Log-linear Models of Contingency Tables:
Multidimensional Tables

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From 2D to 3D tables

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- Tables are harder to visualize
- A plethora of interactions are estimable
- New notation is used to cope with growing complexity
- New concepts: *conditional independence*, *conditional association*, *collapsibility*
- Estimation may become difficult if tables have many zeros (curse of dimensionality)

Today, we’ll discuss these challenges, but only scratch the surface of the literature
Log-linear models for 3D tables

Let’s start with the model under independence,

\[ E(\mu_{ijk}) = n\hat{\pi}_i\hat{\pi}_j\hat{\pi}_k \]
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$$ E(\mu_{ijk}) = n \hat{\pi}_{i..} \hat{\pi}_{.j} \hat{\pi}_{..k} $$

Take logs

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Rewriting in log-linear form

\[ \ln E(\mu_{ijk}) = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z \]
Log-linear models for 3D tables

The saturated model for a 3D table has every two-way and three-way interaction

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\ln E(\mu_{ijk}) = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z \\
+ \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} + \lambda_{ijk}^{XYZ}
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(How can we think intuitively about these terms? What is a three-way interaction?)
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(How can we think intuitively about these terms? What is a three-way interaction?)

As before, the saturated model perfectly fits the data, and has 0 degrees of freedom
Identifying constraints

As in the $I \times K$ case, the parameters of the marginals must sum to 0 for identification

$$\sum_i \lambda_i^X = 0$$

$$\sum_j \lambda_j^Y = 0$$

$$\sum_k \lambda_k^Z = 0$$
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Alternatively, we drop one element of each set of parameters from the model.

That would leave $I - 1$ $\lambda^X$'s, $J - 1$ $\lambda^Y$'s, and $K - 1$ $\lambda^Z$'s.

The actual parameter estimates would look different, but all quantities of interest would stay the same.
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We also need to put identifying constraints on the interactions.

The interactions must sum to 0 over each *index*:
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\[ \sum_{j} \lambda_{j}^{Y} = \sum_{j} \lambda_{i,j}^{XY} = \sum_{j} \lambda_{j,k}^{YZ} = \sum_{j} \lambda_{i,j,k}^{XYZ} = 0 \]

\[ \sum_{k} \lambda_{k}^{Z} = \sum_{k} \lambda_{i,k}^{XZ} = \sum_{i} \lambda_{j,k}^{YZ} = \sum_{k} \lambda_{i,j,k}^{XYZ} = 0 \]

A different way of saying this is that \( \lambda_{\cdot \cdot \cdot \cdot \cdot}^{XYZ} = 0 \), \( \lambda_{\cdot \cdot \cdot \cdot \cdot}^{XYZ} = 0 \), etc.
Identifying constraints

Once again, there are alternative identifying assumptions.

We could drop one row and column from each set of interactions, leaving:

$$(I - 1) \times (J - 1) \quad \lambda_{XY} \text{ terms}$$
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\[(J - 1) \times (K - 1) \quad \lambda^{YZ} \text{ terms}\]
\[(I - 1) \times (J - 1) \times (K - 1) \quad \lambda^{XYZ} \text{ terms}\]

As before, this parameterization yields superficially different estimates, but identical quantities of interest.
Concepts for 3D tables: Conditional independence

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We write this $X \perp Y \mid Z$.
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**Conditional independence:**

$X$ and $Y$ are *conditionally independent given $Z$* if they are independent within any partial table where $Z$ is fixed.

We write this $X \perp Y | Z$

Note this is a weaker condition than joint independence (which is written $X \perp Y$)
Concepts for 3D tables: Collapsibility

If a variable $X$ is jointly or conditionally independent of all other variables, we say the table is *collapsible* over $X$. 
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Once again, we can understand this as omitted variable bias, which is harmless when the omitted variable is uncorrelated with other covariates.
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In a moment, we’ll introduce a useful tool for investigating collapsibility.
Concepts for 3D tables: Conditional association

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→ That is, for fixed $Z = k$, $X$ and $Y$ are *not* independent.

If the association between $X$ and $Y$ is the same for all $Z = k$, then we have homogenous association.
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There are tests for both conditional and homogenous association; see Agresti
Concepts for 3D tables: Association Diagrams

1. Independence
   \[ \begin{align*}
   &X \\
   &\quad \bullet \\
   &\quad \quad \bullet \quad Z \\
   &\quad \bullet \\
   &Y
   \end{align*} \]

2. An XY interaction only
   \[ \begin{align*}
   &X \\
   &\quad \bullet \\
   &\quad \quad \bullet \quad Z \\
   &\quad \bullet \\
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   \end{align*} \]

3. An XZ and XY interaction
   \[ \begin{align*}
   &X \\
   &\quad \quad \downarrow \quad Z \\
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4. XY, YZ, and XZ interactions
   \[ \begin{align*}
   &X \\
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   \end{align*} \]

It helps to draw diagrams of which 2-way interactions are present.
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1. Independence

\[ \begin{array}{c}
X \\
\cdot \\
\cdot \\
\cdot \\
Y \\
\end{array} \quad \begin{array}{c}
X \\
\cdot \\
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It helps to draw diagrams of which 2-way interactions are present.

Variables are:

- jointly independent if no *path* connects them
- conditionally associated if directly linked
- conditionally independent if linked by a chain
Several models can have the same association diagram
Several models can have the same association diagram.

(4) gives both the saturated model, and the saturated model minus the 3-way interaction.
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   \[ Z \]

Several models can have the same association diagram

(4) gives both the saturated model, and the saturated model minus the 3-way interaction

If there are multiple paths between two variables, the variables are conditionally independent given any of the paths
Association diagrams are an easy way to check for collapsibility

- Obviously, singletons are collapsible (joint independence)
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- Obviously, singletons are collapsible (joint independence)
- So are variables at the ends of chains (conditional independence)
- But *not* variables between other variables in a chain (cond. assoc.)
Under independence, we have the model

\[ \ln E(\mu_{ijk}) = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z \]
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$$\ln E(\mu_{ijk}) = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z$$

and the fitted cell probabilities are

$$E(\pi_{ijk}) = \hat{\pi}_i \cdot \hat{\pi}_j \cdot \hat{\pi}_k$$
Concepts for 3D tables: Sufficient marginals

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Now suppose that $X \perp Z$ and $Y \perp Z$, but $X$ and $Y$ are not independent. Then

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If $X$ and $Y$ are conditionally independent given $Z$, we write the LLM

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and the fitted cell probabilities are

$$E(\pi_{ijk}) = \hat{\pi}_{i\cdot k}\hat{\pi}_{\cdot j k}$$

Finally, in the saturated model, the fitted cell probabilities are the observed probabilities,

$$E(\pi_{ijk}) = \hat{\pi}_{ijk}$$
In each of the above examples, the probabilities on the right-hand-side are the “sufficient marginals”.

They are the only proportions we need to observe to estimate the model.
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We could throw the other cells away, as we did in our $2 \times 2$ example of independence.
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As the model gets more complicated, the sufficient marginals include more and more of the actual cells
Notation for 3D tables

It helps to define a more compact notation for multidimensional LLMs

<table>
<thead>
<tr>
<th>Model ( \ln E(\mu_{ijk}) = \ldots )</th>
<th>Notation</th>
<th>Assumptions</th>
</tr>
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<tbody>
<tr>
<td>( \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z )</td>
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Note this notation assumes lower level terms are *included*
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Models that nest all lower order terms sometimes referred to as “hierarchical”

This is a helpful simplifying assumption, but not *logically* required
Fussy points on interaction terms

There is a case where you could omit a lower order term:

1. Theoretically, it should be zero
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A common example from linear regression, if we think fertility depends on GDP/population, but not GDP, we don’t insist on including 1/GDP as a “main effect”
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A common example from linear regression, if we think fertility depends on GDP/population, but not GDP, we don’t insist on including 1/GDP as a “main effect”

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(Sidenote: “main effect” is a misleading term. If interactions are present, they are part of the effect)
Estimating LLM for 3D tables

Estimation of 3+ dimensional tables uses the same MLE as 2D tables.

We could use `loglm` on the table directly.

Or rewrite as a series of observations with categorical explanatory variables, and estimate with `optim`.
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Why? Although Poisson counts can be 0, fitted values from $\exp(\lambda)$, $\lambda > -\infty$ must be positive.
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As with 2D tables, you can often code creative new variables that are potentially more enlightening.
Interpreting parameters

Interpreting the parameters of LLMs with interactions is more difficult
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Key step in modelling a high dimensional data set: reducing the patterns in the data to simple, mostly accurate statements.

Just fitting an LLM is usually not sufficient; you also need to extract summary results from it.
Example: Military Deaths

We examine the deaths of US military personnel in 1999 (the example is from Simonoff 2003)
**Example: Military Deaths**

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The data come in a $5 \times 4 \times 2$ table:

<table>
<thead>
<tr>
<th>Gender</th>
<th>Manner of death</th>
<th>Army</th>
<th>Navy</th>
<th>Air Force</th>
<th>Marines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Accident</td>
<td>158</td>
<td>96</td>
<td>54</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>Illness</td>
<td>56</td>
<td>30</td>
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<td>6</td>
</tr>
<tr>
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</tr>
<tr>
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<td>32</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td></td>
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</tr>
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Note there is no row for combat deaths (there weren't any).
Extrapolating from this table to, say, 2004 would be dangerous.
Example: Military Deaths

We want to know if there are any service or gender related patterns in the different death rates
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Clearly, we need the base terms (more men than women in the data, and more Army than Marines).

But what interactions should we include? Let's explore combinations of

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Note: Simonoff judged models using a corrected \(\text{AIC}_c\) test.
I'm taking his published AICs at face value.

We'll also look at BIC.
Example: Military Deaths

The most complicated feasible model is

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<tr>
<th>Model</th>
<th>df</th>
<th>$G^2$</th>
<th>BIC</th>
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<tbody>
<tr>
<td>$(SM, SG, MG)$</td>
<td>12</td>
<td>5.5</td>
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Can we simplify this?
Example: Military Deaths

The next step is to drop each two-way interaction in turn

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$\text{AIC}_c$ and BIC disagree about the best choice.

Simonoff was using $\text{AIC}_c$, and proceeded with $(SM, MG)$

(He made the substantive conclusion that Gender was conditionally indep of Service)

We will follow his path for now
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BIC picks $(M,SG)$ by a hair. Were Simonoff’s conclusions wrong?

Or do the data even allow us to reject any of the options?
Example: Military Deaths

Neither: we’ve made two (related) common mistakes.

1. We’ve confused statistical and substantive significance
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Only way to tell is to look at the model predictions
These are the observed counts. We’d like to reproduce interesting patterns here or convince ourselves they were spurious.
Military deaths: Fitted proportions under independence

<table>
<thead>
<tr>
<th>Accident</th>
<th>Army</th>
<th>Navy</th>
<th>Air Force</th>
<th>Marines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illness</td>
<td>Army</td>
<td>Navy</td>
<td>Air Force</td>
<td>Marines</td>
</tr>
<tr>
<td>Homicide</td>
<td>Army</td>
<td>Navy</td>
<td>Air Force</td>
<td>Marines</td>
</tr>
<tr>
<td>Suicide</td>
<td>Army</td>
<td>Navy</td>
<td>Air Force</td>
<td>Marines</td>
</tr>
<tr>
<td>Unknown</td>
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</tr>
</tbody>
</table>
Military deaths: Fitted proportions under independence

This is the model under independence. It’s not horrible (interactions must be weak) but maybe those weak interactions are still non-zero.
Simonoff’s preferred model:
Simonoff’s preferred model: Holding Gender independent, we see the Marines have a different pattern of deaths.
Military deaths: Fitted proportions under \((S, GM)\)
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Looks like \((G, SM)\) missed something important: Holding Service independent, we see that women are at higher risk of homicide.
Military deaths: Fitted proportions under $(M, GS)$

- Accident
- Illness
- Homicide
- Suicide
- Unknown

Army  |  Navy  |  Air Force  |  Marines
Holding manner of death independent, we see women are at lower risk in the Marines.
Note that no Marine women were victims of homicide.
Example: Military deaths

The fitted models reveal two interesting interactions:

1. gender × manner: women are relatively more likely to be homicide victims
Example: Military deaths

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1. gender $\times$ manner: women are relatively more likely to be homicide victims

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(Elephant in the middle of the room: “Unknown deaths”? If these are mostly suicides, \(MS\) interaction mostly vanishes)
Concluding thoughts on tabular data

Advantages of tabular presentation:
- Compact presentation of data
Concluding thoughts on tabular data

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Remember there is always an equivalent approach using the standard Poisson or NegBin setup

Remember to check for overdispersion