Superfluid LDA (SLDA)

Local Density Approximation / Kohn-Sham
for Systems with Superfluid Correlations

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Slides will be posted shortly at
http://www.phys.washington.edu/~bulgac/
What I would like to cover

- Brief review of DFT and LDA
- Introduce SLDA
- Apply SLDA to nuclei and neutron stars (vortices)
- Apply SLDA to dilute atomic Fermi gases (vortices)
- Conclusions
Superconductivity and superfluidity in Fermi systems

- Dilute atomic Fermi gases \( T_c \approx 10^{-12} - 10^{-9} \text{ eV} \)
- Liquid \(^3\text{He}\) \( T_c \approx 10^{-7} \text{ eV} \)
- Metals, composite materials \( T_c \approx 10^{-3} - 10^{-2} \text{ eV} \)
- Nuclei, neutron stars \( T_c \approx 10^5 - 10^6 \text{ eV} \)
- QCD color superconductivity \( T_c \approx 10^7 - 10^8 \text{ eV} \)

units (1 eV \( \approx 10^4 \text{ K} \))
Density Functional Theory (DFT)  
Hohenberg and Kohn, 1964

Local Density Approximation (LDA)  
Kohn and Sham, 1965

\[ E_{gs} = E[\rho(\vec{r})] \]

The energy density is typically determined in \textit{ab initio} calculations of infinite homogeneous matter.

\[ E_{gs} = \int d^3r \left\{ \frac{\hbar^2}{2m} \tau(\vec{r}) + \varepsilon[\rho(\vec{r})] \rho(\vec{r}) \right\} \]

\[ \rho(\vec{r}) = \sum_{i=1}^{N} |\psi_i(\vec{r})|^2 \]

\[ \tau(\vec{r}) = \sum_{i=1}^{N} |\vec{\nabla} \psi_i(\vec{r})|^2 \]

Kohn-Sham equations
Extended Kohn-Sham equations

Position dependent mass

\[
E_{gs} = \int d^3r \left\{ \frac{\hbar^2}{2m^* [\rho(\vec{r})]} \tau(\vec{r}) + \varepsilon[\rho(\vec{r})] \rho(\vec{r}) \right\}
\]

\[
\rho(\vec{r}) = \sum_{i=1}^{N} |\psi_i(\vec{r})|^2 \quad \tau(\vec{r}) = \sum_{i=1}^{N} |\nabla \psi_i(\vec{r})|^2
\]

\[
-\nabla \frac{\hbar^2}{2m^* [\rho(\vec{r})]} \nabla \psi_i(\vec{r}) + U(\vec{r})\psi_i(\vec{r}) = \varepsilon_i \psi_i(\vec{r})
\]
Phenomenological nuclear Skyrme EDF

\[
\mathcal{E}_{SK}(x) = \frac{1}{2M} \tau(x) + \frac{3}{8} t_0 [\rho(x)]^2 + \frac{1}{16} t_3 [\rho(x)]^{2+\alpha} + \frac{1}{16} (3t_1 + 5t_2) \rho(x) \tau(x) + \frac{1}{64} (9t_1 - 5t_2) |\nabla \rho(x)|^2 - \frac{3}{4} W_0 \rho(x) \nabla \cdot J(x) + \frac{1}{32} (t_1 - t_2) [J(x)]^2.
\]

One can try to derive it, however, from an \textit{ab initio} (?) lagrangian

\[
\mathcal{L} = \psi^\dagger \left[ i \partial_t + \frac{\nabla^2}{2M} \right] \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[ (\psi \psi)^\dagger (\psi \nabla^2 \psi) + \text{h.c.} \right] + \frac{C_2'}{8} (\psi \nabla \psi)^\dagger \cdot (\psi \nabla \psi) + \cdots,
\]

\[
C_0 = \frac{4 \pi a_s}{M}, \quad C_2 = C_0 \frac{a_s r_s}{2}, \quad \text{and} \quad C_2' = \frac{4 \pi a_p^3}{M}.
\]

Bhattacharyya and Furnstahl, nucl-phys/0408014
\[
\frac{E}{N} = \frac{k_F^2}{2M} \left[ \frac{3}{5} + (g - 1) \left\{ \frac{2}{3\pi} (k_F a_s) + \frac{4}{35\pi^2} (11 - 2 \ln 2) (k_F a_s)^2 + \frac{1}{10\pi} (k_F r_s) (k_F a_s)^2 \right. \\
+ (0.076 + 0.057 (g - 3)) (k_F a_s)^3 \right\} + (g + 1) \frac{1}{5\pi} (k_F a_p)^3 \\
+ (g - 1) (g - 2) \frac{16}{27\pi^3} (4\pi - 3\sqrt{3}) (k_F a_s)^4 \ln (k_F a_s) + \cdots \right].
\]
One can construct however an EDF which depends both on particle density and kinetic energy density and use it in a extended Kohn-Sham approach

\[
E[\rho(x), \tau(x)] = \int d^3x \left\{ \frac{1}{2M} \tau(x) + \nu(x) \rho(x) + \frac{1}{2} \frac{(\nu - 1)}{\nu} \frac{4\pi a_s}{M} [\rho(x)]^2 \\
+ \left( B_2 a_s^2 r_s + B_3 a_p^3 \right) \frac{1}{2M} \rho(x) \tau(x) + \left( 3B_2 a_s^2 r_s - B_3 a_p^3 \right) \frac{1}{8M} [\nabla \rho(x)]^2 \\
+ b_1 \frac{a_s^2}{2M} [\rho(x)]^{7/3} + b_4 \frac{a_s^3}{2M} [\rho(x)]^{8/3} \right\}.
\]

Notice that dependence on kinetic energy density and on the gradient of the particle density emerges because of finite range effects.

Bhattacharyya and Furnstahl, nucl-phys/0408014
The single-particle spectrum of usual Kohn-Sham approach is unphysical, with the exception of the Fermi level. The single-particle spectrum of extended Kohn-Sham approach has physical meaning.

<table>
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<th>$\nu$</th>
<th>$N_F$</th>
<th>$A$</th>
<th>$a_p$</th>
<th>$E/A$</th>
<th>$\langle k_F \rangle$</th>
<th>$\sqrt{\langle r^2 \rangle}$</th>
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Local Density Approximation (LDA)
Kohn and Sham, 1965

\[ E_{gs} = \int d^3r \left\{ \frac{\hbar^2}{2m} \tau(\vec{r}) + \varepsilon[\rho(\vec{r})]\rho(\vec{r}) \right\} \]

\[ \rho(\vec{r}) = \sum_{i=1}^{N} |\psi_i(\vec{r})|^2 \quad \tau(\vec{r}) = \sum_{i=1}^{N} |\nabla \psi_i(\vec{r})|^2 \]

\[ -\frac{\hbar^2}{2m} \Delta \psi_i(\vec{r}) + U(\vec{r})\psi_i(\vec{r}) = \varepsilon_i \psi_i(\vec{r}) \]

Normal Fermi systems only!
However, not everyone is normal!
SLDA - Extension of Kohn-Sham approach to superfluid Fermi systems

\[
E_{gs} = \int d^3 r \varepsilon(\rho(\vec{r}), \tau(\vec{r}), \nu(\vec{r}))
\]

\[
\rho(\vec{r}) = 2 \sum_k |v_k(\vec{r})|^2, \quad \tau(\vec{r}) = 2 \sum_k |\vec{\nabla} v_k(\vec{r})|^2
\]

\[
\nu(\vec{r}) = \sum_k u_k(\vec{r}) v_k^*(\vec{r})
\]

\[
\begin{pmatrix}
T + U(\vec{r}) - \mu \\
\Delta^*(\vec{r})
\end{pmatrix}
\begin{pmatrix}
\Delta(\vec{r}) \\
-(T + U(\vec{r}) - \mu)
\end{pmatrix}
\begin{pmatrix}
u_k(\vec{r}) \\
v_k^*(\vec{r})
\end{pmatrix} = E_k
\begin{pmatrix}
u_k(\vec{r}) \\
v_k^*(\vec{r})
\end{pmatrix}
\]

Mean-field and pairing field are both local fields!
(for sake of simplicity spin degrees of freedom are not shown)

There is a little problem! The pairing field \( \Delta \) diverges.
Why would one consider a local pairing field?

- Because it makes sense physically!
- The treatment is so much simpler!
- Our intuition is so much better also.

\[ r_0 \approx \frac{\hbar}{p_F} = k_F^{-1} \]

radius of interaction  inter-particle separation

\[ \Delta = \omega_D \exp \left( -\frac{1}{|V| N} \right) \ll \varepsilon_F \]

\[ \xi \approx \frac{1}{k_F} \frac{\varepsilon_F}{\Delta} \gg r_0 \]

coherence length  size of the Cooper pair
Nature of the problem

\[ \nu(\vec{r}_1, \vec{r}_2) = \sum_{E_k > 0} v_k^*(\vec{r}_1) u_k(\vec{r}_2) \propto \frac{1}{|\vec{r}_1 - \vec{r}_2|} \]

\[ \Delta(\vec{r}_1, \vec{r}_2) = \frac{1}{2} V(\vec{r}_1, \vec{r}_2) \nu(\vec{r}_1, \vec{r}_2) \]

It is easier to show how this singularity appears in infinite homogeneous matter.

\[ v_k(\vec{r}_1) = v_k \exp(ik \cdot \vec{r}_1), \quad u_k(\vec{r}_2) = u_k \exp(ik \cdot \vec{r}_2) \]

\[ v_k^2 = \frac{1}{2} \left( 1 - \frac{\varepsilon_k - \mu}{\sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}} \right), \quad u_k^2 + v_k^2 = 1, \quad \varepsilon_k = \frac{\hbar^2 k^2}{2m} + U, \quad \Delta = \frac{\hbar^2 \delta}{2m} \]

\[ \nu(r) = \frac{\Delta m}{2\pi^2 \hbar^2} \int_0^\infty dk \frac{\sin(kr)}{kr} \frac{k^2}{\sqrt{(k^2 - k_F^2)^2 + \delta^2}}, \quad r = |\vec{r}_1 - \vec{r}_2| \]
How pairing emerges?

Cooper’s argument (1956)

Gap = 2Δ

Cooper pair
**Pseudo-potential approach**
(appropriate for very slow particles, very transparent, but somewhat difficult to improve)

Lenz (1927), Fermi (1931), Blatt and Weiskopf (1952)
Lee, Huang and Yang (1957)

\[ -\frac{\hbar^2}{m} \Delta \vec{r} \psi (\vec{r}) + V (\vec{r}) \psi (\vec{r}) = E \psi (\vec{r}), \quad V (\vec{r}) \approx 0 \text{ if } r > R \]

\[ \psi (\vec{r}) = \exp (i k \cdot \vec{r}) + \frac{f}{r} \exp (i k r) \approx 1 + \frac{f}{r} + ... \approx 1 - \frac{a}{r} + O(kr) \]

\[ f^{-1} = -\frac{1}{a} + \frac{1}{2} r_0 k^2 - i k, \quad g = \frac{4\pi \hbar^2 a}{m(1 + ika)} + ... \]

if \( kr_0 \ll 1 \) then \( V (\vec{r}) \psi (\vec{r}) \Rightarrow g \delta (\vec{r}) \frac{\partial}{\partial r} [r \psi (\vec{r})] \]

Example: \( \psi (\vec{r}) = \frac{A}{r} + B + ... \Rightarrow \delta (\vec{r}) \frac{\partial}{\partial r} [r \psi (\vec{r})] = \delta (\vec{r}) B \)
The SLDA (renormalized) equations

\[ E_{gs} = \int d^3r \left\{ \varepsilon_N \left[ \rho(\vec{r}), \tau(\vec{r}) \right] + \varepsilon_S \left[ \rho(\vec{r}), \nu(\vec{r}) \right] \right\} \]

\[ \varepsilon_S \left[ \rho(\vec{r}), \nu(\vec{r}) \right] = -\Delta(\vec{r})\nu_c(\vec{r}) = g_{eff}(\vec{r})|\nu_c(\vec{r})|^2 \]

\[ \begin{cases} [h(\vec{r}) - \mu]u_i(\vec{r}) + \Delta(\vec{r})v_i(\vec{r}) = E_iu_i(\vec{r}) \\ \Delta^*(\vec{r})u_i(\vec{r}) - [h(\vec{r}) - \mu]v_i(\vec{r}) = E_iv_i(\vec{r}) \end{cases} \]

\[ \begin{cases} h(\vec{r}) = -\nabla \cdot \frac{\hbar^2}{2m(\vec{r})} \nabla + U(\vec{r}) \\ \Delta(\vec{r}) = -g_{eff}(\vec{r})\nu_c(\vec{r}) \end{cases} \]

\[ \frac{1}{g_{eff}(\vec{r})} = \frac{1}{g[\rho(\vec{r})]} - \frac{m(\vec{r})k_c(\vec{r})}{2\pi^2\hbar^2} \left\{ 1 - \frac{k_F(\vec{r})}{2k_c(\vec{r})} \ln \frac{k_c(\vec{r}) + k_F(\vec{r})}{k_c(\vec{r}) - k_F(\vec{r})} \right\} \]

\[ \rho_c(\vec{r}) = 2 \sum_{E_i \geq 0} \left| v_i(\vec{r}) \right|^2, \quad \nu_c(\vec{r}) = \sum_{E_i \geq 0} v_i^*(\vec{r})u_i(\vec{r}) \]

\[ E_c + \mu = \frac{\hbar^2 k_c^2(\vec{r})}{2m(\vec{r})} + U(\vec{r}), \quad \mu = \frac{\hbar^2 k_F^2(\vec{r})}{2m(\vec{r})} + U(\vec{r}) \]

Position and momentum dependent running coupling constant

Observables are (obviously) independent of cut-off energy (when chosen properly).
The nuclear landscape and the models

The isotope and isotone chains treated by us are indicated with red numbers.

Courtesy of Mario Stoitsov
How to describe nuclei?

Fayans parameterization of the infinite matter calculations

This defines the normal part of the EDF.
Pairing correlations show prominently in the staggering of the binding energies.

*Systems with odd particle number are less bound than systems with even particle number.*
One-neutron separation energies

Volume pairing

\[ g(\vec{r}) = g \]

Volume + Surface pairing

\[ g(\vec{r}) = V_0 \left( 1 - \frac{\rho(\vec{r})}{\rho_c} \right) \]

Normal EDF:

- **SLy4** - Chabanat et al.

- **FaNDF^0** – Fayans
Two-neutron separation energies

![Graph showing two-neutron separation energies for Sn and Pb with SLy4 and FaNDF0 models.](image)

- **Sn**
  - SLy4
  - FaNDF0

- **Pb**
  - SLy4
  - FaNDF0

- **Exp. Volume**: Experimental volume
- **Vol. + Surf.**: Volume + Surface

- **$S_{2n}$ [MeV]**

- **A**
One-nucleon separation energies

- N = 50
  - FaNDFO

- N = 82
  - FaNDFO

- N = 126
  - FaNDFO

- Ca
  - FaNDFO
Anderson and Itoh, *Nature*, 1975

“Pulsar glitches and restlessness as a hard superfluidity phenomenon”

The crust of neutron stars is the only other place in the entire Universe where one can find solid matter, except planets.

- A neutron star will cover the map at the bottom
- The mass is about 1.5 solar masses
- Density $10^{14} \text{ g/cm}^3$
How can one determine the density dependence of the coupling constant $g$? I know two methods.

\[
\varepsilon_S[\rho(\vec{r}), \nu(\vec{r})] = g[\rho(\vec{r})] |\nu(\vec{r})|^2
\]

\[\Delta = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)\]

\[\text{Gorkov and Melik-Barkhudarov, 1961}\]

- In homogeneous very low density matter one can compute the pairing gap as a function of the density. NB this is not a simple BCS result!

- One compute also the energy of the normal and superfluid phases as a function of density, as was recently done by Carlson et al, Phys. Rev. Lett. 91, 050401 (2003) for a Fermi system interacting with an infinite scattering length (Bertsch’s MBX 1999 challenge)

In both cases one can extract from these results the superfluid contribution to the LDA energy density functional in a straight forward manner.
“Screening effects” are significant!

s-wave pairing gap in infinite neutron matter with realistic NN-interactions

from Lombardo and Schulze astro-ph/0012209

These are major effects beyond the naïve HFB when it comes to describing pairing correlations.
\[ \Delta = \left( \frac{2}{e} \right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp \left[ -\frac{\pi}{2 \tan \delta(k_F)} \right] \]

NB! Extremely high relative T_c

Corrected Emery formula (1960)

RG- renormalization group calculation
Vortex in neutron matter

\[
\begin{pmatrix}
  u_{\alpha kn}(\vec{r}) \\
  v_{\alpha kn}(\vec{r})
\end{pmatrix}
= \begin{pmatrix}
  u_\alpha(r) \exp[i(n+1/2)\phi - ikz] \\
  v_\alpha(r) \exp[i(n-1/2)\phi - ikz]
\end{pmatrix}, \quad n - \text{half-integer}
\]

\[\Delta(\vec{r}) = \Delta(r) \exp(i\phi), \quad \vec{r} = (r, \phi, z) \ [\text{cylindrical coordinates}]\]

Oz - vortex symmetry axis

Ideal vortex, Onsager's quantization (one \(\hbar\) per Cooper pair)

\[
\vec{V}_v(\vec{r}) = \frac{\hbar}{2mr^2}(y,-x,0) \iff \frac{1}{2\pi} \oint_C \vec{V}_v(\vec{r}) \cdot d\vec{r} = \frac{\hbar}{2m}
\]
Distances scale with $\lambda_F$

Distances scale with $\xi_F$
Dramatic structural changes of the vortex state naturally lead to significant changes in the energy balance of a neutron star.

- \( \frac{v_S}{v_F} \leq \frac{\Delta}{2\varepsilon_F_{max}} \approx 0.12 \), extremely fast vortical motion,
- \( \frac{\lambda_F}{\xi} \propto \frac{\Delta}{\varepsilon_F} \)

- In low density region \( \varepsilon(\rho_{out})\rho_{out} > \varepsilon(\rho_{in})\rho_{in} \)
which thus leads to a large anti-pinning energy \( E_{pin}^V > 0 \):

\[
E_{pin}^V = \left[ \varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in} \right]V
\]

- The energy per unit length is going to be changed dramatically when compared to previous estimates, by

\[
\frac{\Delta E_{vortex}}{L} \approx \left[ \varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in} \right]\pi R^2
\]

- Specific heat, transport properties are expected to significantly affected as well.

Some similar conclusions have been reached recently also by Donati and Pizzochero, Phys. Rev. Lett. 90, 211101 (2003).
What are the ground state properties of the many-body system composed of spin $\frac{1}{2}$ fermions interacting via a zero-range, infinite scattering-length contact interaction.

In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not.

- systems of bosons are unstable (Efimov effect)
- systems of three or more fermion species are unstable (Efimov effect)

• Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)

• Chang et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided best the theoretical estimates for the ground state energy of such systems.

• Thomas’ Duke group (2002) demonstrated experimentally that such systems are (meta)stable.
Consider Bertsch’s MBX challenge (1999): “Find the ground state of infinite homogeneous neutron matter interacting with an infinite scattering length.”

\[ r_0 \rightarrow 0 \ll \lambda_F \ll |a| \rightarrow \infty \]

- Carlson, Morales, Pandharipande and Ravenhall, PRC 68, 025802 (2003), with Green Function Monte Carlo (GFMC)

\[ \frac{E_N}{N} = \alpha_N \frac{3}{5} \varepsilon_F, \quad \alpha_N = 0.54 \]

- Carlson, Chang, Pandharipande and Schmidt, PRL 91, 050401 (2003), with GFMC

\[ \frac{E_S}{N} = \alpha_S \frac{3}{5} \varepsilon_F, \quad \alpha_S = 0.44 \]

This state is half the way from BCS→BEC crossover, the pairing correlations are in the strong coupling limit and HFB invalid again.
Solid line with open circles – Chang et al. physics/0404115
Dashed line with squares – Astrakharchik et al. cond-mat/0406113
\[ \Delta(2n+1) = E(2n+1) - \frac{1}{2}(E(2n) + E(2n+2)) \]

\[ E_{FG} = \frac{3}{5} \frac{\hbar^2 k_F^2}{2m} \]

Green Function Monte Carlo with Fixed Nodes
S.-Y. Chang, J. Carlson, V. Pandharipande and K. Schmidt
physics/0403041
\[
\Delta_{Gorkov} = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)
\]
\[
\Delta_{BCS} = \frac{8}{e^2} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)
\]

Fixed node GFMC results, S.-Y. Chang et al. (2003)
If $a < 0$ at $T=0$ a Fermi system is a BCS superfluid

$$
\Delta \approx \left( \frac{2}{e} \right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp \left( \frac{\pi}{2k_F a} \right) \ll \varepsilon_F, \quad \text{iff} \quad k_F |a| \ll 1 \text{ and } \xi = \frac{1}{k_F} \frac{\varepsilon_F}{\Delta} \gg \frac{1}{k_F}
$$

If $|a| = \infty$ and $nr_0^3 \ll 1$ a Fermi system is strongly coupled and its properties are universal. Carlson et al. PRL 91, 050401 (2003)

$$
\frac{E_{\text{normal}}}{N} \approx 0.54 \frac{3}{5} \varepsilon_F, \quad \frac{E_{\text{superfluid}}}{N} \approx 0.44 \frac{3}{5} \varepsilon_F \quad \text{and} \quad \xi = O(\lambda_F), \quad \Delta = O(\varepsilon_F)
$$

If $a > 0$ ($a \gg r_0$) and $na^3 \ll 1$ the system is a dilute BEC of tightly bound dimers

$$
\varepsilon_2 = -\frac{\hbar^2}{ma^2} \quad \text{and} \quad n_b a^3 \ll 1, \quad \text{where} \quad n_b = \frac{n_f}{2} \quad \text{and} \quad a_{bb} = 0.6a > 0
$$
SLDA for dilute atomic Fermi gases

Parameters determined from GFMC results of Chang, Carlson, Pandharipande and Schmidt, physics/0404115

\[
E \left|_{\text{GFMC}} \right. = \varepsilon[n] \approx \frac{3}{5} \varepsilon_F \left[ \xi - \frac{\xi}{k_F a} - \frac{5i}{3(k_F a)^2} \right], \quad \xi \approx 0.44, \quad \zeta \approx 1, \quad \nu \approx 1
\]

\[
\Delta_{\text{GFMC}} \approx \varepsilon_F \left( \frac{2}{e} \right)^{7/3} \exp\left( \frac{\pi}{2k_F a} \right), \quad n = \frac{k_F^3}{3\pi^2}, \quad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}, \quad x = \frac{1}{k_F a}
\]

\[
\varepsilon_{\text{SLDA}}[n]n = \varepsilon_{\text{kin}} + \frac{\hbar^2}{m} \beta[x] n^{5/3} + \frac{\hbar^2}{m} \gamma[x] \frac{|\nu|^2}{n^{1/3}} + \text{Renormalization}
\]

Dimensionless coupling constants
Now we are going to look at vortices in dilute atomic gases in the vicinity of the Feshbach resonance.

Why would one study vortices in neutral Fermi superfluids?

They are perhaps just about the only phenomenon in which one can have a true stable superflow!
How can one put in evidence a vortex in a Fermi superfluid?

Hard to see, since density changes are not expected, unlike the case of a Bose superfluid.

However, if the gap is not small one can expect a noticeable density depletion along the vortex core, and the bigger the gap the bigger the depletion!

One can change the magnitude of the gap by altering the scattering length between two atoms with magnetic fields by means of a Feshbach resonance.
The depletion along the vortex core is reminiscent of the corresponding density depletion in the case of a vortex in a Bose superfluid, when the density vanishes exactly along the axis for 100% BEC.

Fermions with $1/k_Fa = 0.3, 0.1, 0, -0.1, -0.5$

Bosons with $na^3 = 10^{-3}$ and $10^{-5}$

Extremely fast quantum vortical motion!

Number density and pairing field profiles

Local vortical speed as fraction of Fermi speed

From Ketterle’s group
Conclusions:

- An LDA-DFT formalism for describing pairing correlations in Fermi systems has been developed. This represents the first genuinely local extension of the Kohn-Sham LDA from normal to superfluid systems - SLDA.
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- An LDA-DFT formalism for describing pairing correlations in Fermi systems has been developed. This represents the first genuinely local extension of the Kohn-Sham LDA from normal to superfluid systems - **SLDA**