CSSS/SOC/STAT 536:

Logistic Regression and Log-linear Models

Log-linear Models of Contingency Tables: Multidimensional Tables

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- Tables are harder to visualize
- A plethora of interactions are estimable
- New notation is used to cope with growing complexity
- New concepts: *conditional independence, conditional association, collapsibility*
- Estimation may become difficult if tables have many zeros (curse of dimensionality)

Today, we'll discuss these challenges, but only scratch the surface of the literature

Log-linear models for 3D tables

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Rewriting in log-linear form

$$\ln \mathrm{E}(\mu_{ijk}) = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z$$

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The saturated model for a 3D table has every two-way *and* three-way interaction

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As before, the saturated model perfectly fits the data, and has 0 degrees of freedom

Identifying constraints

As in the $I \times K$ case, the parameters of the marginals must sum to 0 for identification

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Alternatively, we drop one element of each set of parameters from the model

That would leave $I - 1$ λ^X's, $J - 1$ λ^Y's, and $K - 1$ λ^Z's

The actual parameter estimates would look different,
but all quantities of interest would stay the same

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A different way of saying this is that $\lambda_{\cdot jk}^{XYZ} = 0$, $\lambda_{i \cdot k}^{XYZ} = 0$, etc.

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$(I-1)\times(J-1)\times(K-1)$	λ^{XYZ} terms

As before, this parameterization yields superficially different estimates, but identical quantities of interest

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Note this is a weaker condition than joint independence (which is written $X \perp Y$)

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In a moment, we'll introduce a useful tool for investigating collapsibility

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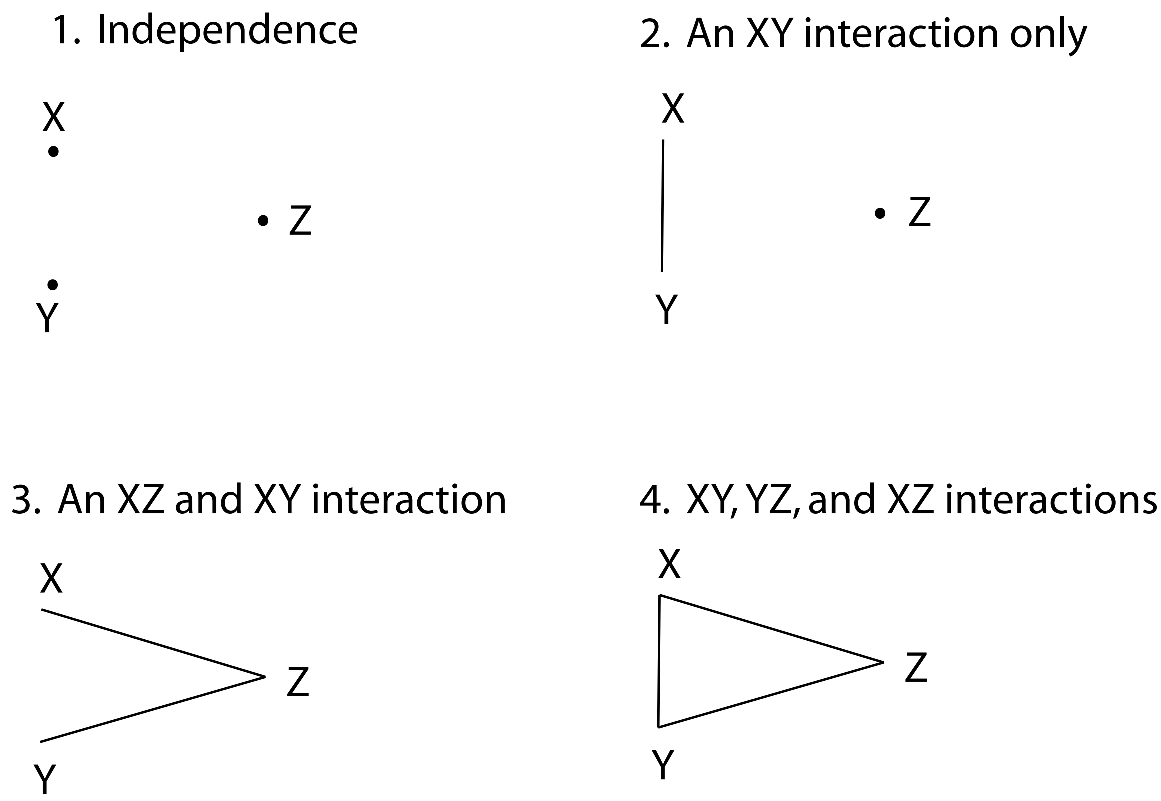
There are tests for both conditional and homogenous association; see Agresti

Concepts for 3D tables: Association Diagrams



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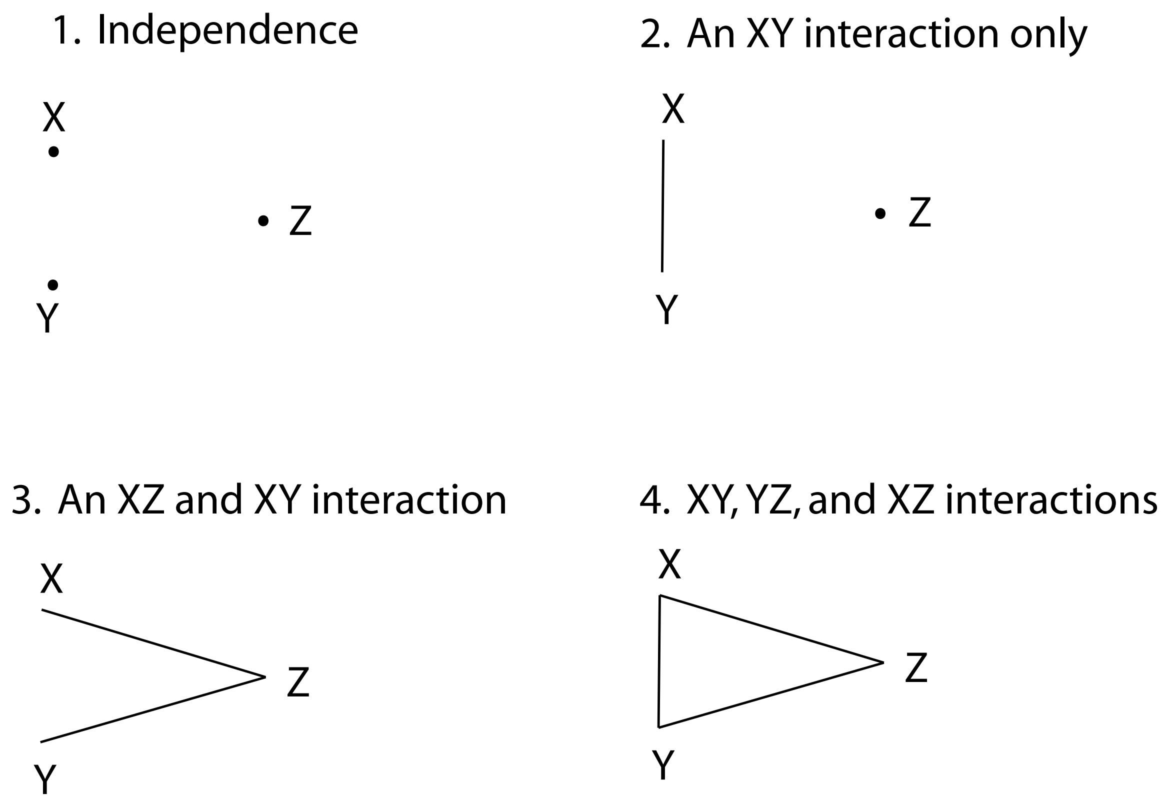

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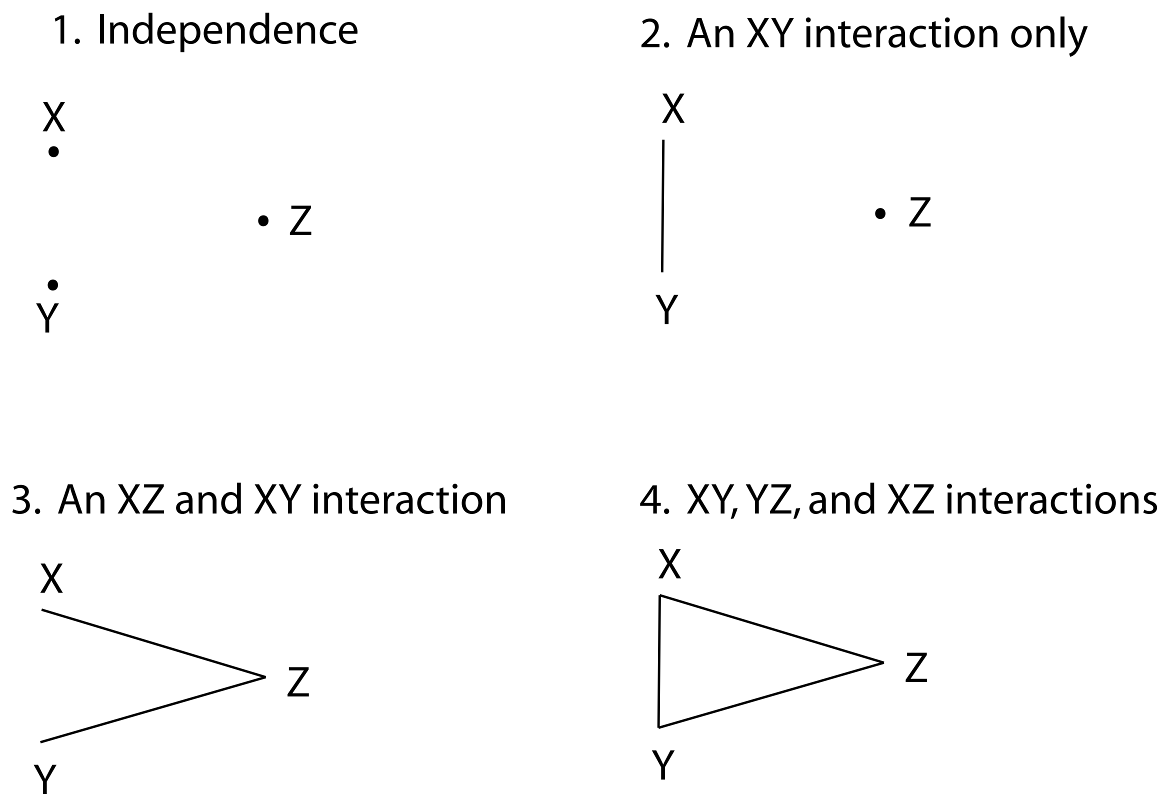

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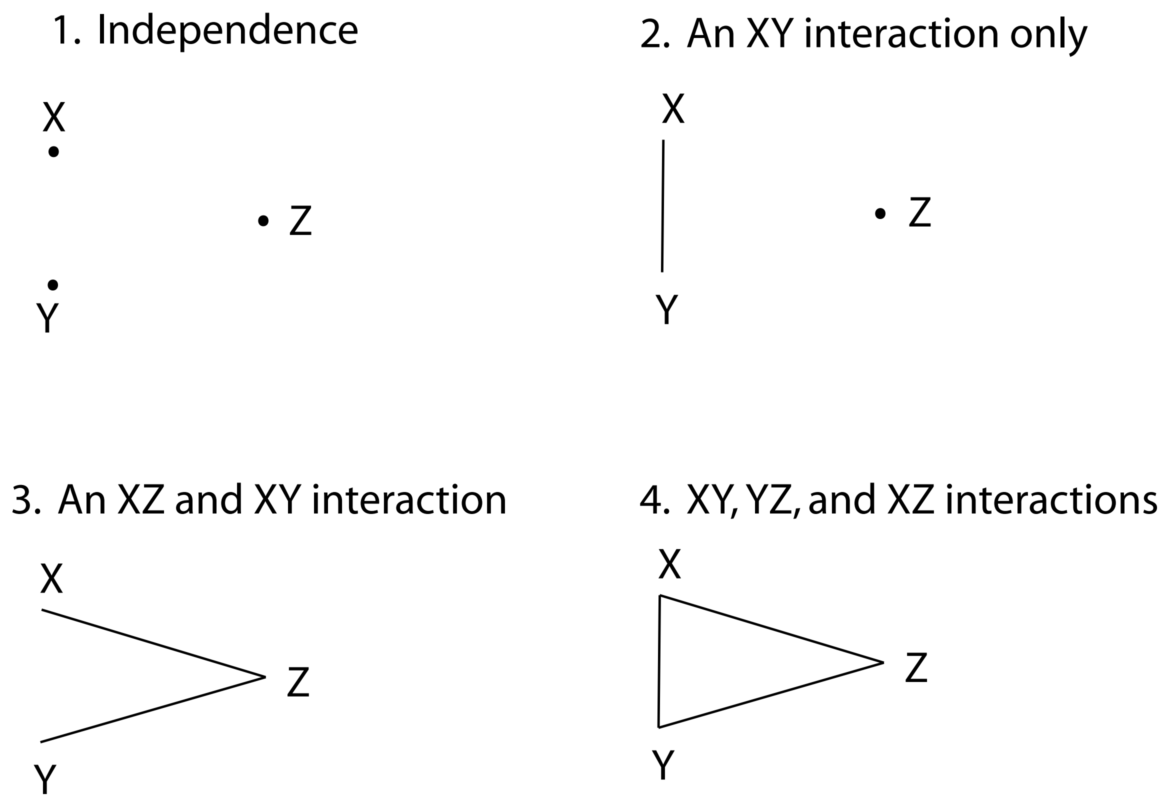

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- conditionally independent if linked by a chain

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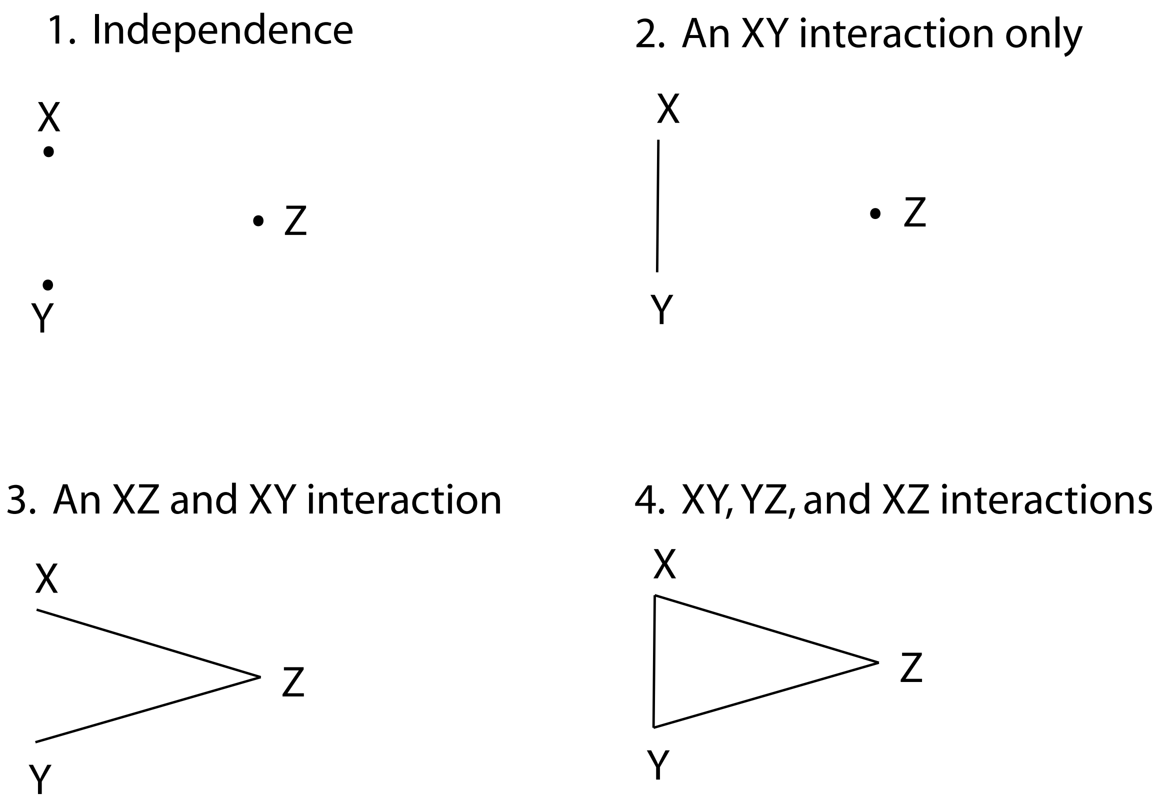

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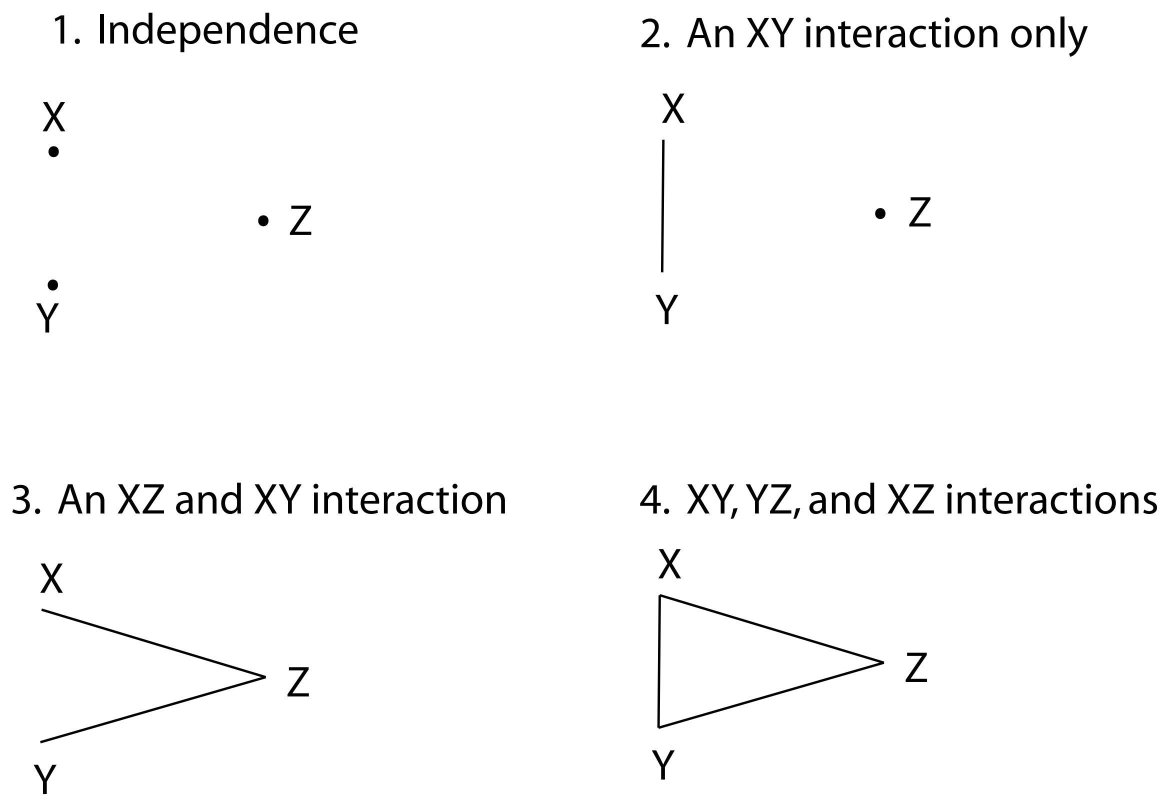

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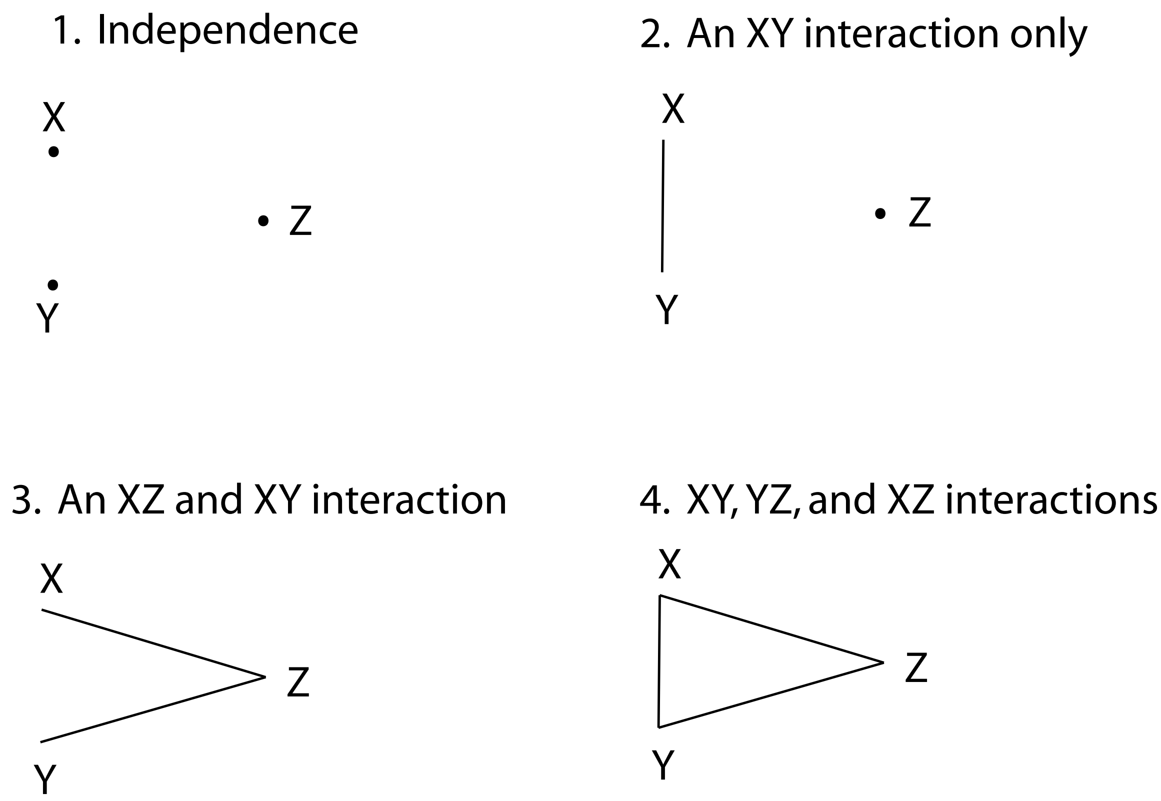

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If there are multiple paths between two variables,
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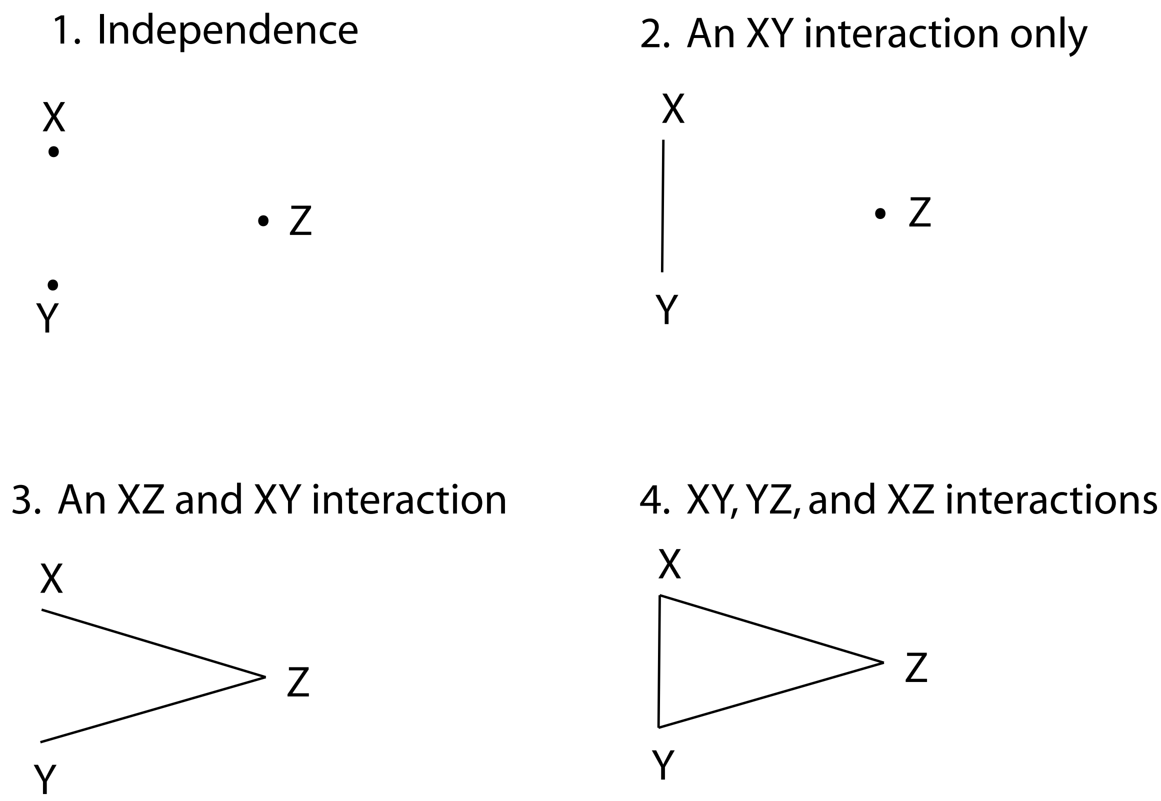

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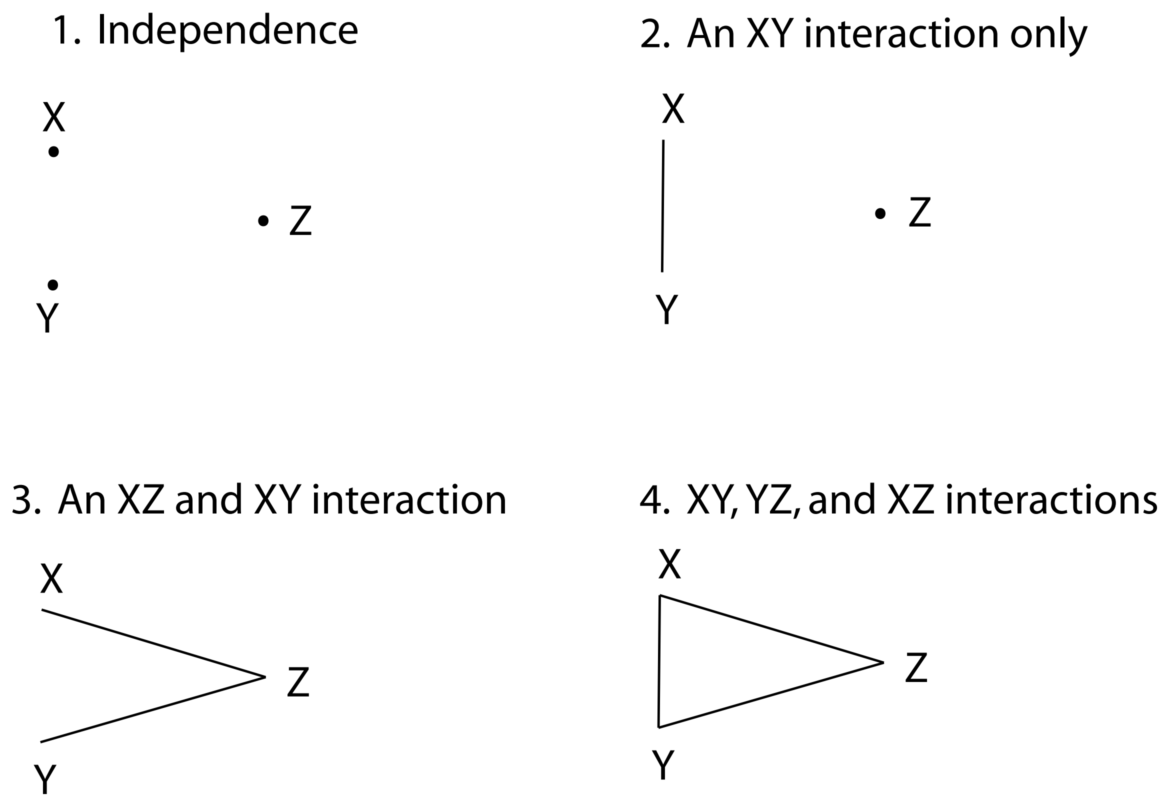

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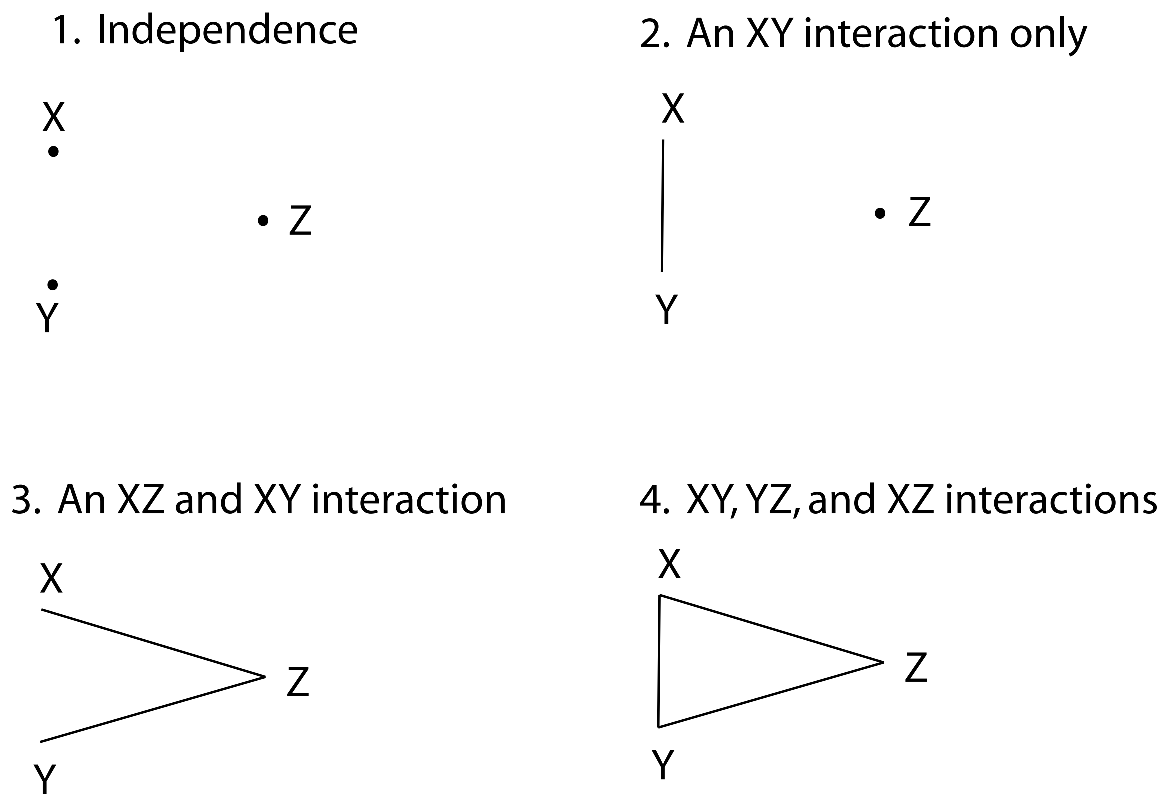

Association diagrams are an easy way to check for collapsibility

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- But *not* variables between other variables in a chain (cond. assoc.)

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Under independence, we have the model

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and the fitted cell probabilities are

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Finally, in the saturated model, the fitted cell probabilities are the observed probabilities,

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As the model gets more complicated,
the sufficient marginals include more and more of the actual cells

Notation for 3D tables

It helps to define a more compact notation for multidimensional LLMs

Model ($\ln \mathrm{E}(\mu_{ijk}) = \ldots$)	Notation	Assumptions
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Models that nest all lower order terms sometimes referred to as "hierarchical"

This is a helpful simplifying assumption, but not *logically* required

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(Sidenote: "main effect" is a misleading term. If interactions are present, they are part of the effect)

Estimating LLM for 3D tables

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Why? Although Poisson *counts* can be 0,
fitted values from $\exp(\lambda)$, $\lambda > -\infty$ must be positive

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As with 2D tables, you can often code creative new variables that are potentially more enlightening

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Key step in modelling a high dimensional data set:
reducing the patterns in the data to simple, mostly accurate statements.

Just fitting an LLM is usually not sufficient;
you also need to extract summary results from it

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We examine the deaths of US military personnel in 1999 (the example is from Simonoff 2003)

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The data come in a $5 \times 4 \times 2$ table:

Gender	Manner of death	Army	Navy	Air Force	Marines
Male	Accident	158	96	54	69
	Illness	56	30	21	6
	Homicide	11	7	2	6
	Suicide	51	32	13	7
	Unknown	24	9	13	27
Female	Accident	15	9	7	3
	Illness	5	4	4	0
	Homicide	5	2	0	1
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Note there is no row for combat deaths (there weren't any). Extrapolating from this table to, say, 2004 would be dangerous.

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(S = Service, G = Gender, M = Manner of death)

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Note: Simonoff judged models using a corrected AIC$_c$ test
I'm taking his published AICs at face value

We'll also look at BIC

Example: Military Deaths

The most complicated feasible model is

Model	df	G^2	BIC	AIC$_c$
(SM,SG,MG)	12	5.5	-74.1	1.93

Can we simplify this?

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The next step is to drop each two-way interaction in turn

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AIC$_c$ and BIC disagree about the best choice.

Simonoff was using AIC$_c$, and proceeded with (SM,MG)

(He made the substantive conclusion that Gender was conditionally indep of Service)

We will follow his path for now

Example: Military Deaths

Next, we try making one of the variables jointly independent

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Simonoff concluded (G,SM) was the best model. Implications:

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Or do the data even allow us to reject any of the options?

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Neither: we've made two (related) common mistakes.

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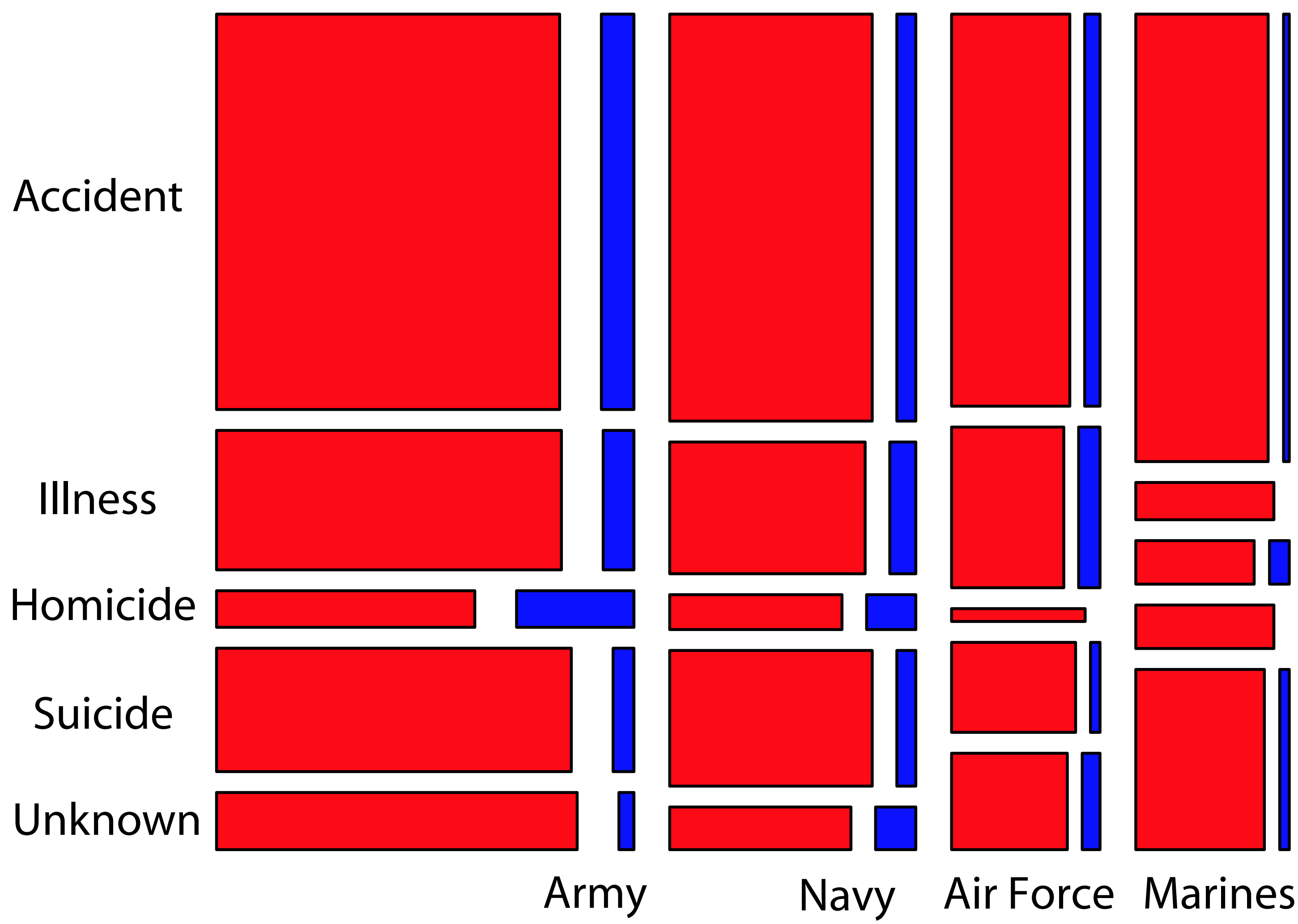
Effects could be confined to particular categories, or be very small but "significant"

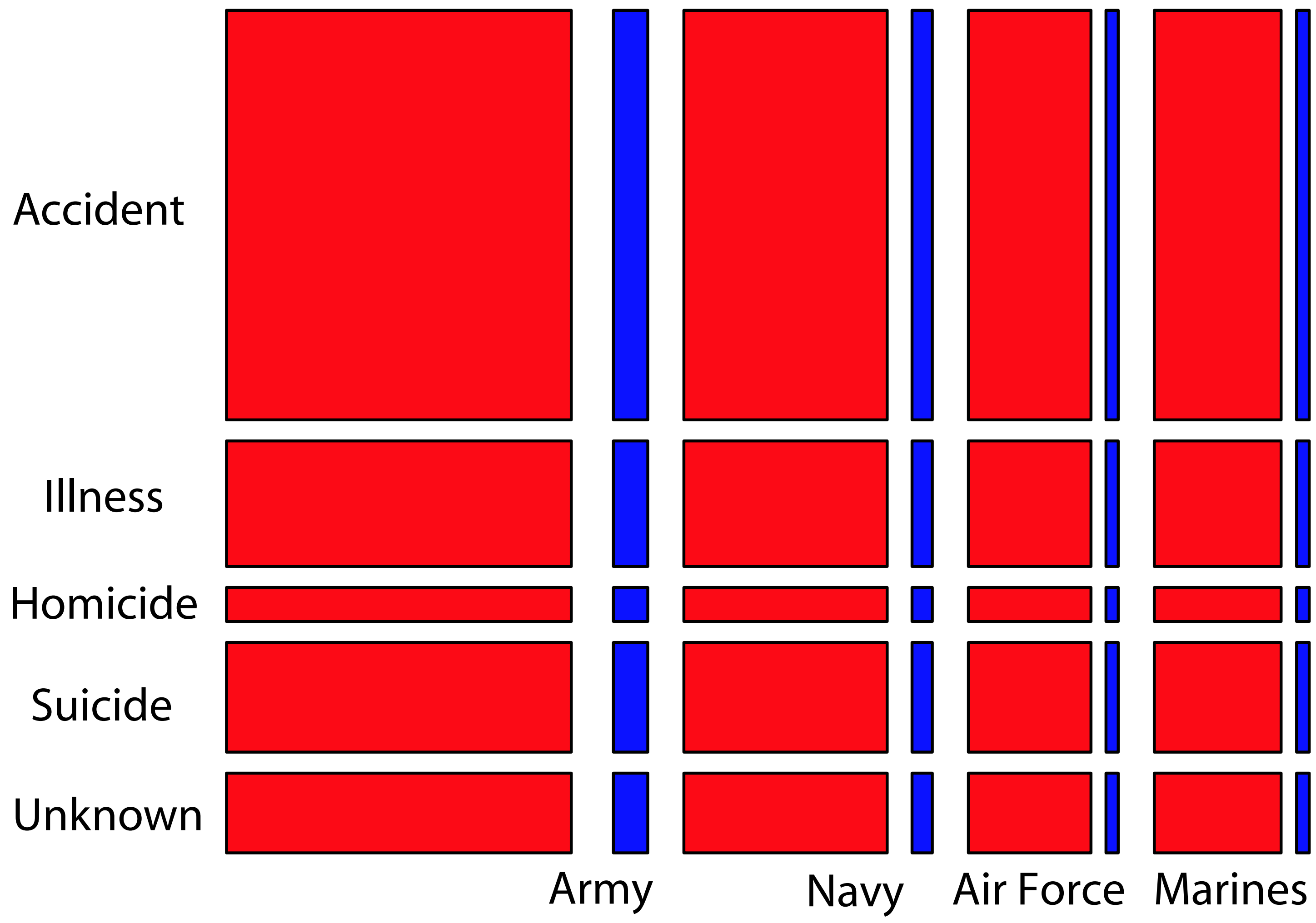
Only way to tell is to look at the model predictions

Military deaths data (Females in Blue)



These are the observed counts. We'd like to reproduce interesting patterns here or convince ourselves they were spurious

Military deaths: Fitted proportions under independence



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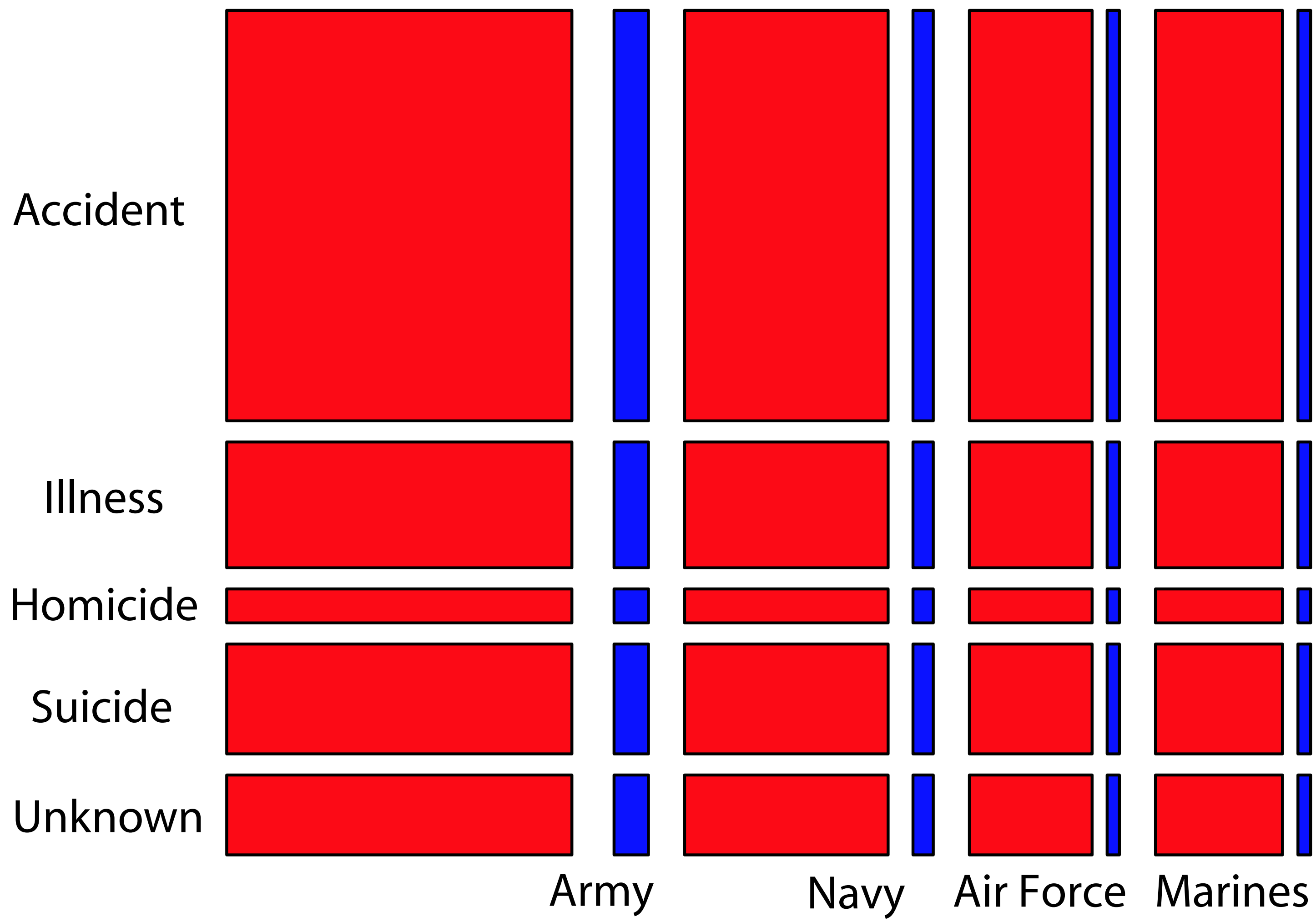
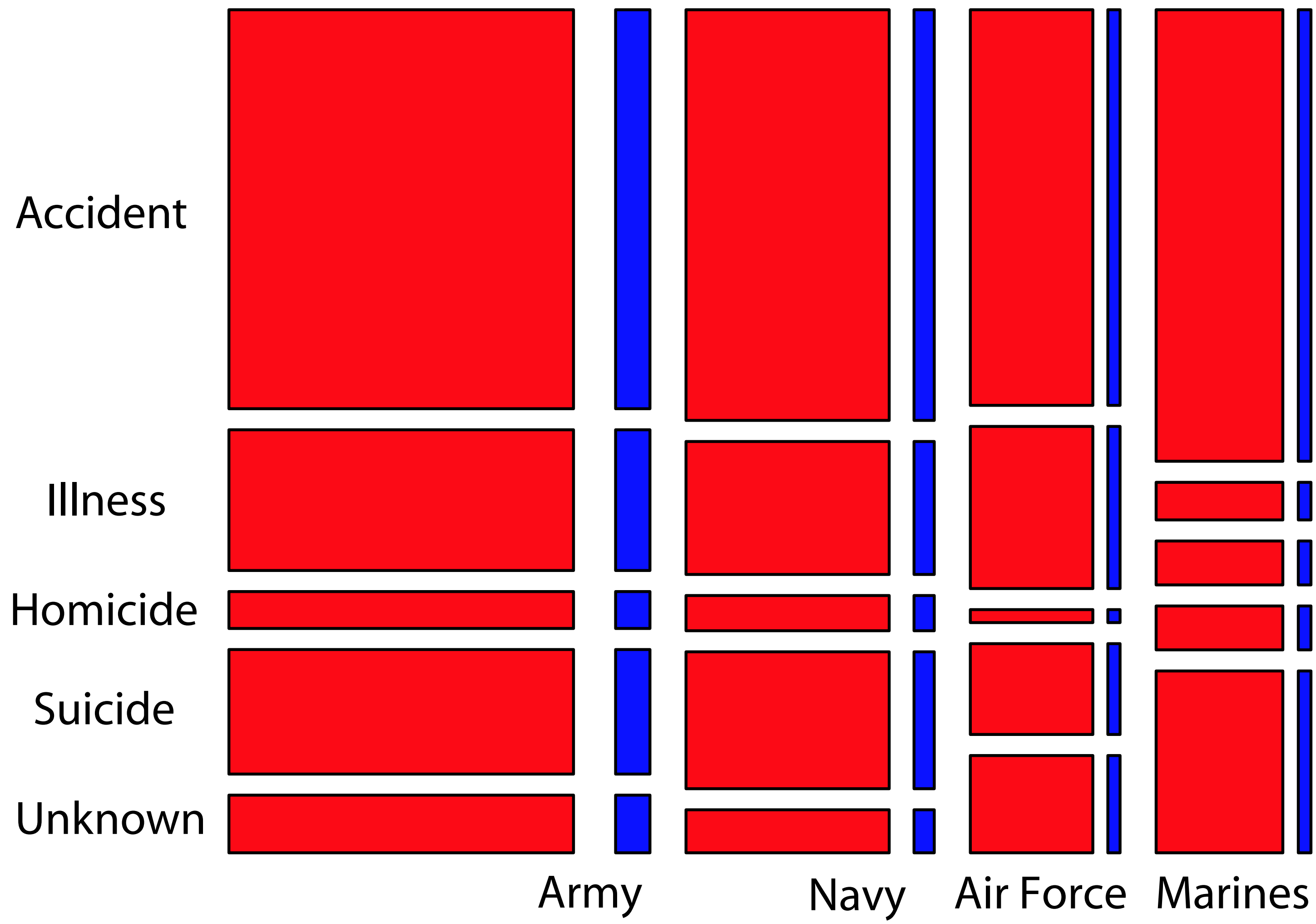


This is the model under independence. It's not horrible (interactions must be weak) but maybe those weak interactions are still non-zero

Military deaths: Fitted proportions under (G, SM)



Simonoff's preferred model:

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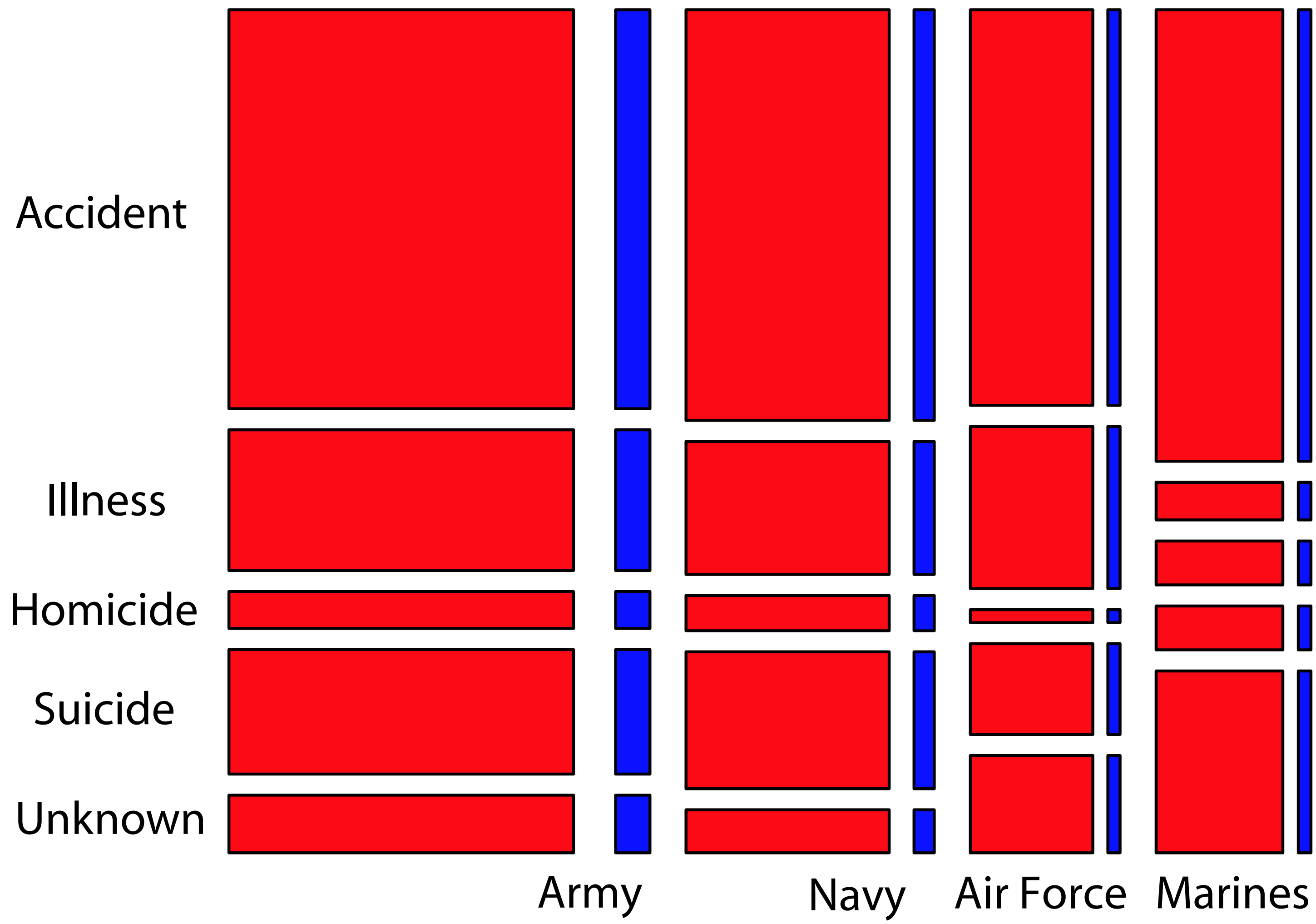

Simonoff's preferred model: Holding Gender independent, we see the Marines have a different pattern of deaths

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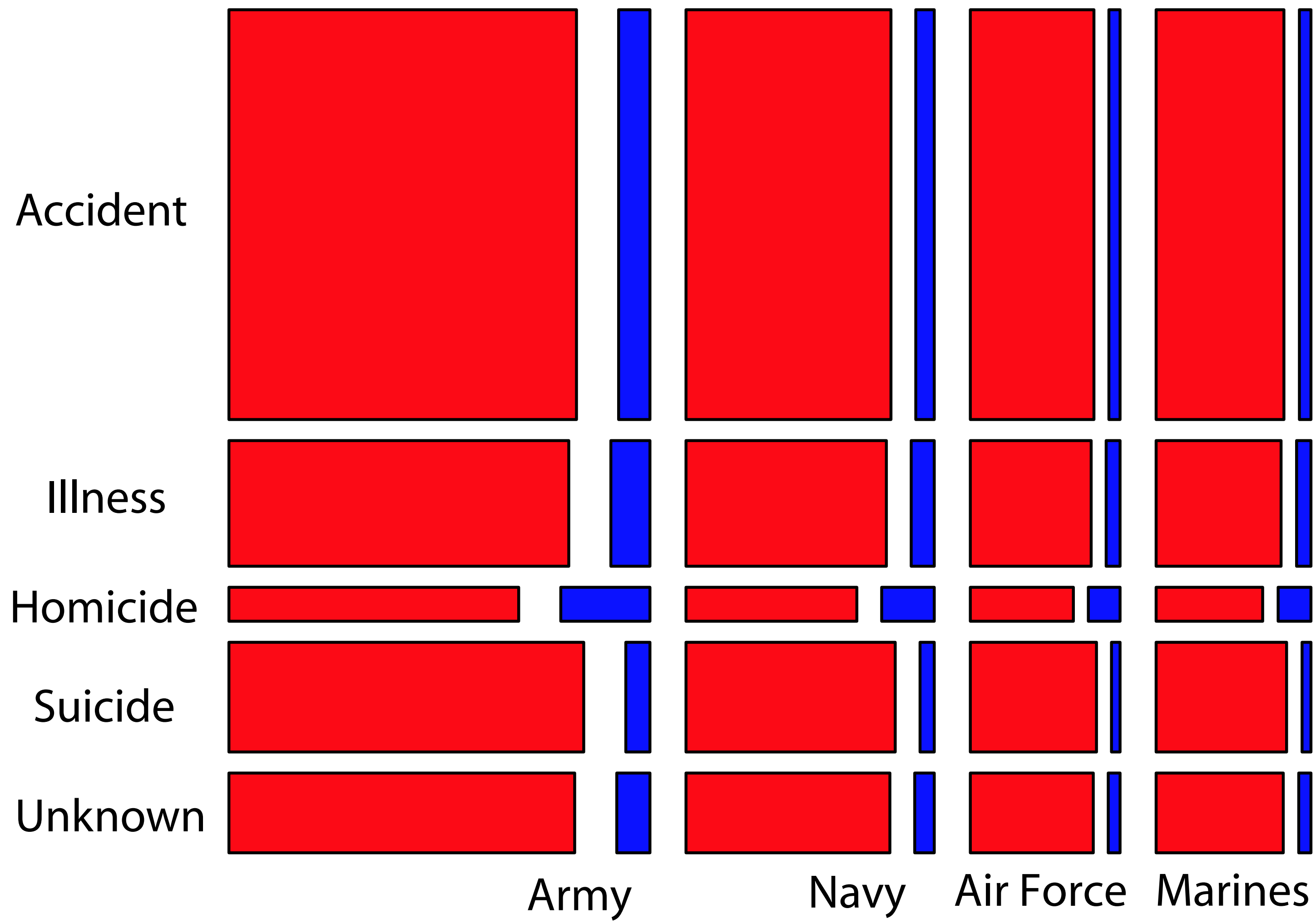

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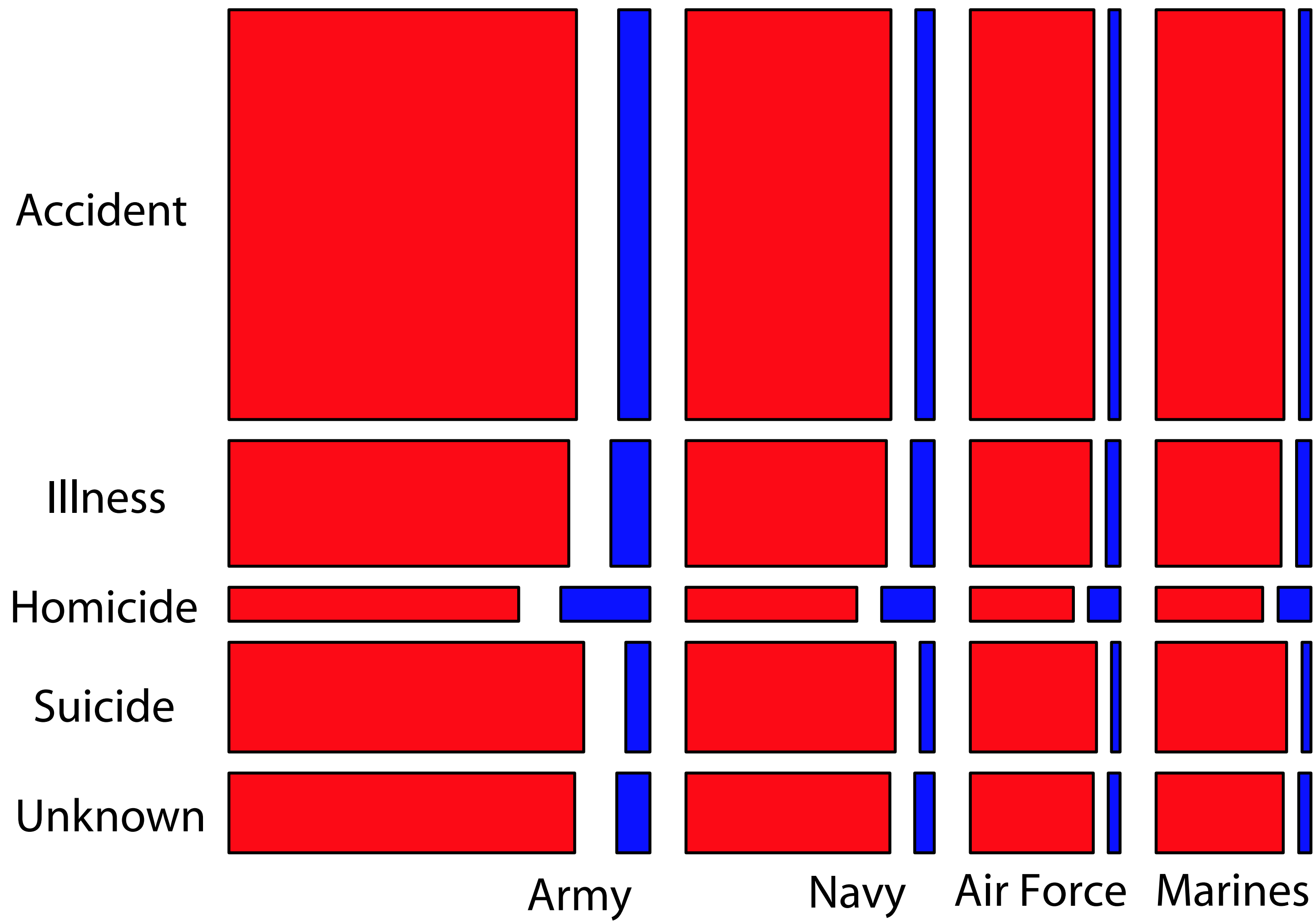
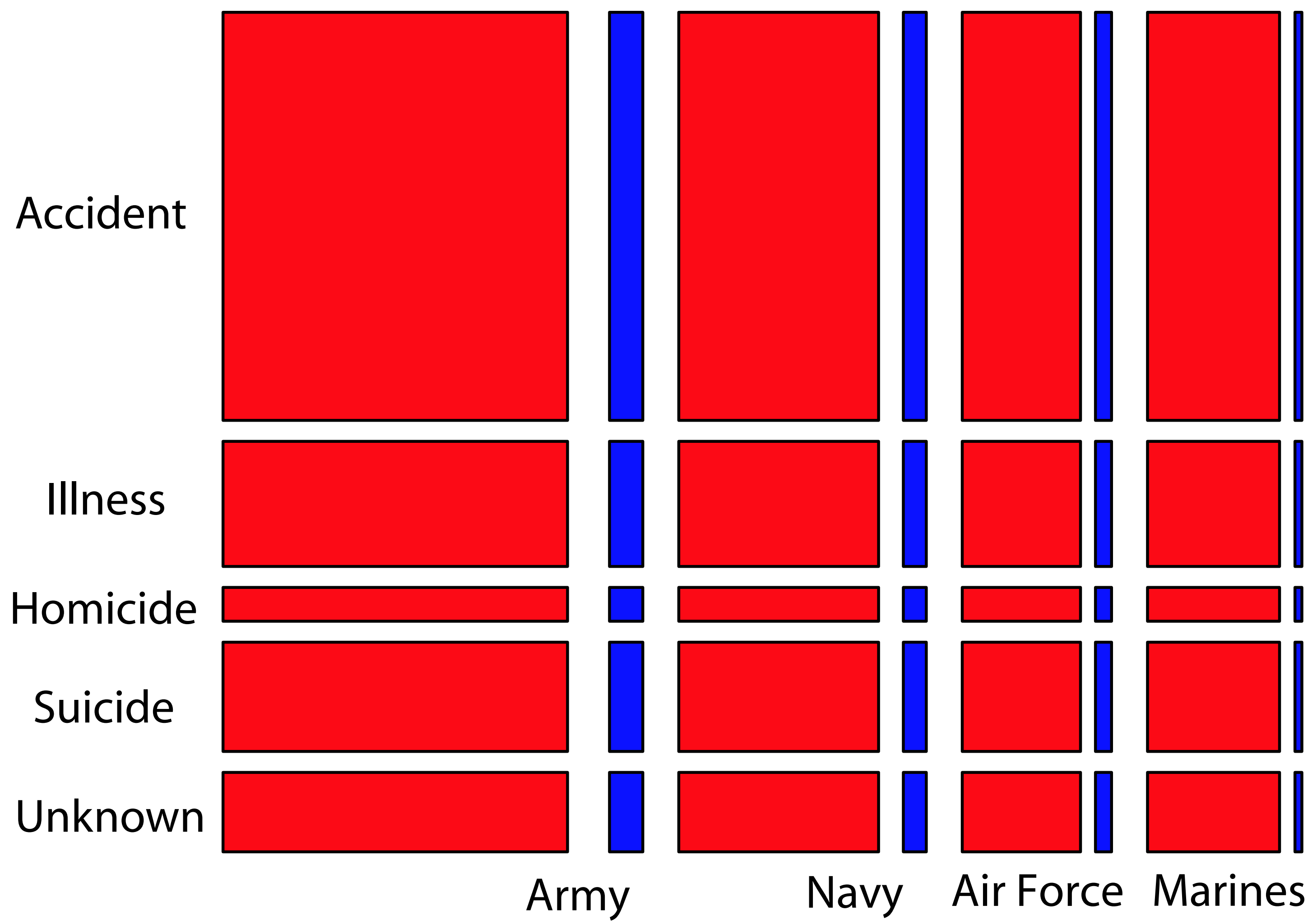


Looks like (G,SM) missed something important: Holding Service independent, we see that women are at higher risk of homicide

Military deaths: Fitted proportions under (M, GS)



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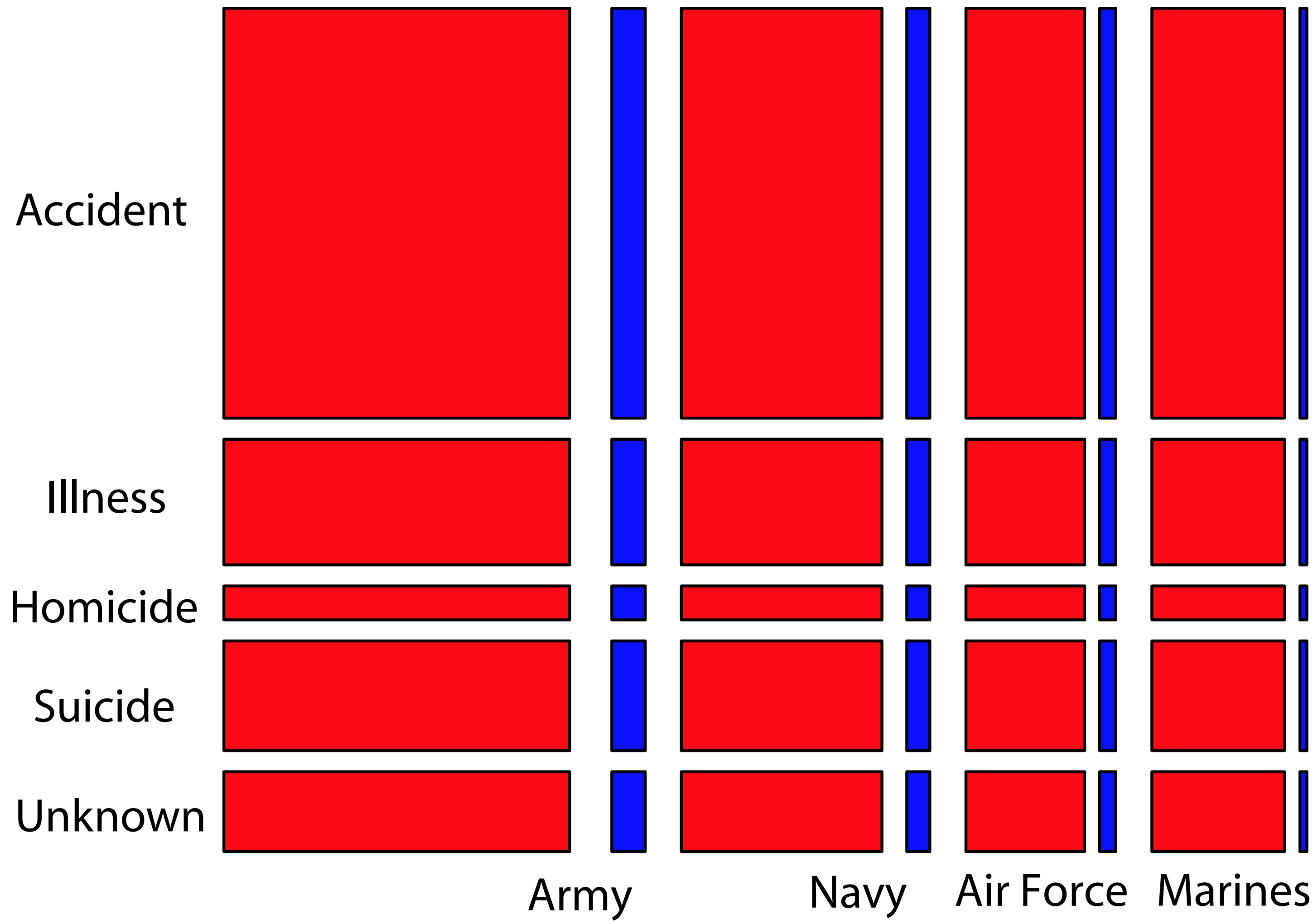

Holding manner of death independent, we see women are at lower risk in the Marines
Note that no Marine women were victims of homicide

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(Elephant in the middle of the room: "Unknown deaths"?
If these are mostly suicides, MS interaction mostly vanishes)

Concluding thoughts on tabular data

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Remember there is always an equivalent approach using the standard Poisson or NegBin setup

Remember to check for overdispersion