Molecular Transitions in Fermi Condensates

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cond-mat/0404687

Abstract
The theoretical underpinnings of two radically different theoretical philosophies will be compared and subsequently these approaches will be confronted with four experimental results:

1) pair magnetic moment,
2) particle loss,
3) cloud spatial size and
4) frequency and frequency shifts of collective excitations

These slides will be posted shortly at
http://www.phys.washington.edu/~bulgac/talks.html#most_recent
How does one decide if one or another theoretical approach is meaningful?

Really, this is a very simple question. One has to check a few things.

😊 Is the theoretical approach based on a sound approximation scheme?

    Well,…, maybe!

😊 Does the particular approach chosen describe known key experimental results, and moreover, does this approach predict new qualitative features, which are later on confirmed experimentally?

😊 Are the theoretical corrections to the leading order result under control, understood and hopefully not too big?

This slide is from another talk of mine on pairing in nuclei and the smiley faces correspond to that situation. The questions are the same, but the smiley faces not quite.
Feshbach resonance

\[ H = \frac{\vec{p}^2}{2\mu_r} + \sum_{i=1}^{2} \left( V_i^{hf} + V_i^Z \right) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \mathbf{K}^d \]

\[ V^{hf} = \frac{a_{hf}}{\hbar^2} \vec{S}^e \cdot \vec{S}^n, \quad V^Z = \left( \gamma_e S_z^e - \gamma_n S_z^n \right) B \]

Tiesinga, Verhaar, Stoof

Regal and Jin
Coupled Fermion-Boson model in a trap

Superconductivity: Lee, Ranning, Fermi atom gas: Timmermans, Holland, Ohashi, and their collaborators

two hyperfine states =

\[
H = \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \left[ \frac{p^2}{2m} + V_{\text{Fermion}}(r) \right] \Psi_\sigma(r) - U \int dr \Psi_{\uparrow}^\dagger \Psi_{\downarrow}^\dagger \Psi_{\downarrow} \Psi_{\uparrow}^\dagger + \Phi^\dagger(r) \left[ \frac{p^2}{2M} + 2 \nu + V_{\text{Boson}}^{\text{trap}}(r) \right] \Phi(r) + g \sum_\sigma \int dr [\Phi^\dagger(r) \Psi_{\downarrow}(r) \Psi_{\uparrow}(r) + h.c.]
\]

Open (triplet) channel
Closed (singlet) channel

Ohashi, Levico 2004
NB The size of the “Feshbach molecule” (closed channel state) is largely B-independent and smaller than the interparticle separation.
\[
\begin{bmatrix} H_{11} & V_{12} \\ V_{21} & H_{22} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = k^2 \begin{bmatrix} u \\ v \end{bmatrix}
\]

- **Open channel**
- **Closed channel**

\[
\begin{align*}
H_{11} u &= k^2 u \\
H_{22} \phi_0 &= \kappa_0^2 \phi_0
\end{align*}
\]

\[
\phi_0(r) = \sqrt{\frac{2}{r_0}} \sin \frac{\pi r}{r_0}, \quad u_0(r) = \sin(kr), \quad v_0(r) \propto \phi_0(r)
\]

\[
V_{12}(r) = g \delta(r-r_1), \quad r_1 = \frac{r_0}{2}
\]

\[
u(r) = \sin(kr) + \frac{g^2 \exp(ikr_>) \sin(kr<)}{k^2 - \kappa_0^2}, \quad \left\{ \begin{array}{l} r_> = \max(r, r_1) \\
 r_< = \min(r, r_1) \end{array} \right.
\]

\[
- \frac{1}{a} = \frac{\kappa_0^2 - k^2}{g^2 r_1}
\]

\[
u(r) = \exp(i\delta) \sin k(r-a), \quad r > r_1
\]

\[
\int_0^R dr |v(r)|^2 = k^2 a^2 \frac{1}{(k^2 a^2 + 1) g^2 r_1} \leq \frac{1}{g^2 r_1} = O(r_0)
\]

\[
\int_{r_0}^{R} dr |u(r)|^2 \approx \int_{r_0}^{R} dr \sin^2(kr + \delta) = O(R) \gg O(r_0)
\]

**Particles in a pair spend most of the time outside the interaction zone, in the triplet state.**

**Halo dimers**
Pandharipande and Bethe, Phys. Rev. C 7, 1312 (1973)

Lowest order constraint variational calculations (LOCV) (applied to liquid $^4$He, liquid $^3$He, neutron gas)

\[ H = -\frac{\hbar^2}{2m} \sum_{i=1}^{N} \Delta_{i} + \sum_{i<j} V(r_{ij}) \]

\[ |\Psi_V\rangle = \prod_{i,j} f(r_{ij}) |\Psi_S\rangle \]

\[ -\frac{\hbar^2}{m} \Delta f(r) + V(r)f(r) = \lambda f(r) \]

\[ n \int_{0}^{R} f^2(r) 4\pi r^2 dr = 1, \quad \frac{df(r)}{dr} \bigg|_{r=R} = 0 \]

\[ n = \frac{k_F^3}{3\pi^2} \]

\[ E_{LOCV} = E_{FG} + \frac{\lambda}{2} \]

This approximate many-body wave function reproduces with great accuracy the exact GFMC results near a Feshbach resonance, see Chang et al, physics/0404115
Theory is now in such a state that it can make verifiable or falsifiable predictions.

Experimental signatures/predictions of the large size pairs/ halo dimer model versus fermion-boson model:

1) Near the Feshbach resonance the pair is in triplet state
   *in agreement with Grimm’s group experiments*
   (fermion-boson model predicts a singlet state)

2) Particle loss is consistent with large spatial size pairs
   *in agreement with Grimm’s and Jin’s groups experiments*
   (fermion-boson model is consistent with small spatial size pairs)

3) Spatial size of the cloud
   *in agreement with Grimm’s group experiment*
   (fermion-boson model disagrees with experiment)

4) Frequencies and frequency shift of the frequencies of the collective oscillations
   (if at the Feshbach resonance the system would be made of small size pairs the frequency would be higher and the frequency shift would be much smaller than observed in experiments, see Pitaevskii and Stringari, PRL 81, 4541 (1998), Braaten and Pearson, PRL 82, 255 (1999))

\[
\frac{\delta \omega_M}{\omega_M} \approx \frac{63 \sqrt{\pi}}{128} \sqrt{n(0)} r_0^3 \ll 1
\]
S. Jochim et al.
PRL 91, 240402 (2003)

Electron spin along B

Electron spin opposite B

see also Petrov et al. cond-mat/0309010
Regal et al. PRL 92, 083201 (2004)
40K (Fermi) atoms in a spherical harmonic trap

Effect of interaction, with and without weak and strong pairing correlations with fixed particle number, \( N = 5200 \).

\[ \hbar w = 0.568 \sim 10^{-12} \text{eV}, \, a = -12.63 \text{nm} \text{ (when finite)} \]

Unpublished, fully self-consistent SLDA (Kohn-Sham generalized to pairing) calculation performed by Yongle Yu in July 2003 and presented in a number of talks (ECT*, Trento, July 2003, and other talks).
The fermion-boson model predict that at resonance the size of the cloud is significantly smaller than the observed one.

Fermi atoms form Cooper-pair bosons, which does not affect the density profile in the BCS regime.

Density profile shrinks in the BEC regime due to the absence of Pauli Principle between molecules associated with the F.R.

Ohashi, Levico 2004

Grimm, Levico 2004
The fermion-boson model predict that at resonance an atomic Fermi cloud consists predominantly of molecules in the closed channel (singlet), which thus have an almost vanishing magnetic moment.

Ohashi and Griffin,

Bruun, cond-mat/0401497
Fermion-boson model would predict here

Grimm’s group experiment

Thomas’ group experiment

Hu et al. cond-mat/0404012, a semi-quantitative analysis (gap and chemical potential inaccurate) assuming a polytropic equation of state

For a more careful analysis, using GFMC equation of state in a trap see Bulgac and Bertsch, cond-mat/0404687
Falco and Stoof,

$\frac{n_m}{2} \approx \frac{1}{2} - \frac{a}{(k_F a)^3}$

Theory declared full victory here! 😊

Regal, Greiner and Jin,

Zwierlein et al.
Fermion superfluidity, more specifically superflow, has not yet been demonstrated unambiguously experimentally. There is lots of circumstantial evidence and facts in agreement with theoretical models assuming its existence. Vortices!? ✔️

Theory is able to make very precise predictions in this regime and the agreement with experiment can be check quantitatively.

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**TABLE I:** Character of the condensate as a function of the inverse scattering length $a^{-1}$ in various in intervals, the approximate boundaries of these intervals being shown in the second row. The total electron spin and spin projection ($S, S_Z$) along the magnetic field for various pairs are shown in the last row.

<table>
<thead>
<tr>
<th>$a^{-1} &gt; 0$</th>
<th>$a^{-1} &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+\infty$</td>
<td>0</td>
</tr>
<tr>
<td>$r_0^{-1}$</td>
<td>$k_F$</td>
</tr>
<tr>
<td>$k_F$</td>
<td>$-\infty$</td>
</tr>
<tr>
<td>molecules</td>
<td>halo dimers</td>
</tr>
<tr>
<td>(+ atoms ?[15])</td>
<td>?</td>
</tr>
<tr>
<td>(0,0)</td>
<td>(1,-1)</td>
</tr>
<tr>
<td>(1,-1)</td>
<td>(1,-1)</td>
</tr>
</tbody>
</table>

**Notes:**
- BCS: Bardeen-Cooper-Schrieffer
  - strong coupling
  - weak coupling
- Vortices!?

✔️ ✔️