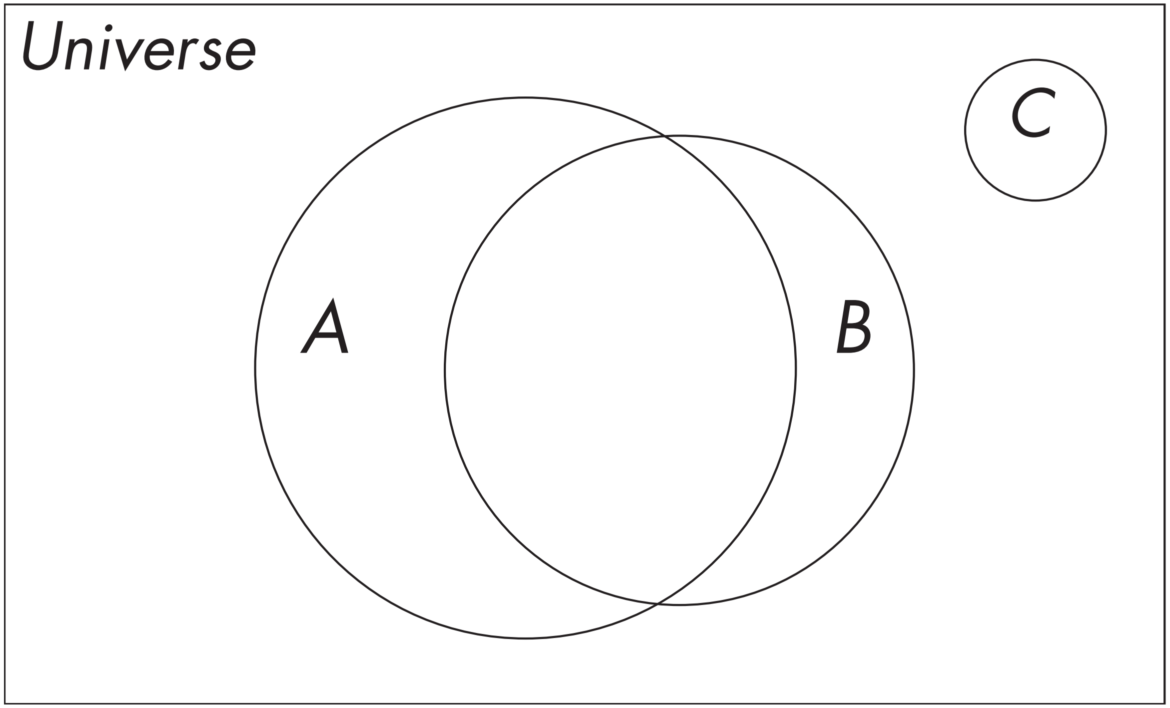

Let's adjust our diagram to better fit our example.

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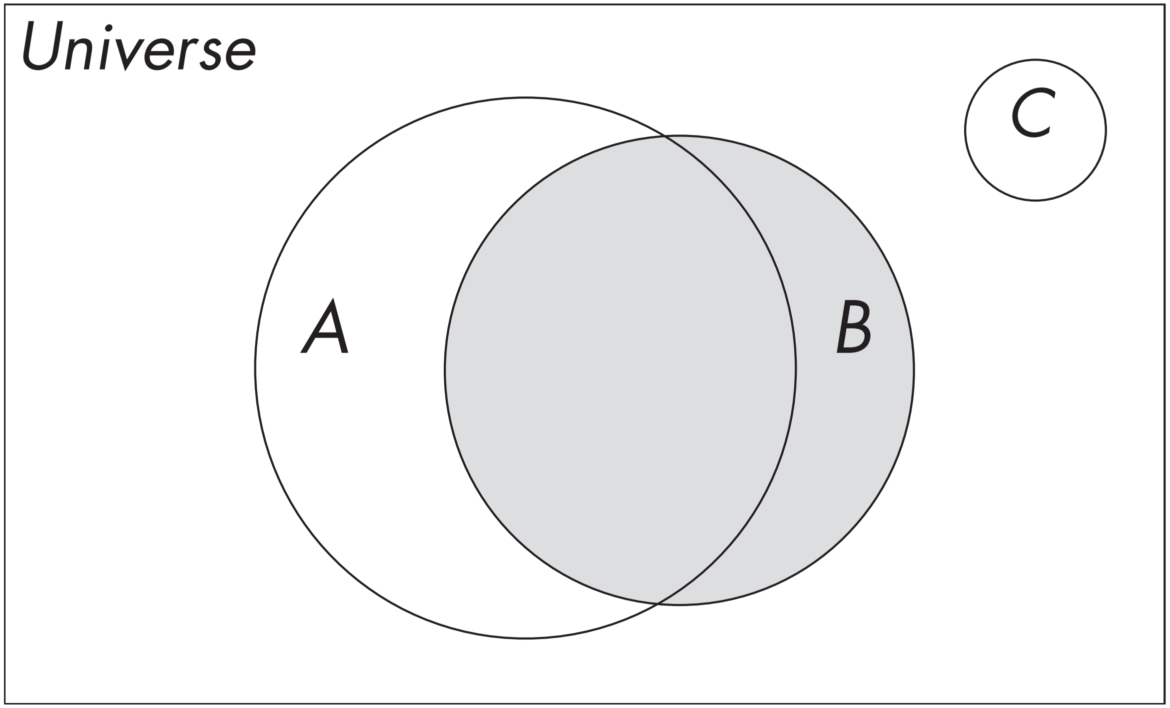

We know event *B* will happen so the set of possible outcomes is limited to those in *B*'s circle.

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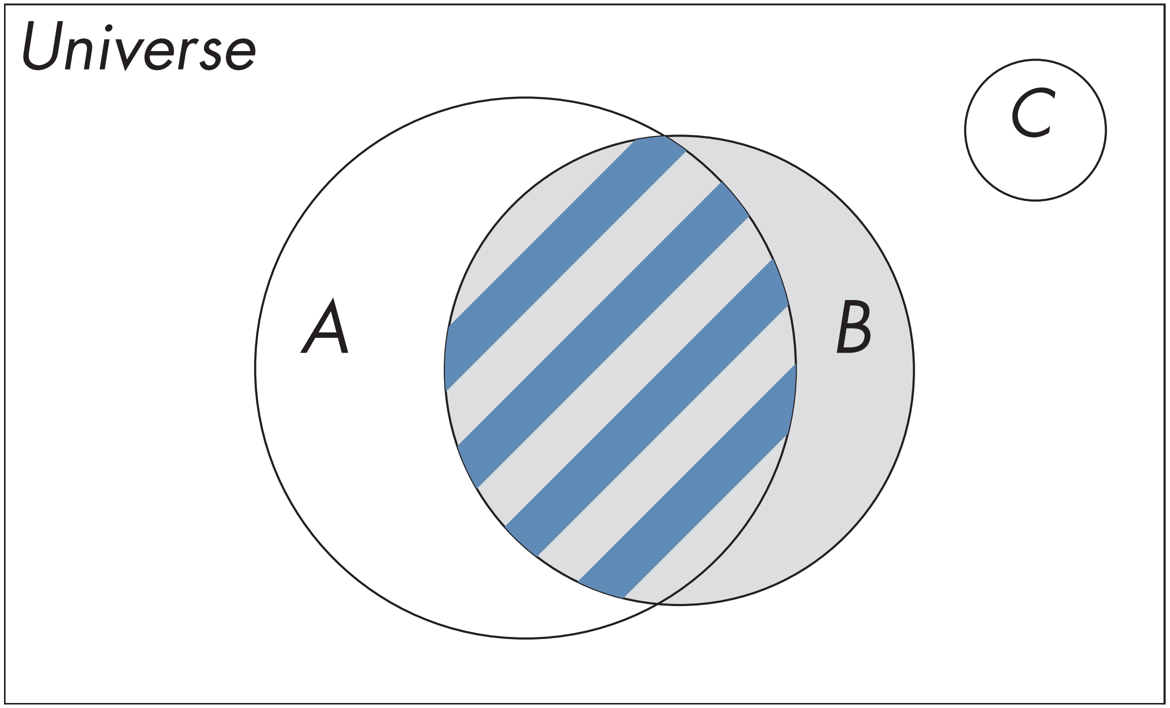

If *B* definitely occurs, what fraction of the time does *A* also occur?

The ratio of the intersection of A and B to the circle B.

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Note that we can re-arrange to find other useful identities:

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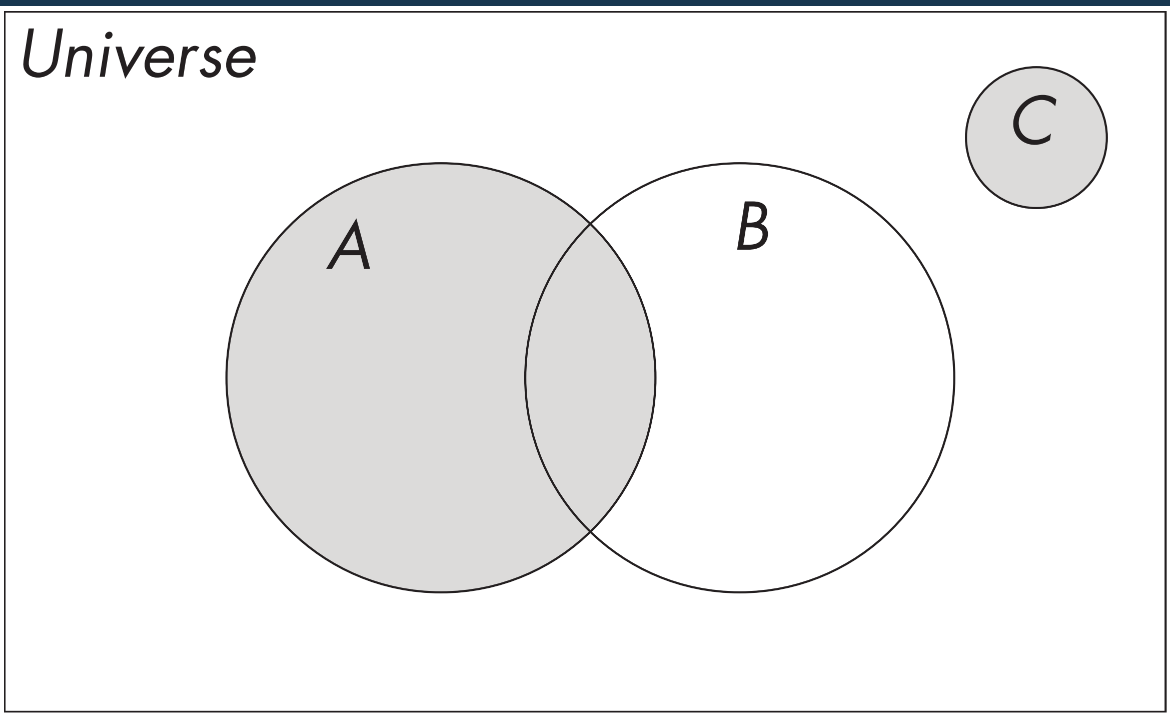

The probability that *A* or *C* occurs is just the sum of their marginal probabilities because they are *disjoint*.

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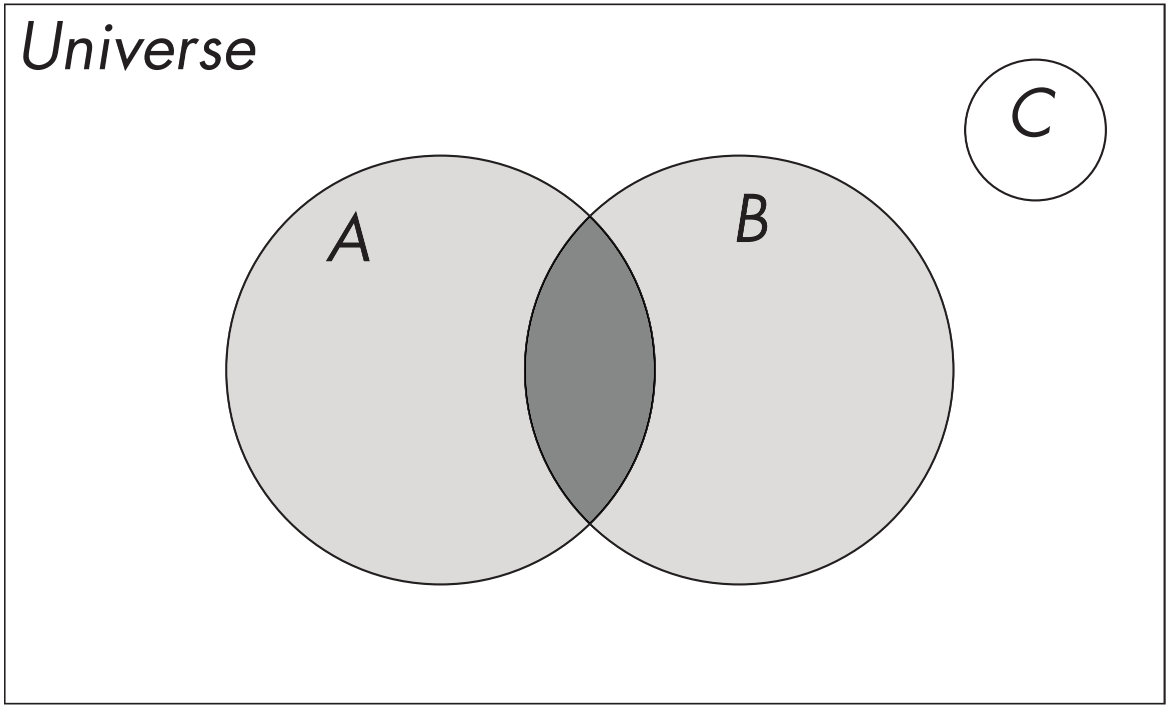

If we try the same trick to find the probability of *A* or *B*, we'll double count their intersection.

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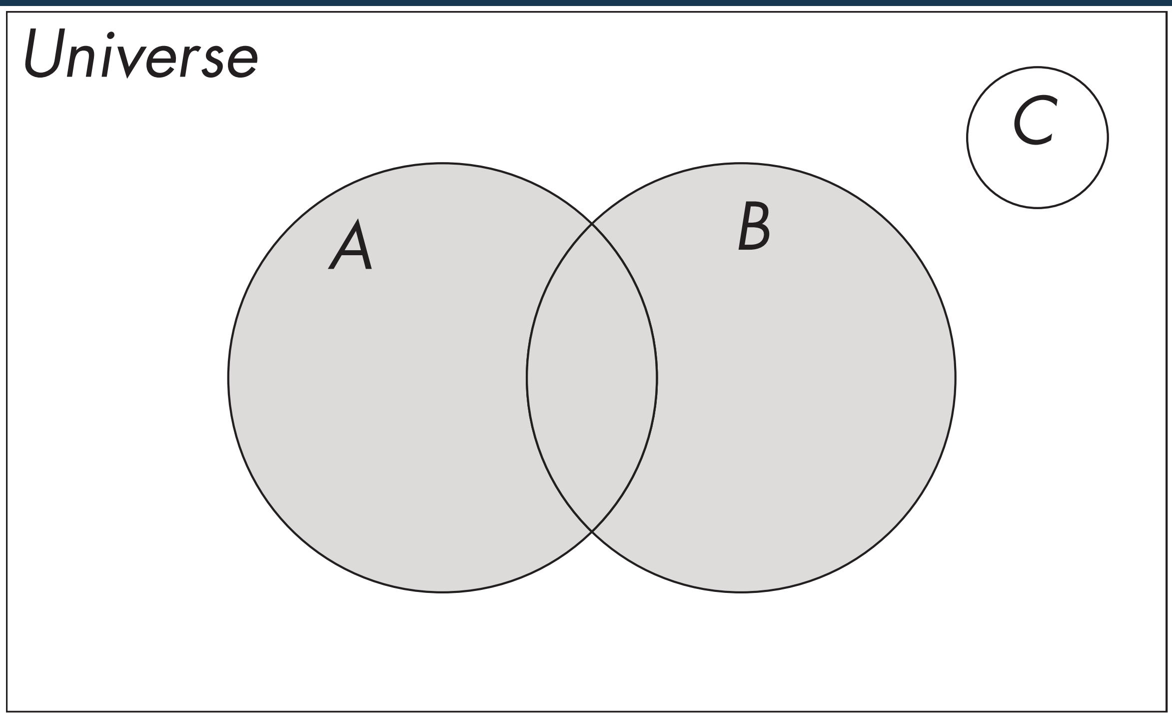

In general, to find the probability of A or *B* we should add their marginal probabilities and subtract their intersection.

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Finally, define *independence* as holding if $\Pr(A \cap B) = \Pr(A)\Pr(B)$

Note that when independence holds,

$$\begin{aligned}
\Pr(A|B) &= \frac{\Pr(A \cap B)}{\Pr(B)} \\
\Pr(A|B) &= \frac{\Pr(A)\Pr(B)}{\Pr(B)} \\
\Pr(A|B) &= \Pr(A)
\end{aligned}$$

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Ω for economic activity	$\rightarrow$	$Y = \$$GDP

This mapping can produce discrete or continous variables

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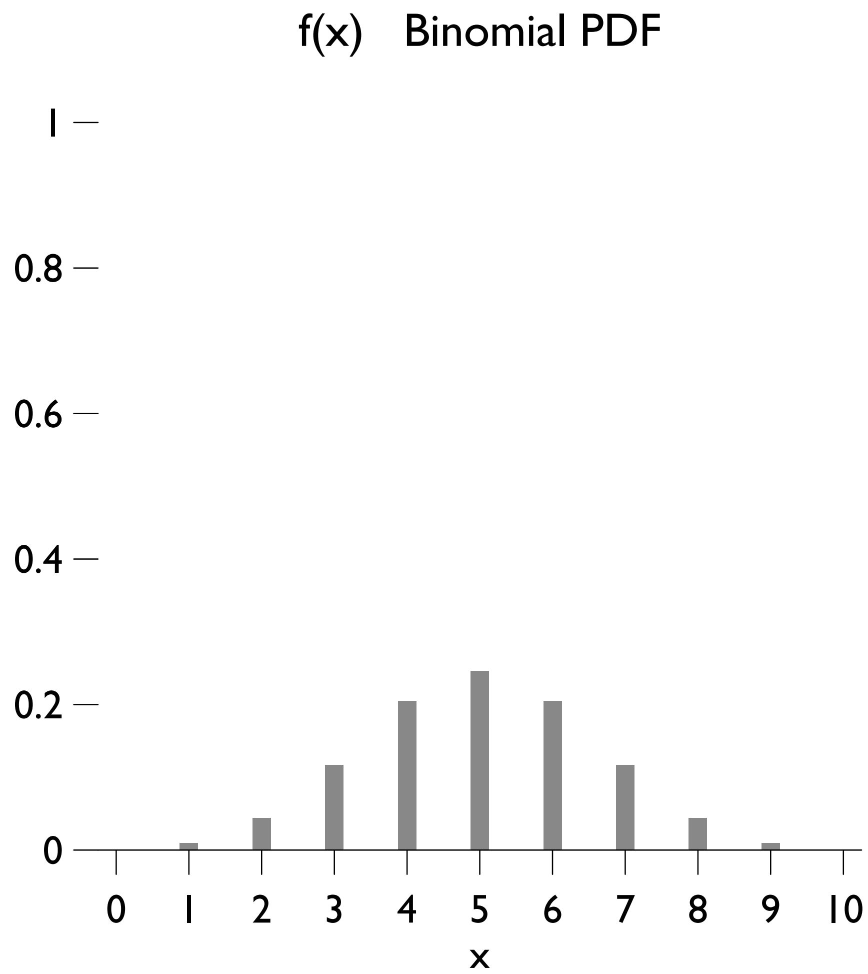

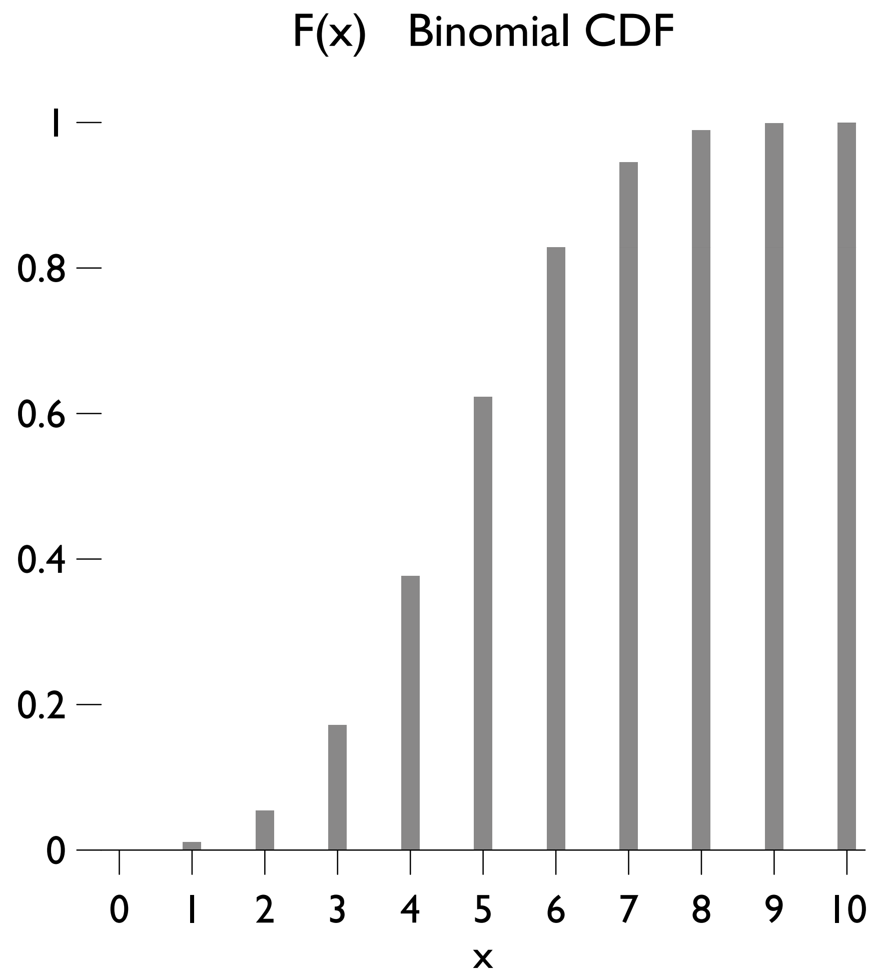

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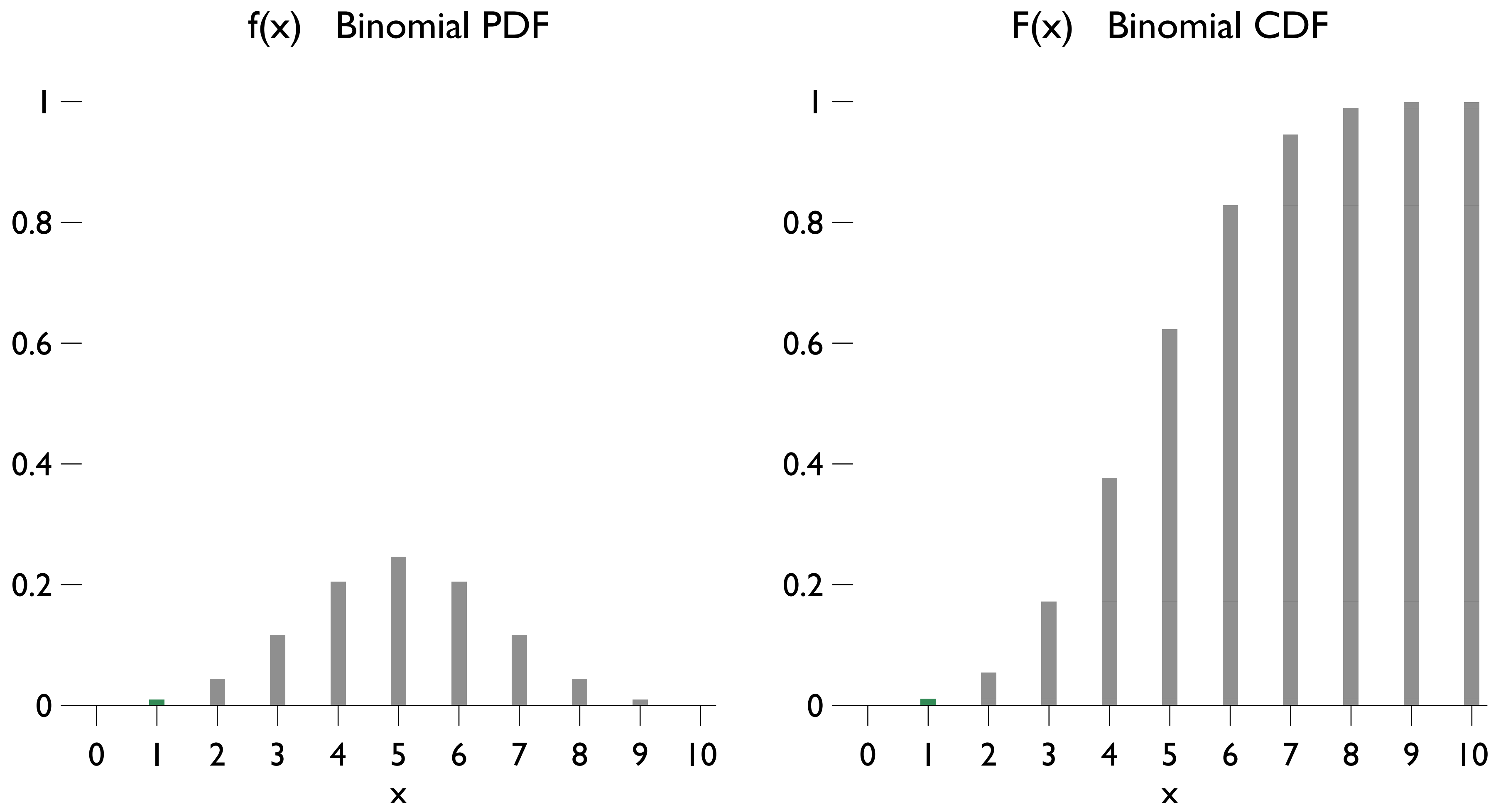

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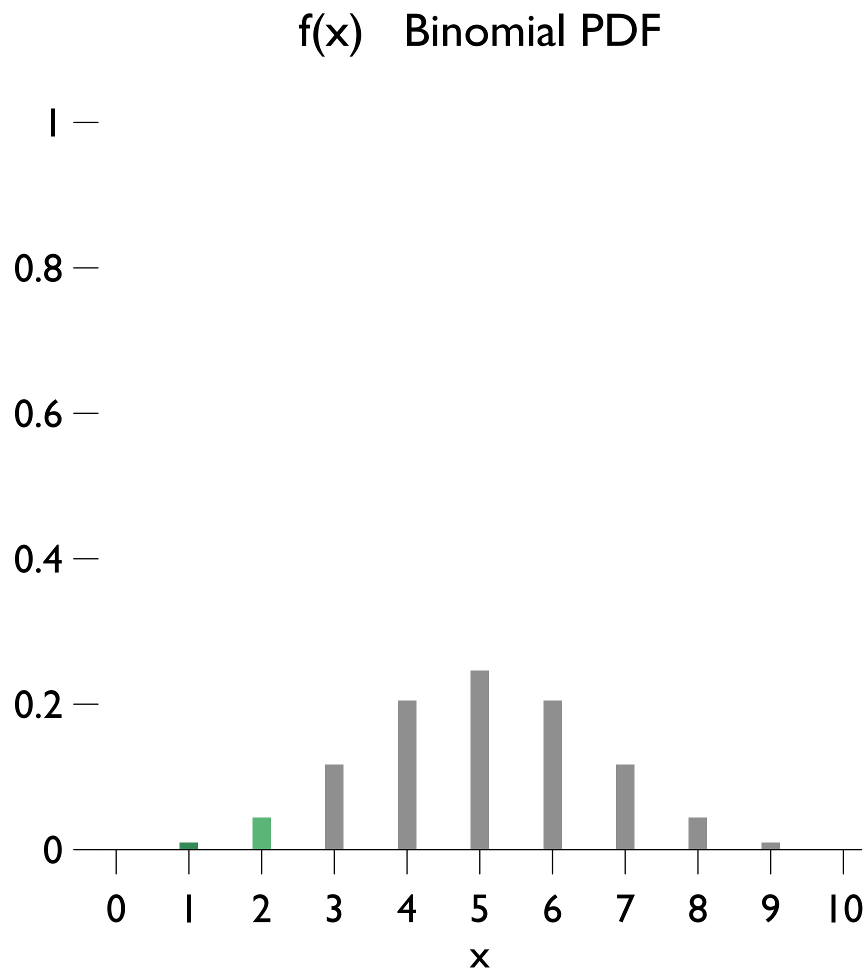

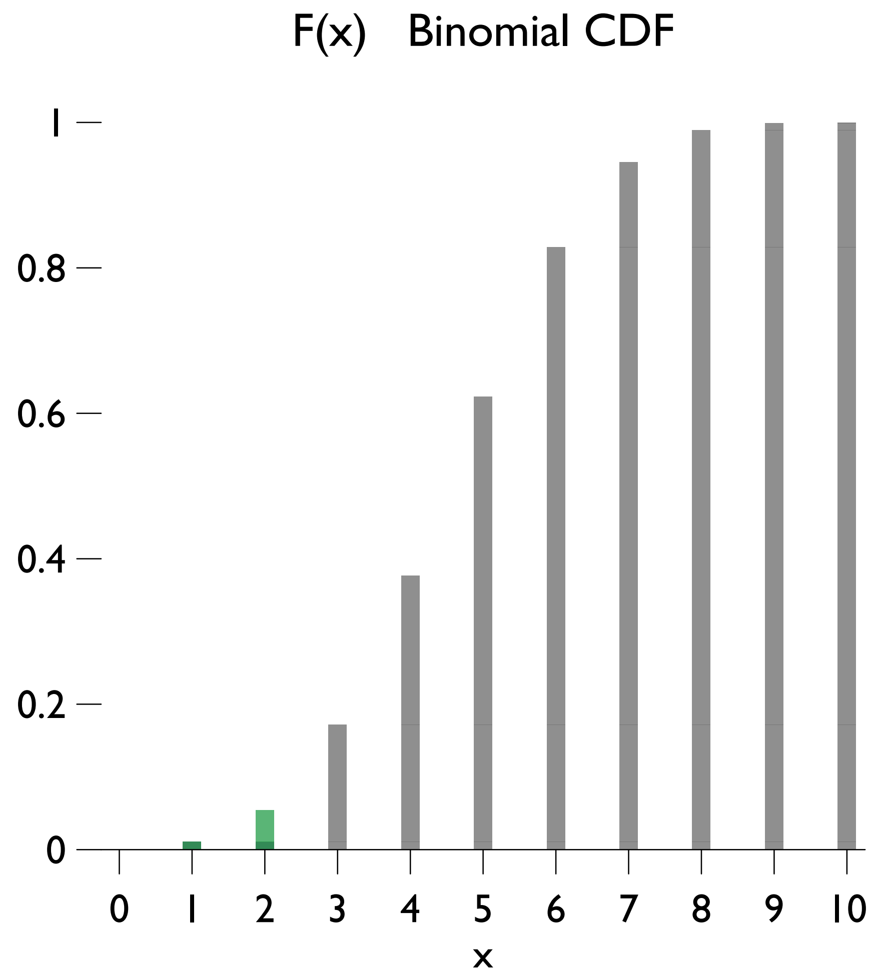

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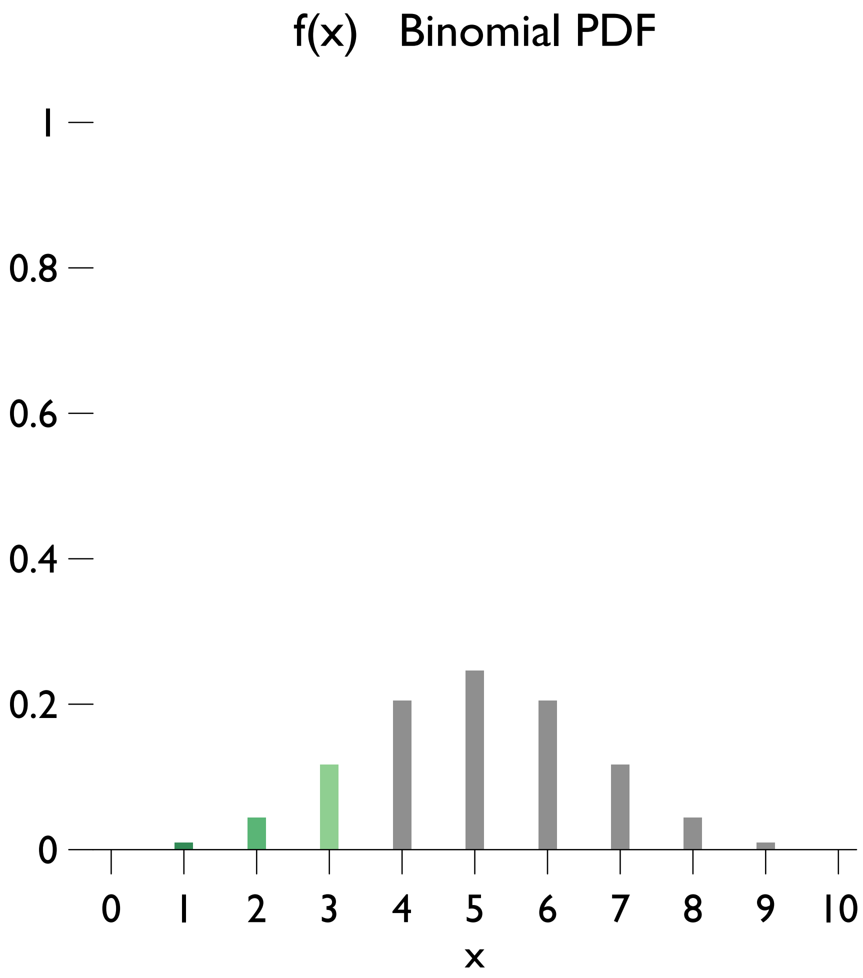

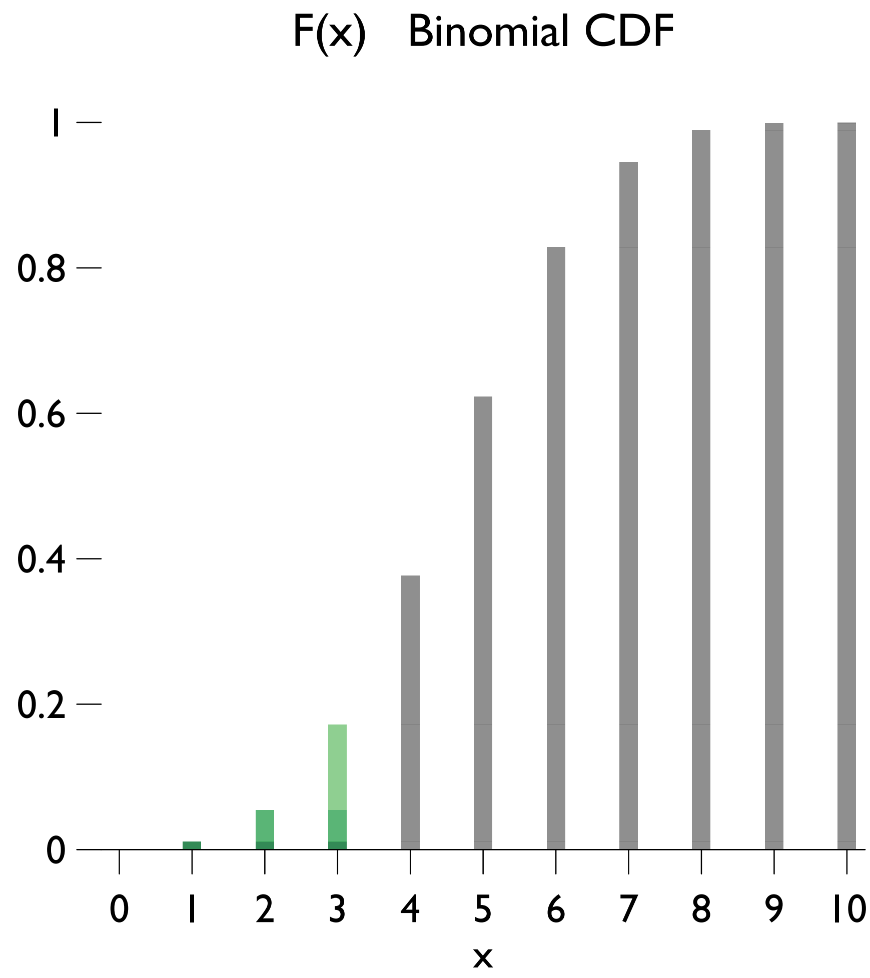

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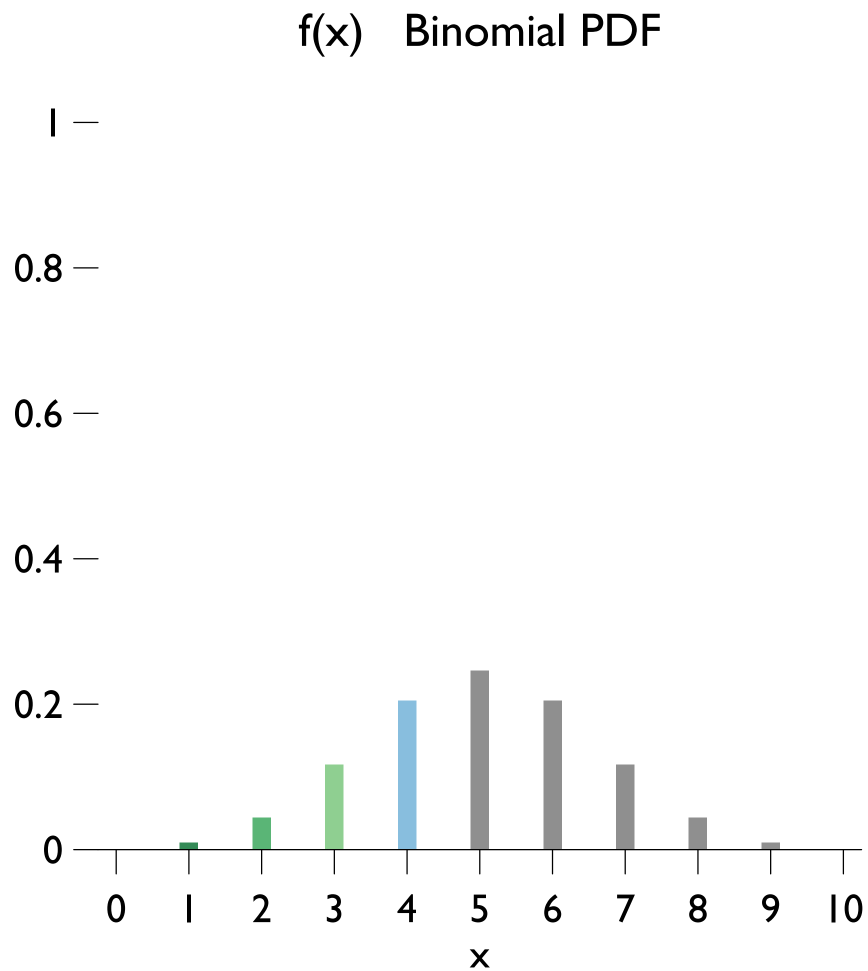

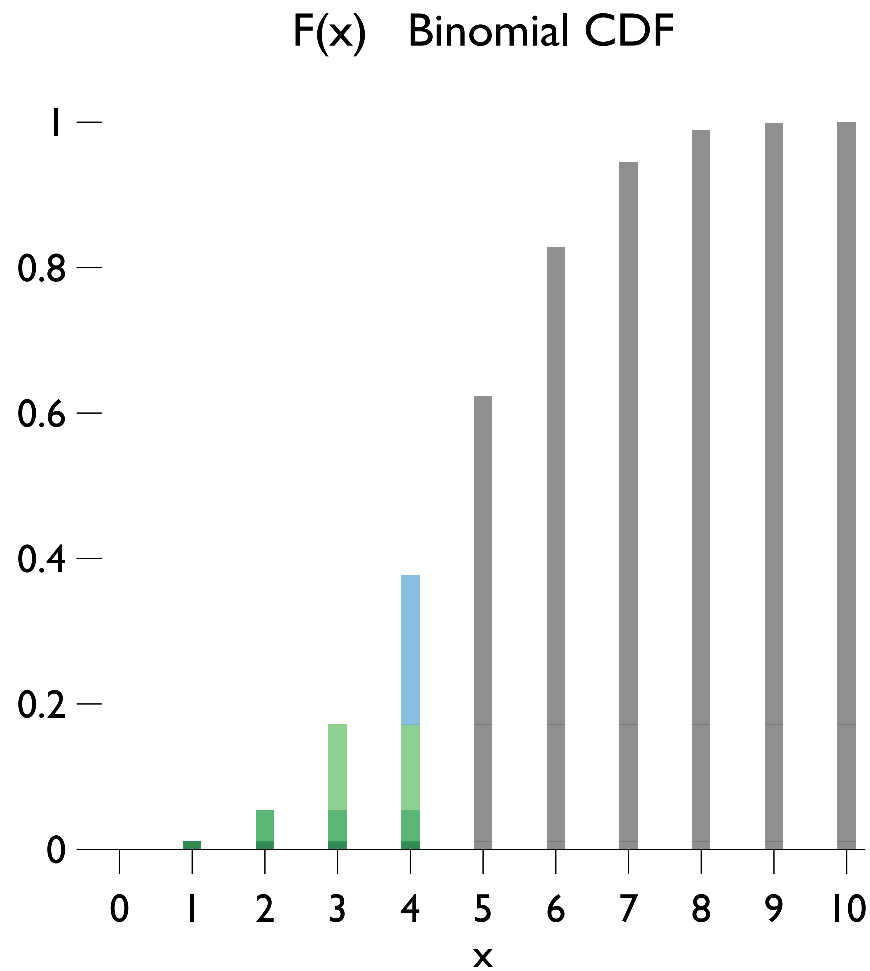

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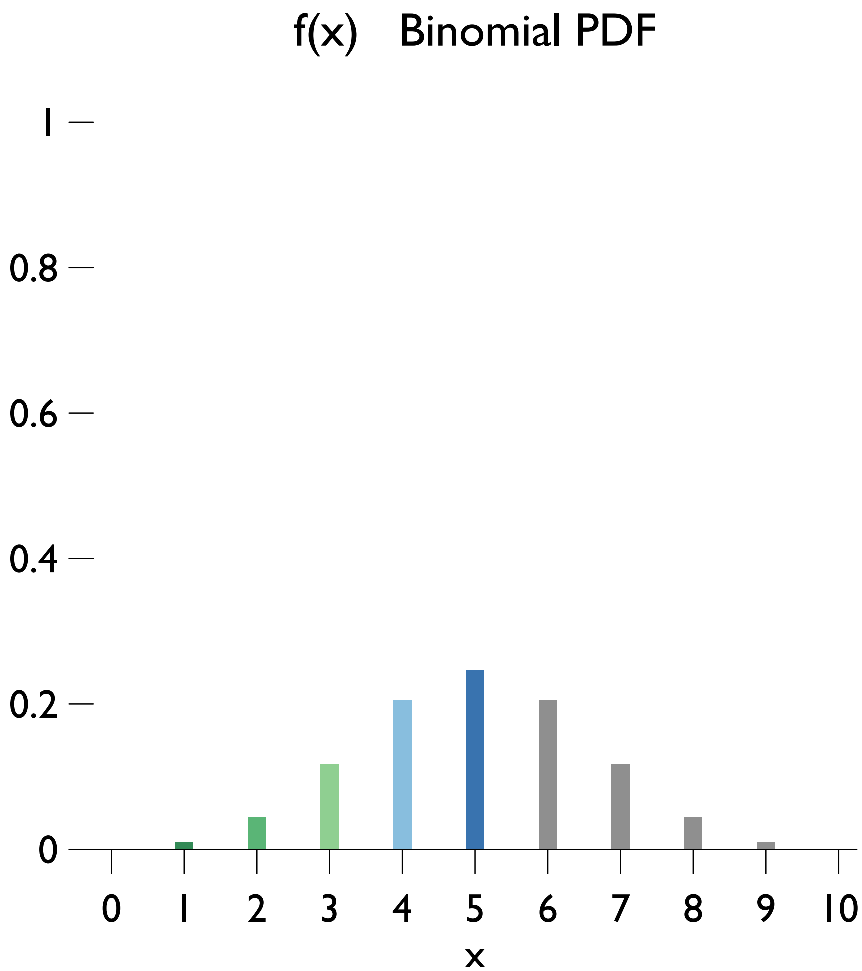

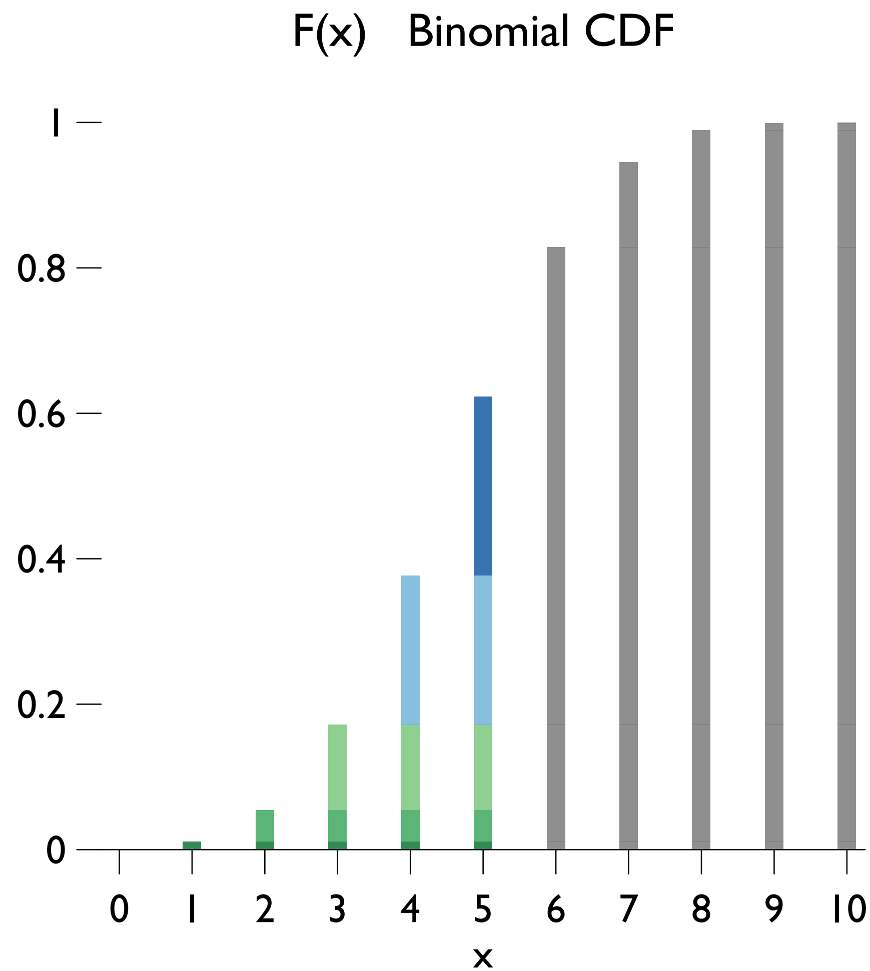

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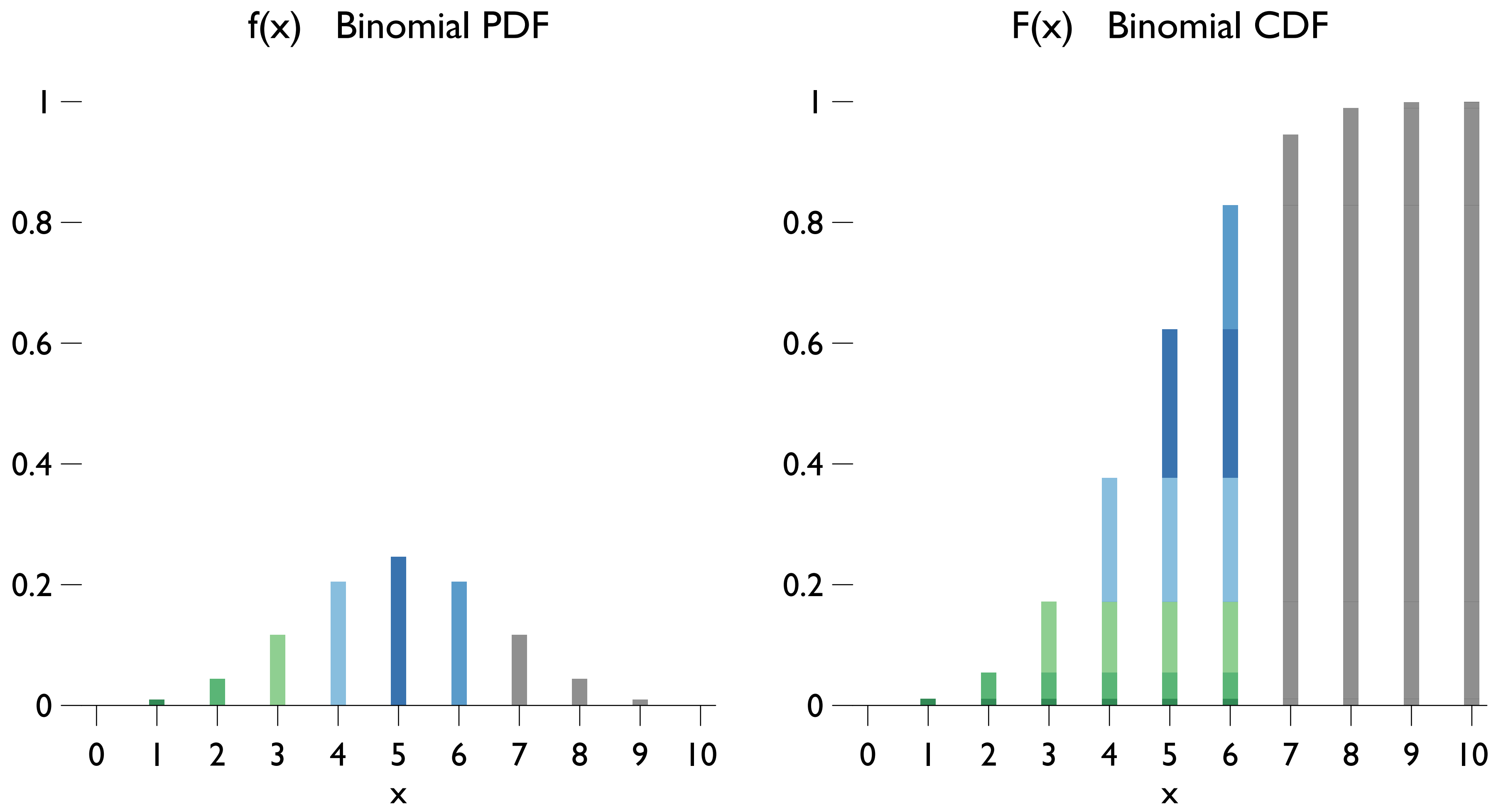

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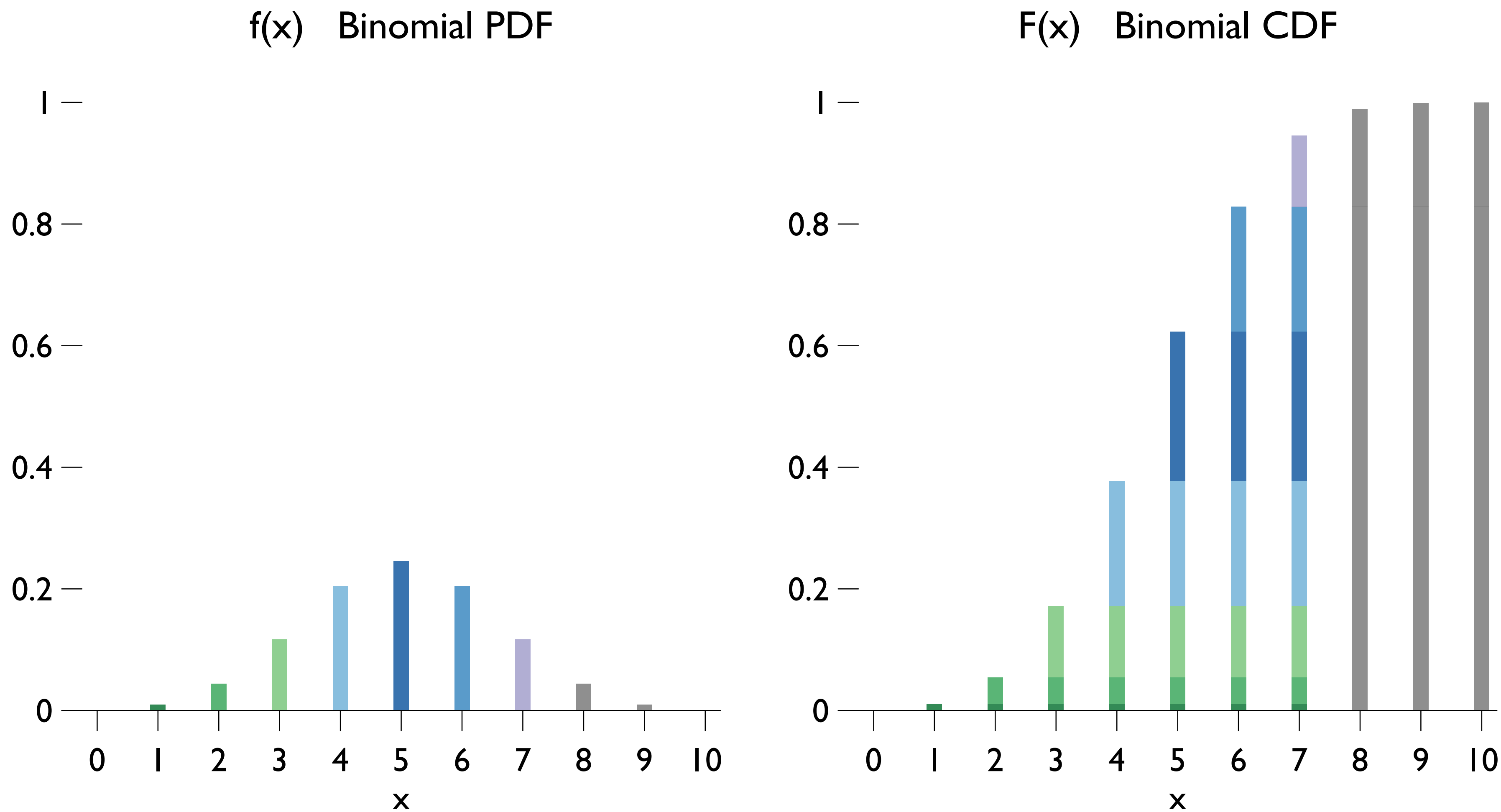

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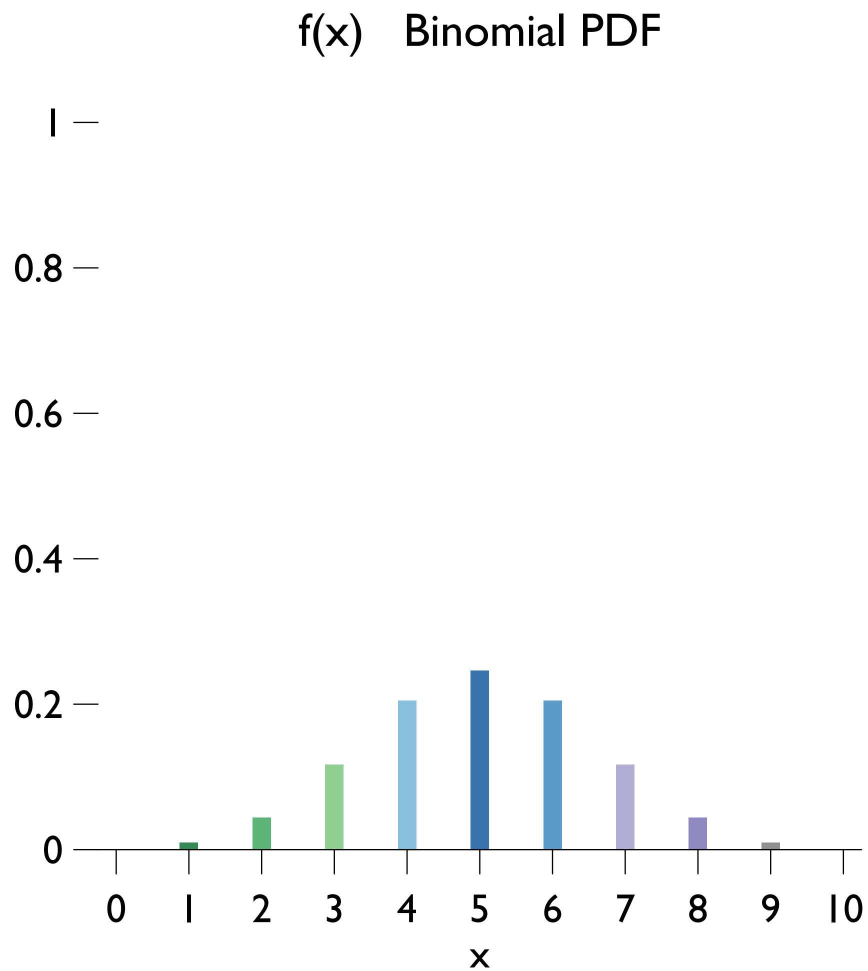

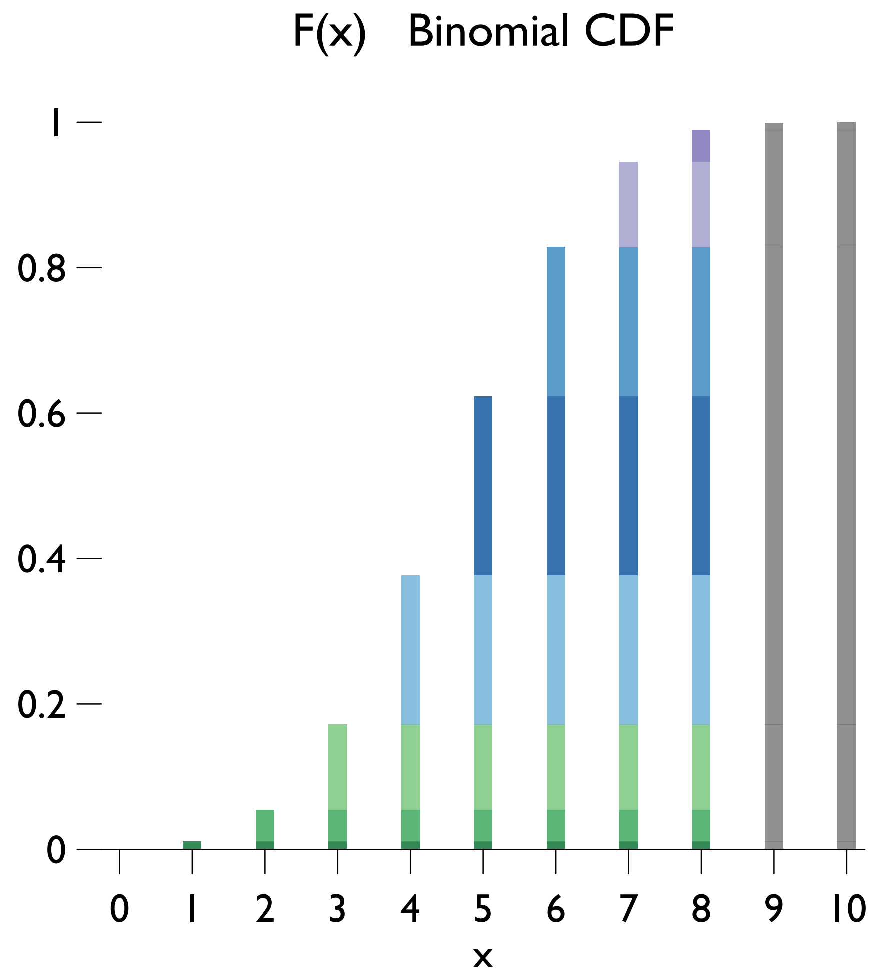

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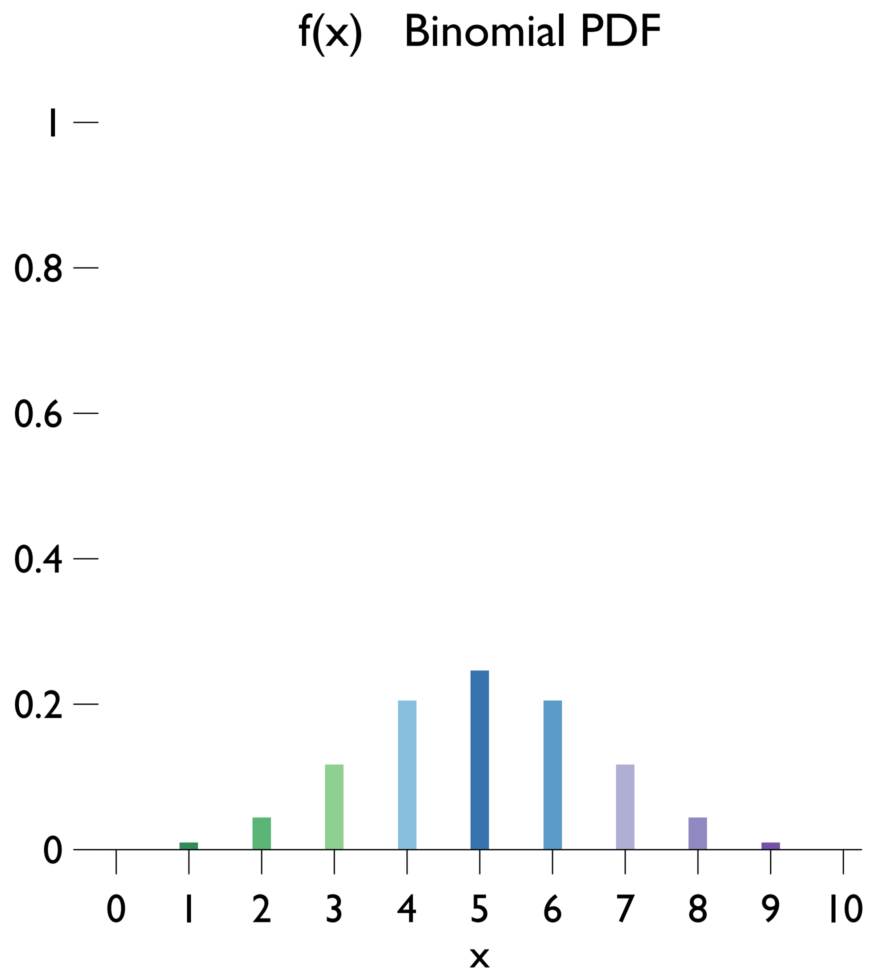

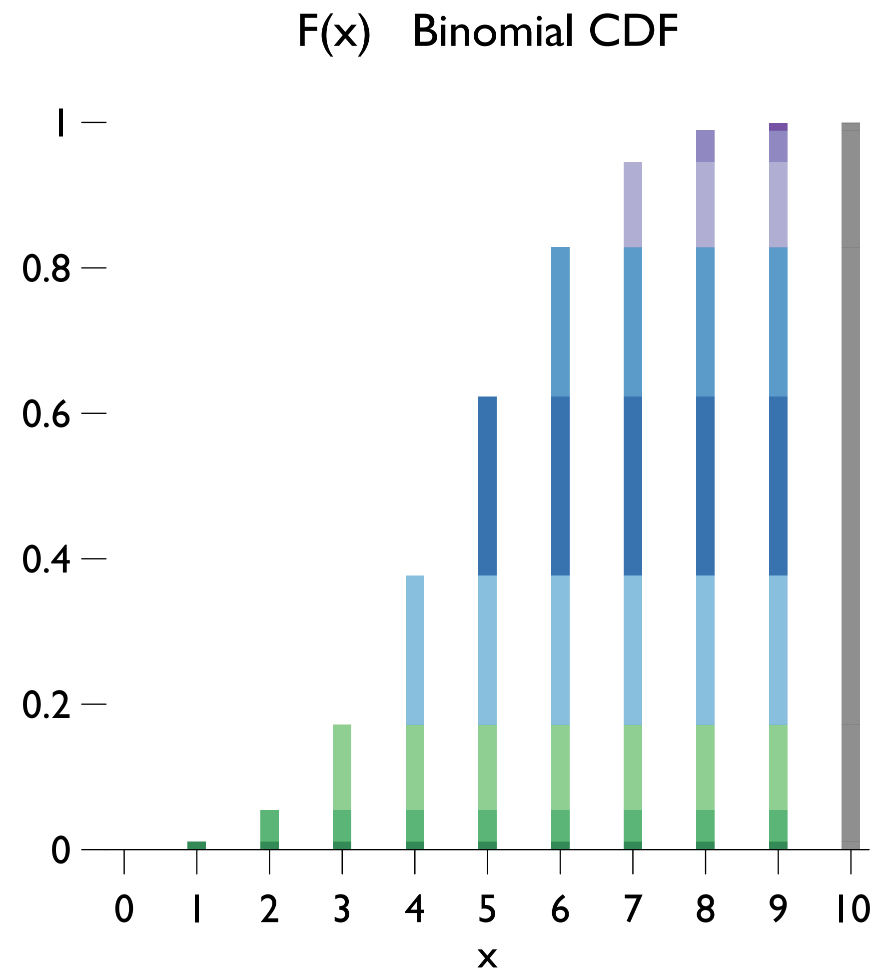

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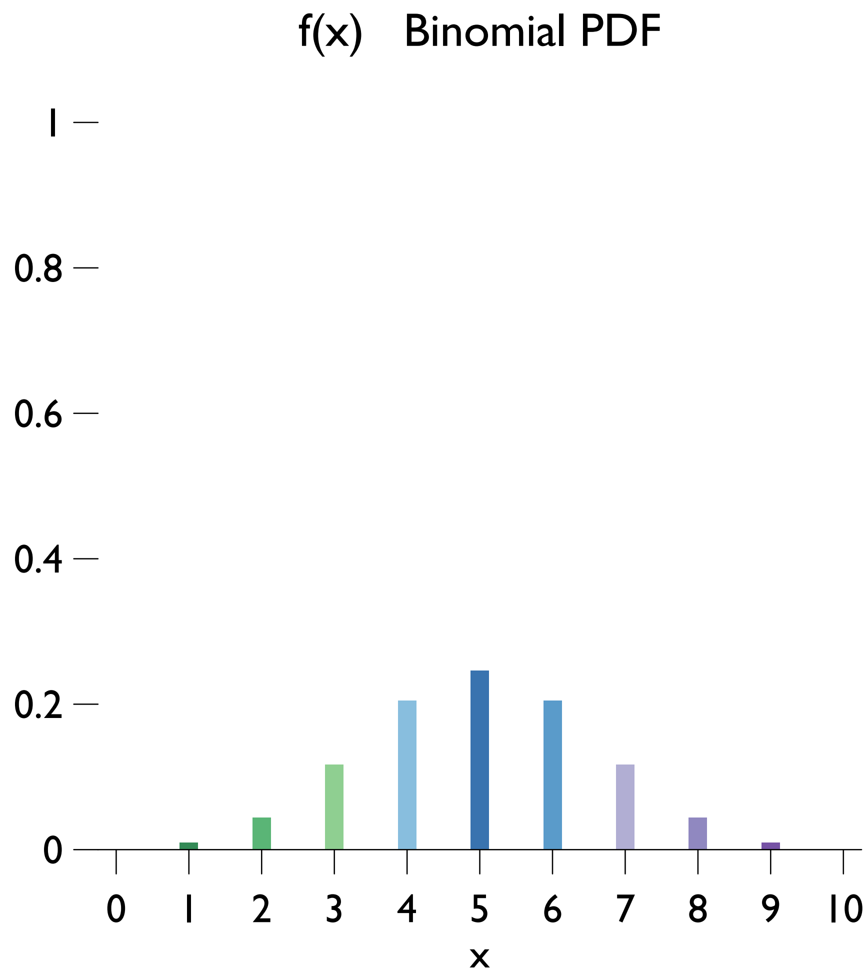

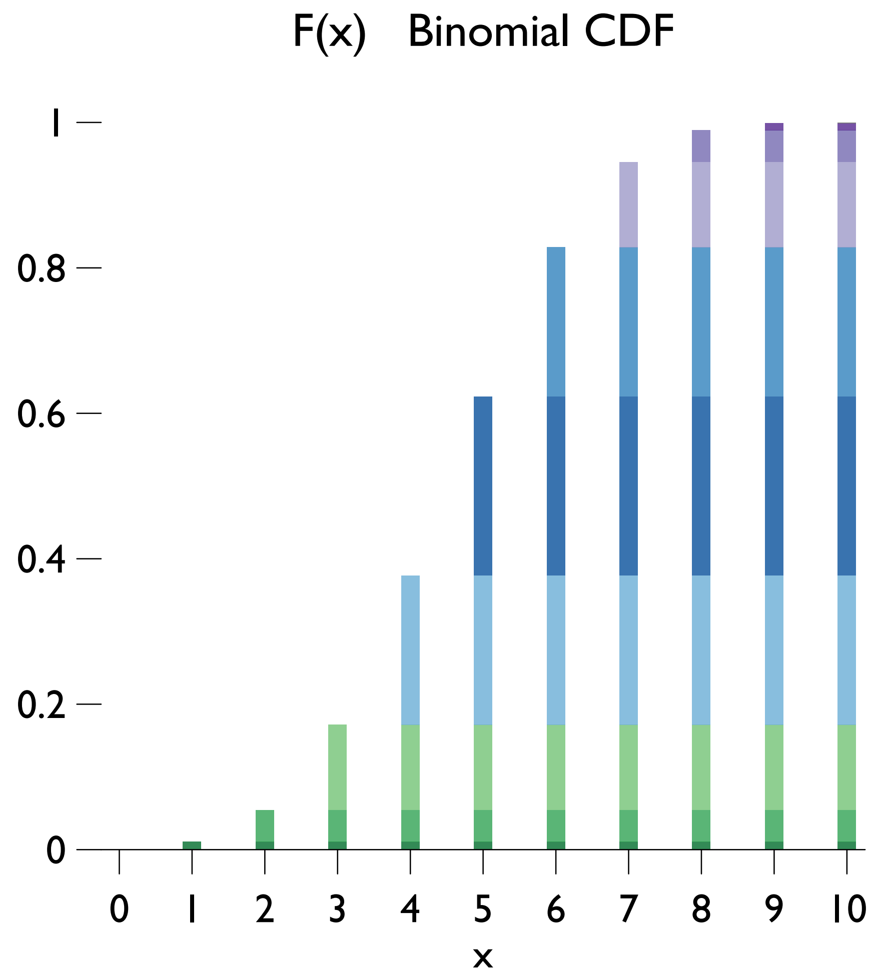

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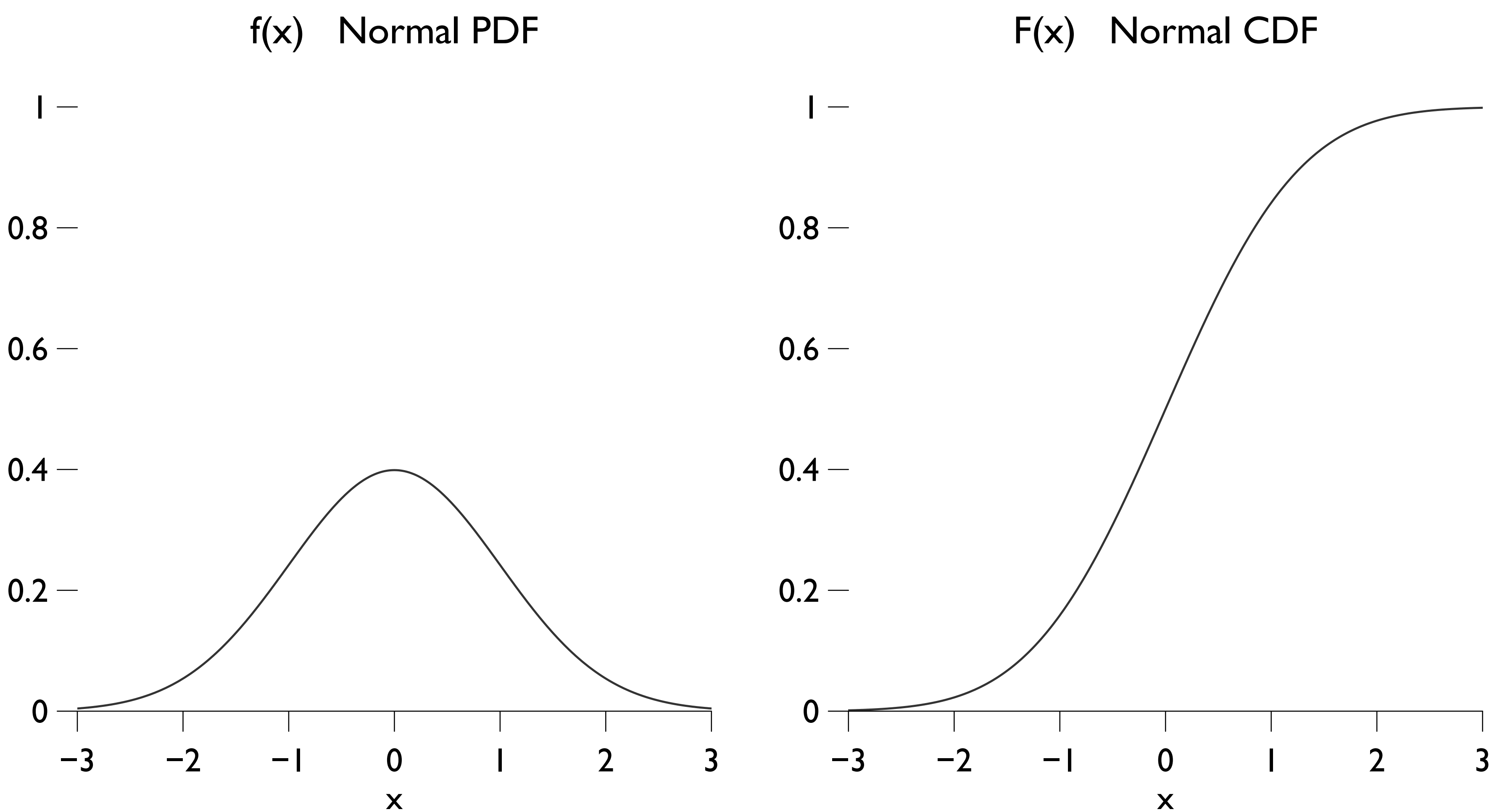

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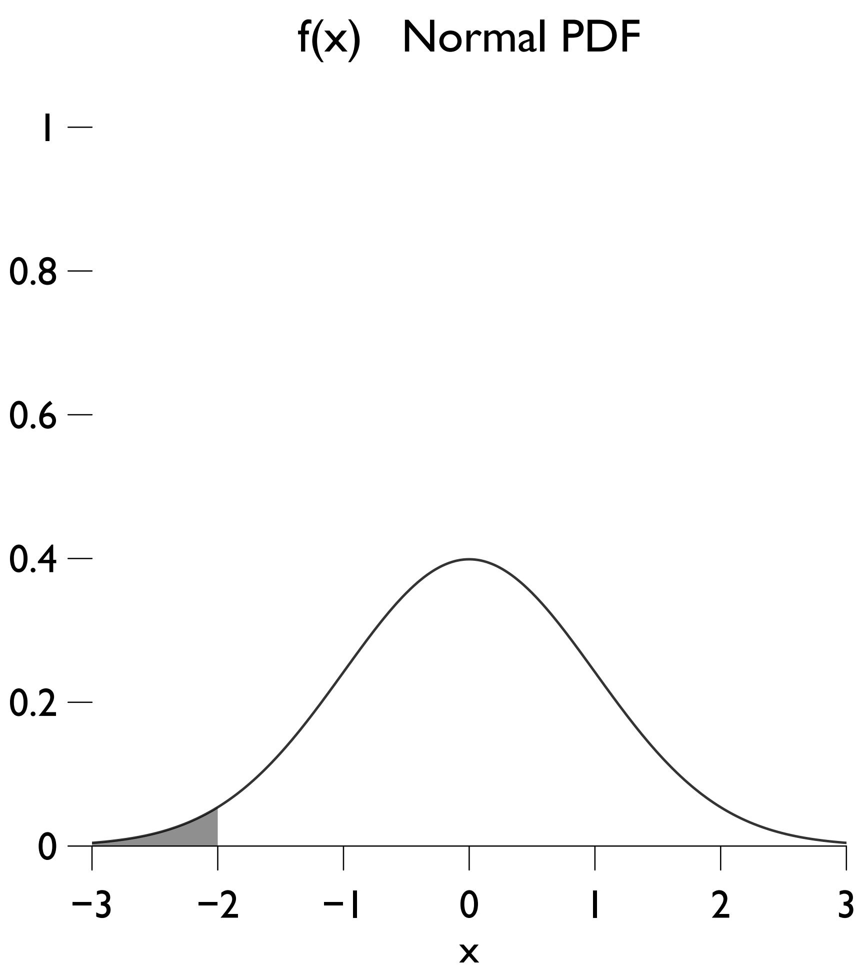

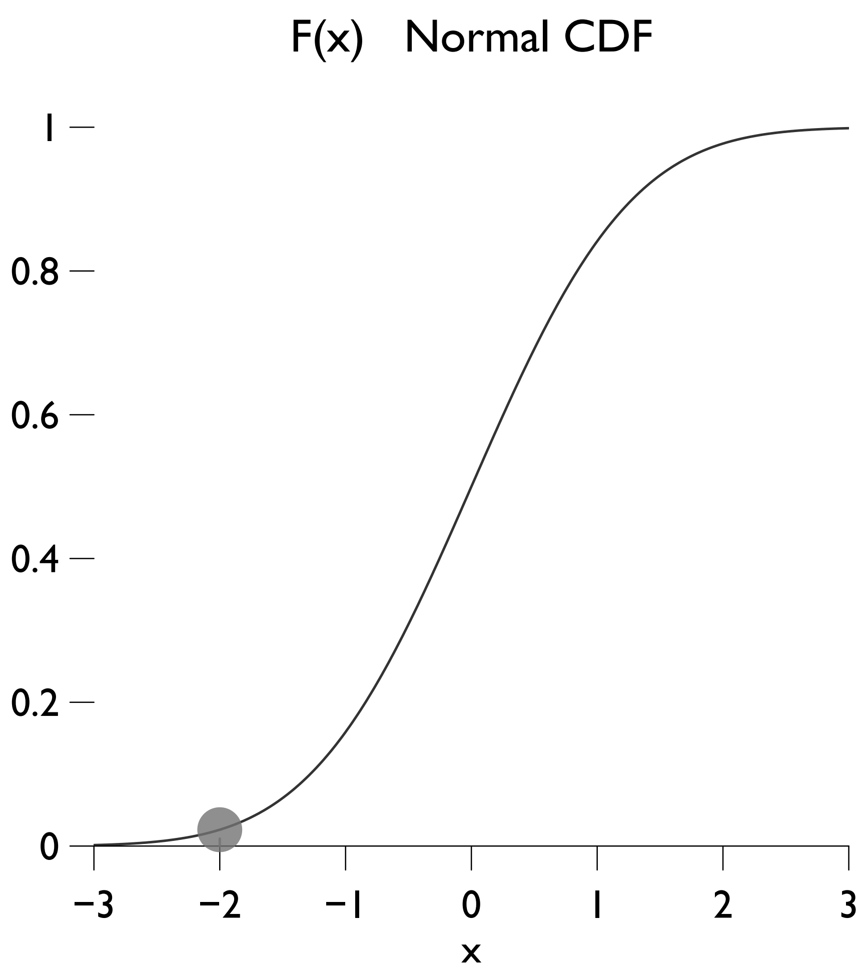

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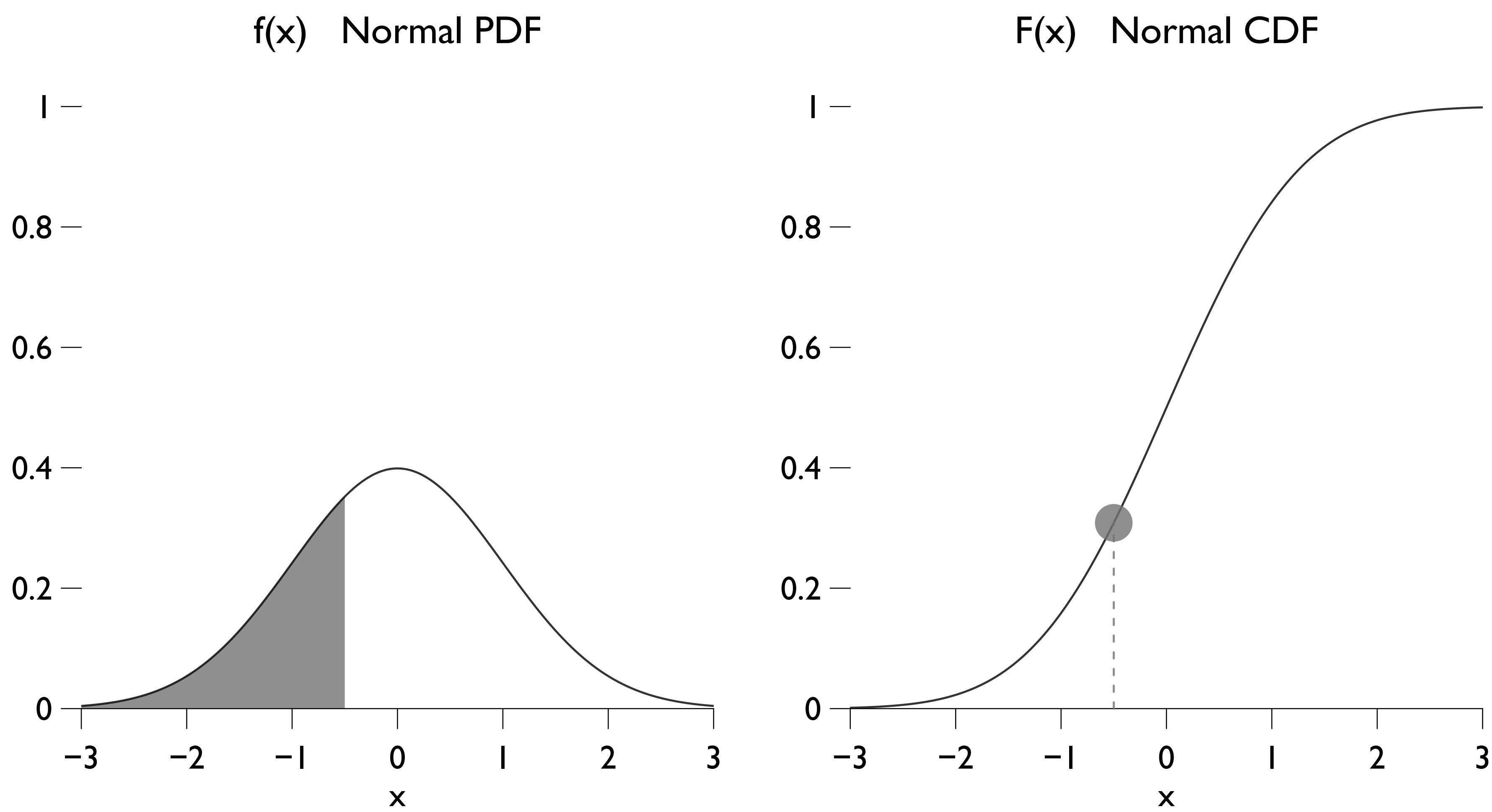

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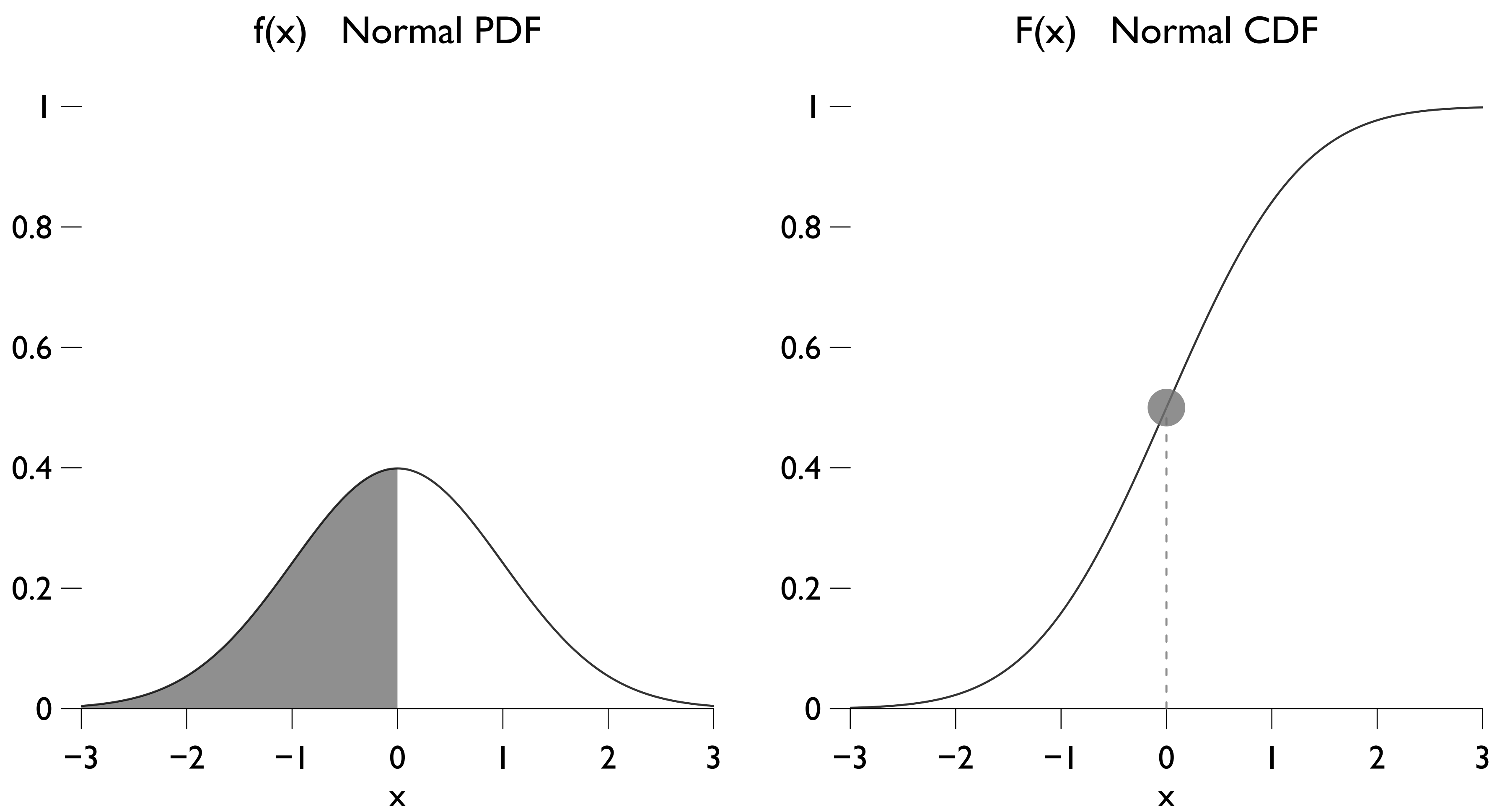

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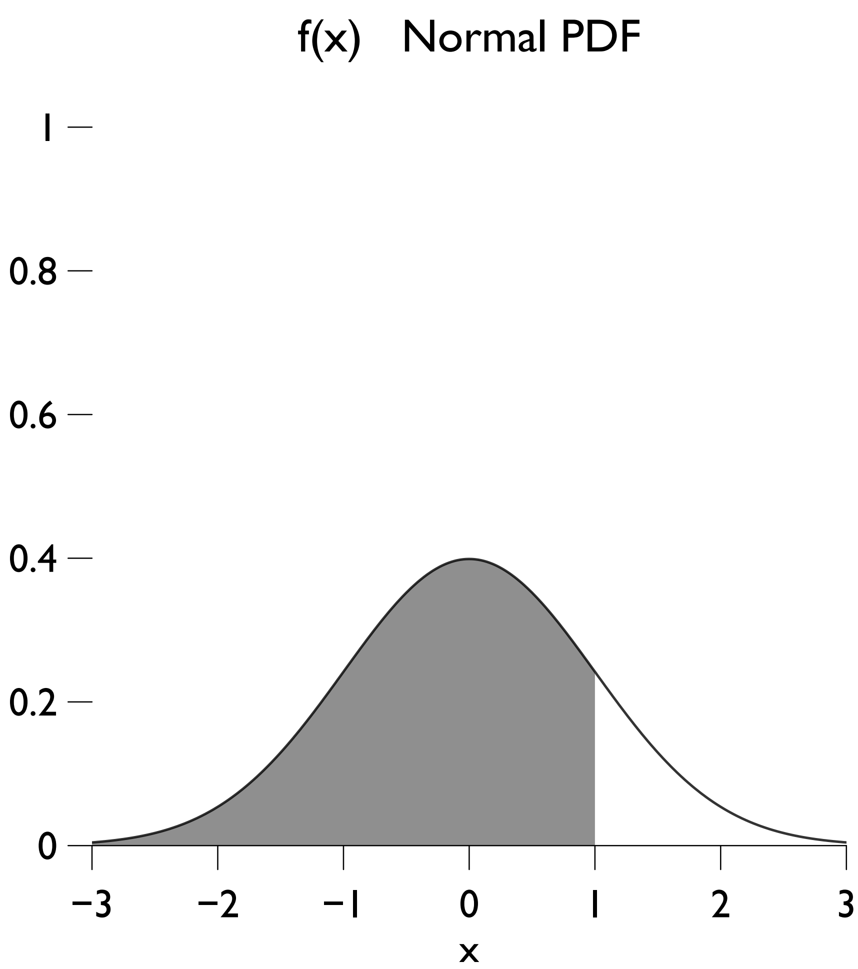

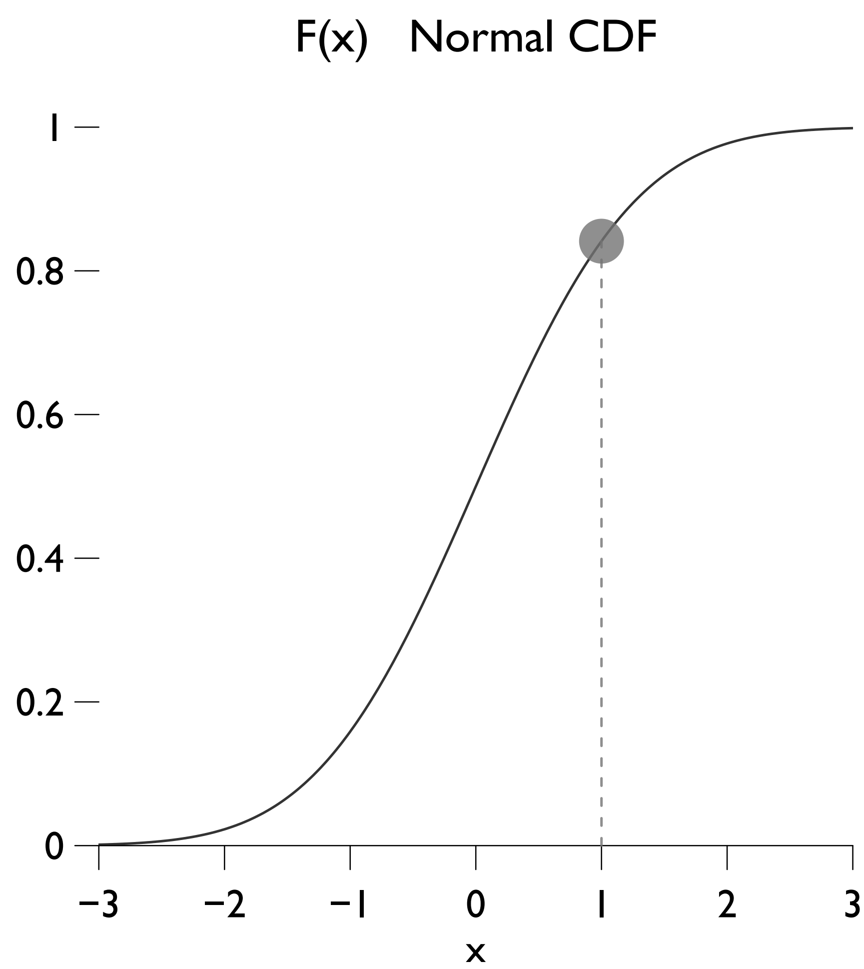

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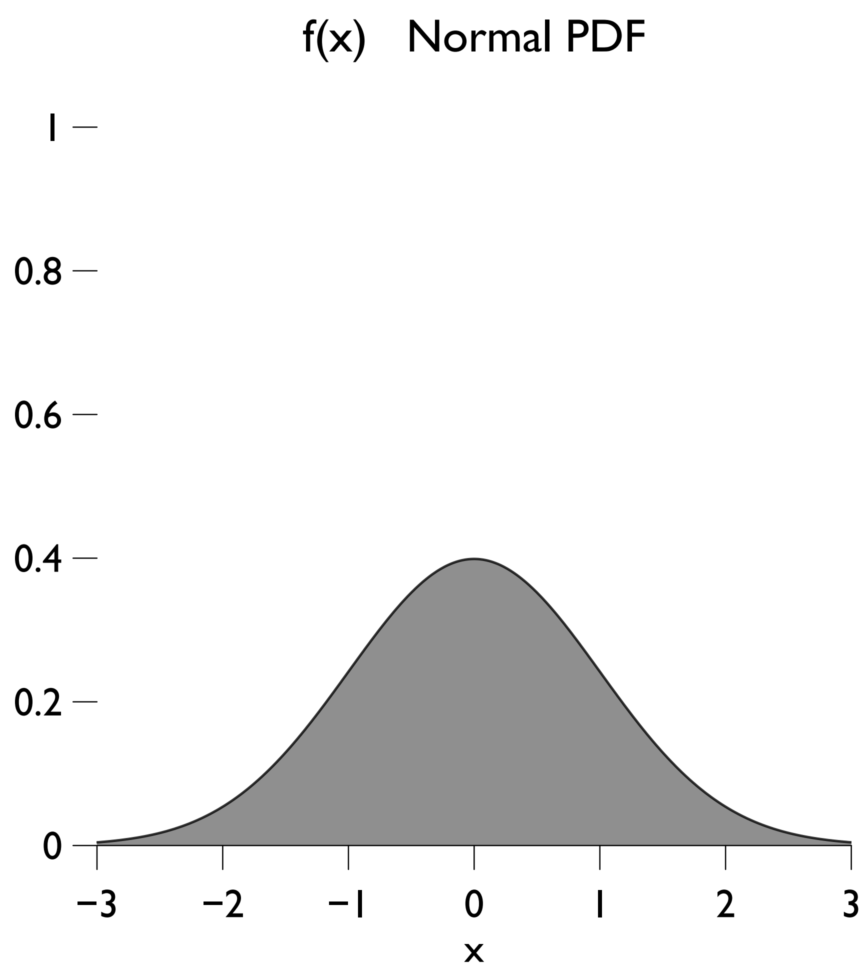

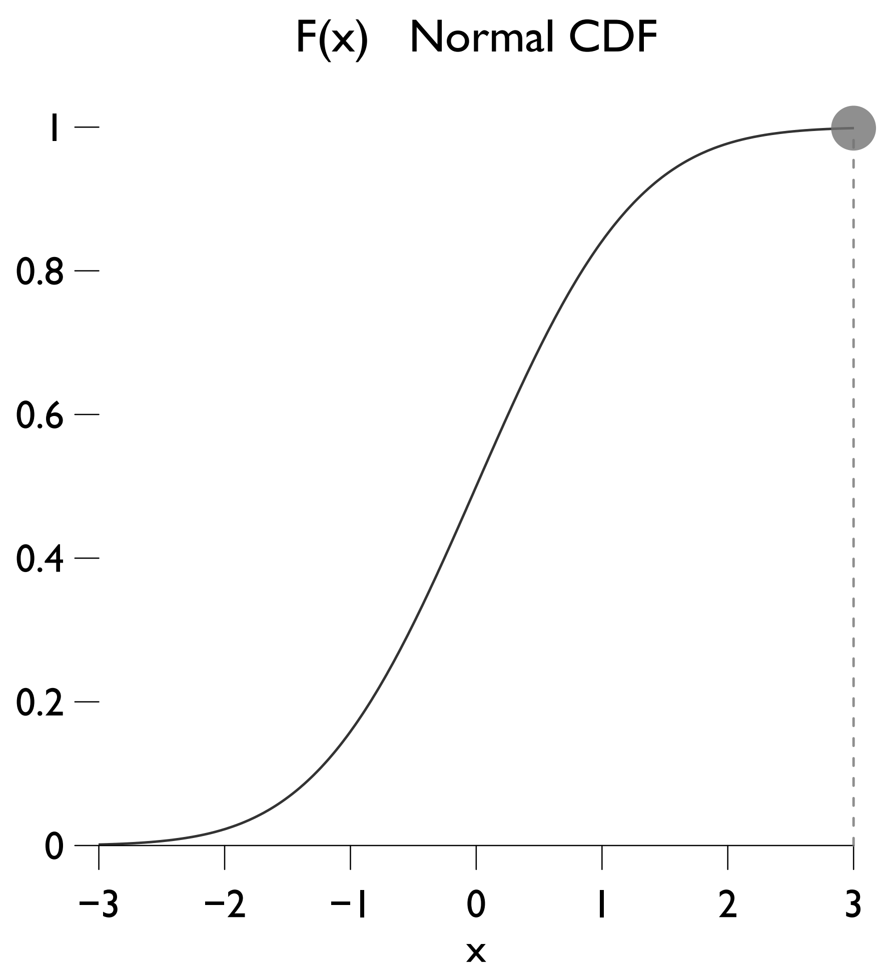

Example probability distributions

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Let's look at a few distributions to see how this might work

Bear in mind the key distinction between continuous and discrete distributions

Let's start with the simplest and most fundamental discrete distribution, the Bernoulli

The Bernoulli distribution

Consider a random variable x with 2 mutually exclusive & exhaustive outcomes

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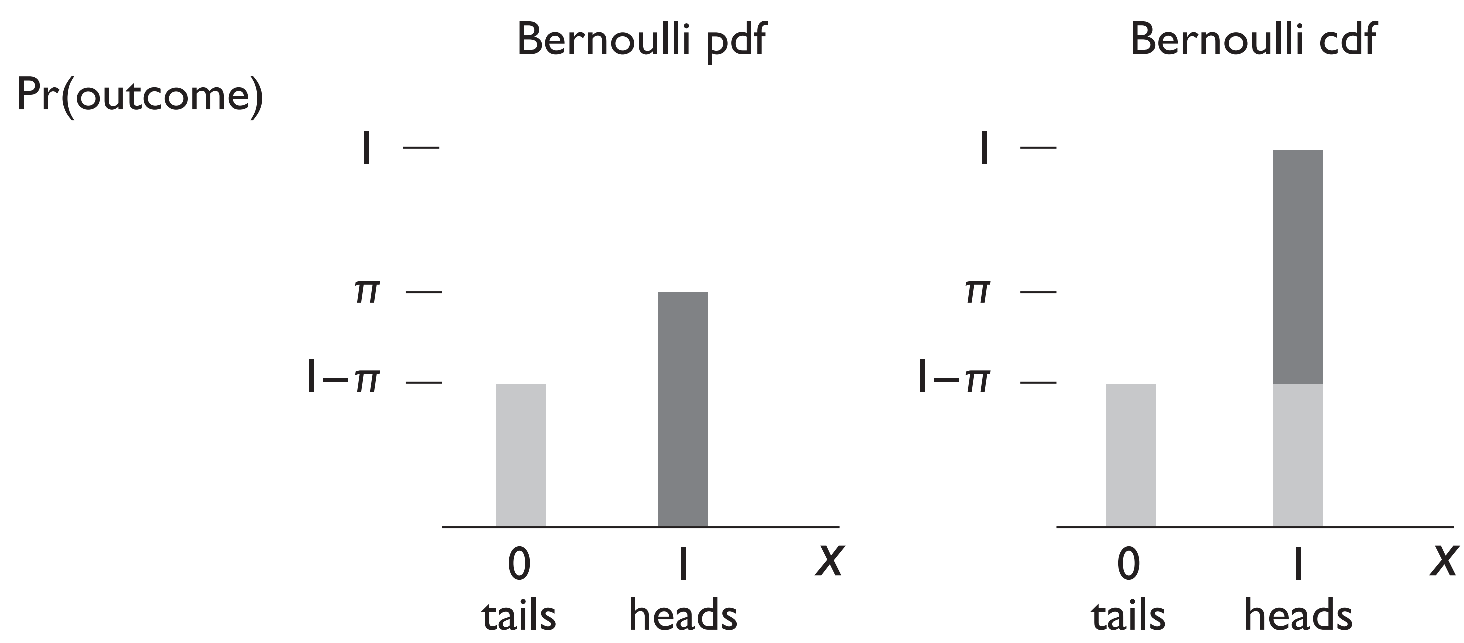
These assumptions create the Bernoulli distribution (pdf and cdf below):



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$$f_{\text{Bern}}(x|\pi) = \begin{cases} 1 - \pi & \text{if } x = 0 \\ \pi & \text{if } x = 1 \end{cases}$$

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If we are clever, we can write it much more conveniently:

$$f_{\text{Bern}}(x|\pi) = \pi^{x}(1 - \pi)^{1-x}$$

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The first two "moments" of a distribution are the expected value and variance:

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How do we come up with a pdf for these assumptions?

Let's model the sum of our unobserved trials as a new random variable, X_i,

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$$f_{\text{Bin}}(X_i|M,\pi) \quad = \quad \#\text{ of ways to get } X_i \quad \times \quad \Pr(\text{getting } X_i)$$

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f_{\text{Bin}}(X_i|M,\pi) &= \#\text{ of ways to get } X_i \quad \times \quad \Pr(\text{getting } X_i) \\
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Similarity to the Bernoulli evident, especially in the moments:

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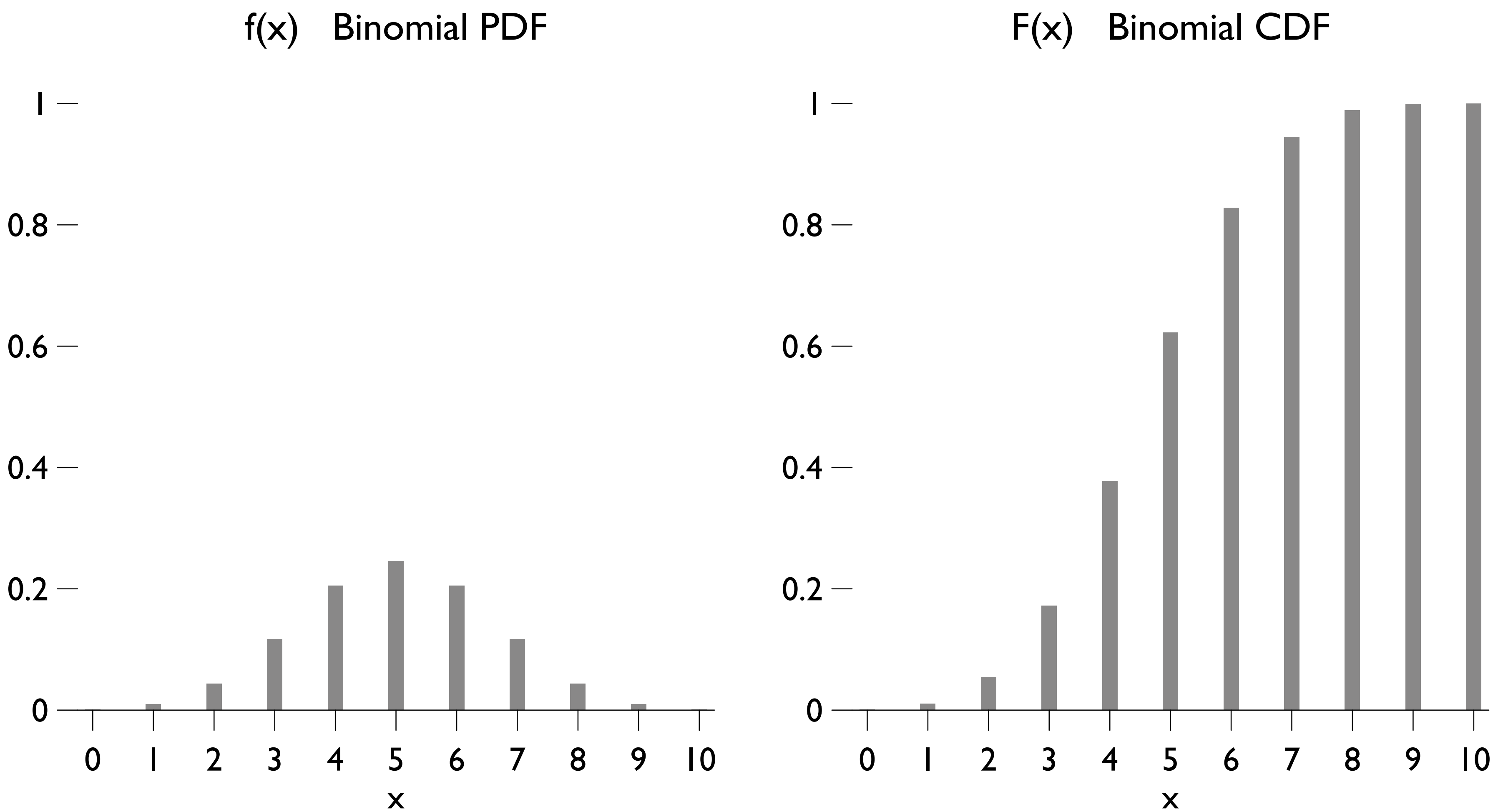
Similarity to the Bernoulli evident, especially in the moments:

$$\mathbb{E}(X) = M\pi \qquad \text{Var}(X) = M\pi(1-\pi)$$

Indeed, the Bernoulli is a special case of the binomial where $M = 1$

The binomial distribution

We've already seen the binomial, as our example discrete distribution



This binomial sums over 10 trials, with each trial having an 0.5 probability of success

The Poisson distribution

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Sometimes assumption 3 is fulfilled exactly (e.g., cosmic radiation)

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But often it's not even close to correct (e.g., phone calls per hour)

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Accepting these asumptions leads to the following distribution (we'll derive later)

$$f_{\text{Pois}}(x|\lambda) = \frac{\exp(-\lambda)\lambda^x}{x!} \qquad \forall x \in \{0, 1, \ldots\}, \qquad 0 \text{ otherwise}$$

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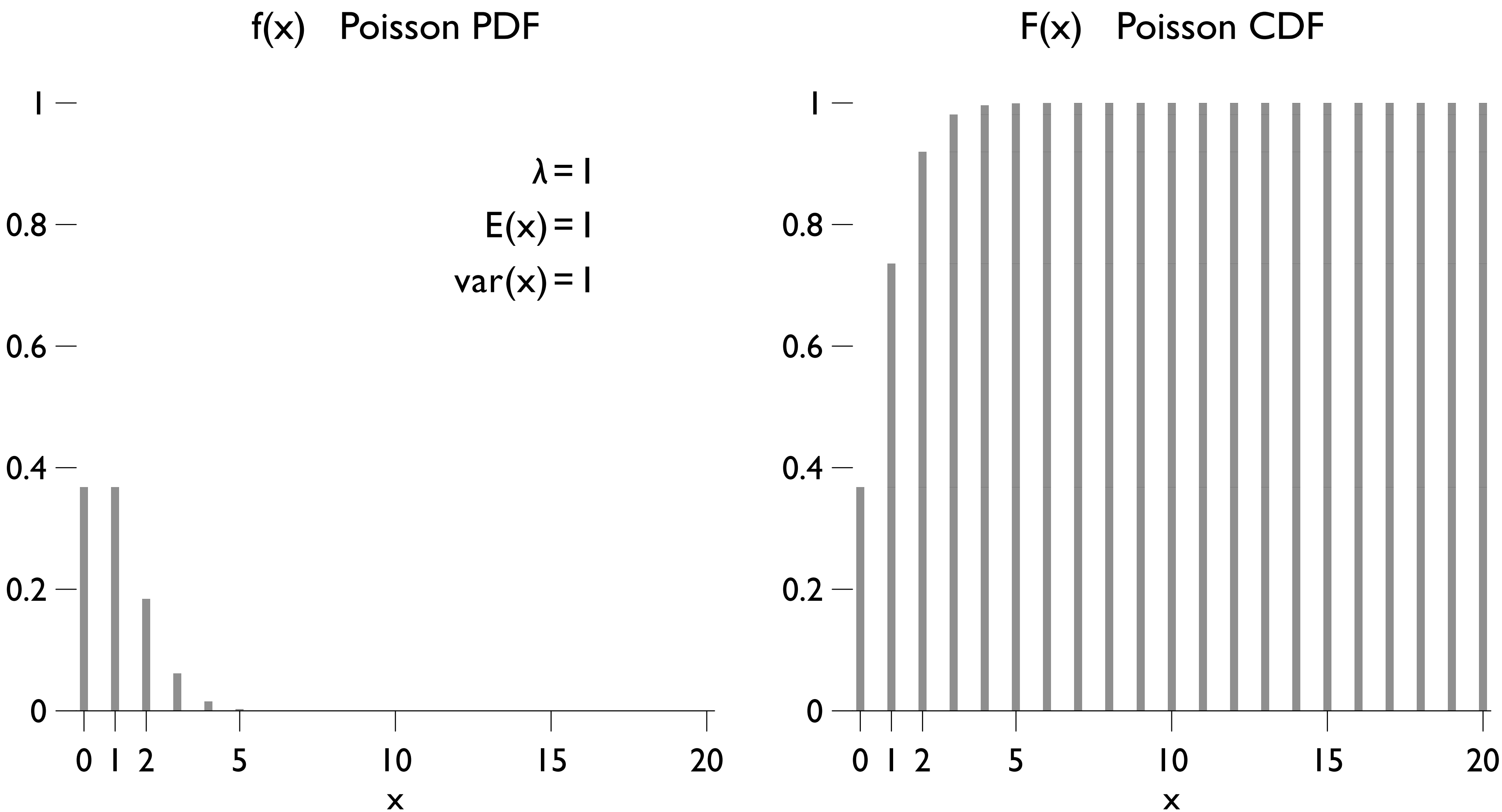
3. We can relax the "equal periods" assumption: just replace λ_i with $t_i\lambda_i$, where i indexes observations and t measures their relative length

Examples of the Poisson distribution



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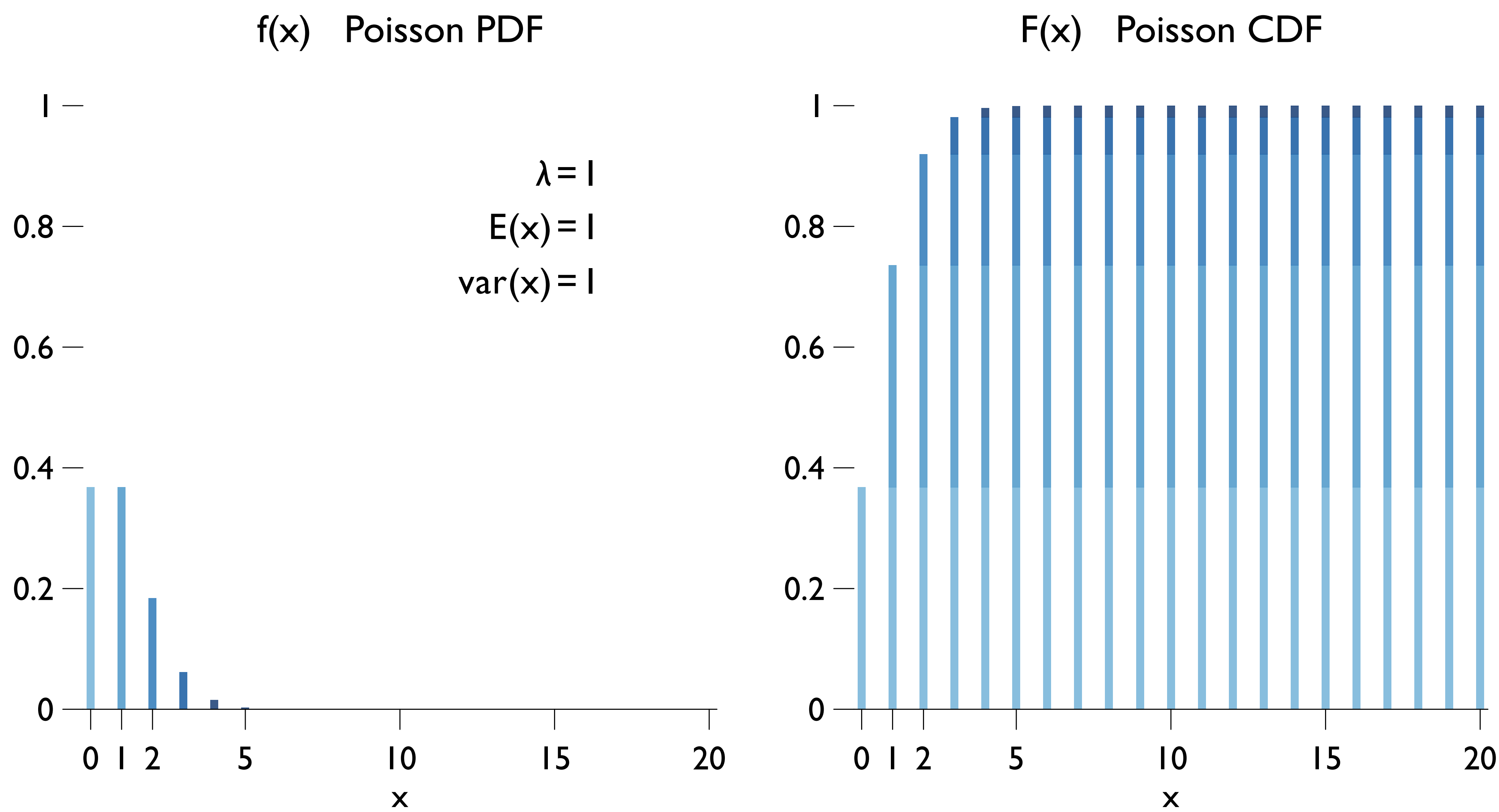

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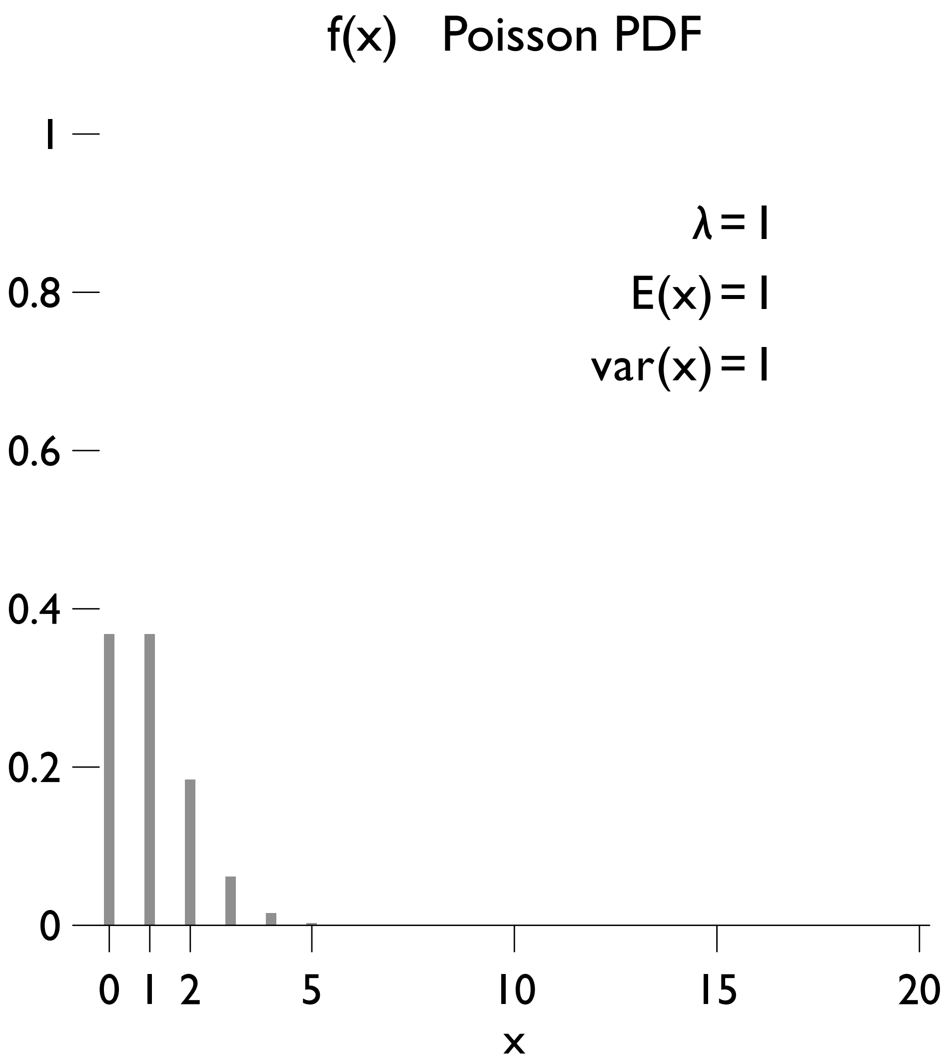

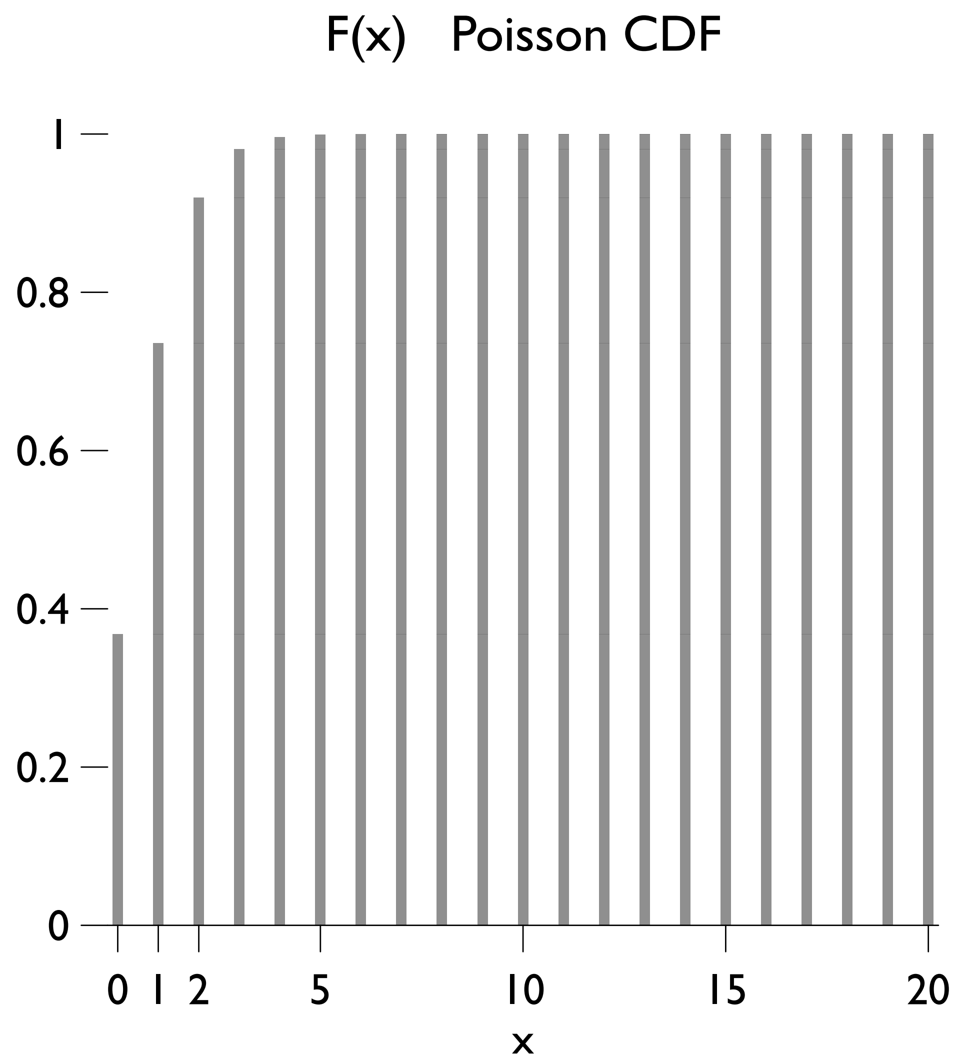

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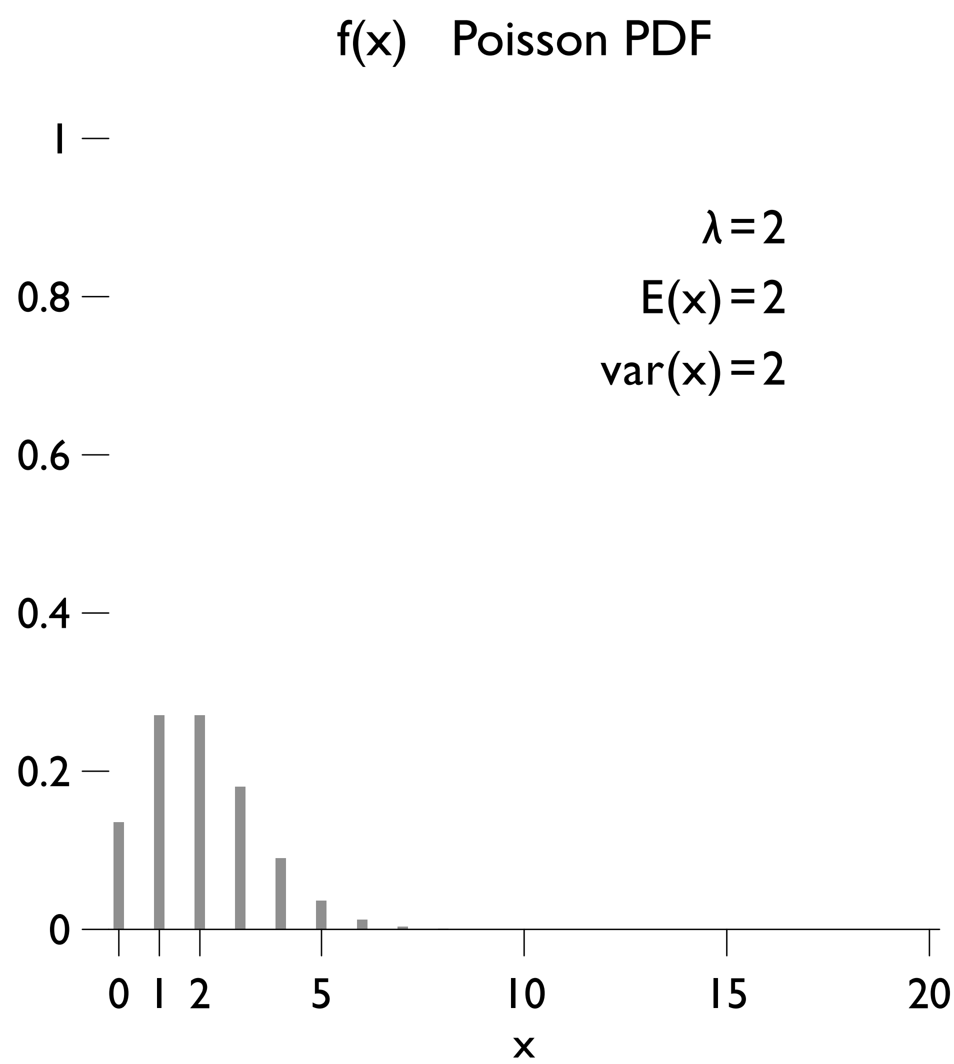

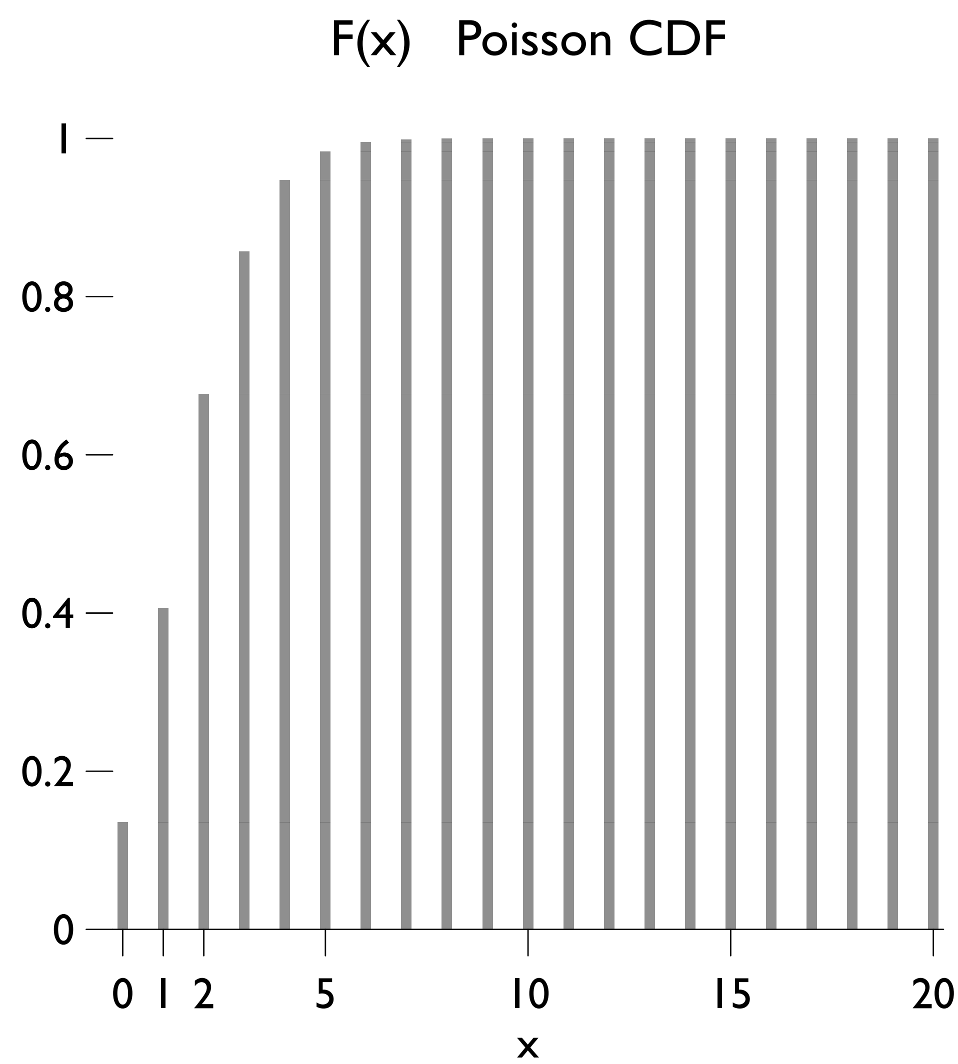

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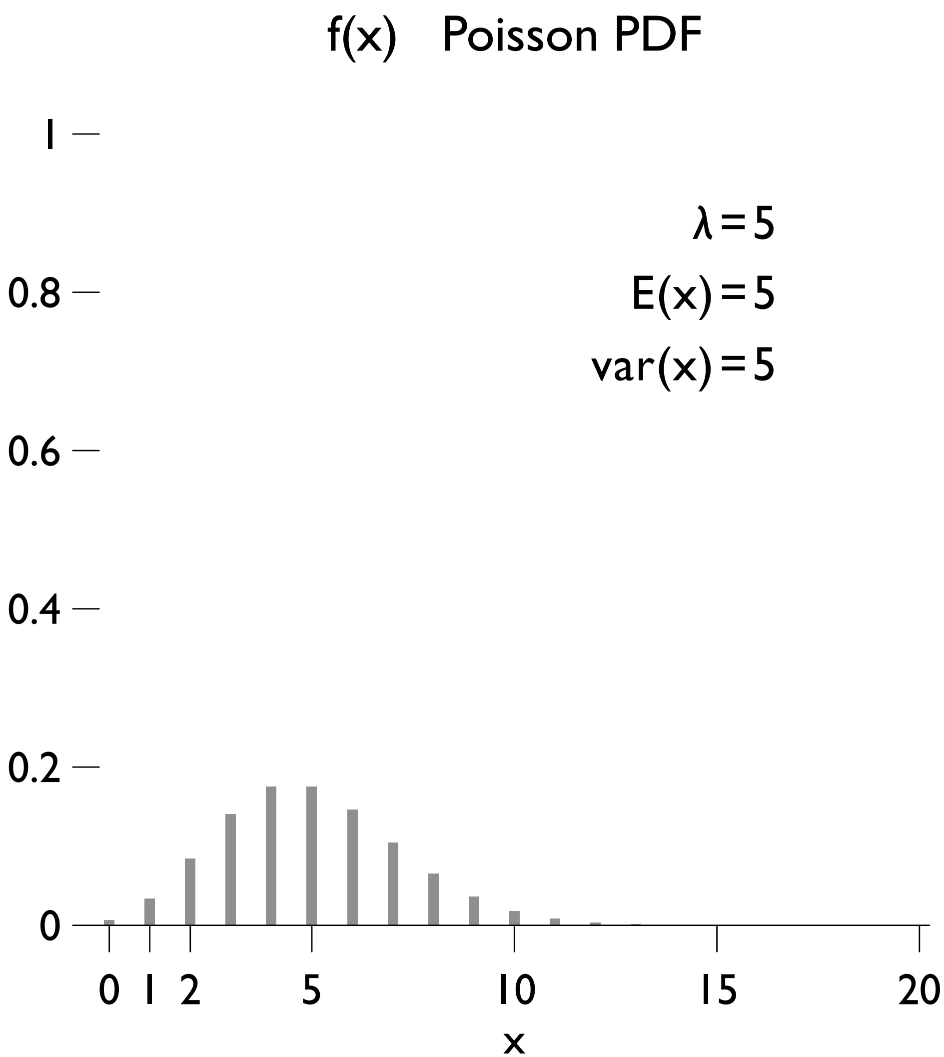

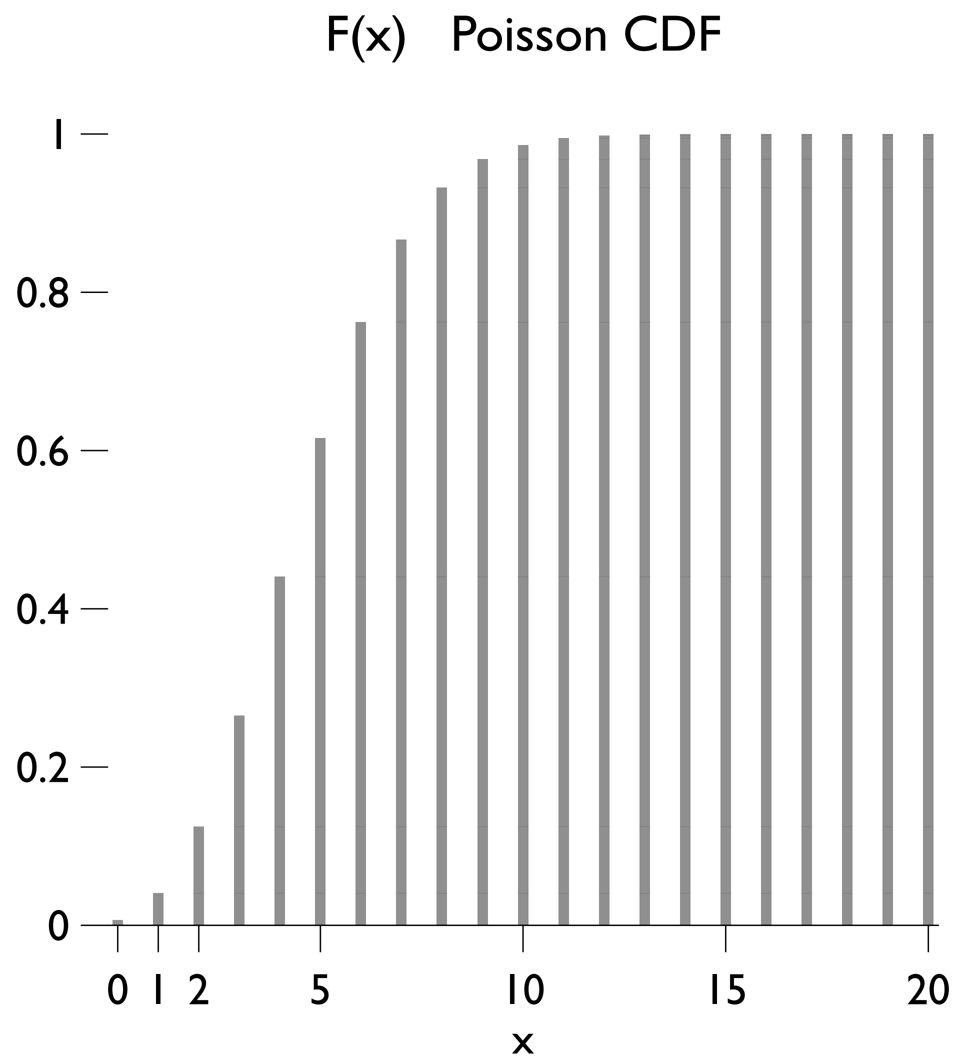

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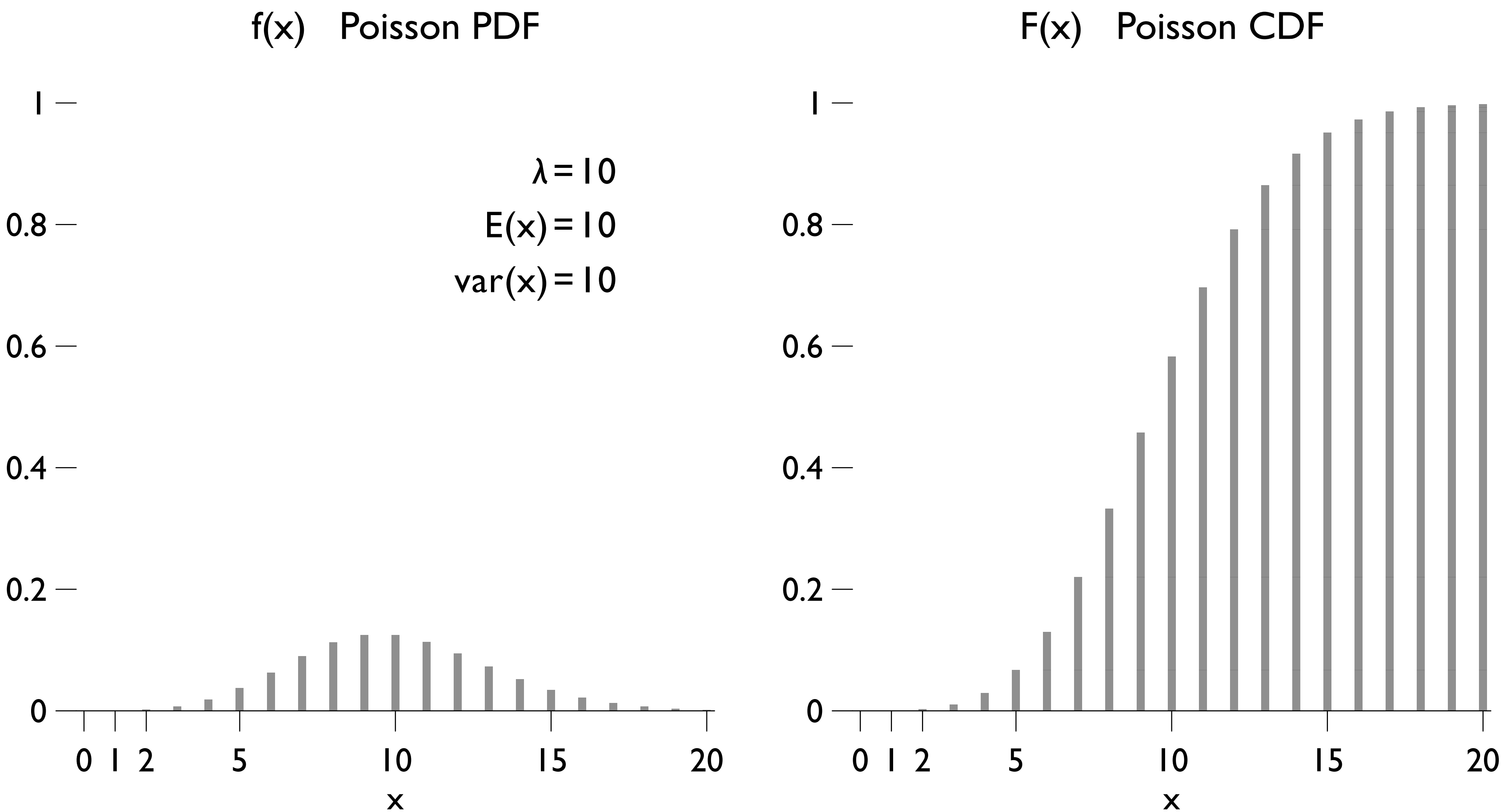
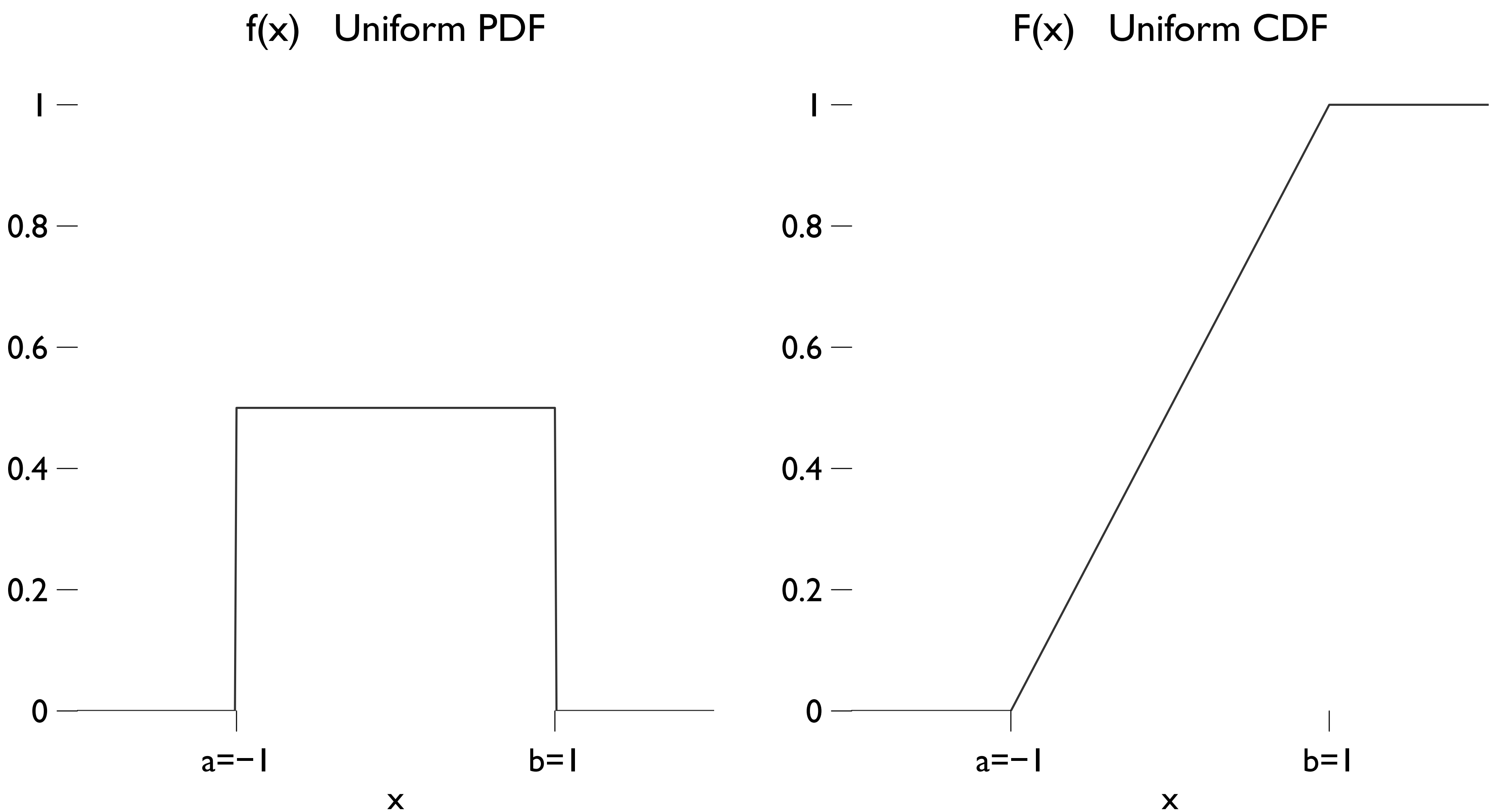


The Uniform distribution

The Uniform distribution is the simplest continuous distribution

Assumes all members of a real interval $[a, b]$ are equally likely



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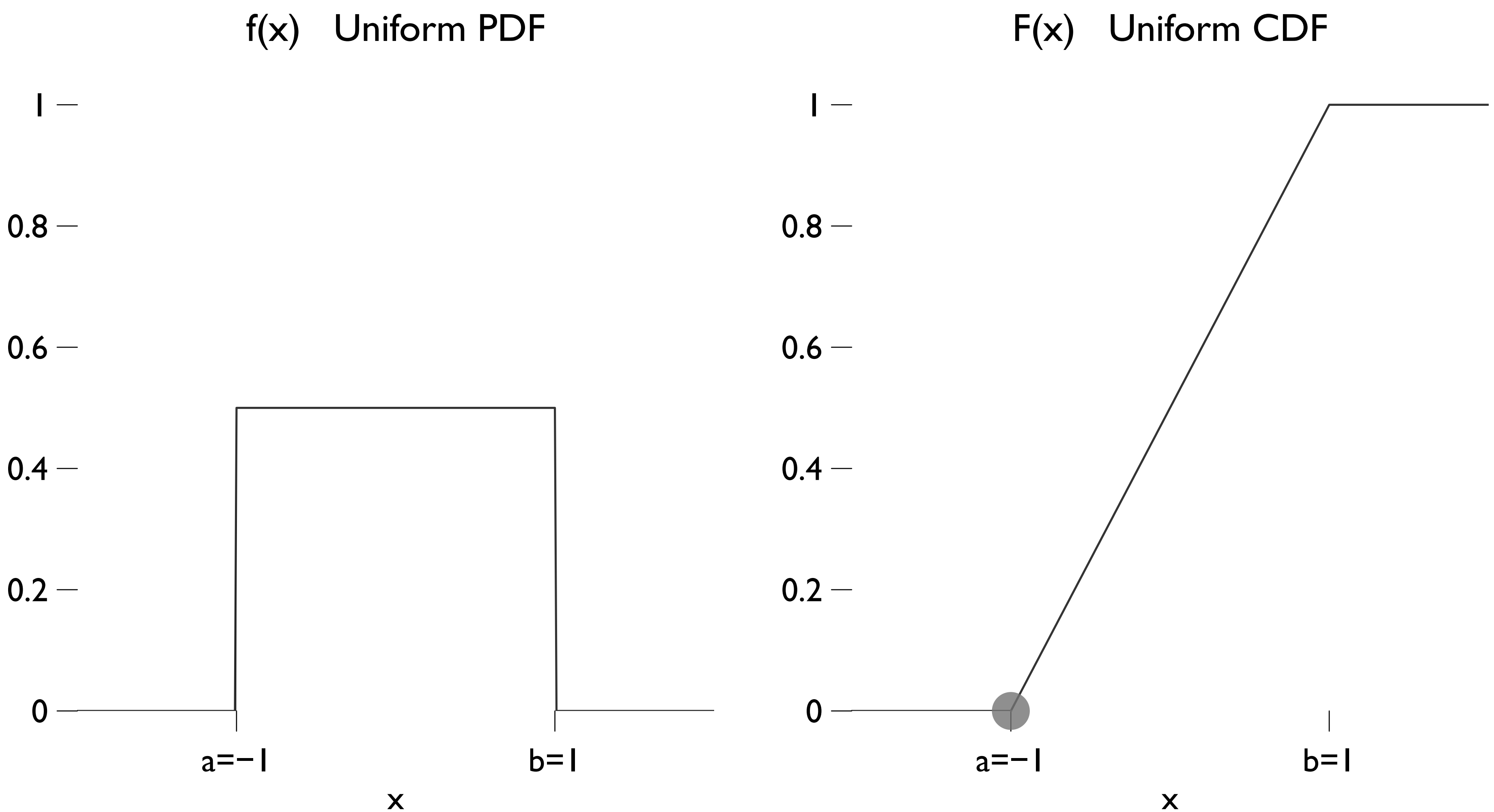

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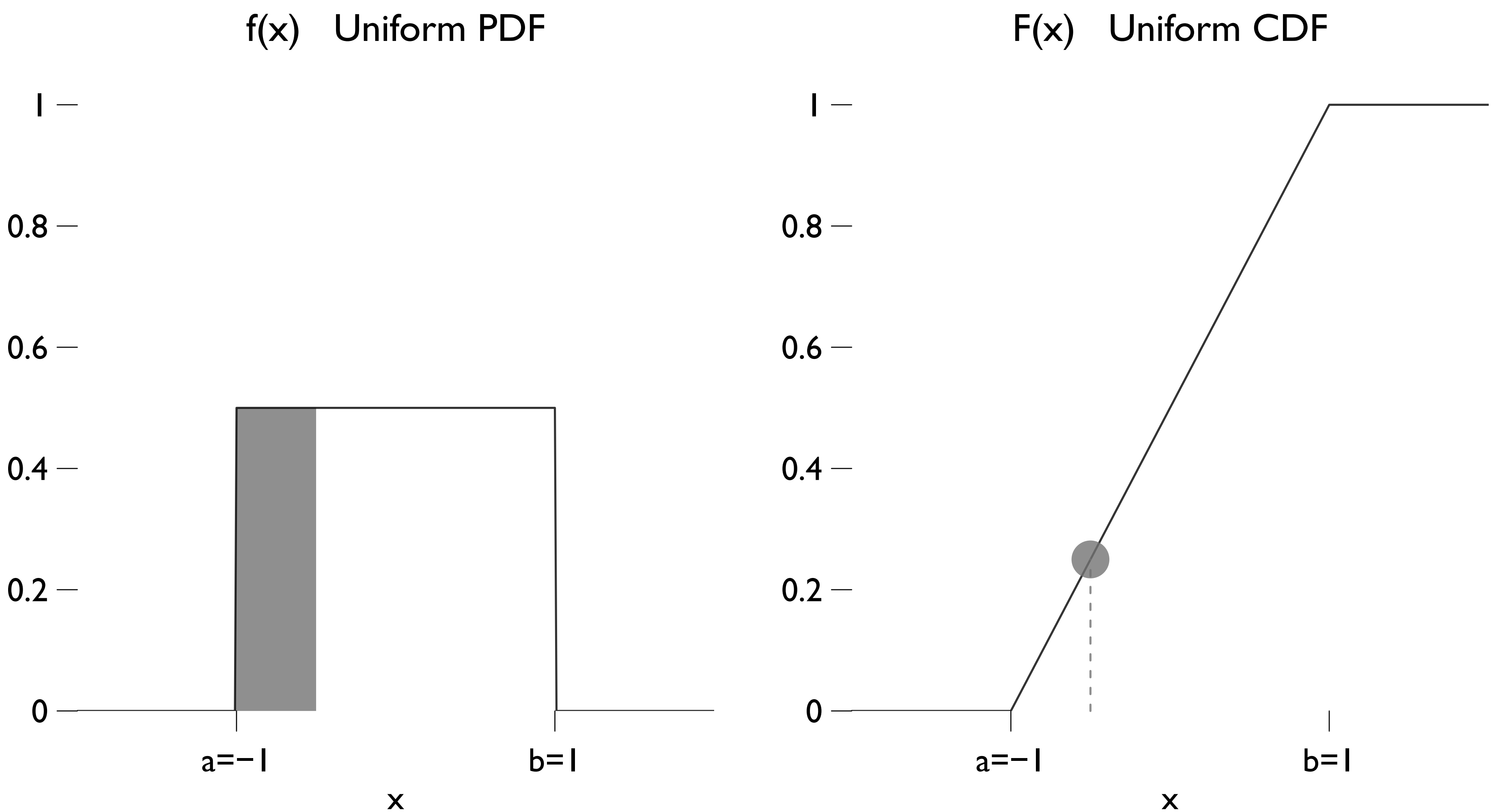

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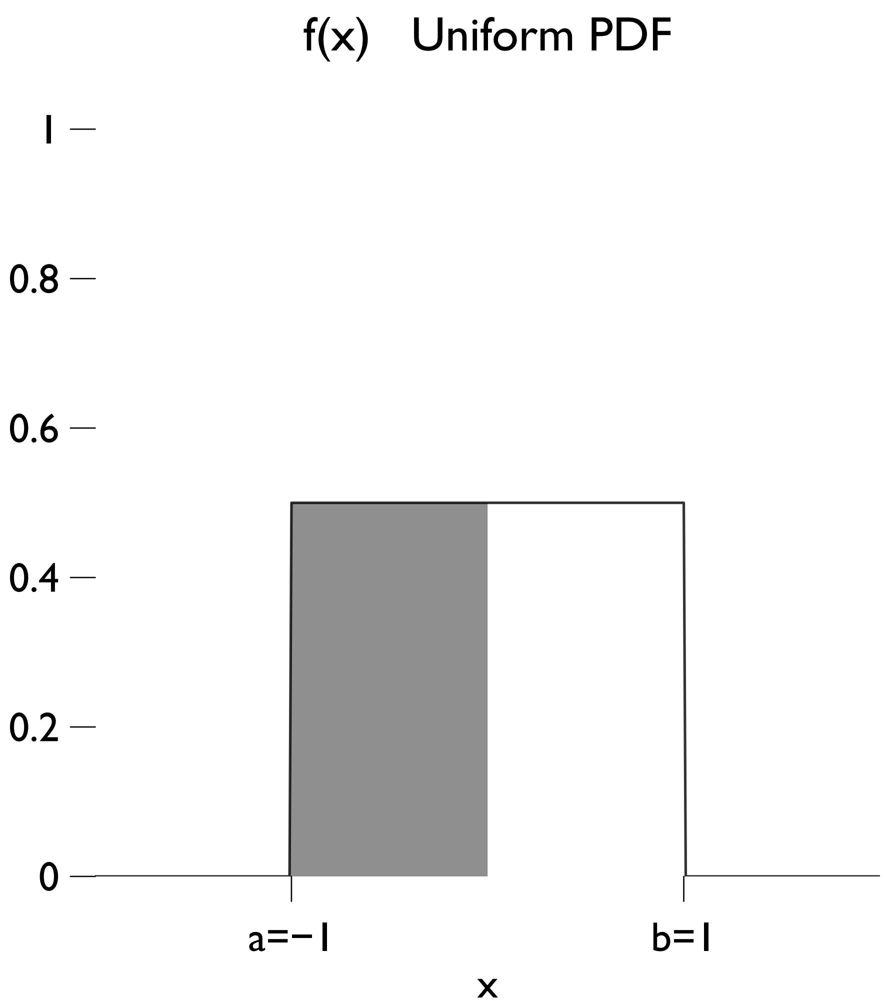

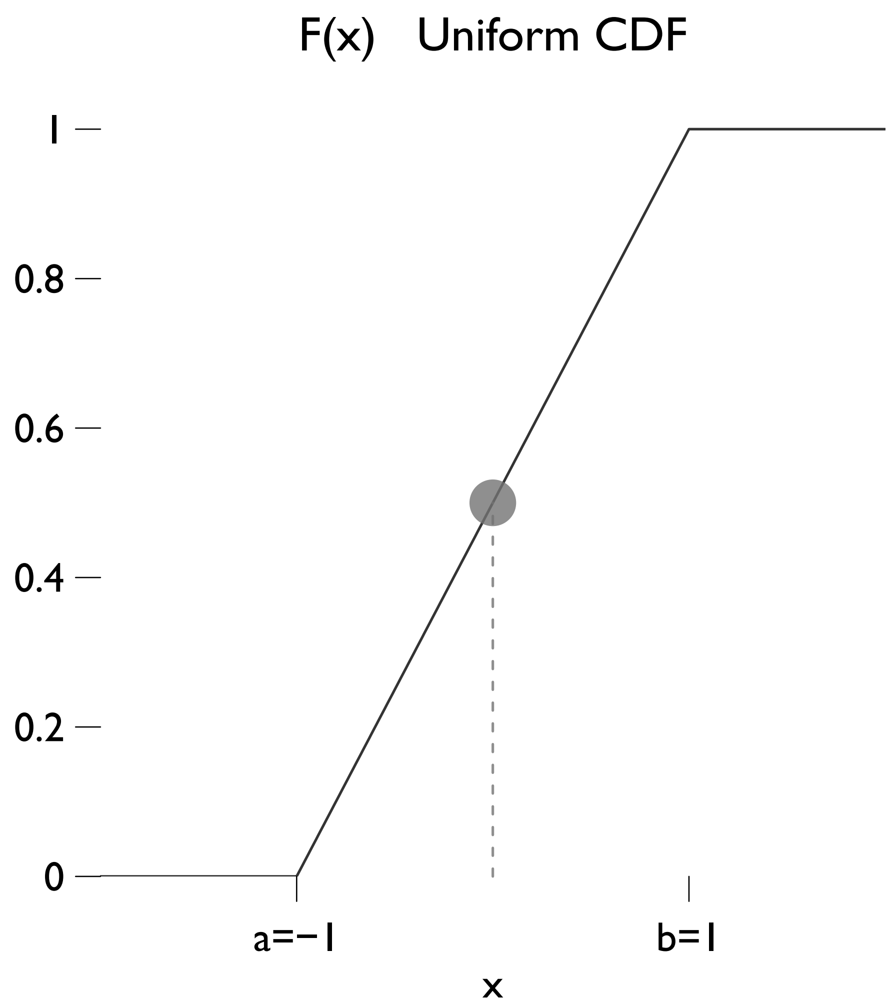

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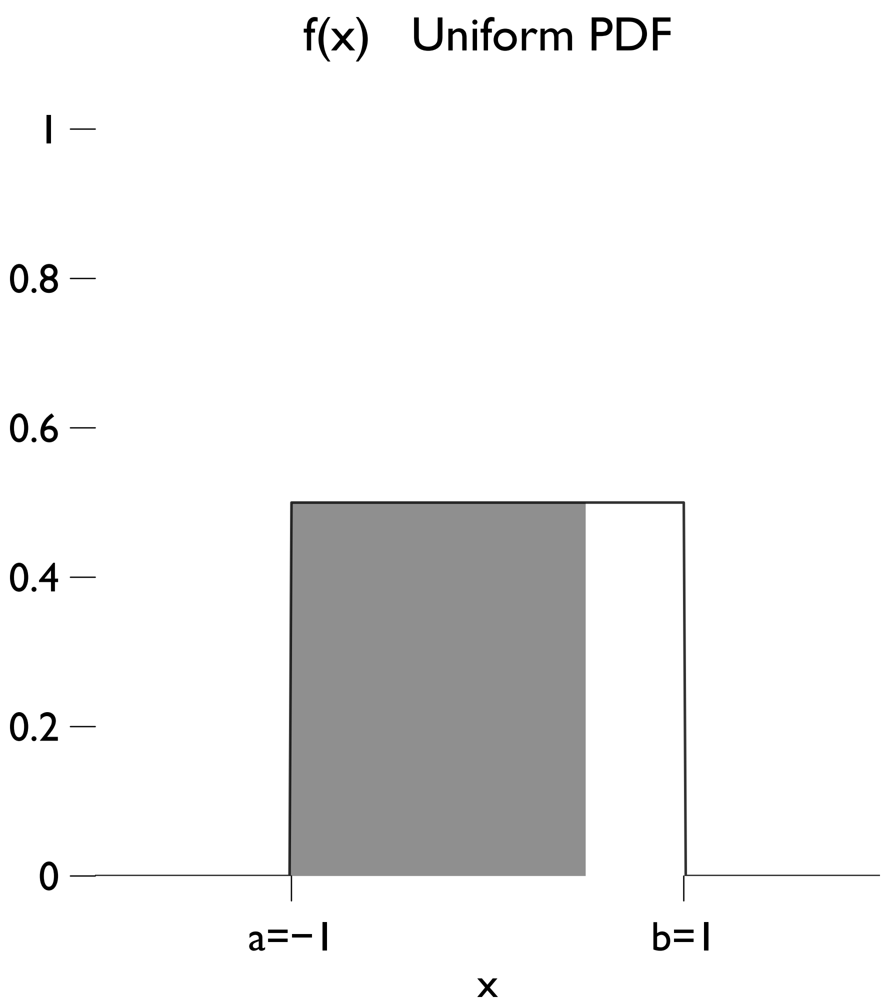

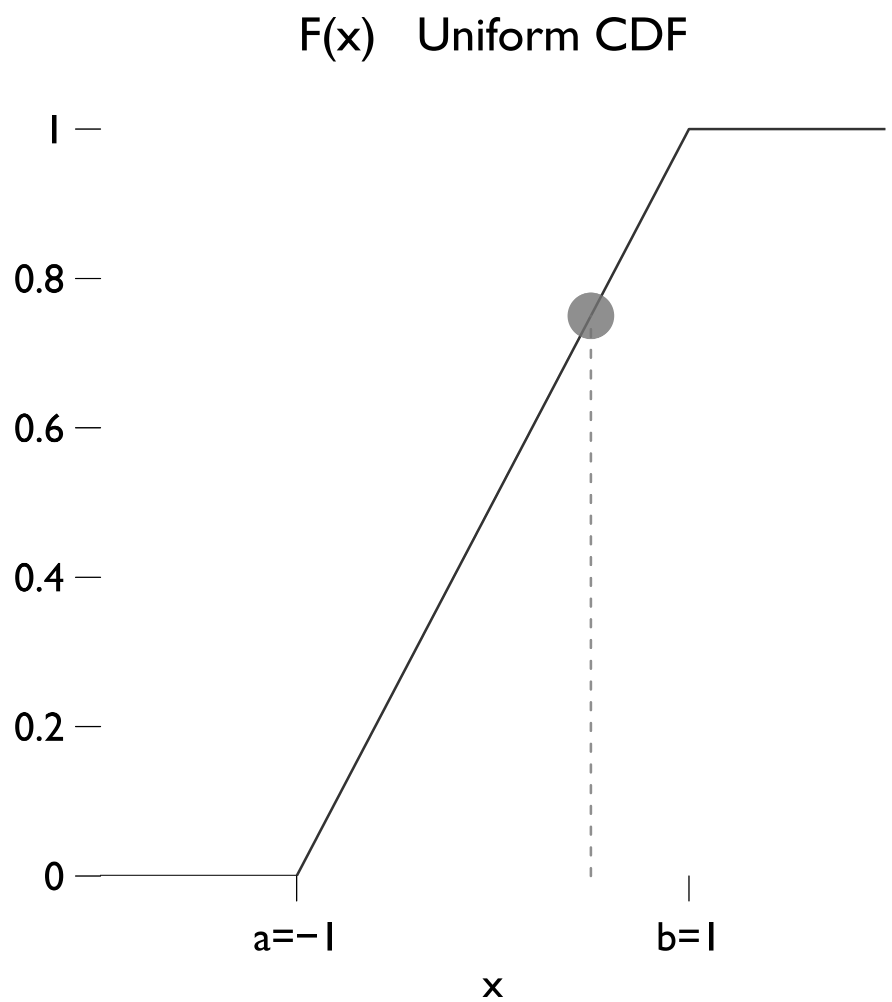

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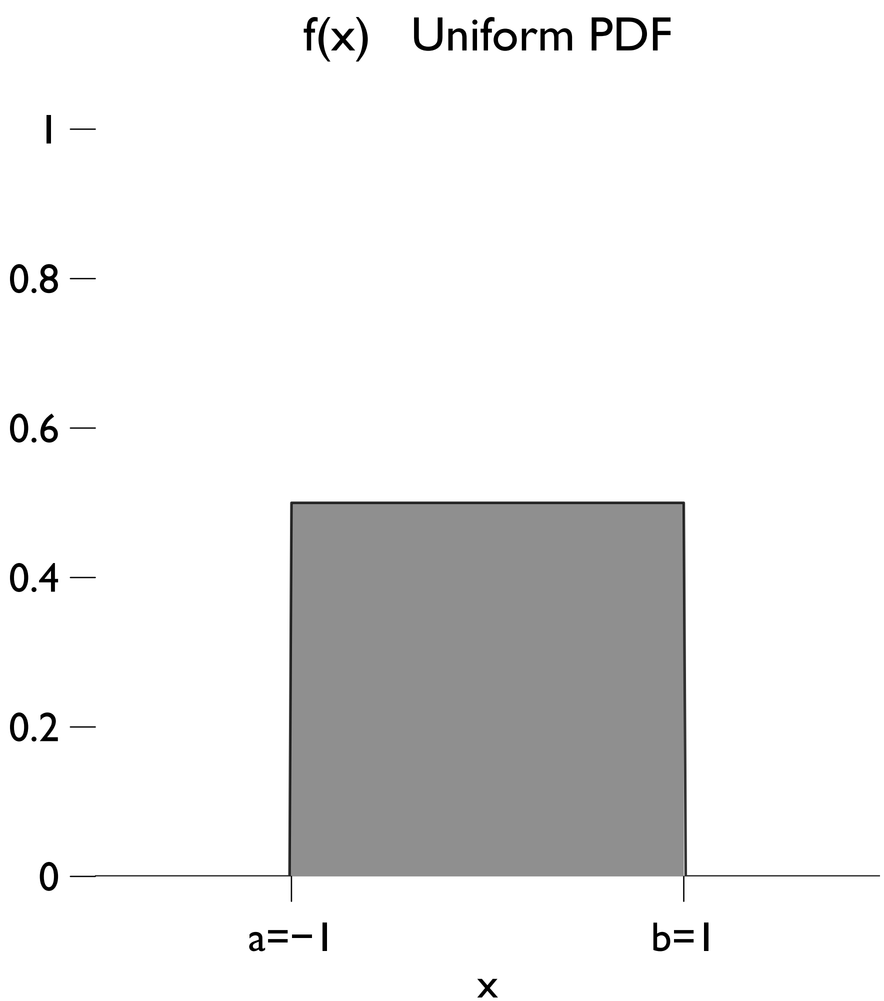

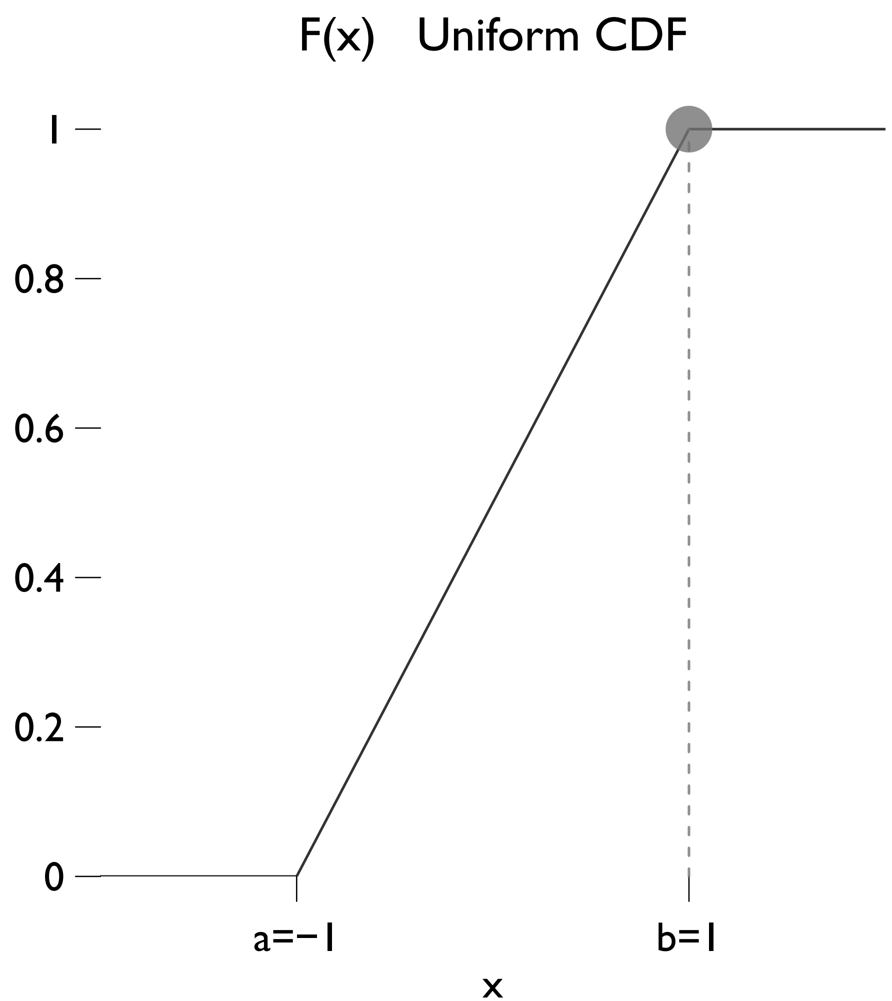

The uniform distribution

Moments of the Uniform distribution

$$\mathbb{E}(x) = \frac{1}{2}(a+b)$$

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- *Why?* Because the choice of a, b is arbitrary and important

The Normal (or Gaussian) distribution

The Central Limit Theorem holds that the sum of a "large" ($N \to \infty$) number of independently distributed random variables is distributed as

$$f_{\mathcal{N}}(x|\mu, \sigma^2) = (2\pi\sigma^2)^{-1/2} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$$

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The cdf of the Normal has no closed form representation (hard integral):

$$F_{\mathcal{N}} = \int f_{\mathcal{N}} = \Phi(x|\mu, \sigma^2)$$

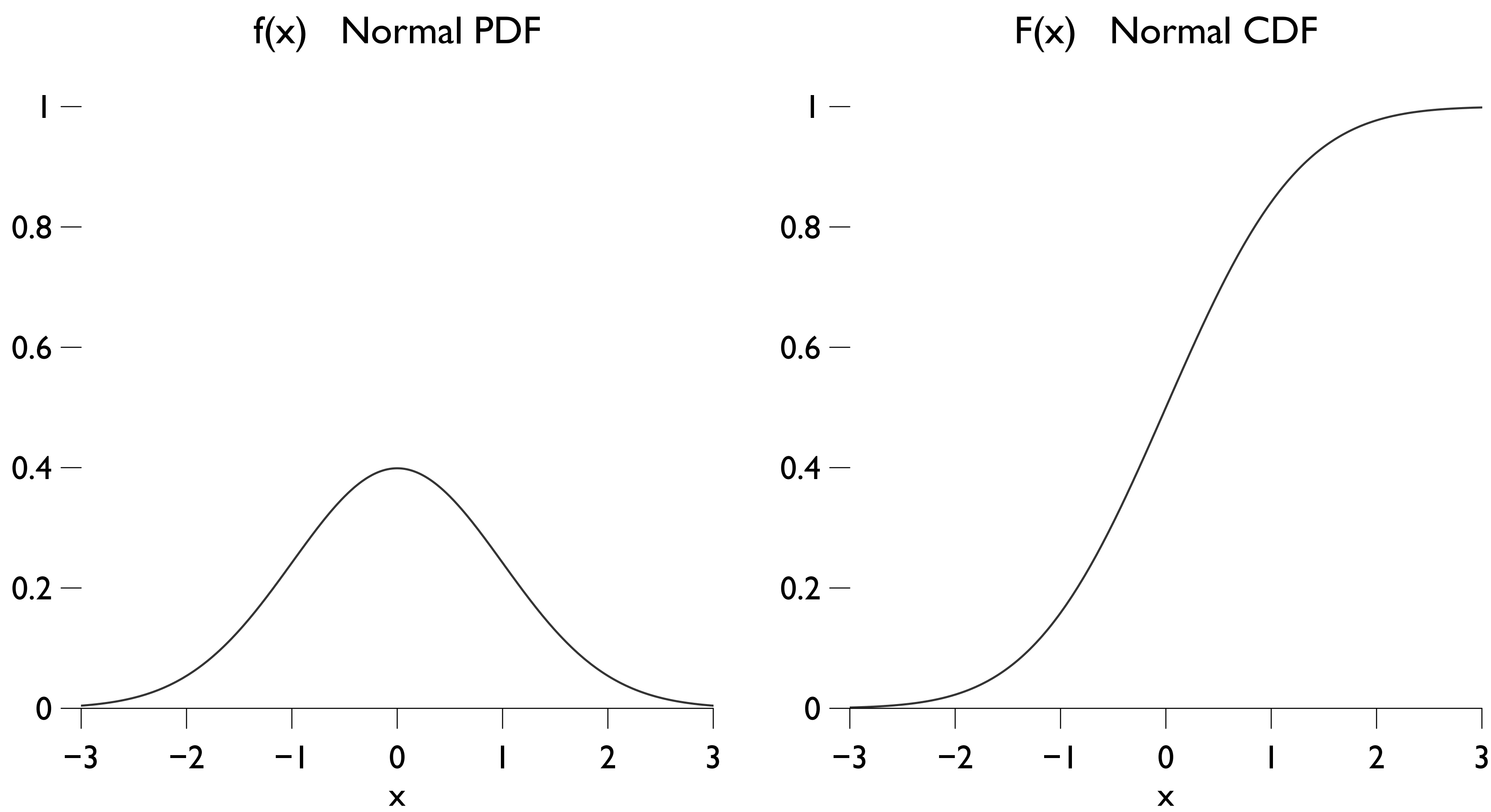
When we need the cdf, we will rely on numerical approximations (quadrature)

The Normal (Gaussian) distribution

We've already seen the Normal, as our example of a continuous distribution

This special case is known as the *Standard Normal distribution*

The Standard Normal has mean 0 and variance 1



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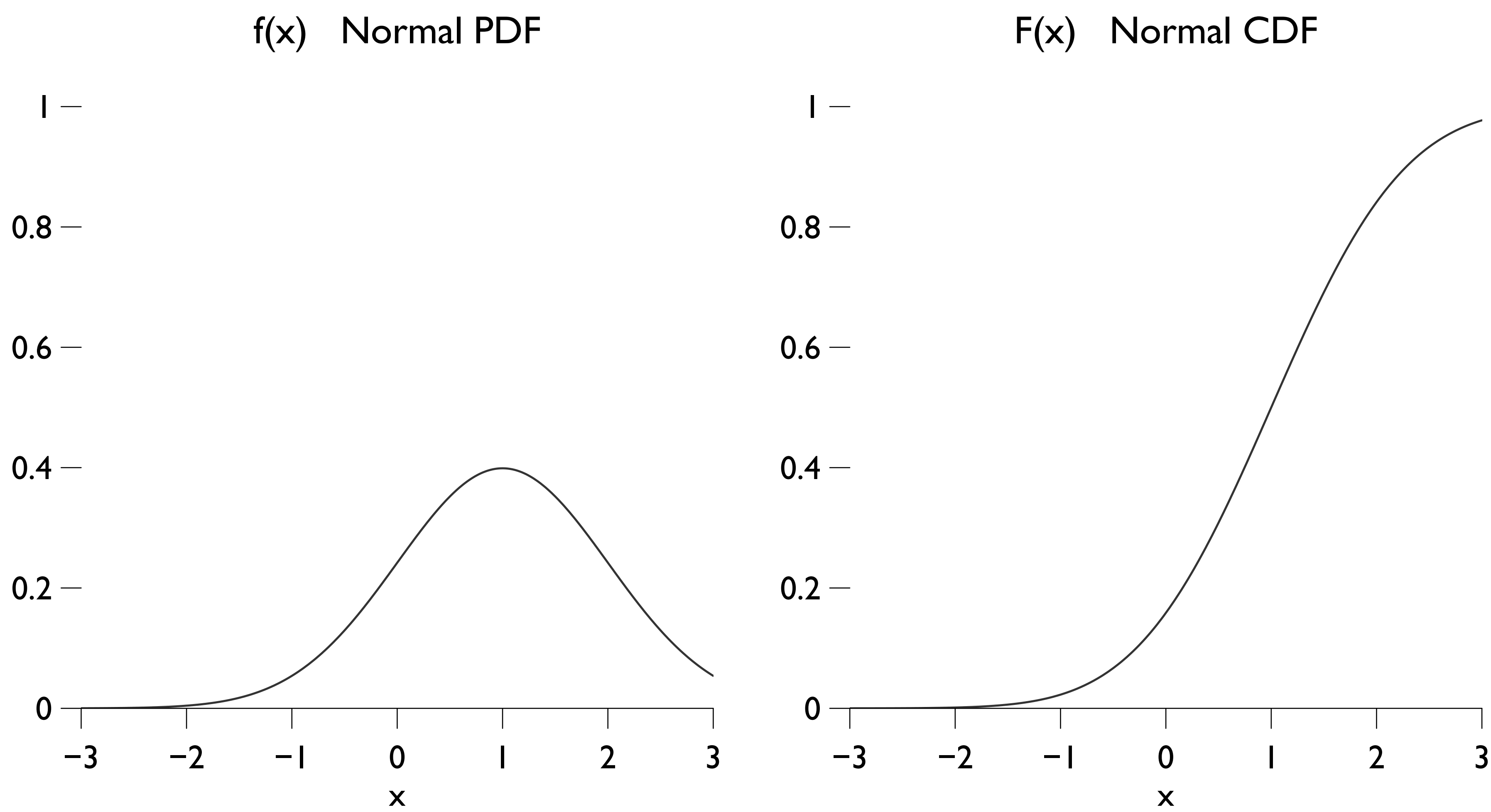
Changing the mean shifts curve's location, but preserves its shape

This Normal has mean 1 and variance 1



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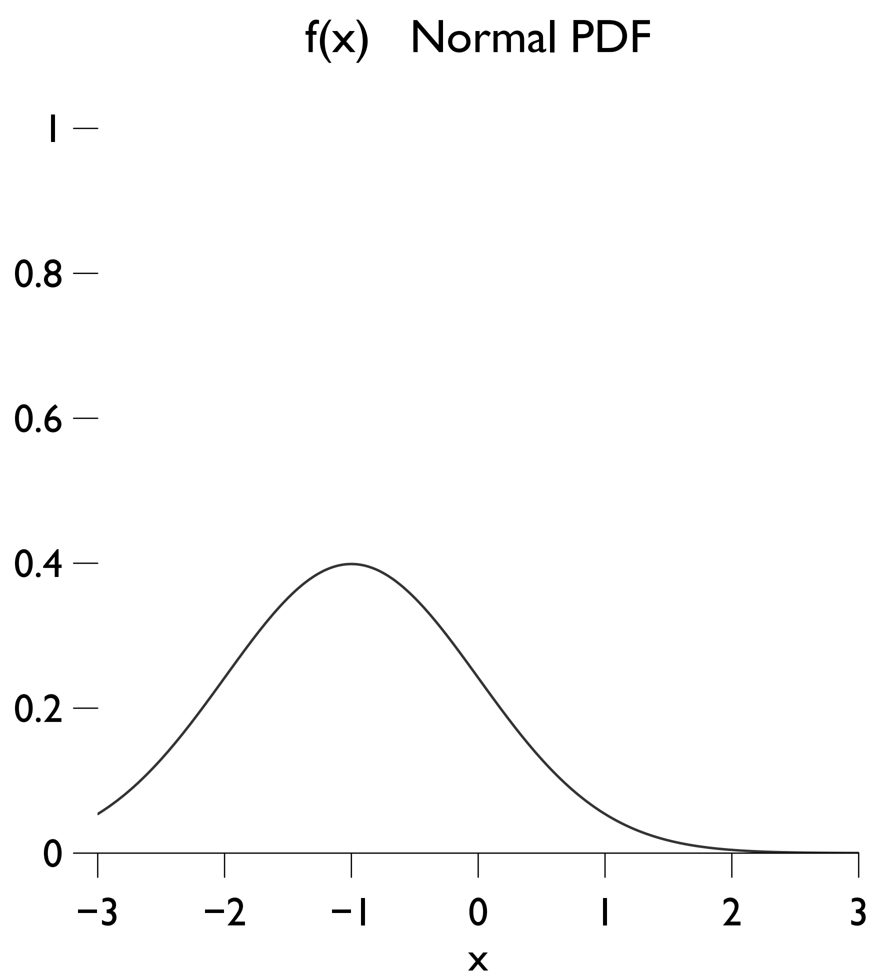

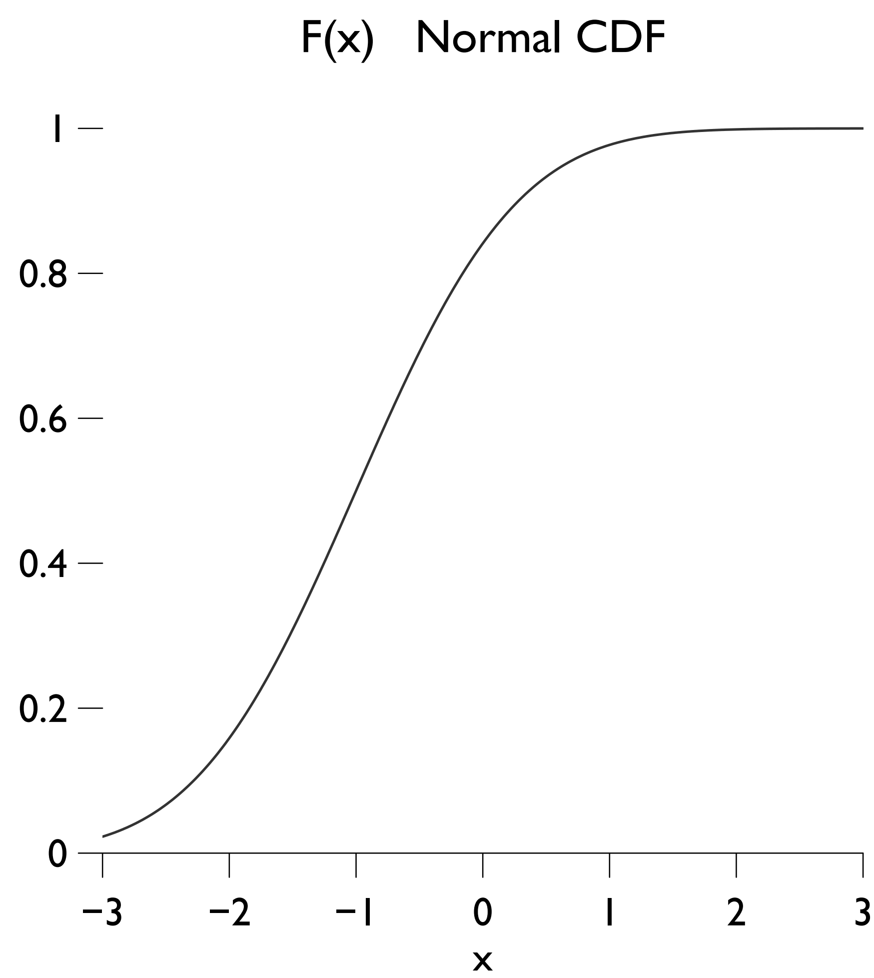

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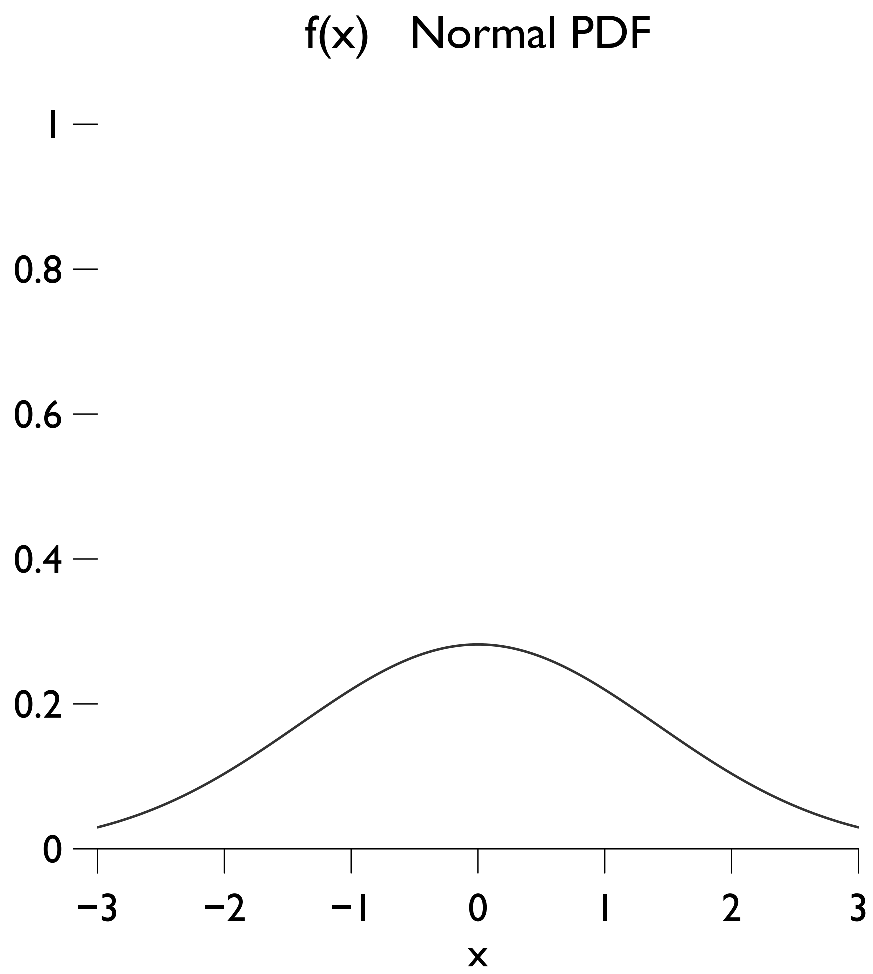
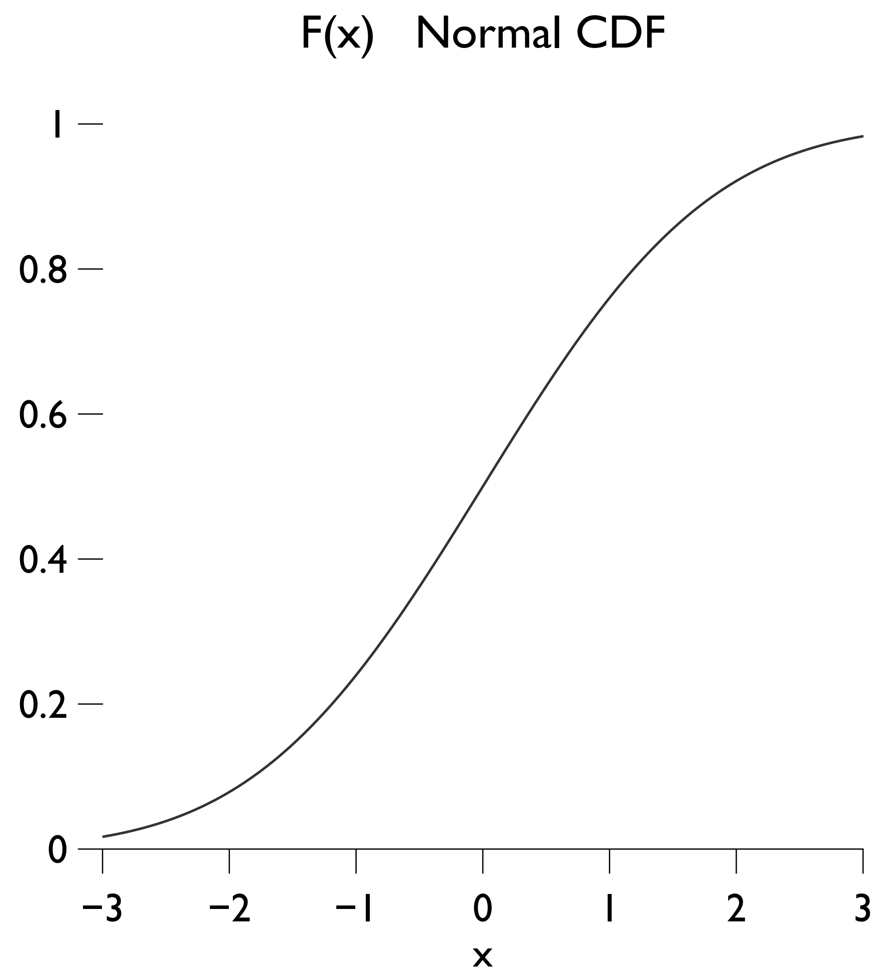
Changing the variance shifts curve's shape, but preserves its location

This Normal has mean 1 and variance 2





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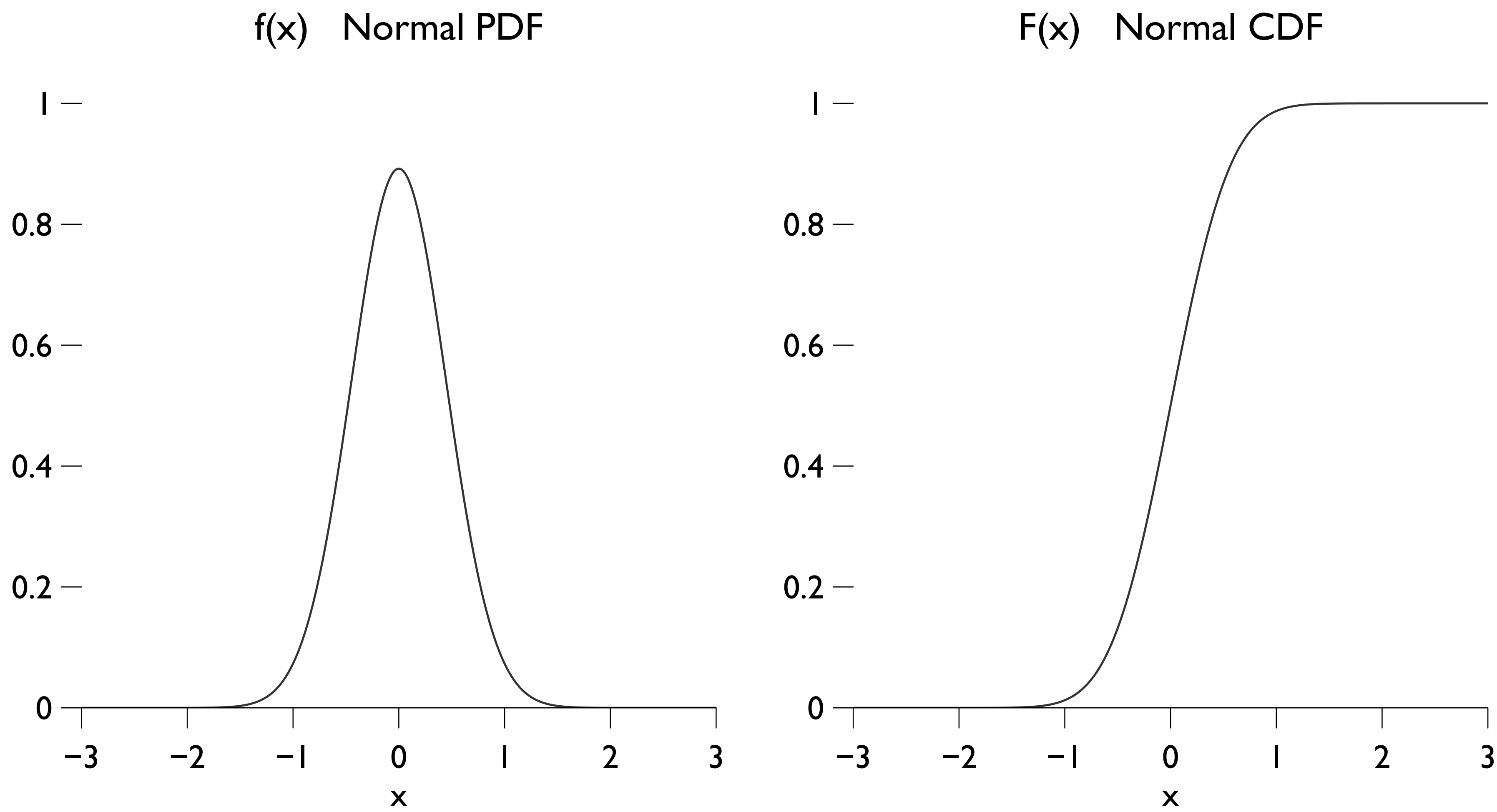
This Normal has mean 1 and variance 0.2



Why R?

Real question: Why programming?

Non-programmers are stuck with package defaults

For your substantive problem, these default settings may be

- inappropriate (not quite the right model, but "close")
- unintelligible (reams of non-linear coefficients and stars)

Programming allows you to match the methods to the data & question

Get better, more easily explained results.

Why R?

Many side benefits:

1. Never forget what you did: The code can be re-run.
2. Repeating an analysis n times? Write a loop!
3. Programming makes data processing/reshaping easy.
4. Programming makes replication easy.

Why R?

R is

- free
- open source
- growing fast
- widely used
- the future for most fields

But once you learn one language, the others are much easier

Introduction to R

R is a calculator that can store lots of information in memory

R stores information as "objects"

```
> x <- 2
> print(x)
[1] 2

> y <- "hello"
> print(y)
[1] "hello"

> z <- c(15, -3, 8.2)
> print(z)
[1] 15.0 -3.0  8.2
```

Introduction to R

```
> w <- c("gdp", "pop", "income")
> print(w)
[1] "gdp"    "pop"    "income"
>
```

Note the assignment operator, <-, not =

An object in memory can be called to make new objects

```
> a <- x^2
> print(x)
[1] 2
> print(a)
[1] 4

> b <- z + 10
> print(z)
[1] 15.0 -3.0  8.2
> print(b)
[1] 25.0  7.0 18.2
```

Introduction to R

```
> c <- c(w,y)
> print(w)
[1] "gdp"    "pop"    "income"
> print(y)
[1] "hello"
> print(c)
[1] "gdp"    "pop"    "income" "hello"
```

Commands (or “functions”) in R are always written `command()`

The usual way to use a command is:

```
output <- command(input)
```

We’ve already seen that `c()` pastes together variables.

A simple example:

```
> z <- c(15, -3, 8.2)
> mz <- mean(z)
> print(mz)
[1] 6.733333
```

Introduction to R

Some commands have multiple inputs. Separate them by commas:

`plot(var1,var2)` plots var1 against var2

Some commands have optional inputs. If omitted, they have default values.

`plot(var1)` plots var1 against the sequence $\{1,2,3,\ldots\}$

Inputs can be identified by their position or by name.

`plot(x=var1,y=var2)` plots var2 against var1

Entering code

You can enter code by typing at the prompt, by cutting or pasting, or from a file

If you haven't closed the parenthesis, and hit enter, `R` let's you continue with this prompt +

You can copy and paste multiple commands at once

You can run a text file containing a program using `source()`, with the name of the file as input (ie, in "")

I prefer the `source()` approach. Leads to good habits of retaining code.

Data types

R has three important data types to learn now

Numeric	`y <- 4.3`
Character	`y <- "hello"`
Logical	`y <- TRUE`

We can always check a variable's type, and sometimes change it:

```
population <- c("1276", "562", "8903")
print(population)
is.numeric(population)
is.character(population)
```

Oops! The data have been read in as characters, or "strings". R does not know they are numbers.

```
population <- as.numeric(population)
```

Some special values

Missing data	`NA`
A "blank"	`NULL`
Infinity	`Inf`
Not a number	`NaN`

Data structures

All R objects have a data type *and* a data structure

Data structures can contain numeric, character, or logical entries

Important structures:

Vector

Matrix

Dataframe

List (to be covered later)

Vectors in R

Vector is R are simply 1-dimensional lists of numbers or strings

Let's make a vector of random numbers:

```
x <- rnorm(1000)
```

x contains 1000 random normal variates drawn from a Normal distribution with mean 0 and standard deviation 1.

What if we wanted the mean of this vector?

```
mean(x)
```

What if we wanted the standard deviation?

```
sd(x)
```

Vectors in R

What if we wanted just the first element?

```
x[1]
```

or the 10th through 20th elements?

```
x[10:20]
```

what if we wanted the 10th percentile?

```
sort(x)[100]
```

Indexing a vector can be very powerful. Can apply to any vector object.

What if we want a histogram?

```
hist(x)
```

Vectors in R

Useful commands for vectors:

`seq(from, to, by)`	generates a sequence
`rep(x,times)`	repeats `x`
`sort()`	sorts a vector from least to greatest
`rev()`	reverses the order of a vector
`rev(sort())`	sorts a vector from greatest to least

Matrices in R

Vector are the standard way to store and manipulate variables in R

But usually our datasets have several variables measured on the same observations

Several variables collected together form a matrix with one row for each observation and one column for each variable

Matrices in R

Many ways to make a matrix in R

```
a <- matrix(data=NA, nrow, ncol, byrow=FALSE)
```

This makes a matrix of `nrow` $\times$ `ncol`, and fills it with missing values.

To fill it with data, substitute a vector of data for NA in the command. It will fill up the matrix column by column.

We could also paste together vectors, binding them by column or by row:

```
b <- cbind(var1, var2, var3)
c <- rbind(obs1, obs2)
```

Matrices in R

Optionally, R can remember names of the rows and columns of a matrix

To assign names, use the commands:

```
colnames(a) <- c("Var1", "Var2")
rownames(a) <- c("Case1", "Case2")
```

Substituting the actual names of your variables and observations (and making sure there is one name for each variable & observation)

Matrices in R

Matrices are indexed by row and column.

We can subset matrices into vectors or smaller matrices

`a[1,1]`	Gets the first element of a
`a[1:10,1]`	Gets the first ten rows of the first column
`a[,5]`	Gets every row of the fifth column
`a[4:6,]`	Gets every column of the 4th through 6th rows

To make a vector into a matrix, use `as.matrix()`

R defaults to treating one-dimensional arrays as vectors, not matrices

Useful matrix commands:

`nrow()`	Gives the number of rows of the matrix
`ncol()`	Gives the number of columns
`t()`	Transposes the matrix

Much more on matrices next week.

Dataframes in R

Dataframes are a special kind of matrix used to store datasets

To turn a matrix into a dataframe (note the extra .):

```
a <- as.data.frame(a)
```

Dataframes always have columns names, and these are set or retrieved using the `names()` command

```
names(a) <- c("Var1","Var2")
```

You can access a variable from a dataframe directly using $:

```
a$Var1
```

Dataframes can also be “attached”, which makes each column into a vector with the appropriate name

```
attach(a)
```

Loading data

There are many ways to load data to R.

I prefer using comma-separated variable files, which can be loaded with `read.csv()`

You can also check the `foreign` library for other data file types

Suppose you load a dataset using

```
data <- read.csv("mydata.csv")
```

You can check out the names of the variables using `names(data)`

And access any variables, such as gdp, using `data$gdp`

Benefits and dangers of `attach()`

If your data have variable names, you can also "attach" the dataset like so:

```
data <- read.csv("mydata.csv")
attach(data)
```

to access all the variables directly through newly created vectors.

Be careful! `attach()` is tricky.

1. If you attach a variable `data$x` in `data` and then modify `x`, the original `data$x` is unchanged.

2. If you have more than one dataset with the same variable names, `attach()` is a bad idea: only one dataset can be attached!

Sometimes `attach()` is handy, but be careful!

Missing data

When loading a dataset, you can often tell R what symbol that file uses for missing data using the option `na.strings=`

So if your dataset codes missings as ., set `na.strings="."`

If your dataset codes missings as a blank, set `na.strings=""`

If your dataset codes missings in multiple ways, you could set, e.g., `na.strings=c(".","","NA")`

Missing data

Many R commands will not work properly on vectors, matrices, or dataframes containing missing data (NAs)

To check if a variables contains missings, use `is.na(x)`

To create a new variable with missings listwise deleted, use `na.omit`

If we have a dataset `data` with NAs at data[15,5] and data[17,3]

```
dataomitted <- na.omit(data)
```

will create a new dataset with the 15th and 17th rows left out

Be careful! If you have a variable with lots of NAs you are not using in your analysis, remove it from the dataset *before* using `na.omit()`

Mathematical Operations

R can do all the basic math you need

Binary operators:

`+   -   *   /   ^`

Binary comparisions:

`<   <=   >   >=   ==   !=`

Logical operators (and, or, not, control-flow and, control-flow not; use parentheses!):

`&   |   !   &&   ||`

Math/stat fns:

`log   exp   mean   median   min   max   sd   var   cov   cor`

Set functions (see `help(sets)`), Trigonometry (see `help(Trig)`),

R follows the usual order of operations; if it doubt, use parentheses

Example 1: US Economic growth

Let's investigate an old question in political economy:

Are there partisan cycles, or tendencies, in economic performance?

Does one party tend to produce higher growth on average?

(Theory: Left cares more about growth vis-a-vis inflation than the Right

If there is partisan control of the economy,
then Left should have higher growth ceteris paribus)

Data from the Penn World Tables (Annual growth rate of GDP in percent)

Two variables:

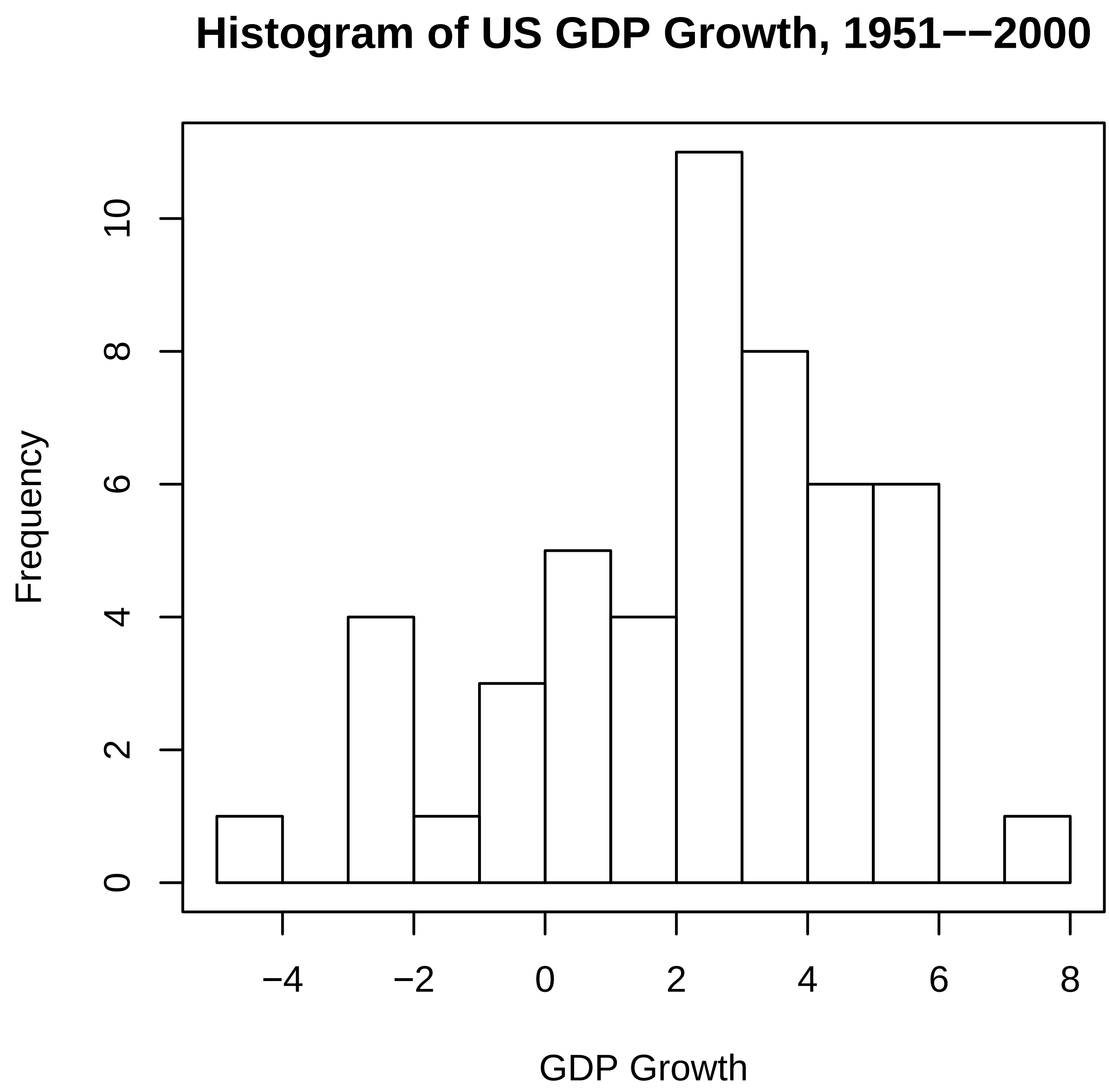
`grgdpch`	The per capita GDP growth rate
`party`	The party of the president (Dem = -1, Rep = 1)

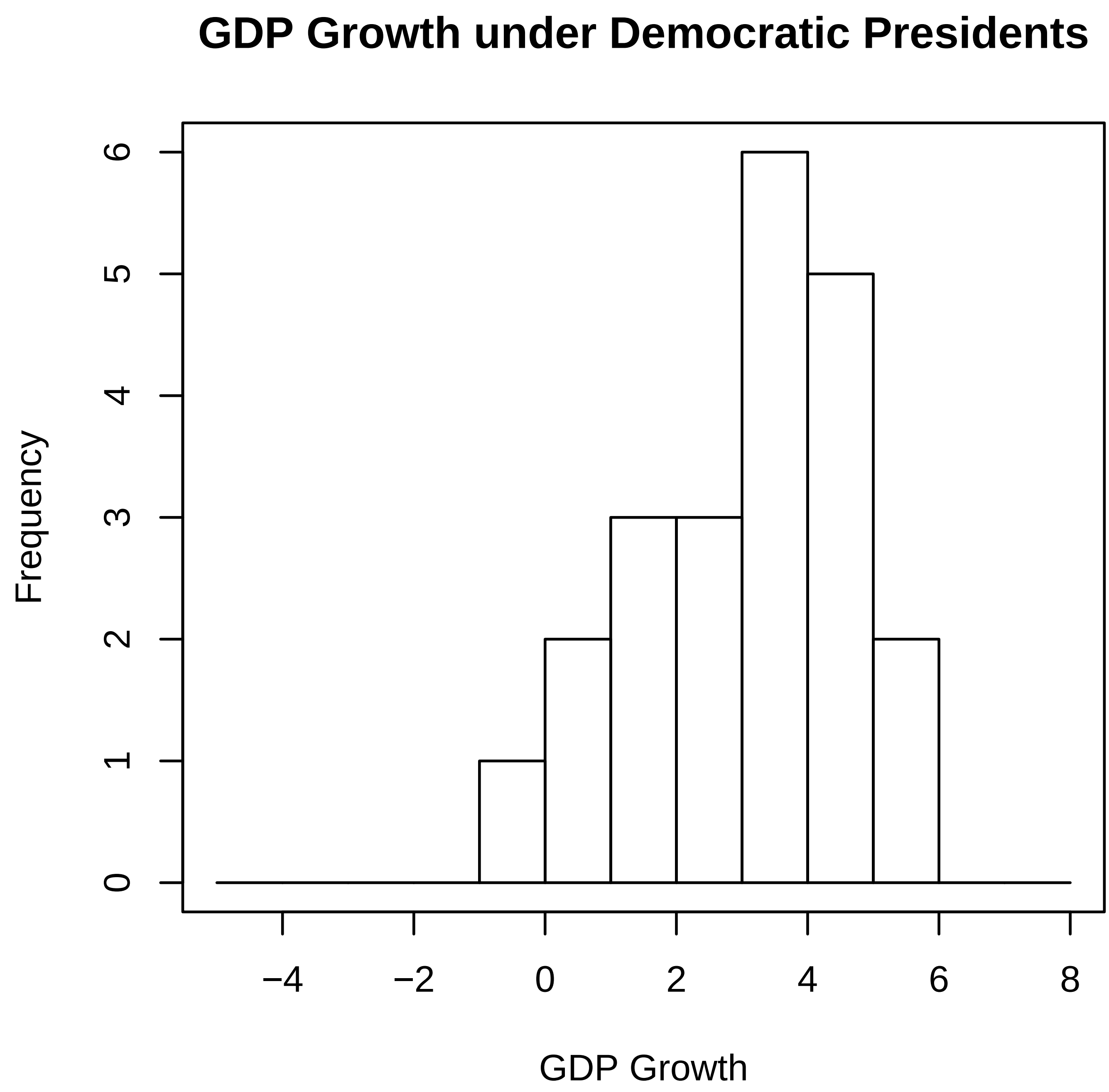
Example 1: US Economic growth

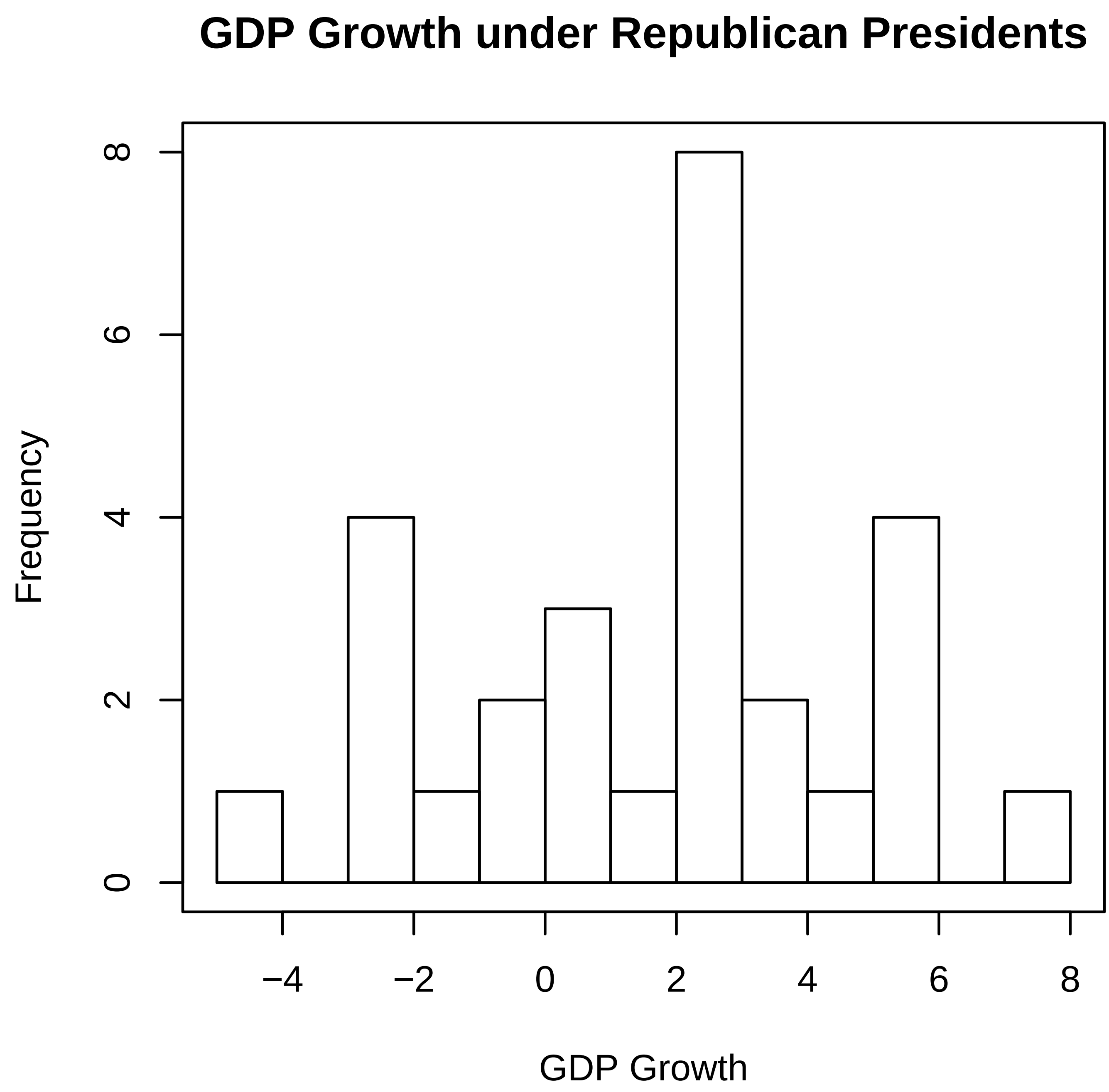
```
# Load data
data <- read.csv("gdp.csv", na.strings="")
attach(data)

# Construct party specific variables
gdp.dem <- grgdpch[party==-1]
gdp.rep <- grgdpch[party==1]

# Make the histogram
hist(grgdpch,
     breaks=seq(-5,8,1),
     main="Histogram of US GDP Growth, 1951--2000",
     xlab="GDP Growth")
```


Histogram of US GDP Growth, 1951−−2000
Frequency
0
2
4
6
8
10
−4
−2
0
2
4
6
8
GDP Growth


GDP Growth under Democratic Presidents
Frequency
0
1
2
3
4
5
6
−4
−2
0
2
4
6
8
GDP Growth


GDP Growth under Republican Presidents
Frequency
0
2
4
6
8
−4
−2
0
2
4
6
8
GDP Growth

```
# Make a box plot
boxplot(grgdpch~as.factor(party),
        boxwex=0.3,
        range=0.5,
        names=c("Democratic\n Presidents",
          "Republican\n Presidents"),
        ylab="GDP growth",
        main="Economic performance of partisan governments")
```

Note the unusual first input: this is an R formula

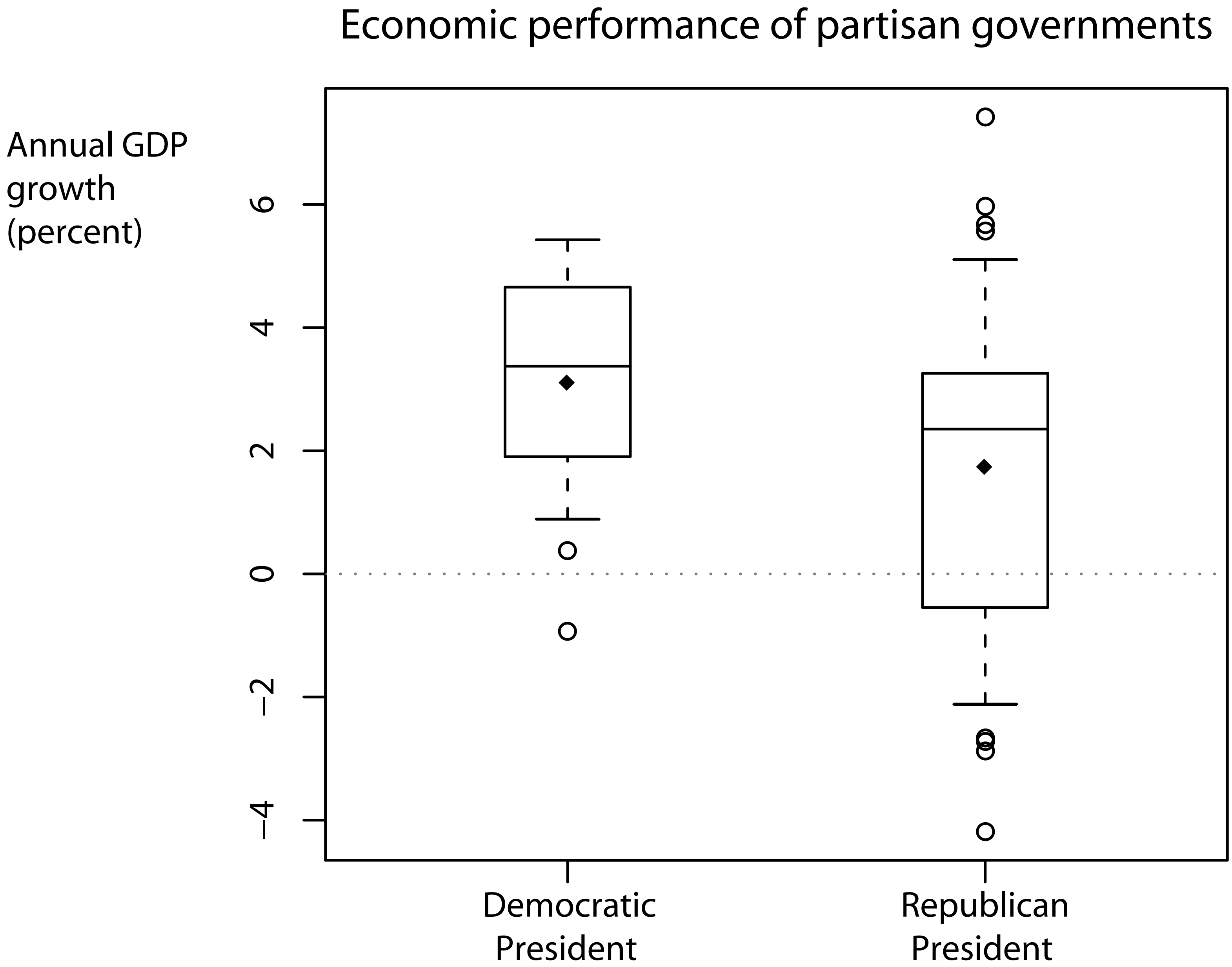
`y~x1+x2+x3`

In this case, `grgdpch` is being "modelled" as a function of `party`

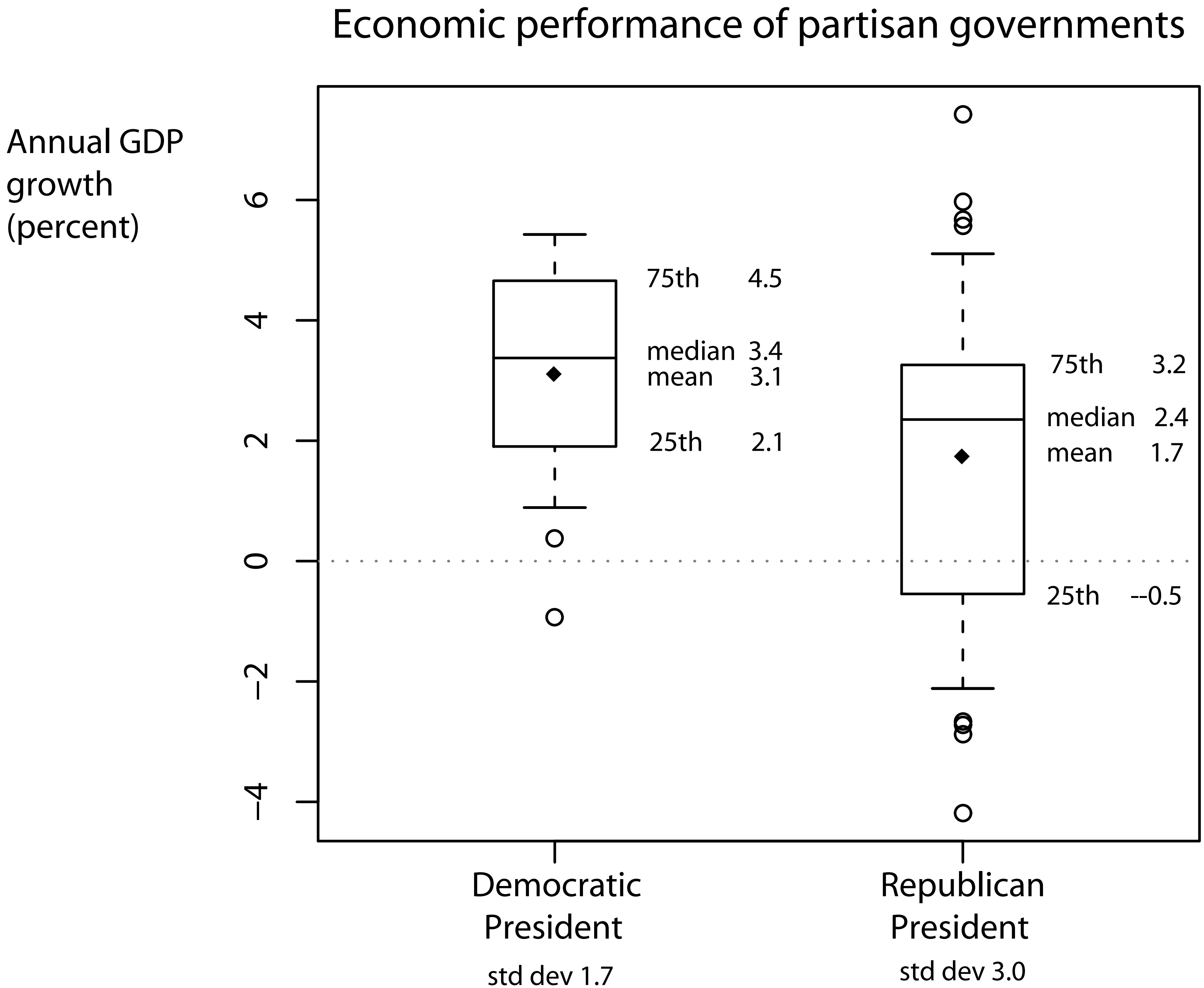
`boxplot()` needs party to be a "factor" or an explicitly categorical variable

Hence we pass boxplot `as.factor(party)`,
which turns the numeric variable into a factor

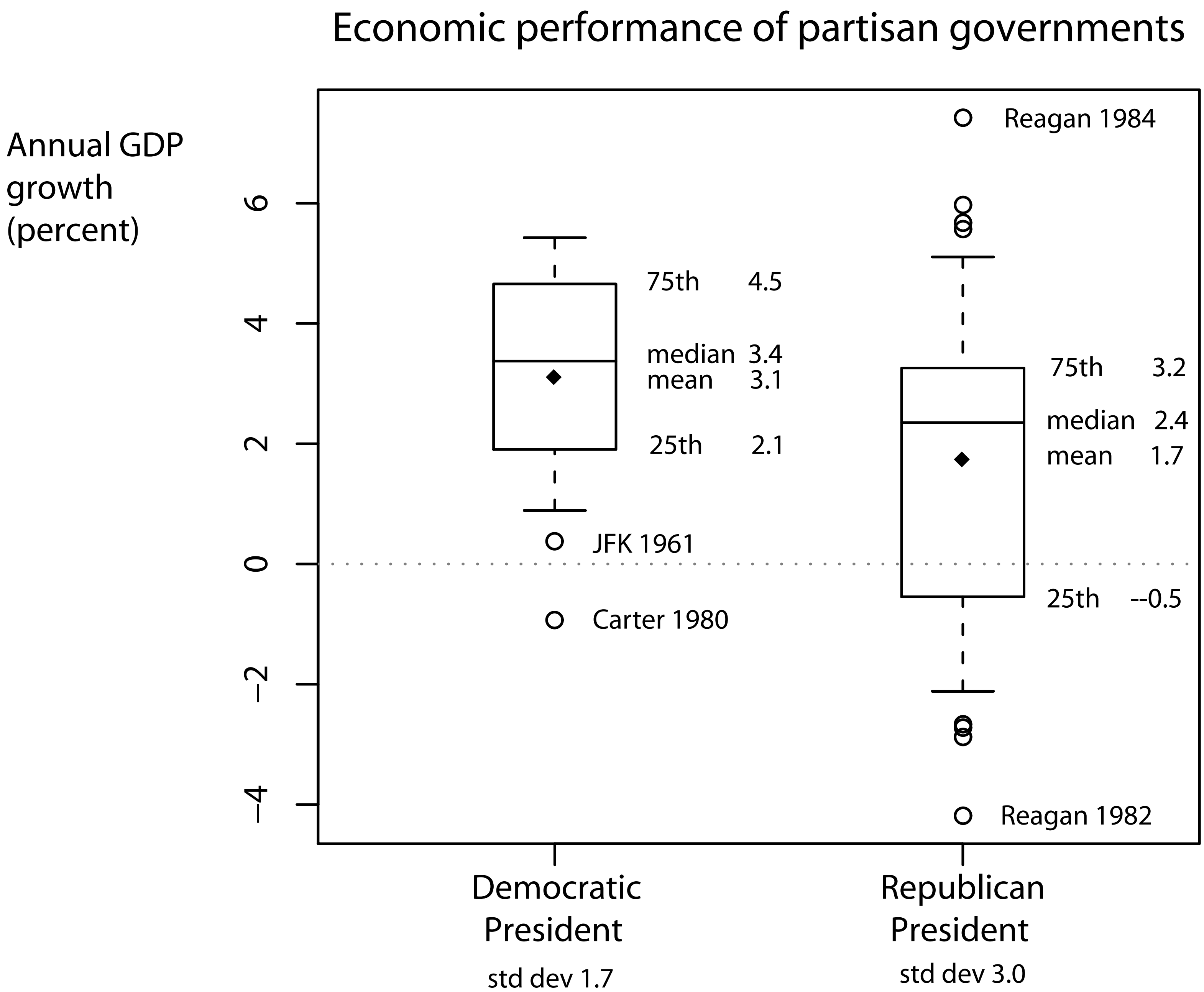
Box plots: Annual US GDP growth, 1951–2000



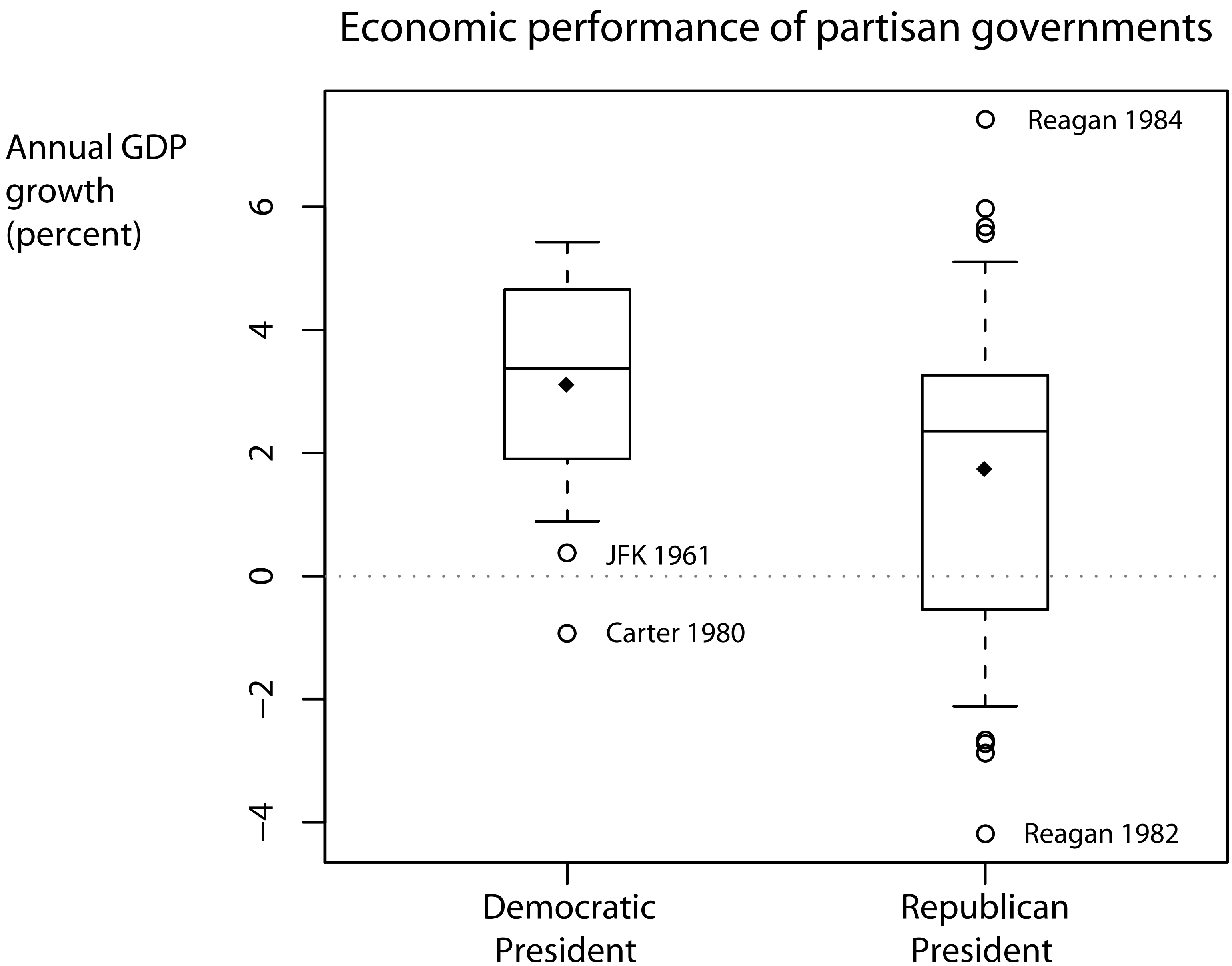
Box plots: Annual US GDP growth, 1951–2000



Box plots: Annual US GDP growth, 1951–2000



Box plots: Annual US GDP growth, 1951–2000



Help!

To get help on a known command `x`, type `help(x)` or `?x`

To search the help files using a keyword string `s`, type `help.search(s)`

Note that this implies to search on the word regression, you should type `help.search("regression")`

but to get help for the command `lm`, you should type `help(lm)`

Hard to use Google directly for R help ("r" is kind of a common letter)

Easiest way to get help from the web: `rseek.org`

Rseek tries to limit results to R topics (not wholly successful)

Installing R on a PC

- Go to the Comprehensive R Archive Network (CRAN)
  `http://cran.r-project.org/`

- Under the heading "Download and Install R", click on "Download R for Windows"

- Click on "base"

- Download and run the R setup program.
  The name changes as R gets updated;
  the current version is "R-3.4.1-win.exe"

- Once you have R running on your computer,
  you can add new libraries from inside R by selecting
  "Install packages" from the Packages menu

Installing R on a Mac

- Go to the Comprehensive R Archive Network (CRAN)
  `http://cran.r-project.org/`

- Under the heading "Download and Install R", click on "Download R for MacOS X"

- Download and run the R setup program.
  The name changes as R gets updated;
  the current version is "R-3.4.1.pkg"

- Once you have R running on your computer,
  you can add new libraries from inside R by selecting
  "Install packages" from the Packages menu

Editing scripts

Don't use Microsoft Word to edit R code!

Word adds lots of "stuff" to text; R needs the script in a plain text file.

Some text editors:

- Notepad: Free, and comes with Windows (under Start → Programs → Accessories). Gets the job done; not powerful.
- TextEdit: Free, and comes with Mac OS X. Gets the job done; not powerful.
- TINN-R: Free and powerful. Windows only.
  `http://www.sciviews.org/Tinn-R/`
- **Emacs:** Free and *very* powerful (my preference). Can use for R, Latex, and any other language. Available for Mac, PC, and Linux.

  For Mac (easy installation): `http://aquamacs.org/`

  For Windows (see the README): `http://ftp.gnu.org/gnu/emacs/windows/`

Editing data

R can load many other packages' data files

See the `foreign` library for commands

For simplicity & universality, I prefer Comma-Separated Variable (CSV) files

Microsoft Excel can edit and export CSV files (under Save As)

R can read them using `read.csv()`

OpenOffice free alternative to Excel (for Windows and Unix):
`http://www.openoffice.org/`

My detailed guide to installing social science software on the Mac:
`http://thewastebook.com/?post=social-science-computing-for-mac`

Focus on steps 1.1 and 1.3 for now; come back later for Latex in step 1.2

Example 2: A simple linear regression

Let's investigate a bivariate relationship

Cross-national data on fertility (children born per adult female) and the percentage of women practicing contraception.

Data are from 50 developing countries.

Source: Robey, B., Shea, M. A., Rutstein, O. and Morris, L. (1992) "The reproductive revolution: New survey findings." *Population Reports.* Technical Report M-11.

Example 2: A simple linear regression

```
# Load data
data <- read.csv("robeymore.csv", na.strings="")
completedata <- na.omit(data)
attach(completedata)

# Transform variables
contraceptors <- contraceptors/100

# Run linear regression
res.lm <- lm(tfr~contraceptors)
print(summary(res.lm))

# Get predicted values
pred.lm <- predict(res.lm)
```

Example 2: A simple linear regression

```
# Make a plot of the data
plot(x=contraceptors,
     y=tfr,
     ylab="Fertility Rate",
     xlab="% of women using contraception",
     main="Average fertility rates & contraception; \n
           50 developing countries",
     xaxp=c(0,1,5)
     )

# Add predicted values to the plot
points(x=contraceptors,y=pred.lm,pch=16,col="red")
```

Example 2: A simple linear regression

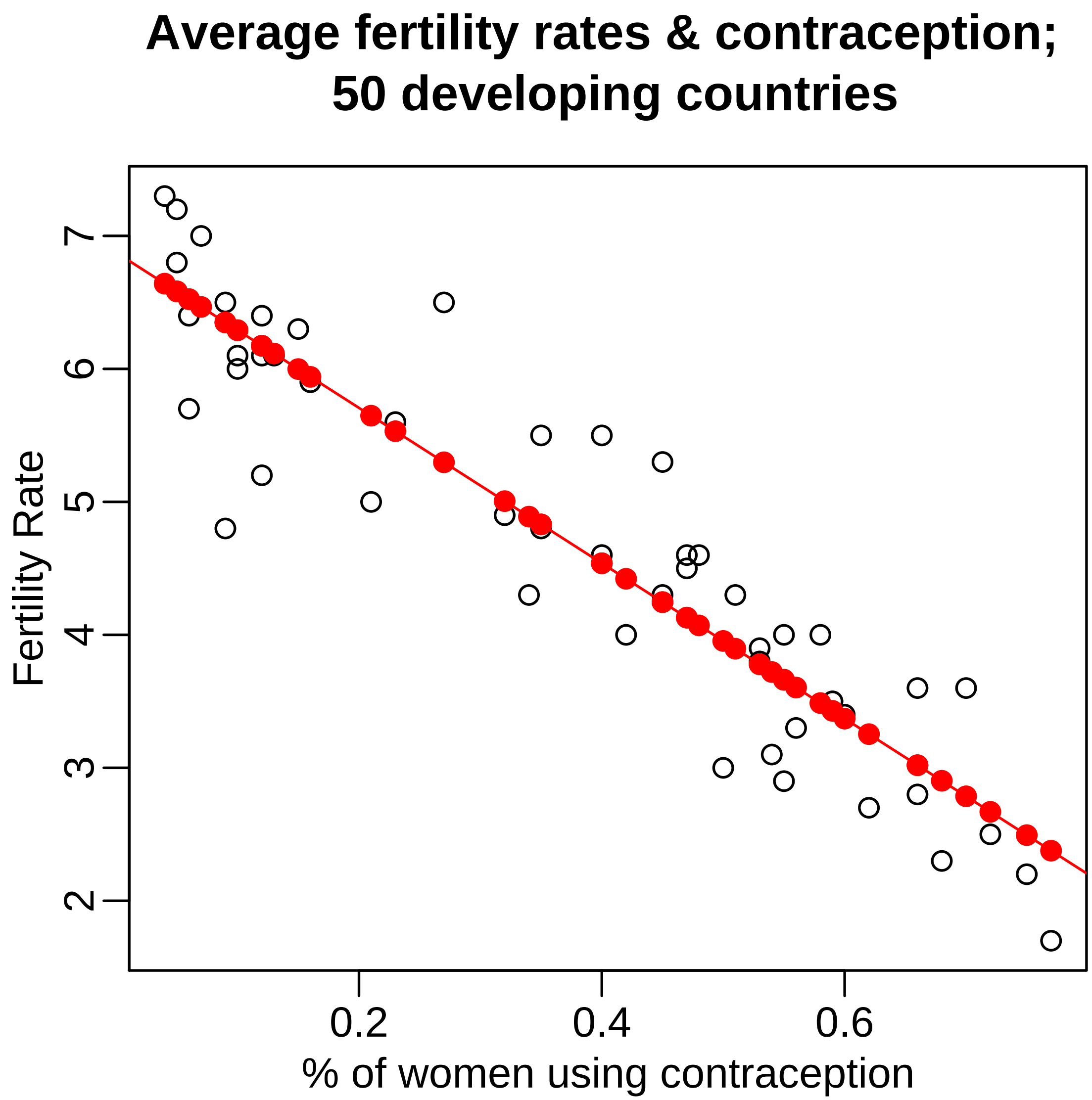
```
> summary(res.lm)

Call:
lm(formula = tfr ~ contraceptors)

Residuals:
     Min       1Q   Median       3Q      Max
-1.54934 -0.30133  0.02540  0.39570  1.20214

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)     6.8751     0.1569   43.83   <2e-16 ***
contraceptors  -5.8416     0.3584  -16.30   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5745 on 48 degrees of freedom
Multiple R-Squared: 0.847,      Adjusted R-squared: 0.8438
F-statistic: 265.7 on 1 and 48 DF,  p-value: < 2.2e-16
```


Average fertility rates & contraception;
50 developing countries
Fertility Rate
2
3
4
5
6
7
0.2
0.4
0.6
% of women using contraception

Matrix Algebra in R

`det(a)`	Computes the determinant of matrix `a`
`solve(a)`	Computes the inverse of matrix `a`
`t(a)`	Takes the transpose of `a`
`a%*%b`	Matrix multiplication of `a` by `b`
`a*b`	Element by element multiplication

An R list is a basket containing many other variables

```
> x <- list(a=1, b=c(2,15), giraffe="hello")

> x$a
[1] 1

> x$b
[1]  2 15

> x$b[2]
[1] 15

> x$giraffe
[1] "hello"

> x[3]
$giraffe
[1] "hello"

> x[["giraffe"]]
[1] "hello"
```

R lists

Things to remember about lists

- Lists can contain any number of variables of any type
- Lists can contain other lists
- Contents of a list can be accessed by name or by position
- Allow us to move lots of variables in and out of functions
- Functions often return lists (only way to have multiple outputs)

lm() basics

```
# To run a regression
res <- lm(y~x1+x2+x3,  # A model formula
          data         # A dataframe (optional)
          )

# To print a summary
summary(res)

# To get the coefficients
res$coefficients

# or
coef(res)

#To get residuals
res$residuals

#or

resid(res)
```

`lm()` basics

```
# To get the variance-covariance matrix of the regressors
vcov(res)

# To get the standard errors
sqrt(diag(vcov(res)))

# To get the fitted values
predict(res)

# To get expected values for a new observation or dataset
predict(res,
        newdata,                   # a dataframe with same x vars
                                   # as data, but new values
        interval = "confidence",   # alternative: "prediction"
        level = 0.95
        )
```

R lists & Object Oriented Programming

A list object in R can be given a special "class" using the `class()` function

This is just a metatag telling other R functions that this list object conforms to a certain format

So when we run a linear regression like this:

```
res <- lm(y~x1+x2+x3, data)
```

The result `res` is a list object of class ``lm''

Other functions like `plot()` and `predict()` will react to `res` in a special way because of this class designation

Specifically, they will run functions called `plot.lm()` and `predict.lm()`

Object-oriented programming:
a function does different things depending on class of input object

Example 3: Party systems & Redistribution

Example 3: Party systems & Redistribution

Cross sectional data on industrial democracies:

povertyReduction	Percent of citizens lifted out of poverty by taxes and transfers
effectiveParties	Effective number of parties
partySystem	Whether the party system is Majoritarian, Proportional, or Unanimity (Switzerland)

Source of data & plot: Torben Iversen and David Soskice, 2002, "Why do some democracies redistribute more than others?" Harvard University.

Considerations:

1. The marginal effect of each extra party is probably diminishing, so we want to log the effective number of parties

2. The party system variable needs to be "dummied out;" there are several ways to do this

Example 3: Party systems & Redistribution

```
# Clear memory of all objects
rm(list=ls())

# Load libraries
library(RColorBrewer)       # For nice colors

# Load data
file <- "iverRevised.csv"
iversen <- read.csv(file,header=TRUE)

# Create dummy variables for each party system
iversen$majoritarian <- as.numeric(iversen$partySystem=="Majoritarian")
iversen$proportional <- as.numeric(iversen$partySystem=="Proportional")
iversen$unanimity <- as.numeric(iversen$partySystem=="Unanimity")

# A bivariate model, using a formula to log transform a variable
model1 <- povertyReduction ~ log(effectiveParties)
lm.res1 <- lm(model1, data=iversen)
summary(lm.res1)
```

Example 3: Party systems & Redistribution

```
Call:
lm(formula = model1, data = iversen)

Residuals:
    Min      1Q  Median      3Q     Max 
-48.907  -4.115   8.377  11.873  18.101 

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)  
(Intercept)              21.80      16.15   1.349   0.2021  
log(effectiveParties)    24.17      12.75   1.896   0.0823 .
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1 

Residual standard error: 19.34 on 12 degrees of freedom
Multiple R-squared:  0.2305,Adjusted R-squared:  0.1664 
F-statistic: 3.595 on 1 and 12 DF,  p-value: 0.08229
```

Example 3: Party systems & Redistribution

```
# A new model with multiple regressors
model2 <- povertyReduction ~ log(effectiveParties) + majoritarian
            + proportional
lm.res2 <- lm(model2, data=iversen)
summary(lm.res2)
```

Example 3: Party systems & Redistribution

```
Call:
lm(formula = model2, data = iversen)

Residuals:
     Min       1Q   Median       3Q      Max
-23.3843  -1.4903   0.6783   6.2687  13.9376

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)             -31.29      26.55  -1.178  0.26588
log(effectiveParties)    26.69      14.15   1.886  0.08867 .
majoritarian             48.95      17.86   2.740  0.02082 *
proportional             58.17      13.52   4.302  0.00156 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 12.37 on 10 degrees of freedom
Multiple R-squared:  0.7378,Adjusted R-squared:  0.6592
F-statistic: 9.381 on 3 and 10 DF,  p-value: 0.002964
```

Example 3: Party systems & Redistribution

```
# A new model with multiple regressors and no constant
model3 <- povertyReduction ~ log(effectiveParties) + majoritarian
    + proportional + unanimity - 1
lm.res3 <- lm(model3, data=iversen)
summary(lm.res3)
```

Example 3: Party systems & Redistribution

```
Call:
lm(formula = model3, data = iversen)

Residuals:
     Min       1Q   Median       3Q      Max
-23.3843  -1.4903   0.6783   6.2687  13.9376

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
log(effectiveParties)    26.69      14.15   1.886   0.0887 .
majoritarian             17.66      12.69   1.392   0.1941
proportional             26.88      21.18   1.269   0.2331
unanimity               -31.29      26.55  -1.178   0.2659
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 12.37 on 10 degrees of freedom
Multiple R-squared:  0.9636,Adjusted R-squared:  0.949
F-statistic: 66.13 on 4 and 10 DF,  p-value: 3.731e-07
```

Example 3: Party systems & Redistribution

```
# A new model with multiple regressors and an interaction
model4 <- povertyReduction ~ log(effectiveParties) + majoritarian
    + proportional + log(effectiveParties):majoritarian
lm.res4 <- lm(model4, data=iversen)
summary(lm.res4)
```

Example 3: Party systems & Redistribution

```
Call:
lm(formula = model4, data = iversen)

Residuals:
     Min       1Q   Median       3Q      Max 
-22.2513   0.0668   2.8532   4.7318  12.9948 

Coefficients:
                                   Estimate Std. Error t value Pr(>|t|)   
(Intercept)                          -14.83      31.42  -0.472  0.64813   
log(effectiveParties)                 16.78      17.39   0.965  0.35994   
majoritarian                          16.34      37.65   0.434  0.67445   
proportional                          56.18      13.70   4.102  0.00267 **
log(effectiveParties):majoritarian    29.55      30.02   0.984  0.35065   
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1 

Residual standard error: 12.39 on 9 degrees of freedom
Multiple R-squared:  0.7633,Adjusted R-squared:  0.6581 
F-statistic: 7.256 on 4 and 9 DF,  p-value: 0.006772
```

Example 3: Party systems & Redistribution

```
# A more efficient way to specify an interaction
model5 <- povertyReduction ~ log(effectiveParties)*majoritarian +
    proportional
lm.res5 <- lm(model5, data=iversen)
summary(lm.res5)
```

Example 3: Party systems & Redistribution

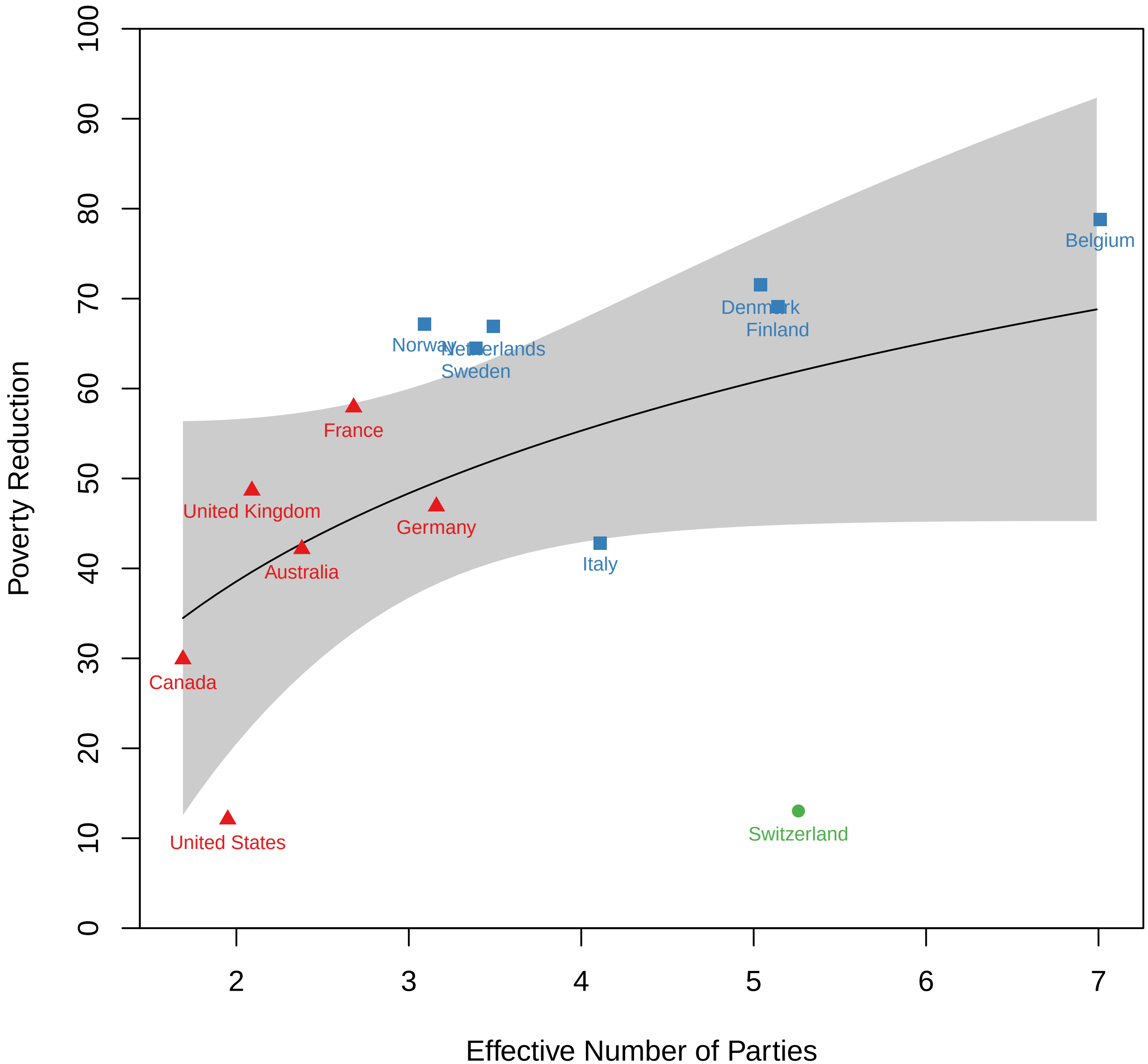
```
Call:
lm(formula = model5, data = iversen)

Residuals:
     Min       1Q   Median       3Q      Max
-22.2513   0.0668   2.8532   4.7318  12.9948

Coefficients:
                                   Estimate Std. Error t value Pr(>|t|)
(Intercept)                          -14.83      31.42  -0.472  0.64813
log(effectiveParties)                 16.78      17.39   0.965  0.35994
majoritarian                          16.34      37.65   0.434  0.67445
proportional                          56.18      13.70   4.102  0.00267 **
log(effectiveParties):majoritarian    29.55      30.02   0.984  0.35065
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 12.39 on 9 degrees of freedom
Multiple R-squared:  0.7633,Adjusted R-squared:  0.6581
F-statistic: 7.256 on 4 and 9 DF,  p-value: 0.006772
```

Plotting a best fit line



Let's turn to the code to see how we can make this plot using R base graphics

R Graphics

R has several graphics systems.

The `base` system

The `grid` system

(`grid` is more powerful, but has a steeper learning curve.
See Paul Murrel's book on R Graphics for an introduction.)

Focus here on `base`

R Graphics: Devices

Everything you draw in `R` must be drawn on a canvas

Must create the canvas before you draw anything

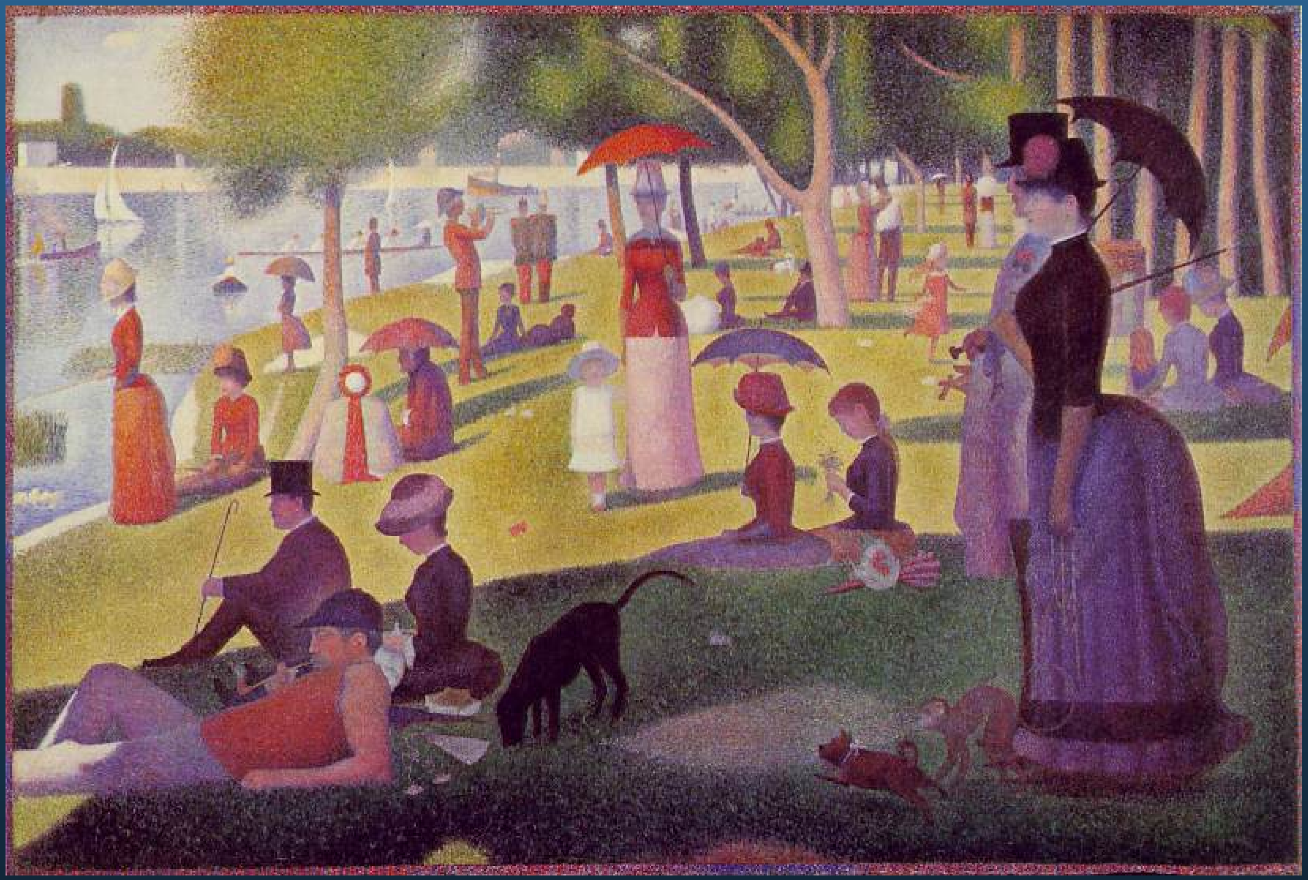
Computer canvasses are **devices** you draw to

Devices save graphical input in different ways

Sometimes to the disk, sometimes to the screen

Most important distinction: raster vs. vector devices

Vector vs. raster



Pointalism = raster graphics. Plot each pixel on an n by m grid.

Vector vs. raster

Pixel = Point = Raster

Good for pictures. Bad for drawings/graphics/cartoons.

(Puzzle: isn't everything raster? In display, yes. Not in storage)

Advantages of vector:

- Easily manipulable/modifiable groupings of objects
- Easy to scale objects larger or smaller/ Arbitrary precision
- Much smaller file sizes
- Can always convert to raster (but not the other way round, at least not well)

Disadvantages:

- A photograph would be really hard to show (and huge file size)
- Not web accessible. Convert to PNG or PDF.

Some common graphics file formats

	Lossy	Lossless
Raster	.gif, .jpeg	.wmf, .png, .bmp
Vector –		.ps, .eps, .pdf, .ai, .wmf

Lossy means during file compression, some data is (intentionally) lost

Avoid lossy formats whenever possible

Some common graphics file formats

In `R`, have access to several formats:

win.metafile()	wmf, Windows media file
pdf()	pdf, Adobe portable data file
postscript()	postscript file (printer language)
quartz()	opens a screen; Mac only
windows()	opens a screen; PC only
x11()	opens a screen; works on all machines

Latex, Mac or Unix users can't use wmf

`windows(record=TRUE)` let's you cycle thru old graphs with arrow keys

High-level graphics commands

In R, High level graphics commands:

- produce a standard graphic type
- fill in lots of details (axes, titles, annotation)
- have many configurable parameters
- have varied flexibility

You don't need to use HLCs to make R graphics.

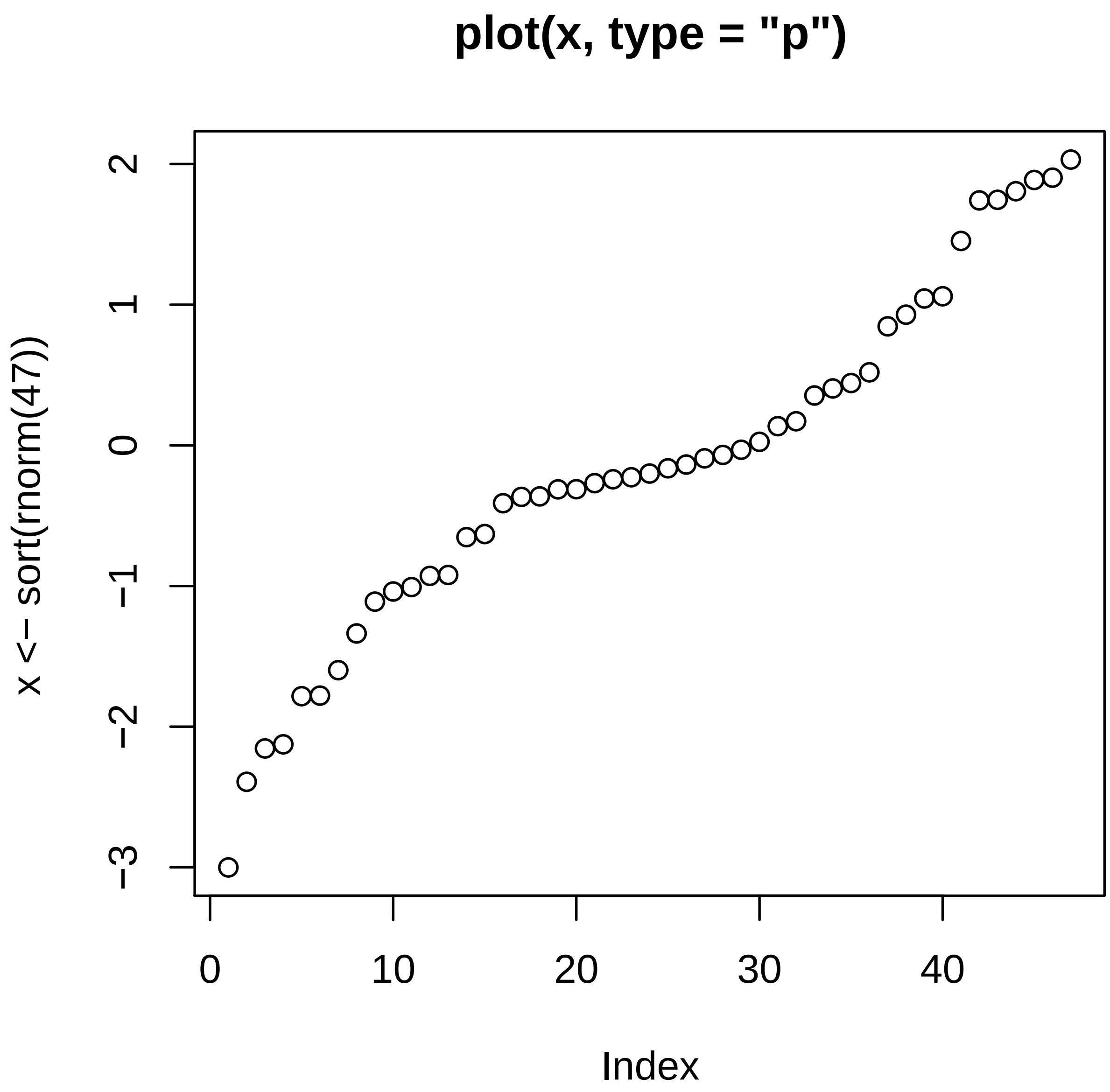
Could use primitive commands to do each task above

Using low levels commands gives more control but takes more time

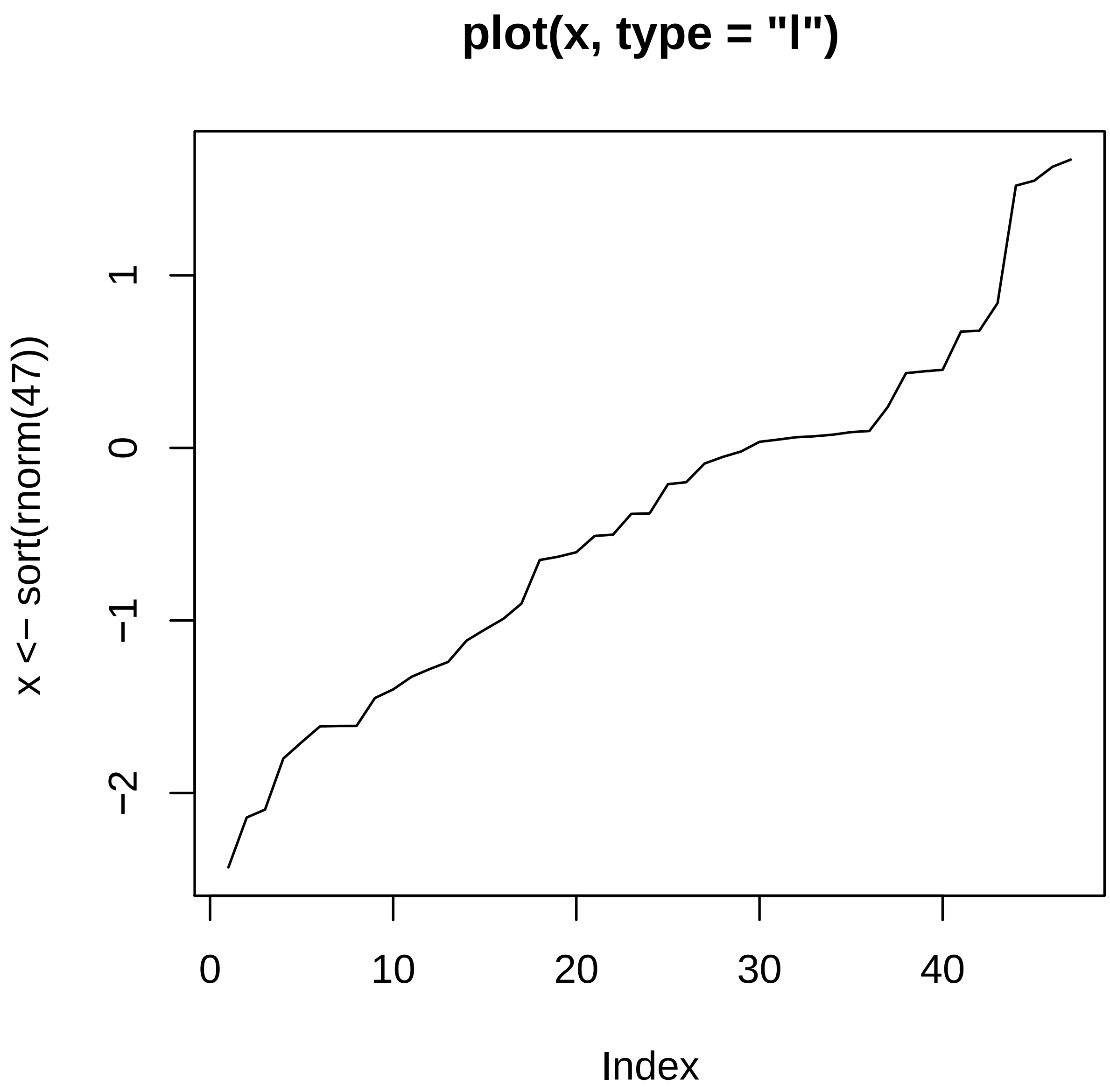
Some major high-level graphics commands

Graphic	Base command	Lattice command
scatterplot	plot()	xyplot()
line plot	plot(. . . ,type="l")	xyplot(. . . ,type="l")
Bar chart	barplot()	barchart()
Histogram	hist()	histogram()
Smoothed histograms	plot() after density()	densityplot()
boxplot	boxplot()	bwplot()
Dot plot	dotchart()	dotplot()
Contour plots	contour()	contourplot()
image plot	image()	levelplot()
3D surface	persp()	wireframe()
3D scatter	scatterplot3d()*	cloud()
conditional plots	coplot()	xyplot()
Scatterplot matrix		splom()
Parallel coordinates		parallel()
Star plot	stars()	
Stem-and-leaf plots	stem()	
ternary plot	ternaryplot() in vcd	
Fourfold plot	fourfoldplot() in vcd	
Mosaic plots	mosaicplot() in vcd	

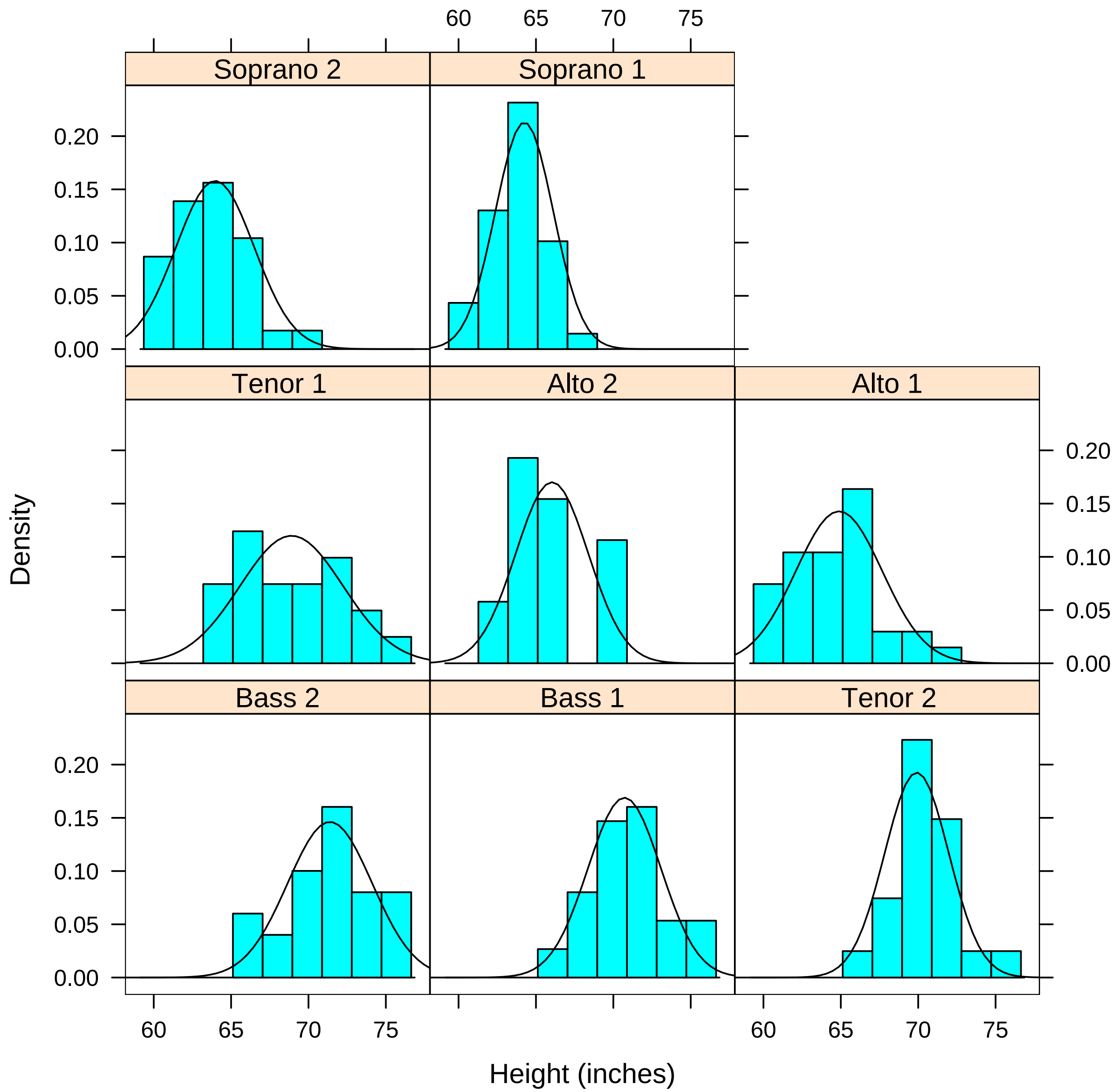
Scatterplot: `plot()`



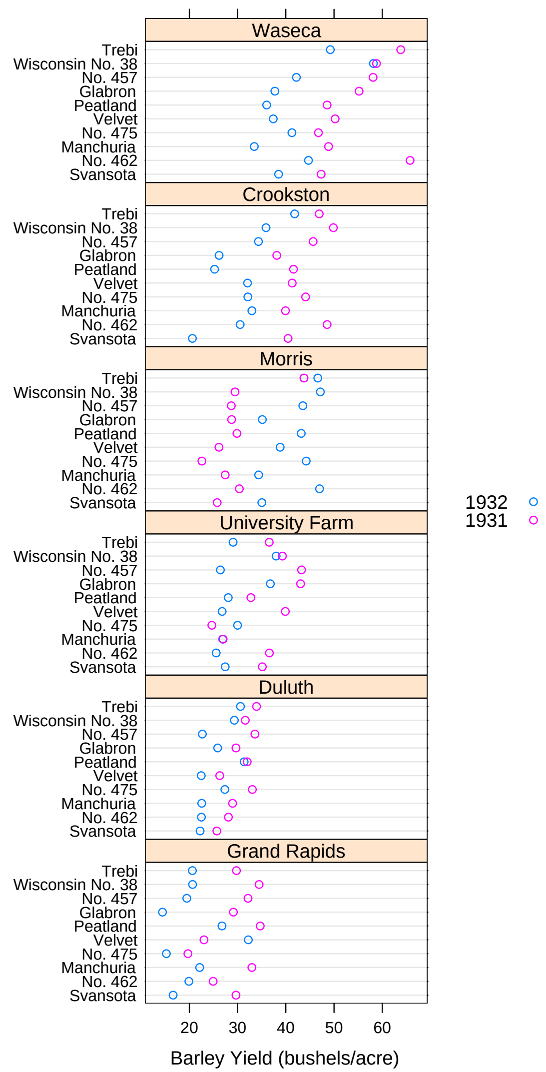
Line plot: `plot(...,type="l")`

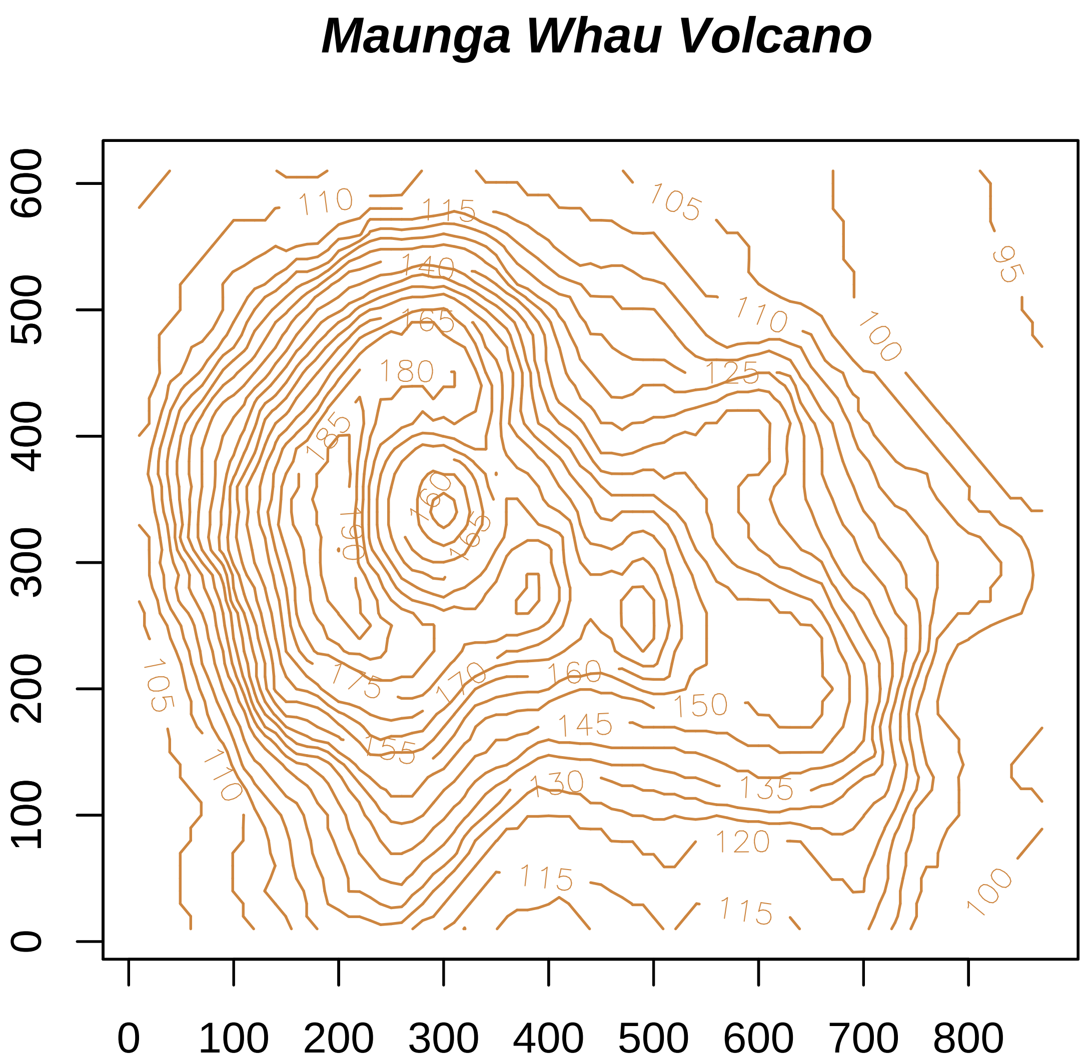


(Smoothed) Histograms: `densityplot()` & others



Dot plot: `dotplot()`





Maunga Whau Volcano
0
100
200
300
400
500
600
700
800
0
100
200
300
400
500
600
95
100
105
110
115
120
125
130
135
140
145
150
155
160
165
170
175
180
185
190

Image plot: `image()`

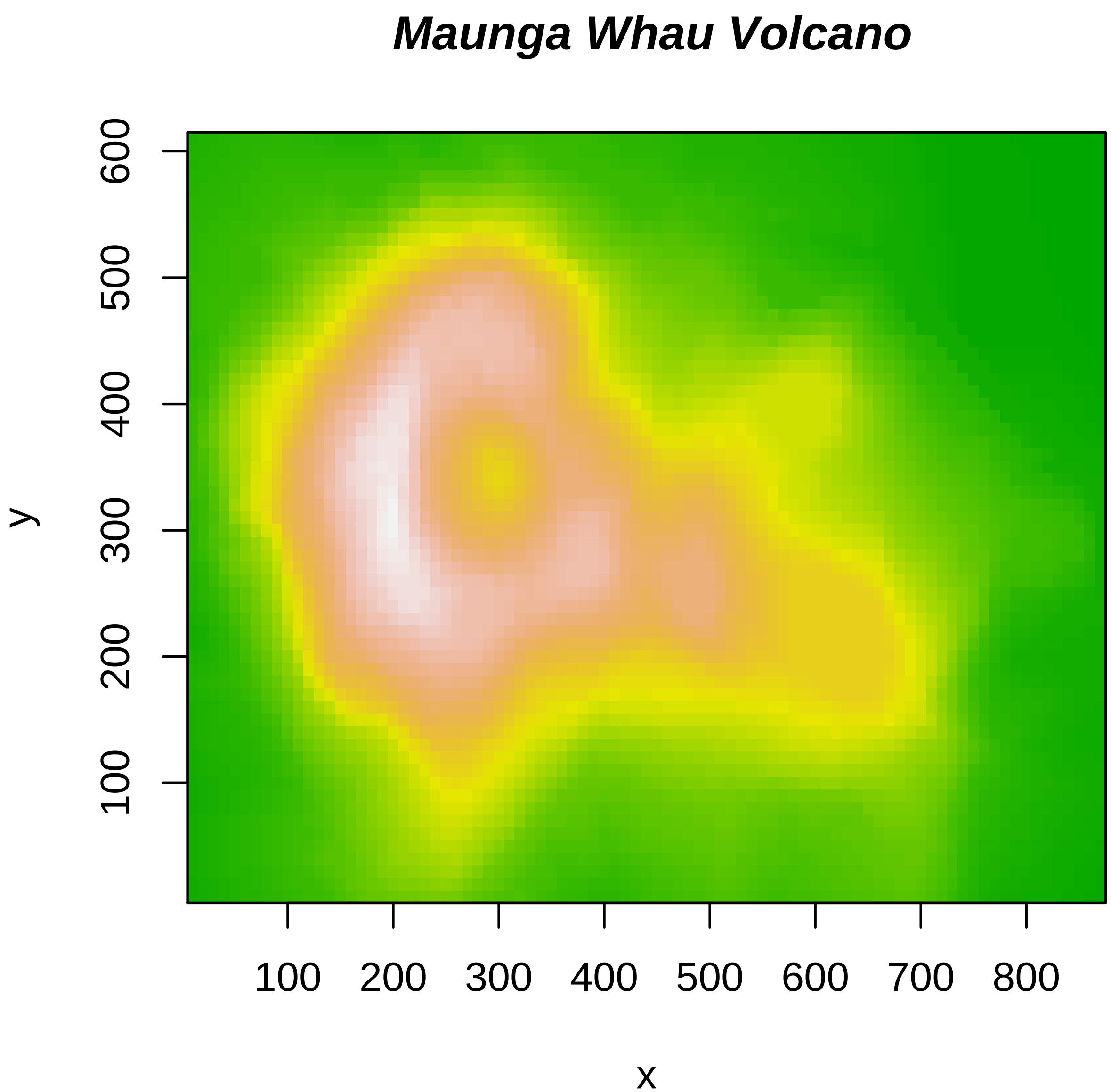
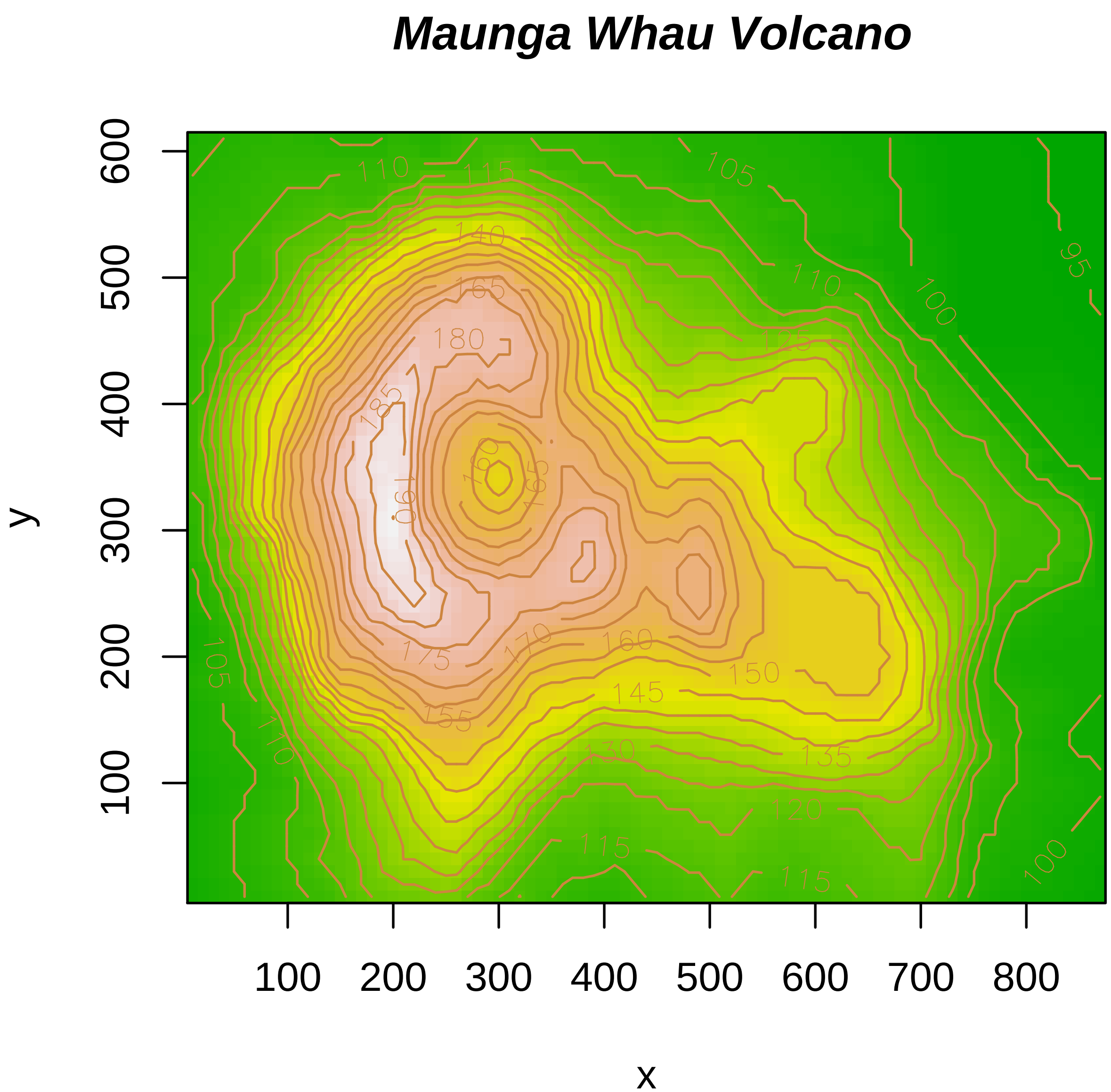
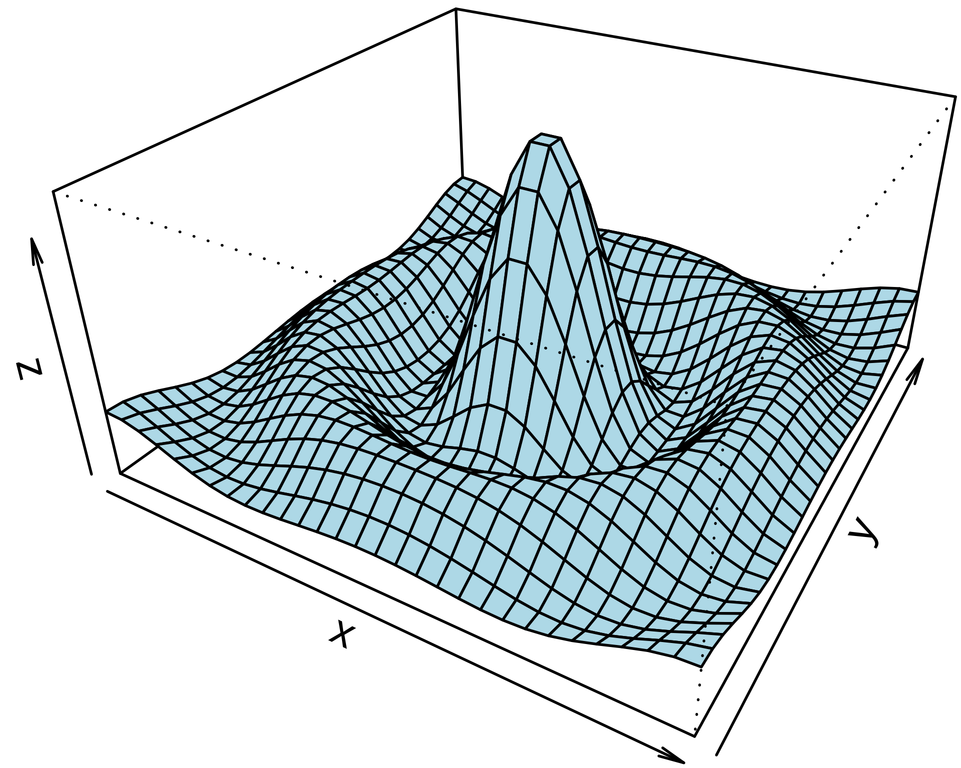


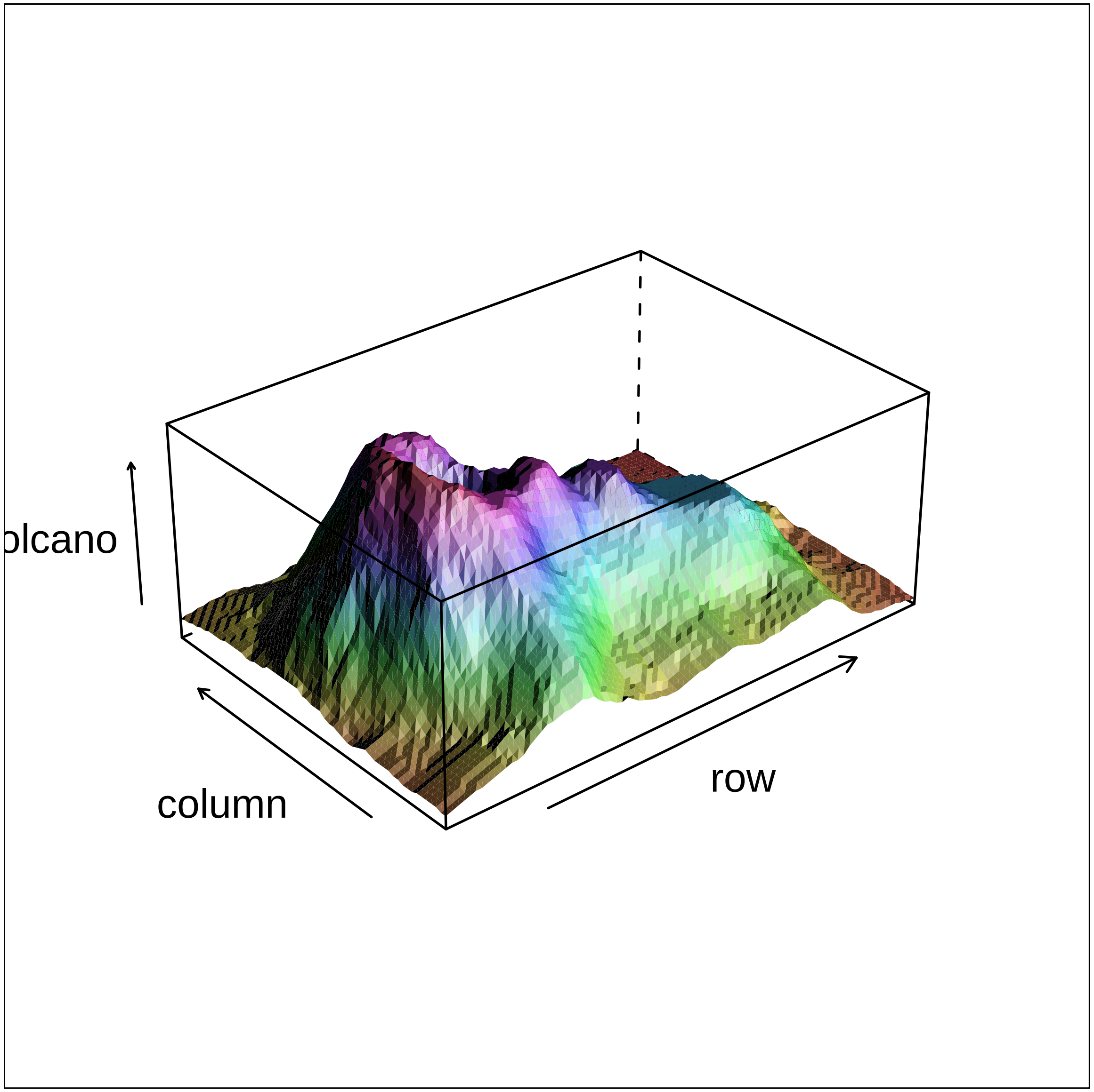
Image plot with contours: `contour(...,add=TRUE)`



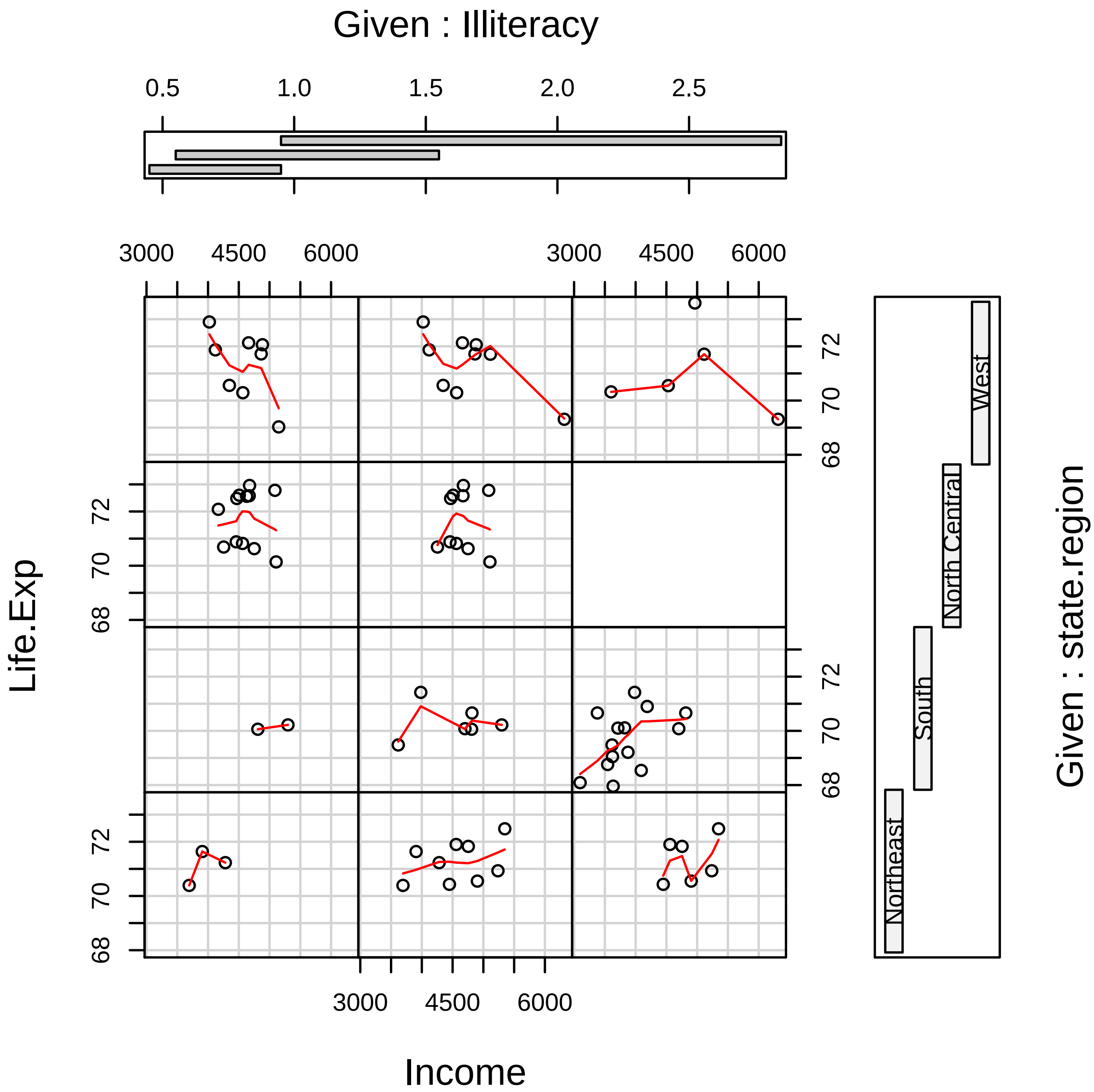
3D surface: `persp()`



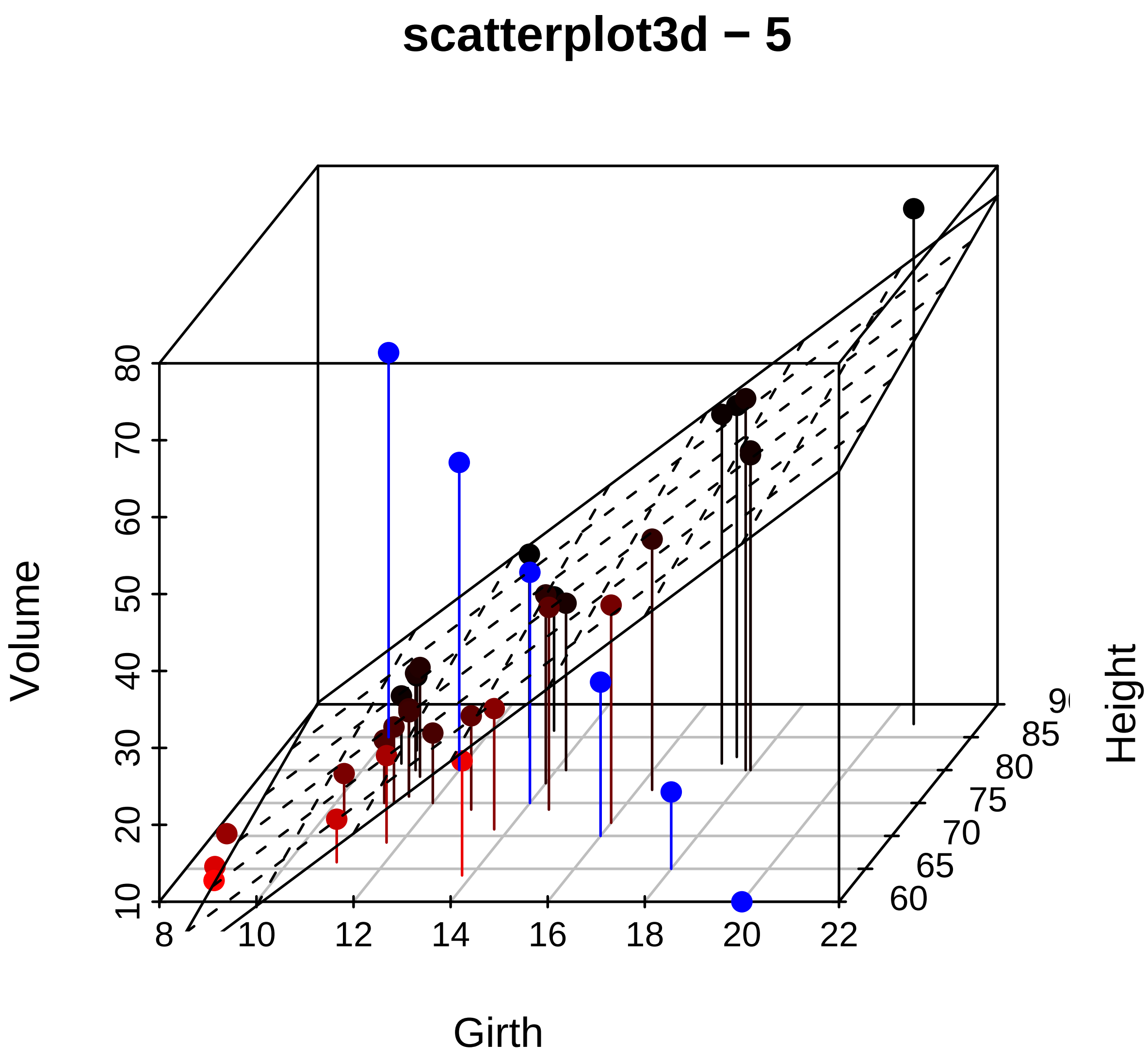
3D surface: `wireframe()`



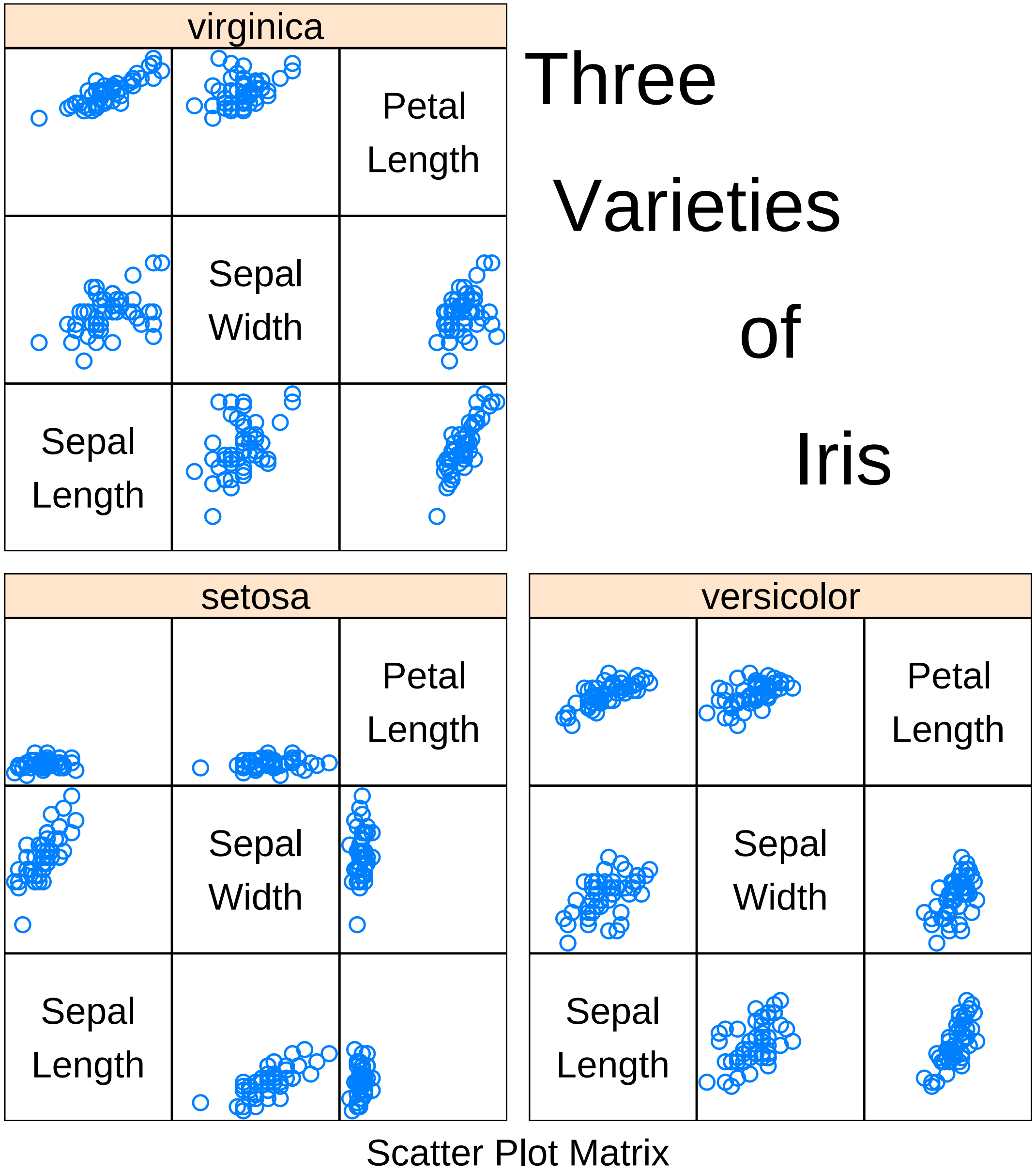
Conditional plots: `coplot()`



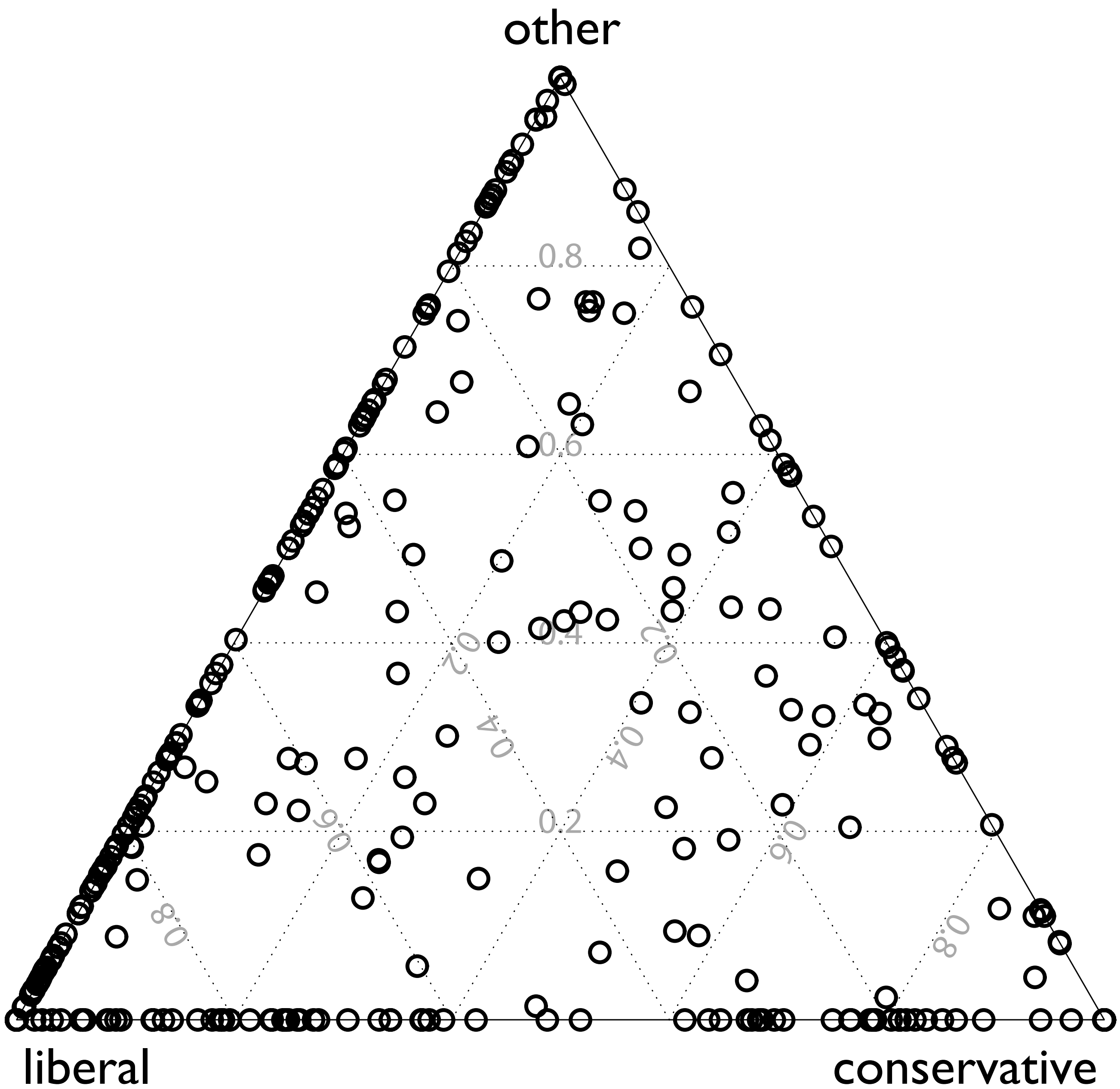
3D scatter: `scatterplot3d()` in own library



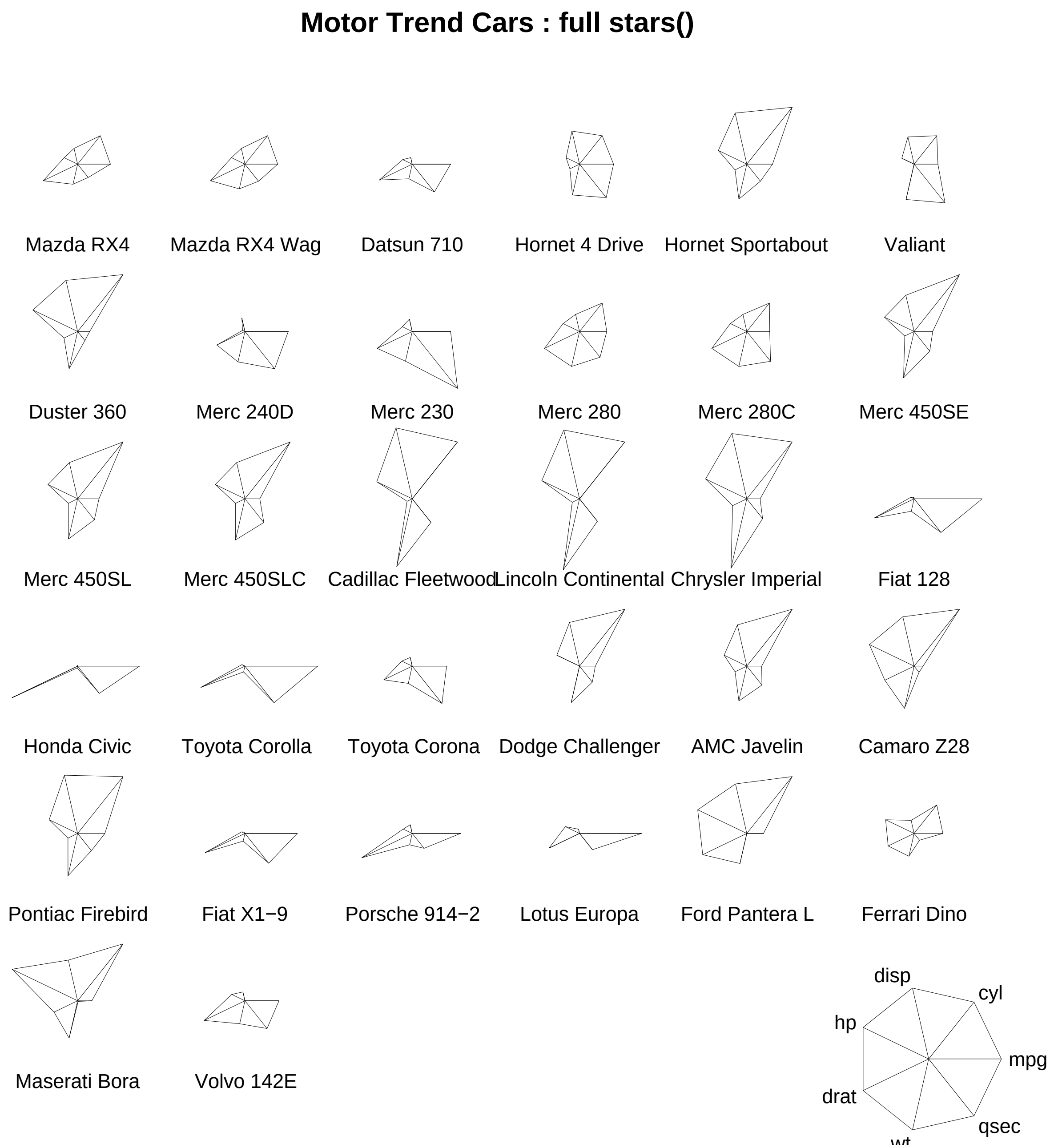
Scatterplot matrix: `splom()`



Ternary plot: `ternaryplot()` in `vcd`



Star plot: stars()



Stem-and-leaf plot

```
stem> stem(log10(islands))

  The decimal point is at the |

  1 | 1111112222233444
  1 | 5555556666667899999
  2 | 3344
  2 | 59
  3 |
  3 | 5678
  4 | 012
```

Basic customization

For any given high-level plotting command, there are many options listed in help

```
barplot(height, width = 1, space = NULL,
        names.arg = NULL, legend.text = NULL, beside = FALSE,
        horiz = FALSE, density = NULL, angle = 45,
        col = NULL, border = par("fg"),
        main = NULL, sub = NULL, xlab = NULL, ylab = NULL,
        xlim = NULL, ylim = NULL, xpd = TRUE,
        axes = TRUE, axisnames = TRUE,
        cex.axis = par("cex.axis"), cex.names = par("cex.axis"),
        inside = TRUE, plot = TRUE, axis.lty = 0, offset = 0, ...)
```

Just the tip of the iceberg: notice the `...`

This means you can pass other, unspecified commands throough `barplot`

Basic customization

The most important (semi-) documented parameters to send through `...` are settings to `par()`

Most base (traditional) graphics options are set through `par()`

`par()` has no effect on lattice or grid graphics

Consult `help(par)` for the full list of options

Some key examples, grouped functionally

`par()` settings

Customizing text size:

cex	Text size (a multiplier)
cex.axis	Text size of tick numbers
cex.lab	Text size of axes labels
cex.main	Text size of plot title
cex.sub	Text size of plot subtitle

note the latter will multiply off the basic `cex`

`par()` settings

More text specific formatting

font	Font face (bold, italic)
font.axis	etc
srt	Rotation of text in plot (degrees)
las	Rotation of text in margin (degrees)

Note the distinction between text in the plot and outside.

Text in the plot is plotted with `text()`

Text outside the plot is plotted with `mtext()`, which was designed to put on titles, etc.

`par()` settings

Formatting for most any object

bg	background color
col	Color of lines, symbols in plot
col.axis	Color of tick numbers, etc

The above expect colors (see `colors()` for a list of names

`par()` settings

Formatting for lines and symbols

lty	Line type (solid, dashed, etc)
lwd	Line width (default too large; try really small, e.g., 0)
pch	Data symbol type; see `example(points)`

You will very often need to set the above

More `par()` settings

Formatting for axes

lab	Number of ticks
xaxp	Number of ticks for xaxis
tck,tcl	Length of ticks relative to plot/text
mgp	Axis spacing: axis title, tick labels, axis line

These may seem trivial, but affect the aesthetics of the plot & effective use of space

R defaults to excessive mgp, which looks ugly & wastes space

`par()` settings

More formating for axes

The following commands are special:
they are primitives in `par()` that can't be set inside the `...` of high-level commands

You must set them with `par()` first

usr	Ranges of axes, (xmin, xmax, ymin, ymax)
xlog	Log scale for x axis?
ylog	Log scale for y axis?

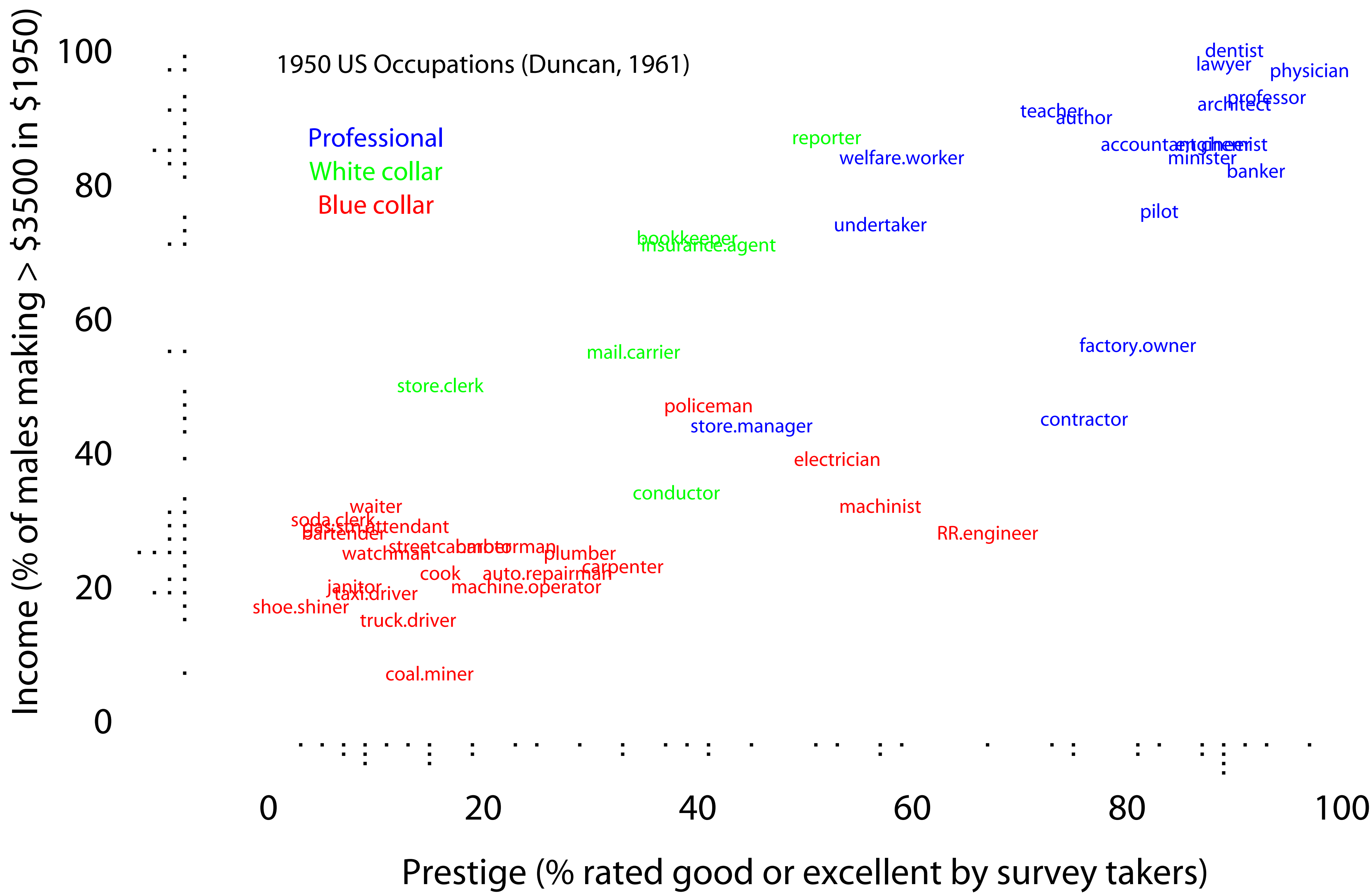
You can also make a logged axis by hand, as we will do now

Scatterplot: Occupational Prestige & Income

Classic data from sociology. Three variables

- Prestige of occupations, as rated by surveys
- Income of occupations (averaged across males)
- Type of occupation (blue collar, white collar, professional)

Data is in `R`. Look for `Duncan`.



1950 US Occupations (Duncan, 1961)
Professional
White collar
Blue collar
Income (% of males making > $3500 in $1950)
Prestige (% rated good or excellent by survey takers)
0
20
40
60
80
100
dentist
lawyer
physician
professor
architect
teacher
author
reporter
welfare.worker
accountant
engineer
chemist
minister
banker
pilot
undertaker
bookkeeper
insurance.agent
mail.carrier
factory.owner
store.clerk
policeman
store.manager
contractor
electrician
conductor
machinist
waiter
soda.clerk
gas.stn.attendant
bartender
RR.engineer
watchman
streetcar.motorman
barber
plumber
carpenter
cook
auto.repairman
janitor
taxi.driver
machine.operator
shoe.shiner
truck.driver
coal.miner

```
> lm.res <- lm(prestige~income+education)
> summary(lm.res)

Call:
lm(formula = prestige ~ income + education)

Residuals:
     Min       1Q   Median       3Q      Max
-29.5380  -6.4174   0.6546   6.6051  34.6412

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -6.06466    4.27194  -1.420    0.163
income       0.59873    0.11967   5.003 1.05e-05 ***
education    0.54583    0.09825   5.555 1.73e-06 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 13.37 on 42 degrees of freedom
Multiple R-Squared: 0.8282,     Adjusted R-squared:  0.82
F-statistic: 101.2 on 2 and 42 DF,  p-value: < 2.2e-16
```

To find the t-statistics & p-values, use the `summary()` command.

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -6.06466    4.27194  -1.420    0.163
income       0.59873    0.11967   5.003 1.05e-05 ***
education    0.54583    0.09825   5.555 1.73e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Note `1.05e-05` $= 0.0000105$

Or, you could calculate yourself:

```
lm.out <- lm(prestige~income+education) # run linear regression
betas <- lm.out$coefficients            # retrive the of betas
vcmat <- vcov(lm.out)                   # retrieve the var-cov matrix
ses <- sqrt(diag(vcmat))                # calc a vector of ses
tstats <- betas/ses                     # calc vector of tstats
pvalues <- 2*(1-pt(tstats,42))          # calc p-values
```

Confidence intervals for regression coefficients

Standard errors, t-tests, and p-values take expertise to read

They are also subject to misinterpretation

(E.g., smaller p-values do *not* imply a bigger substantive effect)

CIs turn the standard errors into something everyone can easily understand

To get the $100(1-\alpha)\%$ confidence interval for $\hat{\beta}_1$,

$$\begin{aligned}
\hat{\beta}_1^{\text{lower}} &= \hat{\beta}_1 - t_{\alpha/2,n-k-1}\hat{\sigma}_{\hat{\beta}_1} \\
\hat{\beta}_1^{\text{upper}} &= \hat{\beta}_1 + t_{\alpha/2,n-k-1}\hat{\sigma}_{\hat{\beta}_1}
\end{aligned}$$

Confidence intervals for regression coefficients

How to calculate CIs for coefficients in `R`

By hand:

```
lower.95 <- betas - qt(0.025,42)*ses
upper.95 <- betas + qt(0.025,42)*ses
```

Why are we using `qt`? Why `0.025`?

The easy way:

```
library(stats)
confint(lm.out,level=0.95)

                  2.5 %    97.5 %
(Intercept) -14.6857892 2.5564634
education     0.3475521 0.7441158
income        0.3572343 0.8402313
```

Confidence intervals for regression coefficients

Using confidence intervals, we can improve the initial summary table:

		95% Conf Interval	
Variable	Estimate	Lower	Upper
Income	0.60	[0.36,	0.84]
Education	0.55	[0.38,	0.74]
Intercept	-6.06	[-14.69,	2.46]
N	45		
s.e.r.	13.4	(this is $\hat{\sigma}_\varepsilon$)	
R^2	0.83	(this line optional)	

Table 1: Determinants of occupational prestige. Entries are linear regression parameters and their 95 percent confidence intervals.

Think about everything you put in these tables:

- what readers need to see to fully understand your results
- what superfluous R output you can delete
- how to make the results clear for as large an audience as possible

Substantive & statistical significance

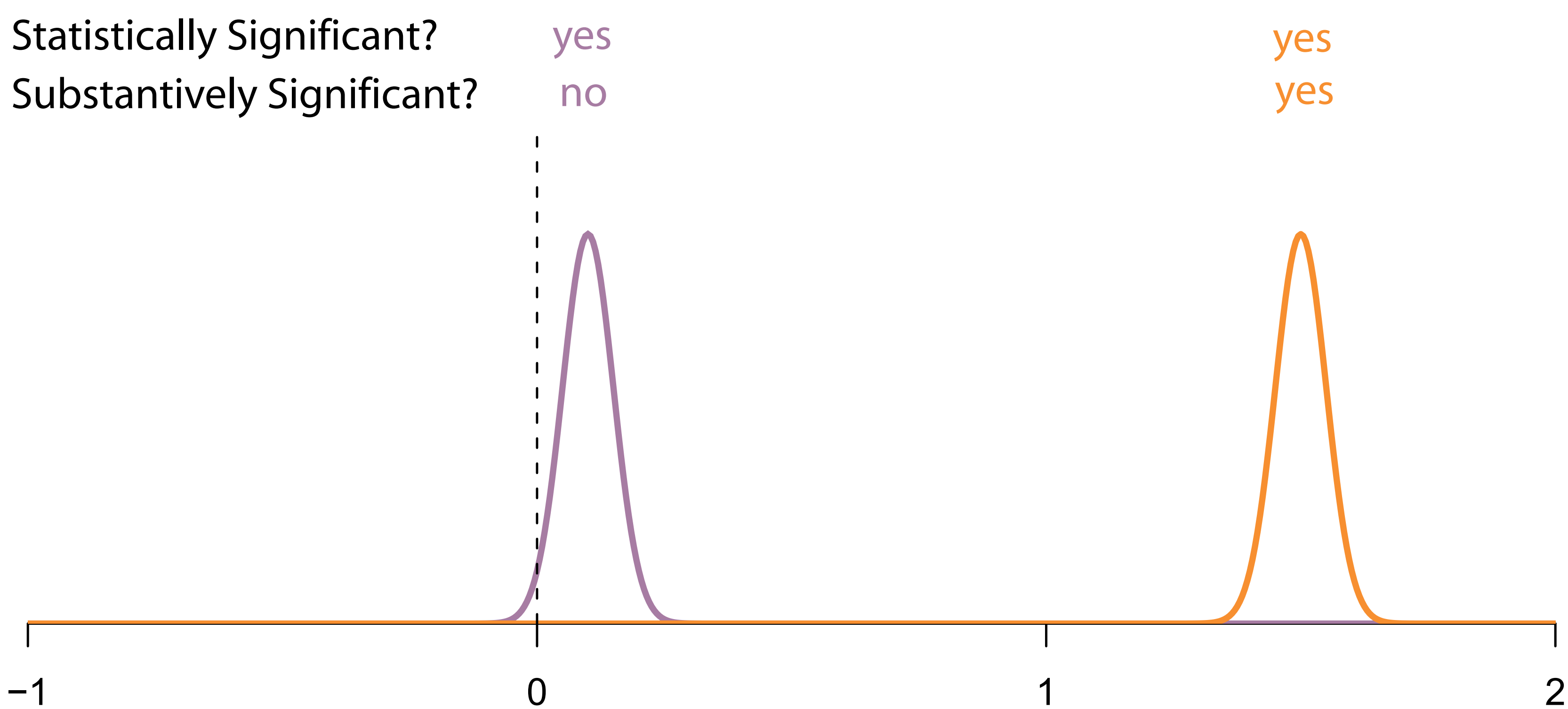
Don't over interpret p-values

They only show statistical significance

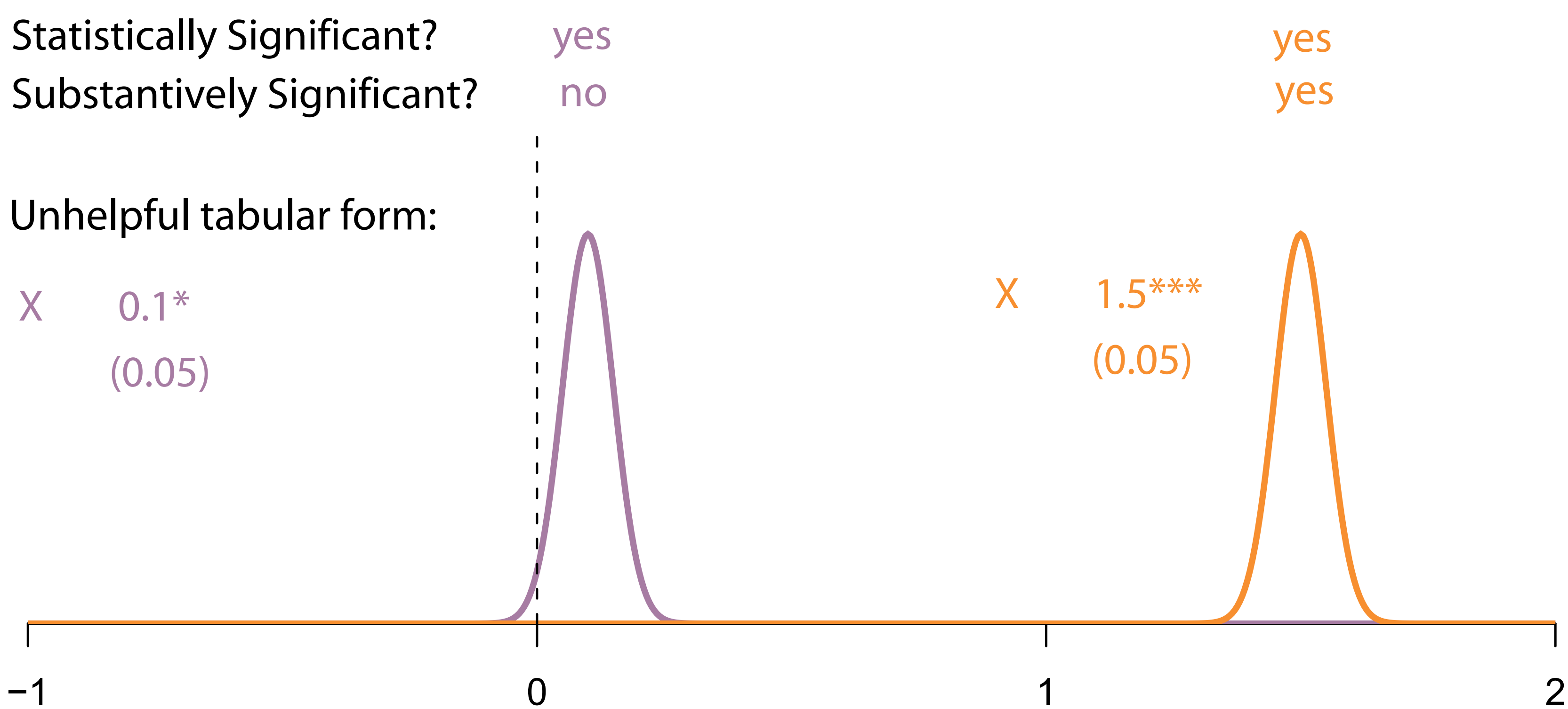
Statistical and substantive significance can interact

A look at some hypothetical distributions of $\hat{\beta}_1$ helps frame the possibilities

Perils of stargazing



Perils of stargazing

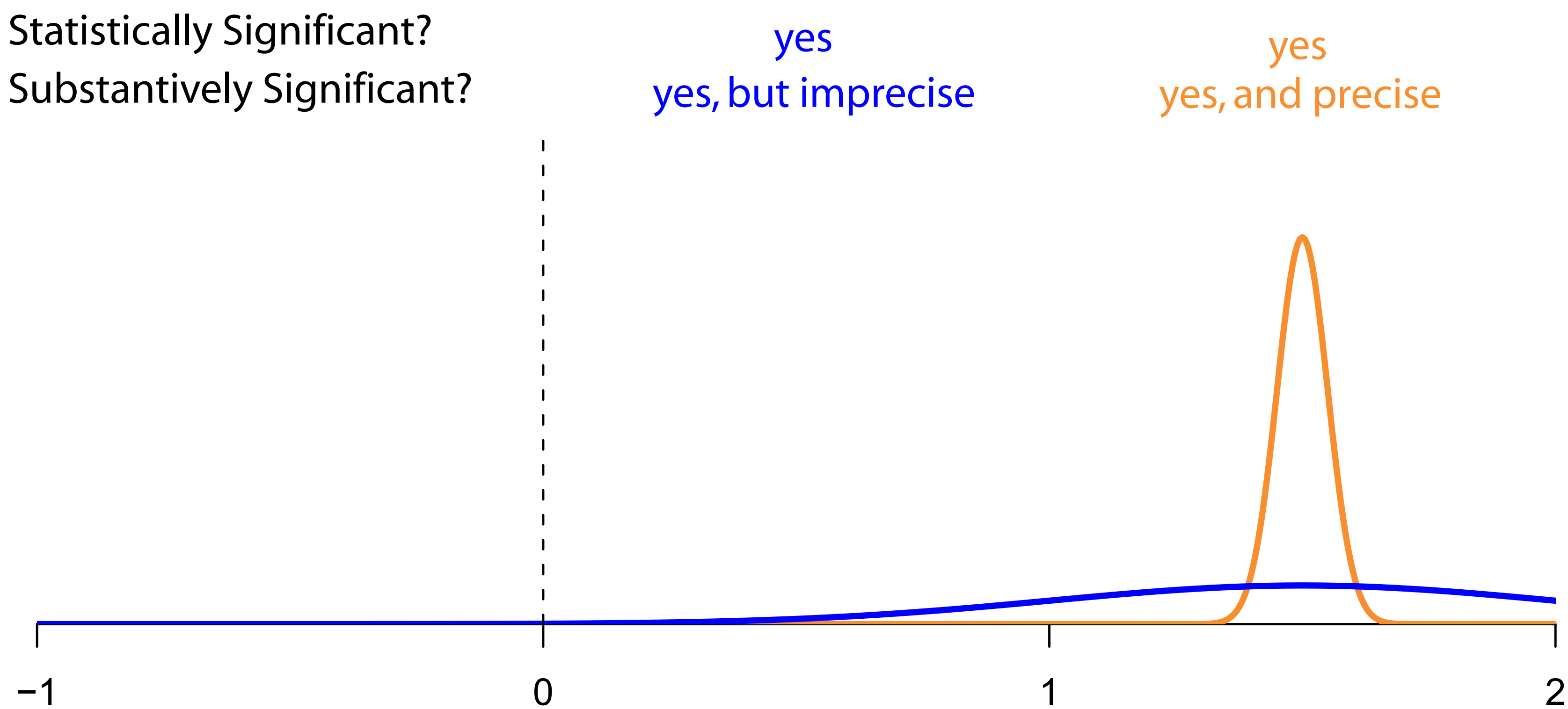


These estimated β's will both be starred in regression output.

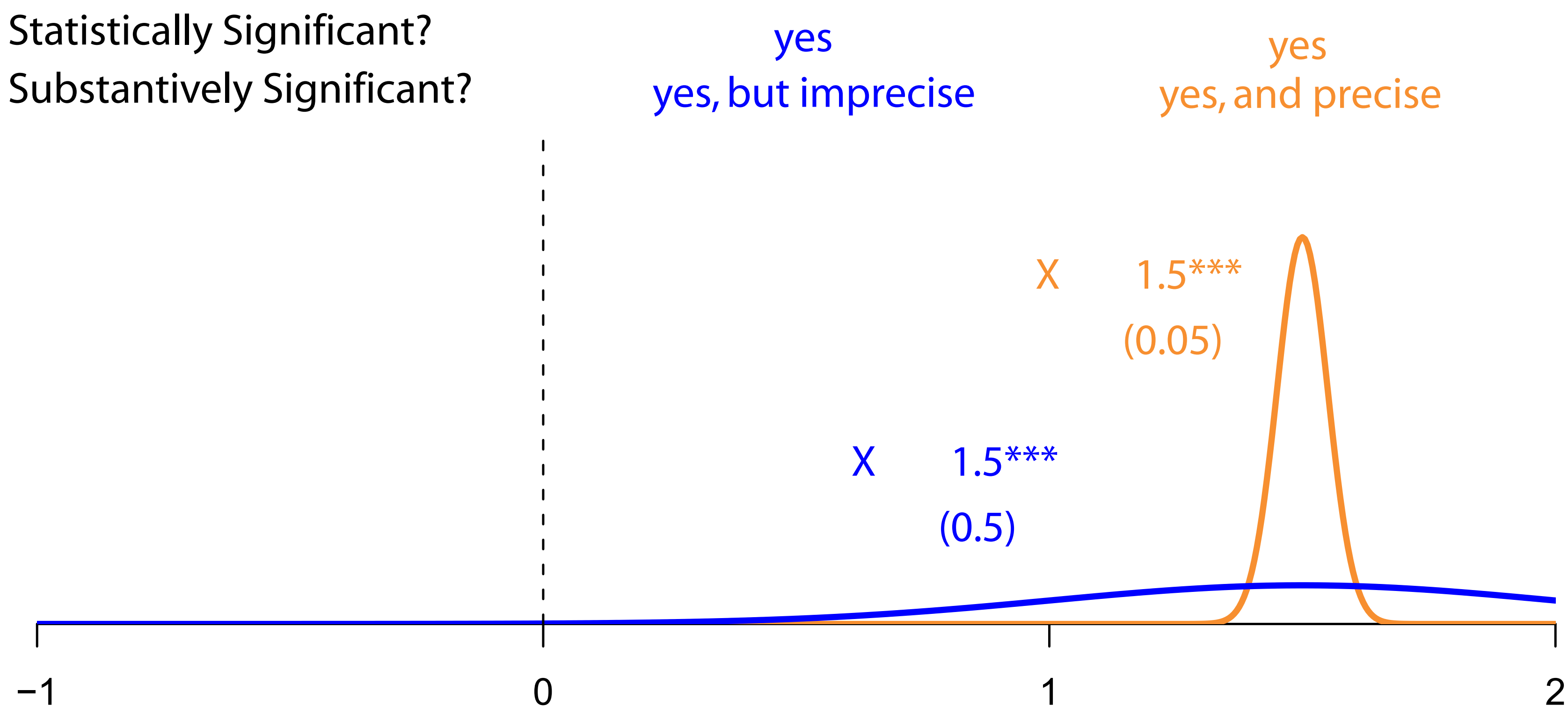
Often, only the estimate to the right will be significant in a substantive sense

The estimate on the left is a precise zero

Perils of stargazing



Perils of stargazing



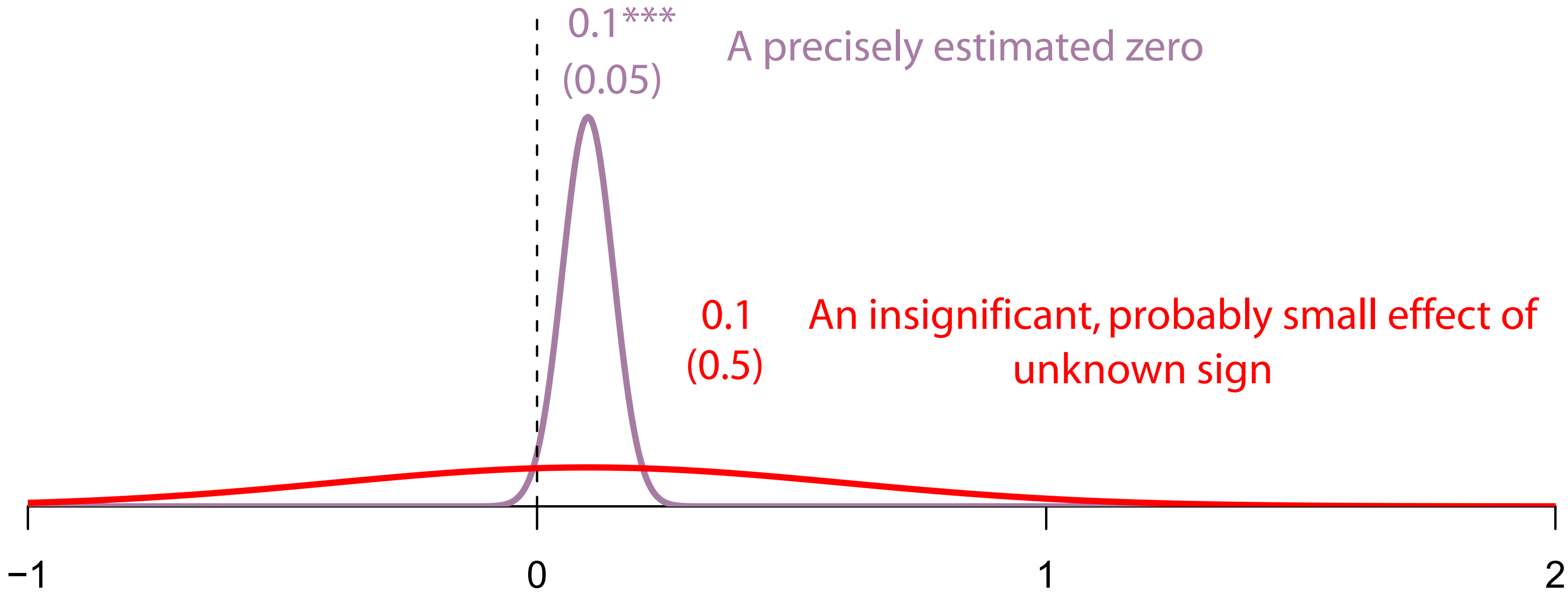
These estimated β's will both be heavily starred in regression output.

They are both substantively significant as well, with identical point estimates

But the orange curve is much more precisely estimated

The blue estimate may be much smaller or larger. Best shown with a CI

Perils of stargazing



How do you verify a null effect? Precise zeros

Sometimes, researchers mistake the precise zero for a positive effect

Confidence interval for expected values

We can calculate the CIs around $\hat{Y}$ as well.

For example, what is the 95% CI around $\widehat{\text{Prestige}}_c$ in:

$$\widehat{\text{Prestige}}_c = \hat{\beta}_0 + \hat{\beta}_1 \text{Income}_c + \hat{\beta}_2 \text{Education}_c$$

The uncertainty in each estimate will "combine" to form the uncertainty in $\widehat{\text{Prestige}}_c$.

In this example,

$$\widehat{\text{Prestige}}_c \quad = \quad -6.1 \quad 0.60 \times \text{Income}_c \quad + \quad 0.55 \times \text{Education}_c$$

Confidence interval for expected values

We can calculate the CIs around $\hat{Y}$ as well.

For example, what is the 95% CI around $\widehat{\text{Prestige}}_c$ in:

$$\widehat{\text{Prestige}}_c = \hat{\beta}_0 + \hat{\beta}_1 \text{Income}_c + \hat{\beta}_2 \text{Education}_c$$

The uncertainty in each estimate will "combine" to form the uncertainty in $\widehat{\text{Prestige}}_c$.

In this example,

$$\begin{array}{ccccccc} \widehat{\text{Prestige}}_c & = & -6.1 & 0.60 \times \text{Income}_c & + & 0.55 \times \text{Education}_c \\ & & [-14.7, 2.6] & [0.36, 0.84] & & [0.35, 0.74] \end{array}$$

Confidence interval for expected values

We can calculate the CIs around $\hat{Y}$ as well.

For example, what is the 95% CI around $\widehat{\text{Prestige}}_c$ in:

$$\widehat{\text{Prestige}}_c = \hat{\beta}_0 + \hat{\beta}_1 \text{Income}_c + \hat{\beta}_2 \text{Education}_c$$

The uncertainty in each estimate will "combine" to form the uncertainty in $\widehat{\text{Prestige}}_c$.

In this example,

$$\begin{array}{ccccccc} \widehat{\text{Prestige}}_c & = & -6.1 & 0.60 \times \text{Income}_c & + & 0.55 \times \text{Education}_c \\ & & [-14.7, 2.6] & [0.36, 0.84] & & [0.35, 0.74] \\ & & & & & \\ 47.7 & = & -6.1 & 0.60 \times 41.9 & + & 0.55 \times 52.6 \end{array}$$

Confidence interval for expected values

We can calculate the CIs around $\hat{Y}$ as well.

For example, what is the 95% CI around $\widehat{\text{Prestige}}_c$ in:

$$\widehat{\text{Prestige}}_c = \hat{\beta}_0 + \hat{\beta}_1 \text{Income}_c + \hat{\beta}_2 \text{Education}_c$$

The uncertainty in each estimate will "combine" to form the uncertainty in $\widehat{\text{Prestige}}_c$.

In this example,

$$\begin{array}{ccccccc} \widehat{\text{Prestige}}_c & = & -6.1 & 0.60 \times \text{Income}_c & + & 0.55 \times \text{Education}_c \\ & & [-14.7, 2.6] & [0.36, 0.84] & & [0.35, 0.74] \\ & & & & & \\ 47.7 & = & -6.1 & 0.60 \times 41.9 & + & 0.55 \times 52.6 \\ [43.7, 51.7] & & [-14.7, 2.6] & [0.36, 0.84] & & [0.35, 0.74] \end{array}$$

In words, when income and education are held at their means, we expect that presitge will equal 47.7 with a 95 % CI of 43.7 to 51.7.

Confidence interval for expected values

How do we calculate confidence intervals around $\hat{y}$ in `R`?

1. Estimate the model
2. Choose hypothetical values of the covariate at which you want to calculate $\hat{y}$ and it's CI.
3. Use the `predict()` function to obtain the expected y and it's CI

Some examples:

```
# To get CIs around all the fitted values
res <- lm(y~x+z)
pred <- predict(res,interval="confidence",level=0.95)
yhat <- pred[,1]
yhat.lower <- pred[,2]
yhat.upper <- pred[,3]
```

Confidence interval for expected values

```
# To get CIs for yhat given a set of hypothetical x & z values
res <- lm(y~x+z)
xhyp <- seq(min(x),max(x),0.01)
zhyp <- rep(mean(z),length(xhyp))
hypo <- data.frame(x=xhyp,z=zhyp)
pred <- predict(res,newdata=hypo,interval="confidence",level=0.95)
yhat <- pred[,1]
yhat.lower <- pred[,2]
yhat.upper <- pred[,3]
```

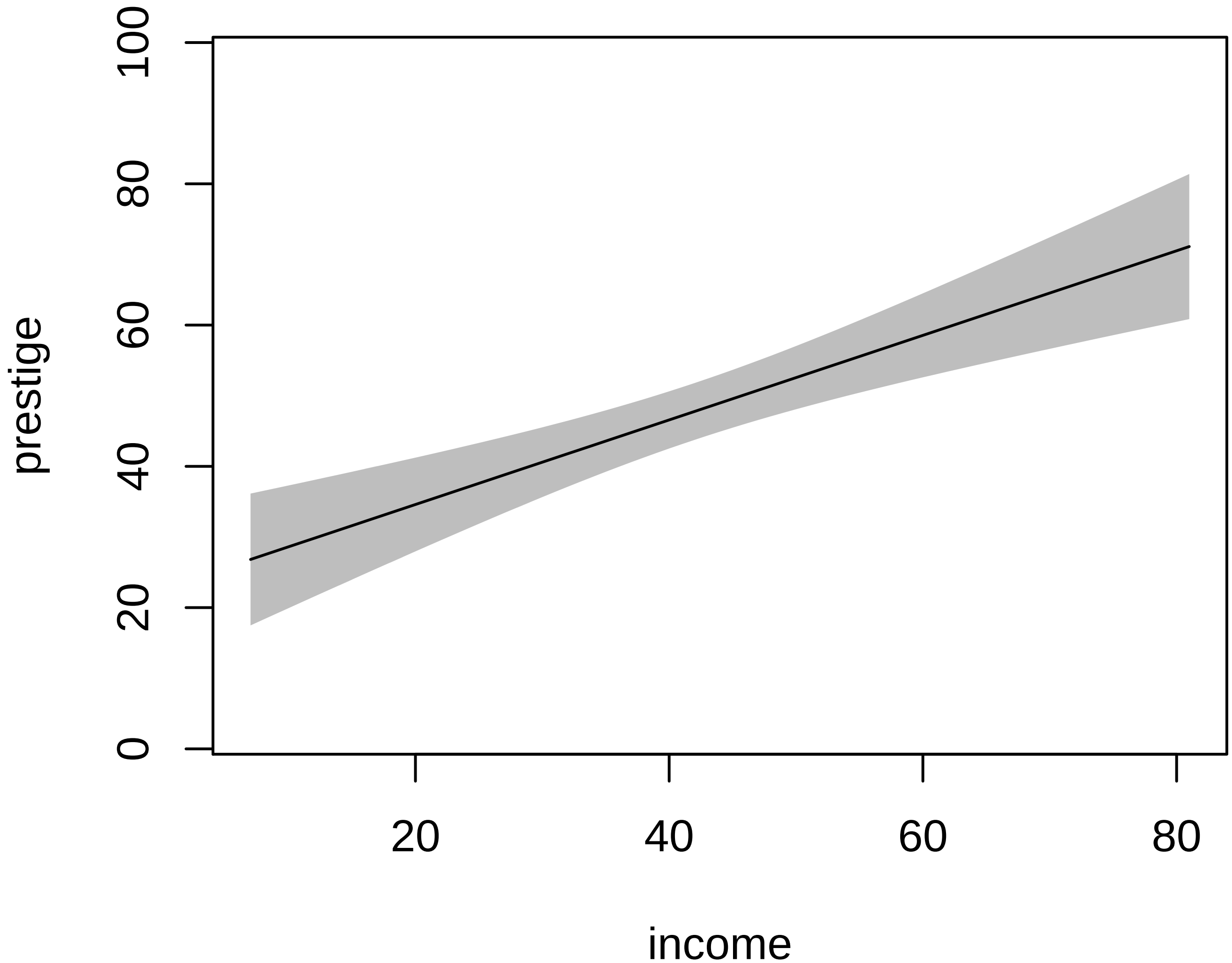
The code above is very useful for adding confidence intervals to a plot.

We can run through a sequence of possible x values, holding z constant,
and predict y and it's confidence interval,
then plot the confidence interval as an envolpe around y

The just add the upper and lower bounds:

```
lines(x=xhyp,y=yhat.lower,lty="dashed")
lines(x=xhyp,y=yhat.upper,lty="dashed")
```

Confidence interval for expected values



Confidence interval for expected values

Interpretation:
All we can say with 95 percent confidence is that the line
– the relation b/w prestige and income –
lies in this envelope

Very useful to show, especially if the relationship is curved in some way

I prefer shaded regions to dotted lines. (lots of lines gets confusing)

You can make shaded regions using the `polygon()` command

Just be sure to plot the polygon before you add any points or lines, so it shows up behind them

Complete code for above figure

```
# Load the occupation data
library(car)
data(Duncan)
attach(Duncan)

# Regress prestige on education & income
lm.out <- lm(prestige~education+income)

# To get CIs for yhat given a set of hypothetical income & education
xhyp <- seq(min(income),max(income),1)
zhyp <- rep(mean(education),length(xhyp))
hypo <- data.frame(income=xhyp,education=zhyp)
pred <- predict(lm.out,
                newdata=hypo,
                interval="confidence",
                level=0.95)
yhat <- pred[,1]
yhat.lower <- pred[,2]
yhat.upper <- pred[,3]

pdf("yhatexample.pdf",horizontal=FALSE,width=5,height=4.5)
```

```r
plot(y=prestige,x=income,type="n")

# Make the x-coord of a confidence envelope polygon
xpoly <- c(xhyp,
           rev(xhyp),
           xhyp[1])

# Make the y-coord of a confidence envelope polygon
ypoly <- c(yhat.lower,
           rev(yhat.upper),
           yhat.lower[1])

# Choose the color of the polygon
col <- "gray"

# Plot the polygon first, before the points & lines
polygon(x=xpoly,
        y=ypoly,
        col=col,
        border=FALSE
        )
```

```
# Plot the fitted line
lines(x=xhyp,y=yhat)

dev.off()
```