Signatures of Superfluidity in Dilute Fermi Gases near a Feshbach Resonance

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These slides will be posted shortly at
http://www.phys.washington.edu/~bulgac/

Topics

- Brief/incomplete survey of theory and experiment
- Superfluid LDA (SLDA)
- Vortex structure
- Collective oscillations
- Atom – Molecule mixtures

Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not.

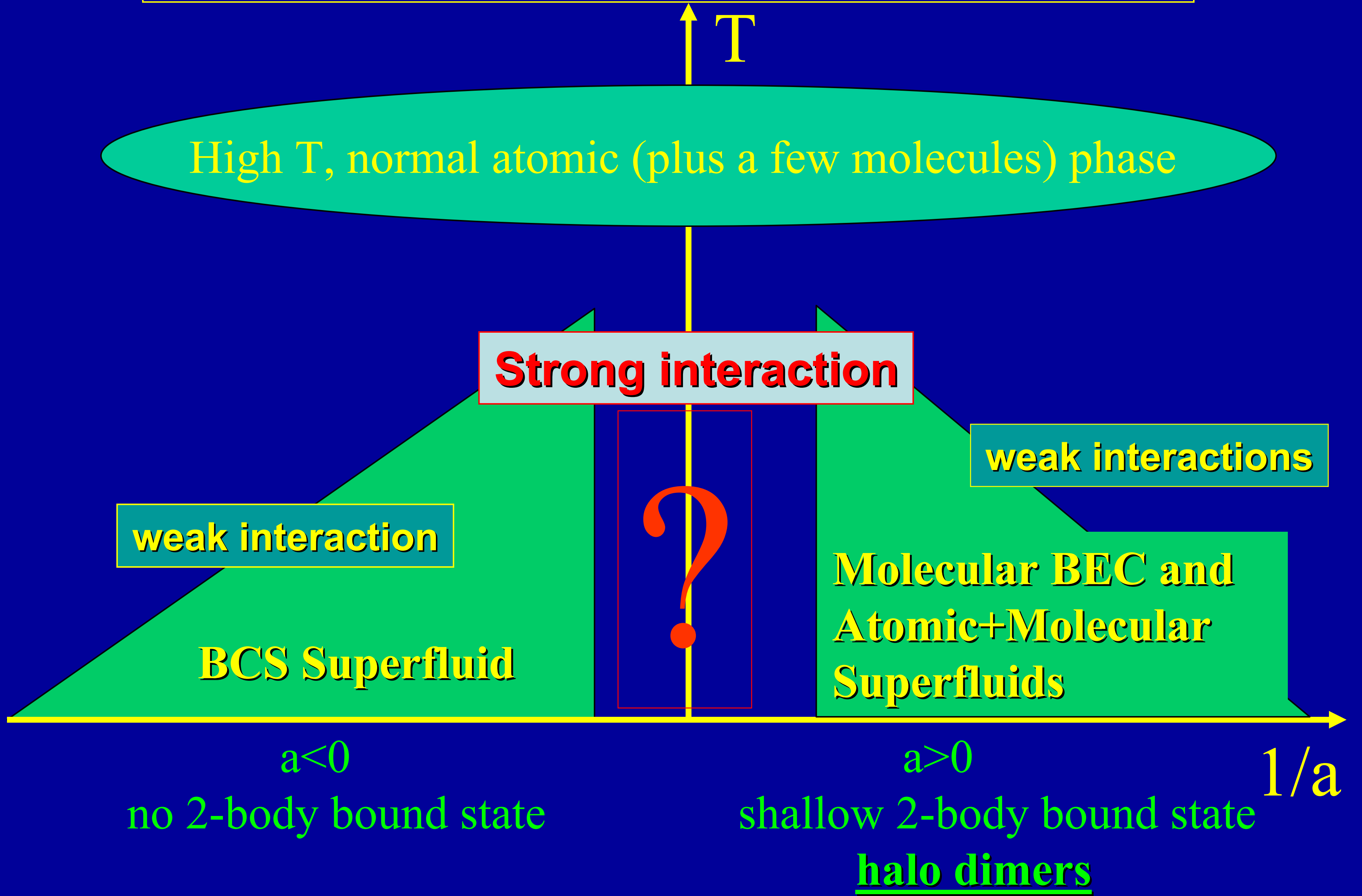
- ***systems of bosons are unstable (Efimov effect)***
- ***systems of three or more fermion species are unstable (Efimov effect)***

- **Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)**

- **Carlson et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided best the theoretical estimates for the ground state energy of such systems.**

- **Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.**

Expected phases of a two species dilute Fermi system
BCS-BEC crossover



Density Functional Theory (DFT)
Hohenberg and Kohn, 1964

$$E_{gs} = \int d^3 r \varepsilon[\rho(\vec{r})]$$

particle density

Local Density Approximation (LDA)
Kohn and Sham, 1965

$$E_{gs} = \int d^3 r \varepsilon[\rho(\vec{r}), \tau(\vec{r})]$$

$$\rho(\vec{r}) = \sum_{i=1}^{N} |\psi_i(\vec{r})|^2$$

$$\tau(\vec{r}) = \sum_{i=1}^{N} |\vec{\nabla}\psi_i(\vec{r})|^2$$

Normal Fermi systems only!

Superfluid LDA (SLDA)

number and kinetic densities

$$n(\vec{r}) = 2\sum_{0<E_k<E_c} \left|\mathrm{v}_k(\vec{r})\right|^2, \qquad \tau(\vec{r}) = 2\sum_{0<E_k<E_c} \left|\vec{\nabla}\mathrm{v}_k(\vec{r})\right|^2$$

anomalous density

$$\nu(\vec{r}) = \sum_{0<E_k<E_c} \mathrm{v}_\mathrm{k}^*(\vec{r})\mathrm{u}_k(\vec{r})$$

Divergent!

$$\mathrm{E} = \int d^3r \left\{ \frac{\hbar^2}{2m}\tau(\vec{r})n(\vec{r}) + \frac{\hbar^2}{m}\beta\left[x(\vec{r})\right]n(\vec{r})^{5/3} - \Delta\left[x(\vec{r})\right]\nu^*(\vec{r}) \right\}$$

Cutoff and position running coupling constant!

$$\Delta\left[x(\vec{r})\right] = -\frac{\hbar^2}{m}\frac{\gamma_{eff}\left[x(\vec{r})\right]}{n(\vec{r})^{1/3}}\nu(\vec{r}), \qquad x(\vec{r}) = \frac{1}{k_F(\vec{r})a}$$

$$\begin{pmatrix} T+U(\vec{r})-\mu & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -(T+U(\vec{r})-\mu) \end{pmatrix} \begin{pmatrix} \mathrm{u}_k(\vec{r}) \\ \mathrm{v}_k(\vec{r}) \end{pmatrix} = E_i \begin{pmatrix} \mathrm{u}_k(\vec{r}) \\ \mathrm{v}_k(\vec{r}) \end{pmatrix}$$

Bogoliubov-de Gennes like equations.
Correlations are however included by default!

Chang, Pandharipande, Carlson and Schmidt physics/0404115

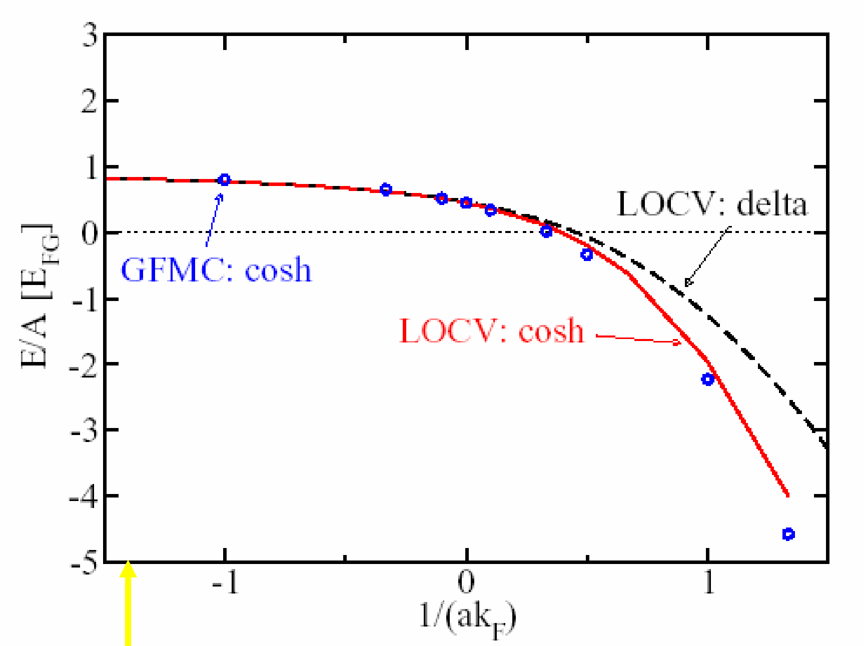
$$r_0 \ll \frac{1}{n^{1/3}} \ll |a|$$

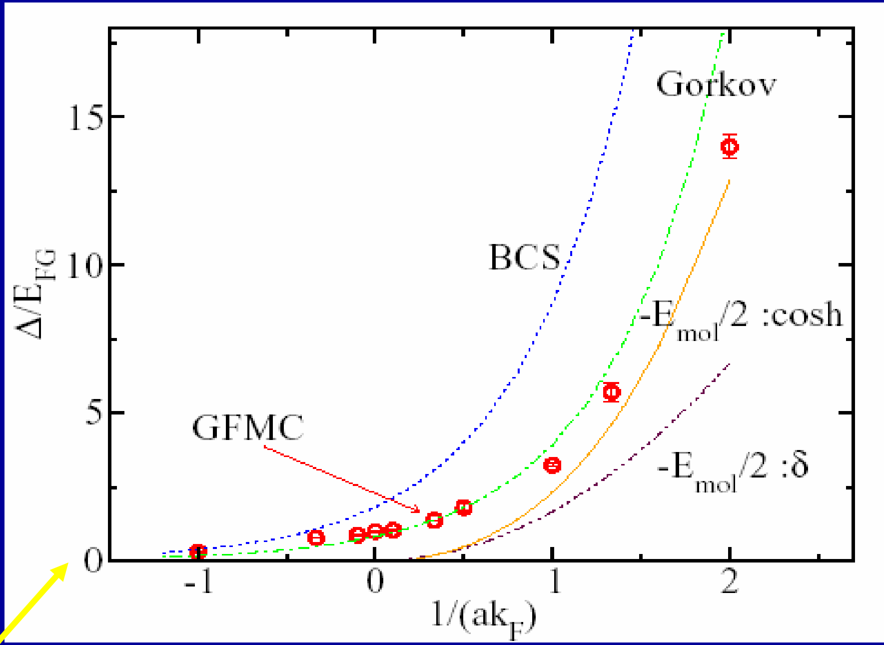
$$\left.\frac{E}{N}\right|_{GFMC} = \varepsilon[n] \approx \frac{3}{5}\varepsilon_F\left[\xi - \frac{\varsigma}{k_F a} - \frac{5\iota}{3(k_F a)^2}\right], \qquad \xi \approx 0.44, \qquad \varsigma \approx 1, \qquad \iota \approx 1$$

$$\Delta_{GFMC} \approx \varepsilon_F\left(\frac{2}{e}\right)^{7/3}\exp\left(\frac{\pi}{2k_F a}\right), \qquad n = \frac{k_F^3}{3\pi^2}, \quad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}, \quad x = \frac{1}{k_F a}$$

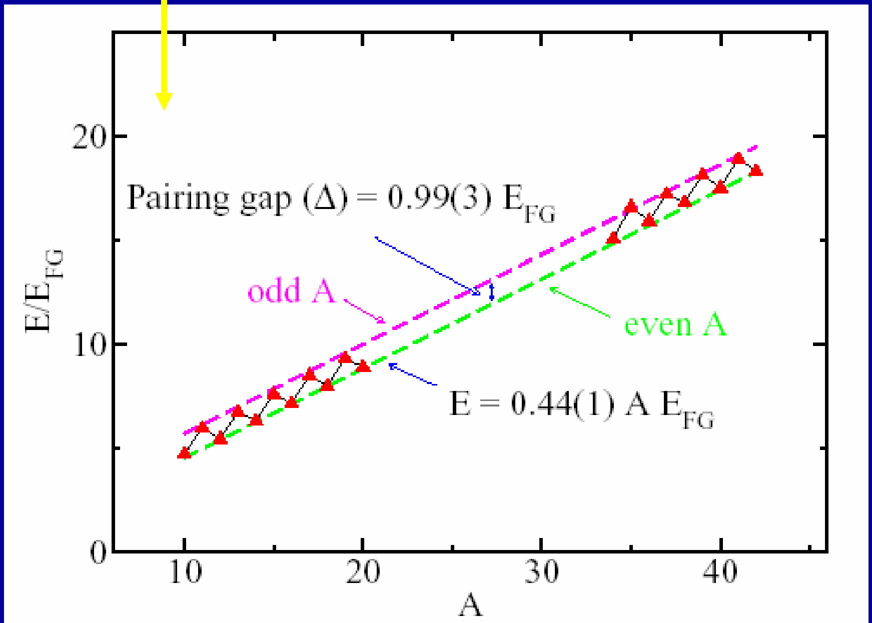
$$\varepsilon_{SLDA}[n]n = \varepsilon_{kin} n + \frac{\hbar^2}{m}\beta[x]\, n^{5/3} + \frac{\hbar^2}{m}\gamma[x]\,\frac{|\nu|^2}{n^{1/3}} \qquad + \quad \text{Renormalization}$$

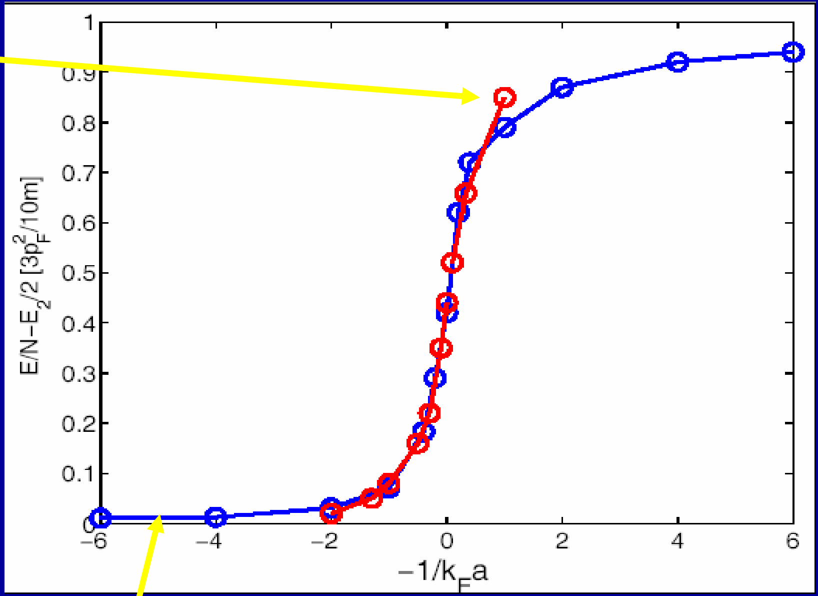
Dimensionless coupling constants





Chang *et al.* physics/0404115





Astrakharchik et al, cond-mat/0406113

Vortex in fermion matter

$$\begin{pmatrix} u_{\alpha\ kn}(\vec{r}) \\ v_{\alpha\ kn}(\vec{r}) \end{pmatrix} = \begin{pmatrix} u_{\alpha}(\rho)\exp[i(n+1/2)\phi - ikz] \\ v_{\alpha}(\rho)\exp[i(n-1/2)\phi - ikz] \end{pmatrix},$$ n - half-integer

$\Delta(\vec{r}) = \Delta(\rho)\exp(i\phi),$ $\vec{r} = (\rho, \phi, z)$ [cyllindrical coordinates]

Oz - vortex symmetry axis

Ideal vortex, Onsager's quantization (one $\hbar$ per Cooper pair)

$$\vec{V}_{v}(\vec{r}) = \frac{\hbar}{2m\rho^2}(y, -x, 0) \quad \Leftarrow \quad \frac{1}{2\pi}\oint_C \vec{V}_{v}(\vec{r})\cdot d\vec{r} = \frac{\hbar}{2m}$$

How can one put in evidence a vortex in a Fermi superfluid?

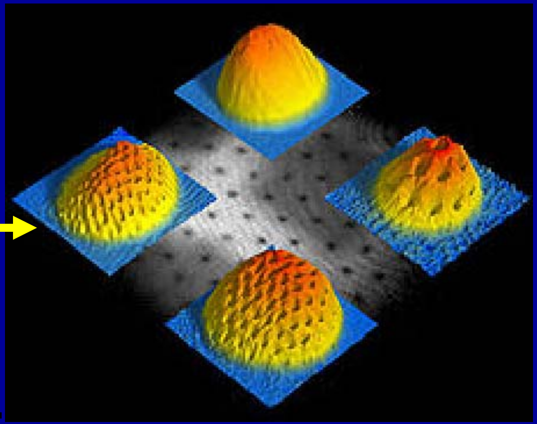
Hard to see, since density changes are not expected, unlike the case of a Bose superfluid.

However, if the gap is not small, one can expect a noticeable density depletion along the vortex core, and the bigger the gap the bigger the depletion, due to an <u>extremely fast vortical motion</u>.

$$\frac{v_s}{v_F} < \frac{\Delta}{2\varepsilon_F} \propto \frac{T_c}{T_F}$$

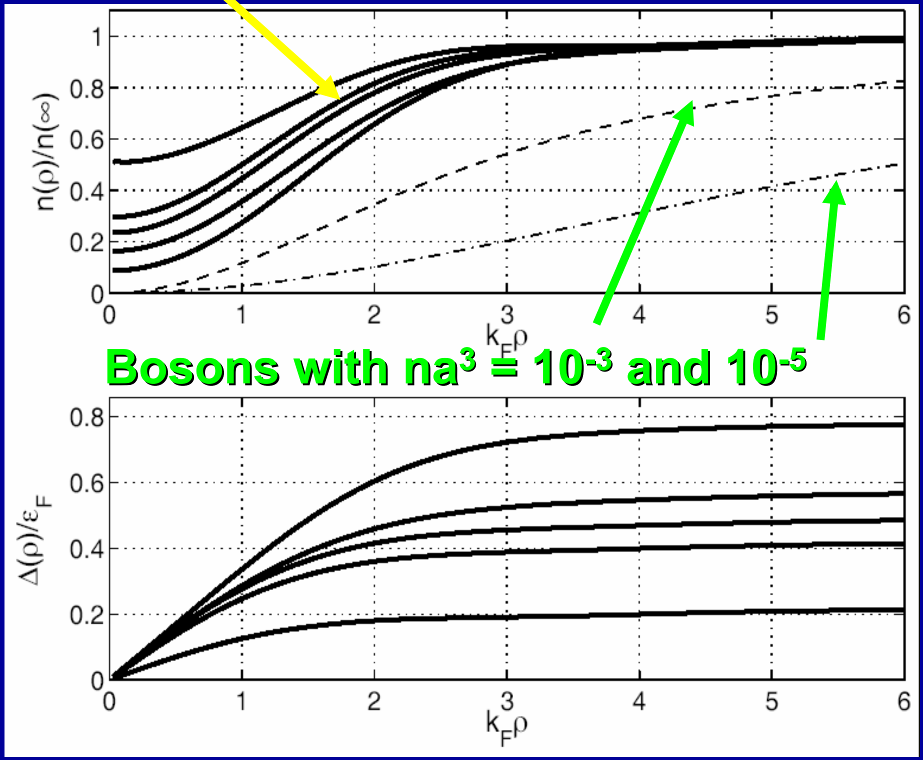
NB T_c unknown in the strong coupling limit!

The depletion along the vortex core is reminiscent of the corresponding density depletion in the case of a vortex in a Bose superfluid, when the density vanishes exactly along the axis for 100% BEC.

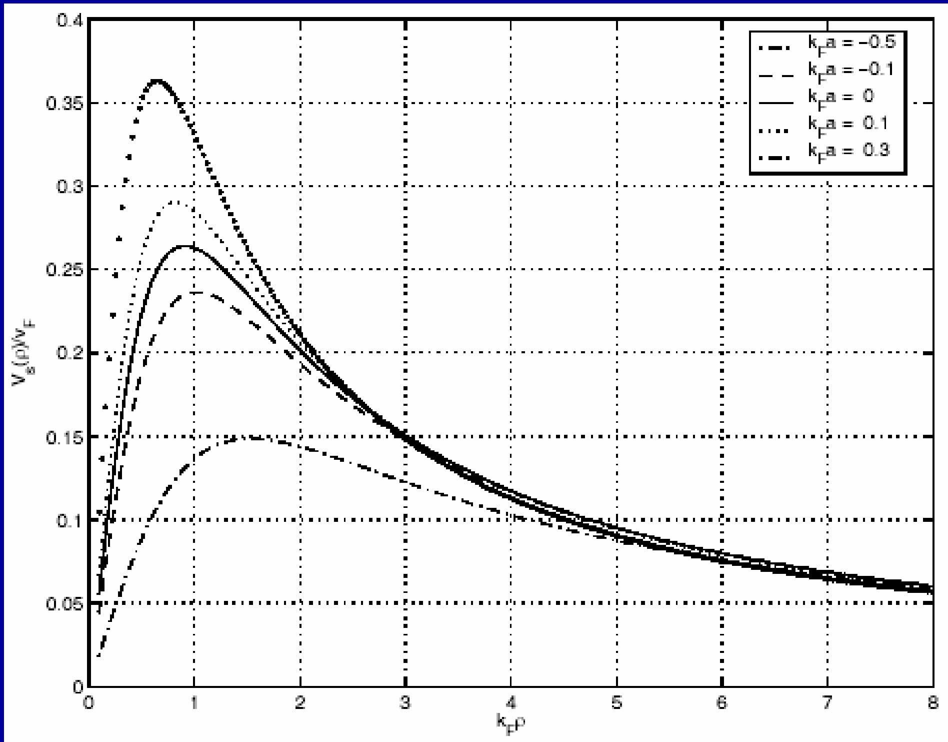


From Ketterle's group

Fermions with $1/k_Fa = 0.3, 0.1, 0, -0.1, -0.5$



Number density and pairing field profiles

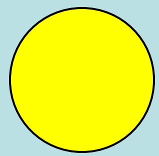
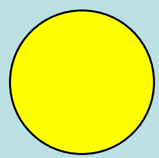
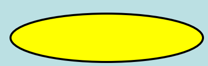


Extremely fast quantum vortical motion!

Local vortical speed as fraction of Fermi speed

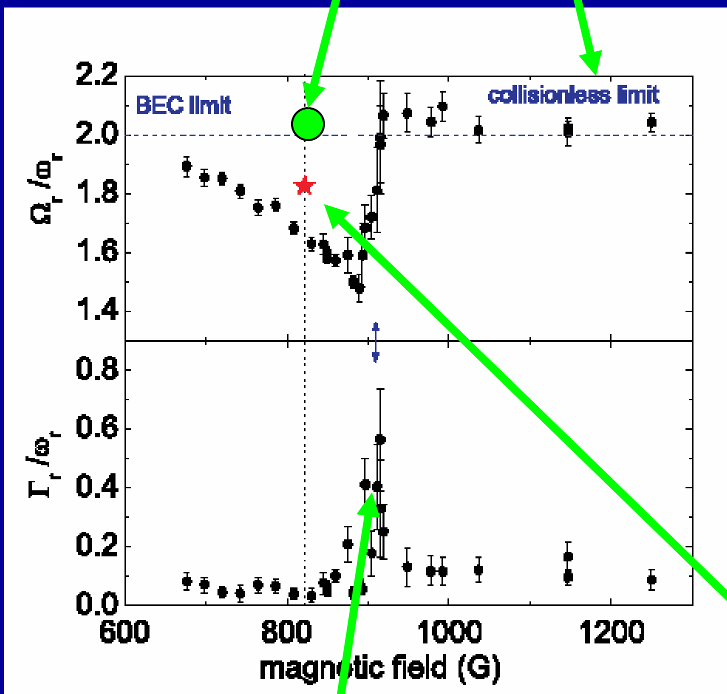
Sound in infinite fermionic matter

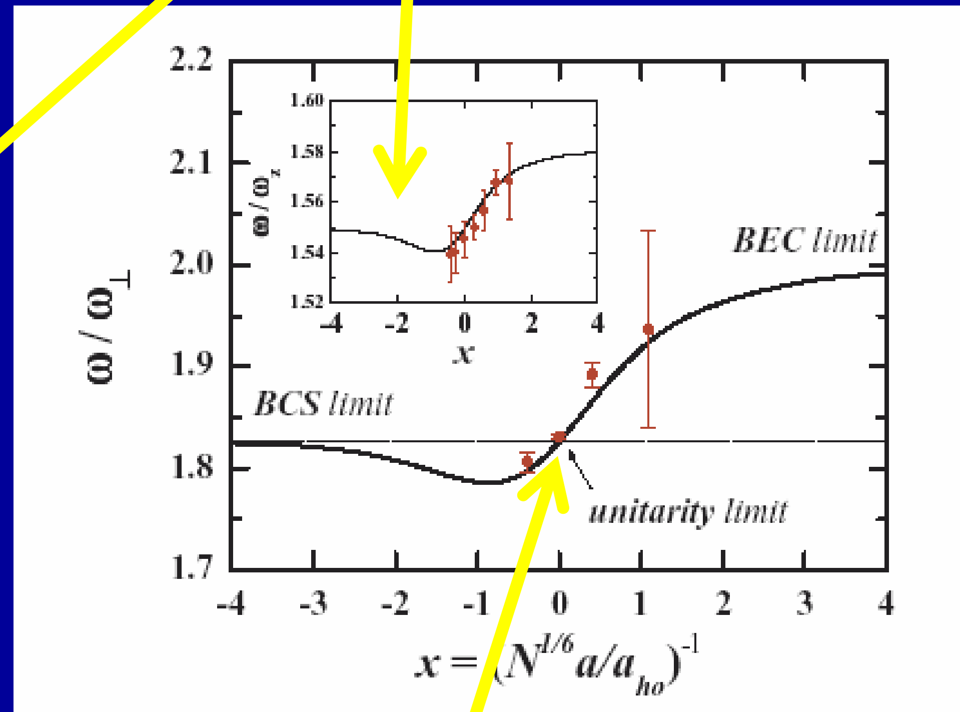
$$\omega = \mathrm{v}_s k$$

	Local shape of Fermi surface	Sound velocity	
Collisional Regime - high T! **Compressional mode**	Spherical 	$\mathrm{v}_s \approx \frac{\mathrm{v}_F}{\sqrt{3}}$	First sound
Superfluid collisionless- low T! **Compressional mode**	Spherical 	$\mathrm{v}_s \approx \frac{\mathrm{v}_F}{\sqrt{3}}$	Anderson-Bogoliubov sound
Normal Fermi fluid collisionless - low T! **Incompressional mode**	 Elongated along propagation direction	$\mathrm{v}_\mathrm{s} = s\mathrm{v}_\mathrm{F}$ $s > 1$	Landau's zero sound

Diabatic (approximate) frequency
Landau's zero sound regime

Grimm's group experiment





Thomas' group experiment

Transition region from
Anderson-Bogoliubov sound
to Landau zero sound

Adiabatic frequency
Anderson-Bogoliubov sound

$$\varepsilon(n) = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}\left[\xi - \frac{\varsigma}{k_F a} - \frac{5\iota}{(k_F a)^2} + O\left(\frac{1}{(k_F a)^3}\right)\right]$$

$$\xi \approx 0.44, \qquad \varsigma \approx 1, \qquad \iota \approx 1$$

$$U = \frac{m\omega_0^2 (x^2 + y^2 + \lambda^2 z^2)}{2}$$

$$\frac{\delta\omega^2}{\omega^2} = \frac{\varsigma}{\xi}\frac{1}{k_F(0)a}K$$

Adiabatic regime
Spherical Fermi surface

Perturbation theory result using GFMC equation of state in a trap

TABLE II: Results for K.

trap type	mode	f_1	ω	K
spherical	dipole	z	ω_0	0
$\lambda = 1$	monopole	$1 - 2r^2$	$2\omega_0$	$\frac{256}{525\pi}$
	quadrupole	xy	$\sqrt{2}\omega_0$	0
axial	$M = \pm 2$	$xy,\ x^2 - y^2$	$\sqrt{2}\omega_0$	0
$\lambda \ll 1$	$M = \pm 1$	$xz,\ yz$	ω_0	0
	radial	$x^2 + y^2 + \frac{2}{5}\lambda^2 z^2 - \frac{2}{5}$	$\sqrt{\frac{10}{3}}\omega_0$	$\frac{1024}{2625\pi}$
	axial	$1 - 6\lambda^2 z^2$	$\sqrt{\frac{12}{5}}\lambda\omega_0$	$\frac{256}{2625\pi}$

Frequency shifts in these modes might carry information about possible atom-halo dimer mixture

Consider now a dilute mixture of fermionic atoms and (bosonic) dimers at temperatures smaller than the dimer binding energy ($a>0$ and $a \gg r_0$)

$$\frac{E}{V} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} n_f + \frac{\pi\hbar^2 a}{m} n_f^2 + \frac{3.537\pi\hbar^2 a}{m} n_f n_b + \frac{0.6\pi\hbar^2 a}{m} n_b^2 + \varepsilon_2 n_b + \text{corrections}$$

$$n_f = \frac{k_F^3}{3\pi^2}, \qquad \varepsilon_2 = -\frac{\hbar^2}{ma^2}$$

Even though atoms repel there is BCS pairing!

$$U_{fbf}(q,\omega) = U_{fb}^2 \frac{2n_b\varepsilon_q}{\hbar^2\omega^2 - \varepsilon_q(\varepsilon_q + 2n_b U_{bb})}$$

$$U_{bb} = \frac{4\pi\hbar^2 a_{bb}}{m_b}, \qquad \varepsilon_q = \frac{\hbar^2 q^2}{2m_b}$$

in coordinate representation at $\omega = 0$

$$U_{fbf}(r) = -\frac{U_{fb}^2}{U_{bb}} \frac{1}{4\pi\xi_b^2 r} \exp\left(-\frac{r}{\xi_b}\right)$$

coherence/healing length

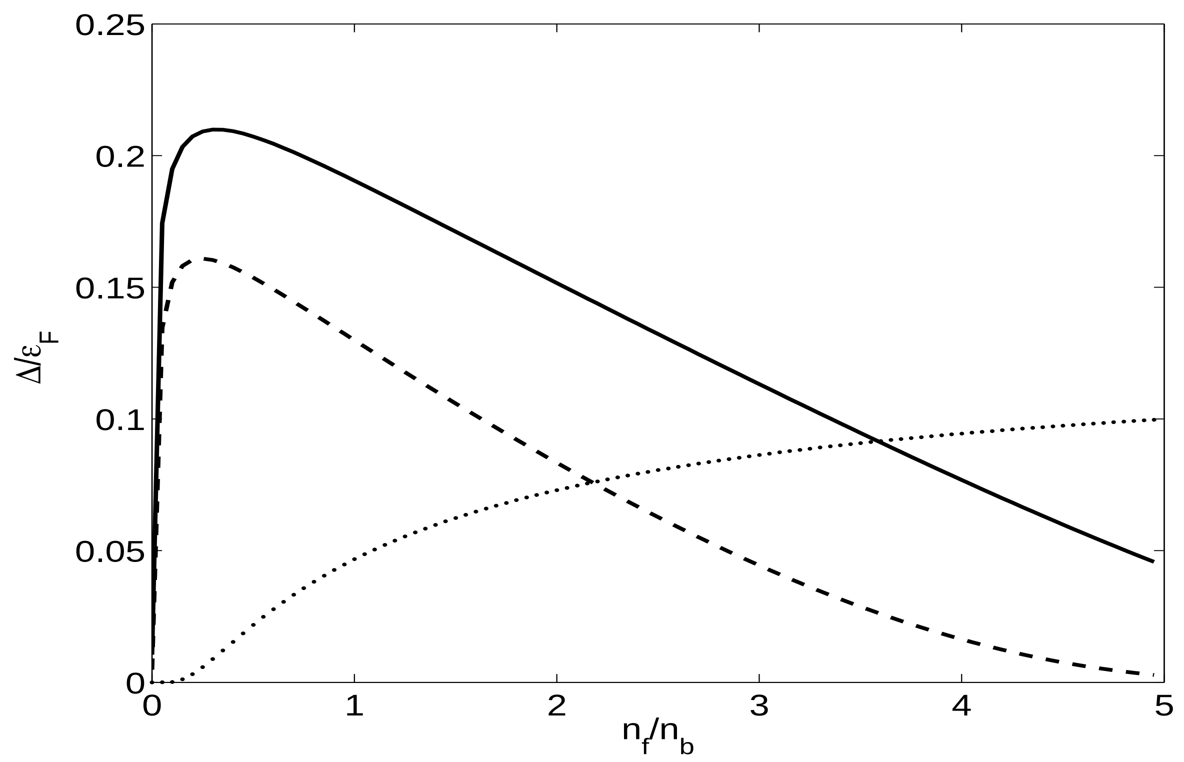
One can show that pairing is typically weak in dilute systems!

Induced fermion-fermion interaction

Bardeen *et al.* (1967),
Heiselberg *et al.* (2000),
Bijlsma *et al.* (2000)
Viverit (2000),
Viverit and Giorgini (2000)

The atom-dimer mixture can potentially be a system where relatively strong coupling pairing can occur.

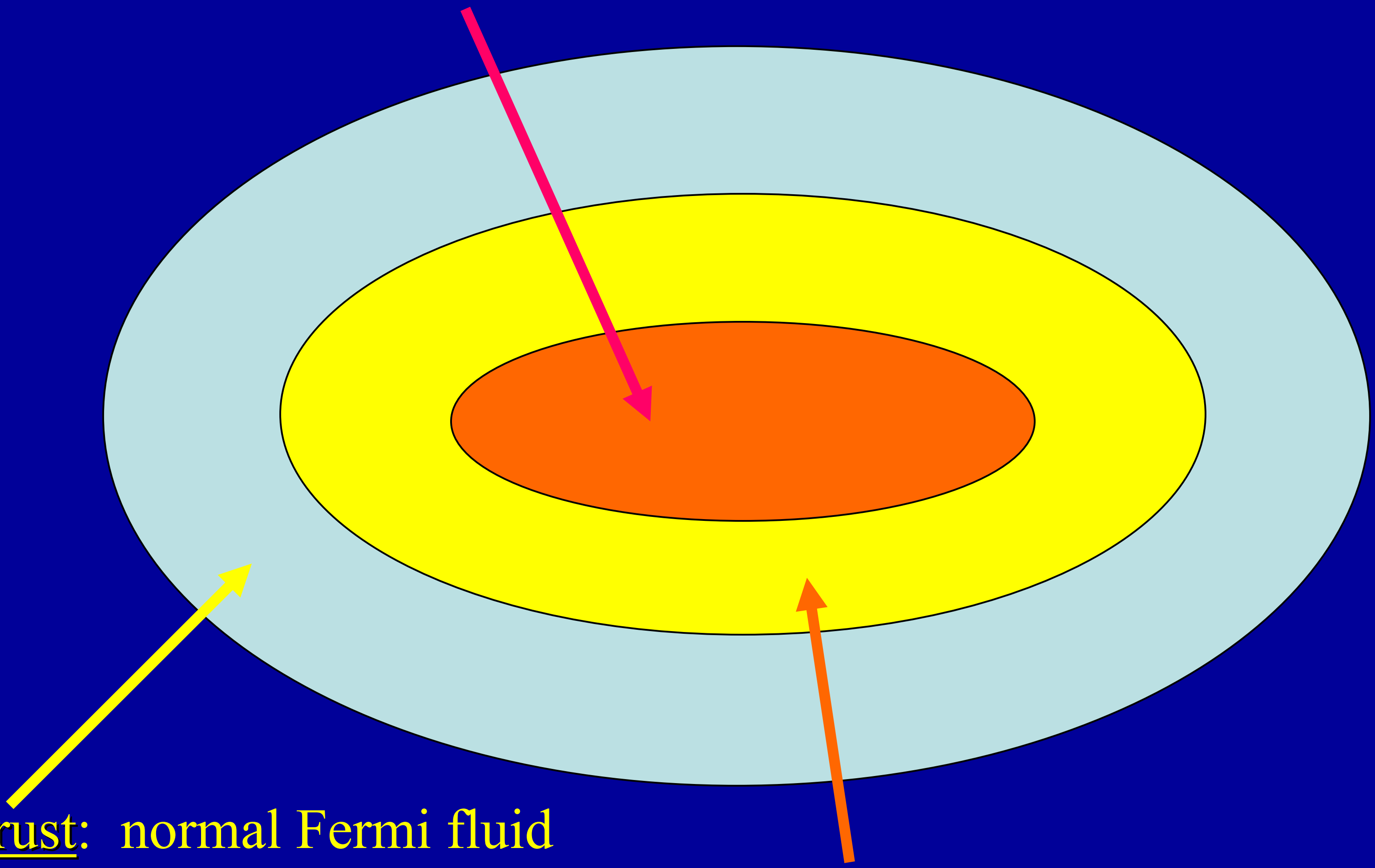
$$\Delta = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left[\frac{2}{\pi k_F a}\left(1 - 5.21\frac{\ln\left(1 + 4k_F^2 \xi_b^2\right)}{4k_F^2 \xi_b^2}\right)^{-1}\right]$$



$n_b\,a^3 = 0.064$ (solid line)
$n_b\,a^3 = 0.037$ (dashed line)
p-wave pairing (dots)

How this atomic-molecular cloud really looks like in a trap?

Core: Molecular BEC



Crust: normal Fermi fluid

Mantle: Molecular BEC + Atomic Fermi Superfluid

Everything s made of one kind of atoms only, in two different hyperfine states.

Main conclusions

TABLE I: Character of the condensate as a function of the inverse scattering length a^{-1} in various in intervals, the approximate boundaries of these intervals being shown in the second row. The total electron spin and spin projection (S, S_Z) along the magnetic field for various pairs are shown in the last row.

$a^{-1} > 0$			$a^{-1} < 0$	
$+\infty$	r_0^{-1}	k_F $\quad$ 0	0 $\quad$ k_F	$-\infty$
molecules	halo dimers (+ atoms ?[15])	?	BCS strong coupling	BCS weak coupling
(0,0)	(1,-1)	(1,-1)	(1,-1)	(1,-1)

Theory: easy easy hard hard easy

✓ **Fermion superfluidity, more specificaly <u>superflow,</u> has not yet been demonstrated unambiguously experimentally. There is lots of circumstantial evidence and facts in agreement with theoretical models assuming its existence. <u>Vortices!?</u>**

✓ **Theory is able to make very precise predictions in this regime and the agreement with experiment can be check quantitatively.**