Molecular Transitions in Fermi Condensates

A. Bulgac and G.F. Bertsch, cond-mat/0404301
cond-mat/0404687

Abstract

The theoretical underpinnings of two radically different theoretical philosophies will be compared and subsequently these approaches will be confronted with four experimental results:

1) pair magnetic moment,
2) particle loss,
3) cloud spatial size and
4) frequency and frequency shifts of collective excitations

These slides will be posted shortly at
http://www.phys.washington.edu/~bulgac/talks.html#most_recent

How does one decide if one or another theoretical approach is meaningful?

Really, this is a very simple question. One has to check a few things.

😐 Is the theoretical approach based on a sound approximation scheme?

Well,…, maybe!

☺ Does the particular approach chosen describe known key experimental results, and moreover, does this approach predict <u>new qualitative features</u>, which are later on confirmed experimentally?

☹ Are the theoretical corrections to the leading order result under control, understood and hopefully not too big?

This slide is from another talk of mine on pairing in nuclei and the smiley faces correspond to that situation.
The questions are the same, but the smiley faces not quite.

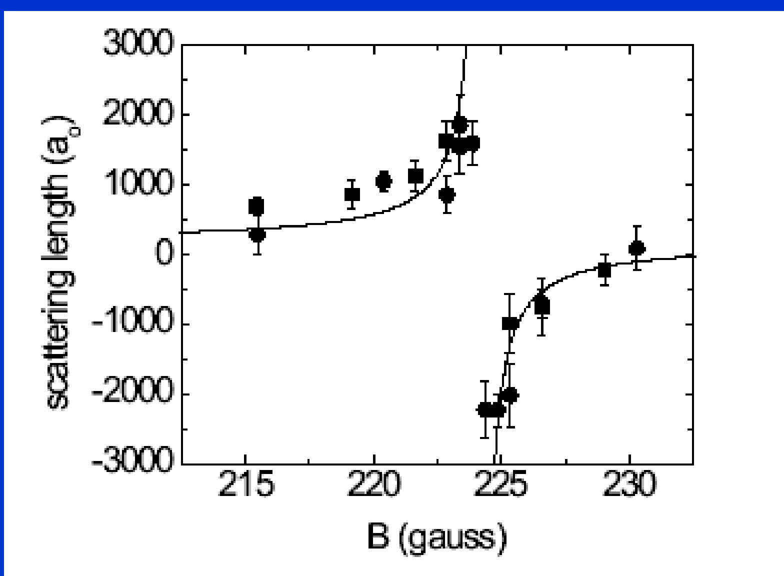
Feshbach resonance

$$H = \frac{\vec{p}^{\,2}}{2\mu_r} + \sum_{i=1}^{2} (V_i^{hf} + V_i^{Z}) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \cancel{V^d}$$

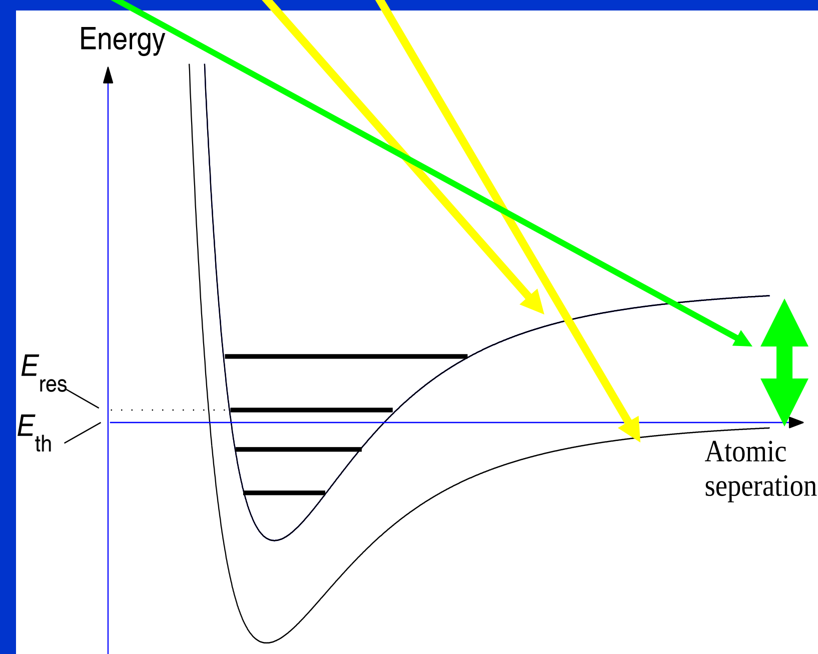
$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{S}^e \cdot \vec{S}^n, \quad V^Z = (\gamma_e S_z^e - \gamma_n S_z^n)B$$

Channel coupling

Tiesinga, Verhaar, Stoof
Phys. Rev. A47, 4114 (1993)



Regal and Jin
Phys. Rev. Lett. 90, 230404 (2003)



Coupled Fermion-Boson model in a trap

Superconductivity: Lee, Ranninger,
Fermi atom gas: Timmermans, Holland, Ohashi, and their collaborators

two hyperfine states = $\downarrow \; \uparrow$

$$H - \sum_{\sigma} \int dr \Psi^{\dagger}_{\sigma}(r)[\frac{p^2}{2m} + V^{Fermion}_{trap}(r)]\Psi_{\sigma}(r) - U\int dr \Psi^{\dagger}_{\uparrow}\Psi^{\dagger}_{\downarrow}\Psi_{\downarrow}\Psi_{\uparrow}$$

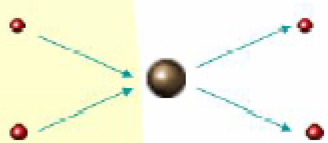
Open (triplet) channel

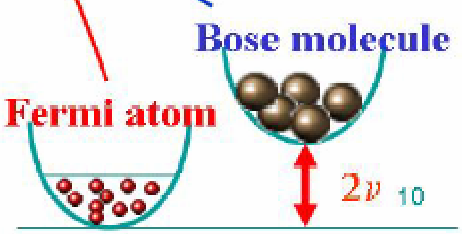
$$+\Phi^{\dagger}(r)[\frac{p^2}{2M} + 2\nu + V^{Boson}_{trap}(r)]\Phi(r)$$

Closed (singlet) channel

$$+g\sum_{\sigma}\int dr[\Phi^{\dagger}(r)\Psi_{\downarrow}(r)\Psi_{\uparrow}(r) + h.c.]$$

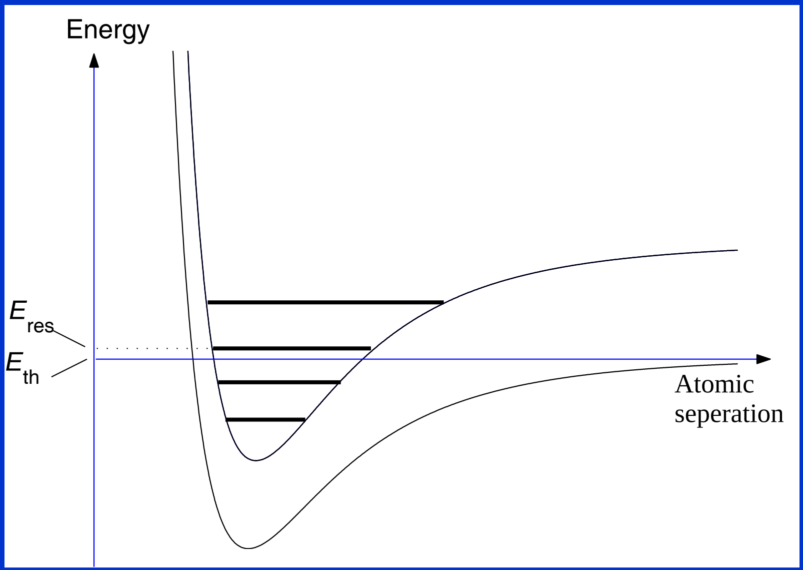
Feshbach resonance

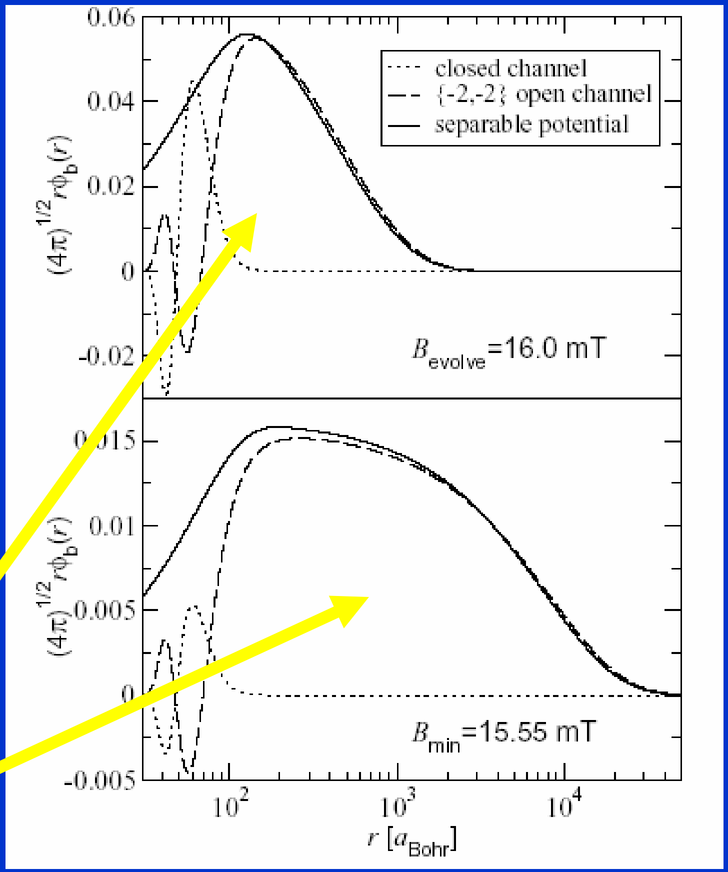




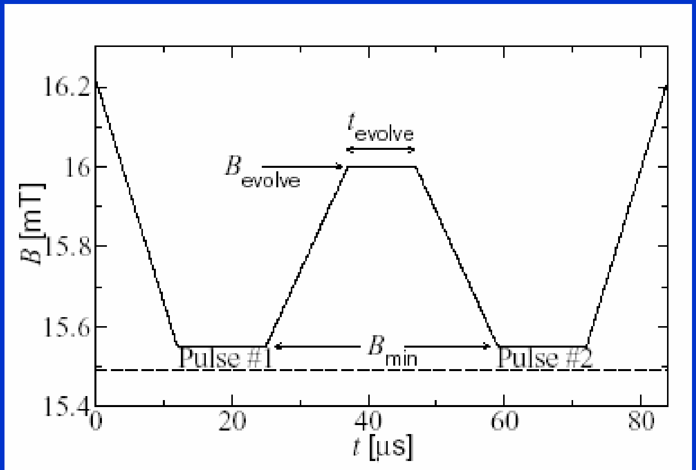
Ohashi, Levico 2004

Köhler, Gasenzer, Jullienne and Burnett
PRL 91, 230401 (2003), inspired by
Braaten, Hammer and Kusunoki
cond-mat/0301489





Halo dimer



$$\frac{P(r > r_0)}{P(r < r_0)} \propto \frac{a}{r_0} \quad >> \ 1$$

NB The size of the "Feshbach molecule" (closed channel state) is largely B-independent and smaller than the interparticle separation.

$$\begin{bmatrix} H_{11} & V_{12} \\ V_{21} & H_{22} \end{bmatrix}\begin{bmatrix} \mathrm{u} \\ \mathrm{v} \end{bmatrix} = k^2 \begin{bmatrix} \mathrm{u} \\ \mathrm{v} \end{bmatrix}$$

← Open channel

← Closed channel

$$\begin{cases} H_{11}\mathrm{u} = k^2\mathrm{u} \\ H_{22}\phi_0 = \kappa_0^2\phi_0 \end{cases}$$

$$\phi_0(r) = \sqrt{\frac{2}{r_0}}\sin\frac{\pi r}{r_0}, \qquad \mathrm{u}_0(r) = \sin(kr), \qquad \mathrm{v}_0(r) \propto \phi_0(r)$$

$$V_{12}(r) = g\delta(r - r_1), \qquad r_1 = \frac{r_0}{2}$$

$$\mathrm{u}(r) = \sin(kr) + \frac{g^2\exp(ikr_>)\sin(kr_<)}{k^2 - \kappa_0^2}, \qquad \begin{cases} r_> = \max(r, r_1) \\ r_< = \min(r, r_1) \end{cases}$$

$$\begin{cases} -\dfrac{1}{a} = \dfrac{\kappa_0^2 - k^2}{g^2 r_1} \\ \mathrm{u}(r) = \exp(i\delta)\sin k(r - a), \qquad r > r_1 \end{cases}$$

$$\begin{cases} \displaystyle\int \mathrm{d}r\,|\mathrm{v}(r)|^2 = \frac{k^2a^2}{(k^2a^2+1)g^2r_1} \le \frac{1}{g^2r_1} = \mathrm{O}(r_0) \\ \displaystyle\int_0^R \mathrm{d}r\,|\mathrm{u}(r)|^2 \approx \int_{r_0}^R \mathrm{d}r\sin^2(kr+\delta) = \mathrm{O}(R) \gg \mathrm{O}(r_0) \end{cases}$$

← Closed channel wf (singlet)

← Open channel wf (triplet)

$$\left.\frac{d\ln \mathrm{u}(r)}{dr}\right|_{r=r_0} = -\frac{1}{a}$$

$$r_0 \approx \left(\frac{C_6 m}{\hbar^2}\right)^{1/4}$$

$$nr_0^3 \ll 1, \quad a \gg r_0, \qquad \mathrm{O}(k_F r_0) \ll 1$$

$$\mathrm{O}(\kappa_0^2) = \mathrm{O}(g^2) = \frac{2\mu_B B_0 m}{\hbar^2}$$

$$r_0 \ll R \approx \frac{\lambda_F}{2}$$

Particles in a pair spend most of the time outside the interaction zone, in the triplet state.

Halo dimers

Pandharipande and Bethe, Phys. Rev. C 7, 1312 (1973)

Lowest order constraint variational calculations (LOCV)

(applied to liquid ^{4}He, liquid ^{3}He, neutron gas)

$$\mathrm{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_i + \sum_{i<j} V(r_{ij})$$

$$\left|\Psi_V\right\rangle = \prod_{i,j} f(r_{ij})\left|\Psi_S\right\rangle$$

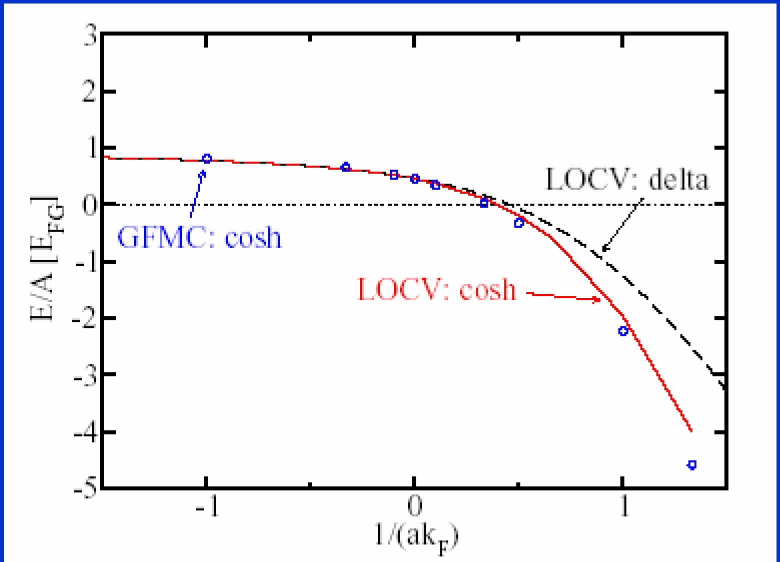
$$-\frac{\hbar^2}{m}\Delta f(r) + V(r) f(r) = \lambda f(r)$$

$$\frac{n}{2}\int_0^R f^2(r) 4\pi r^2 dr = 1, \qquad \left.\frac{df(r)}{dr}\right|_{r=R} = 0$$

$$n = \frac{k_F^3}{3\pi^2}$$

$$E_{LOCV} = E_{FG} + \frac{\lambda}{2}$$

This approximate many-body wave function reproduces with great accuracy the exact GFMC results near a Feshbach resonance, see Chang *et al*, physics/0404115



Theory is now in such a state that it can make verifiable or falsifiable predictions.

Experimental signatures/predictions of the large size pairs/ halo dimer model versus fermion-boson model:

1) Near the Feshbach resonance the pair is in triplet state
in agreement with Grimm's group experiments
(fermion-boson model predicts a singlet state)

2) Particle loss is consistent with large spatial size pairs
in agreement with Grimm's and Jin's groups experiments
(fermion-boson model is consistent with small spatial size pairs)

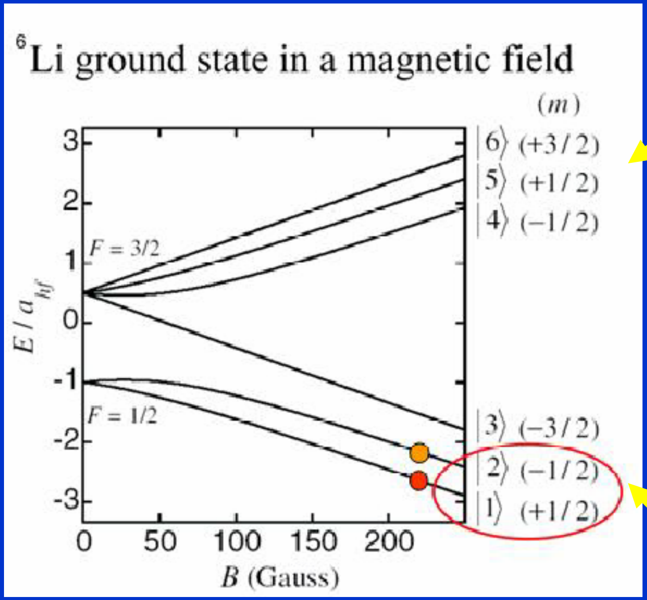
3) Spatial size of the cloud
in agreement with Grimm's group experiment
(fermion-boson model disagrees with experiment)

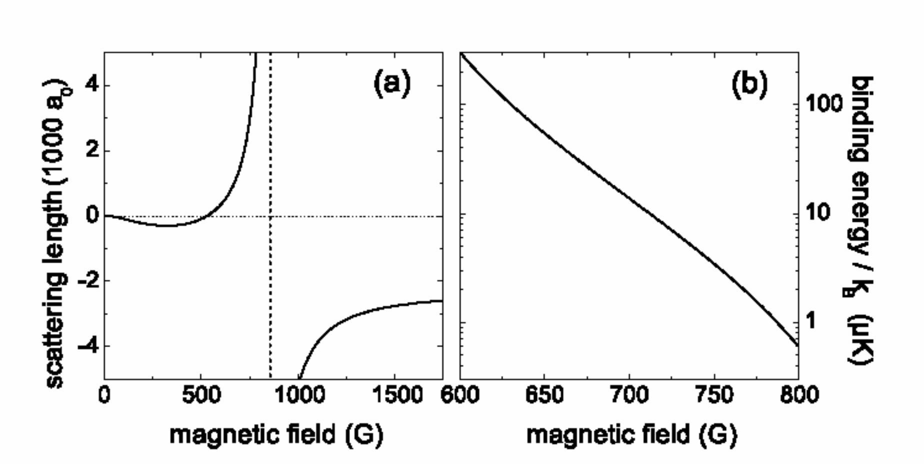
4) Frequencies and frequency shift of the frequencies of the collective oscillations
(if at the Feshbach resonance the system would be made of small size pairs the frequency would be higher and the frequency shift would be much smaller than observed in experiments, see Pitaevskii and Stringari, PRL 81, 4541 (1998), Braaten and Pearson, PRL 82, 255 (1999))

$$\frac{\delta\omega_M}{\omega_M} \approx \frac{63\sqrt{\pi}}{128}\sqrt{n(0)r_0^3} \ll 1$$

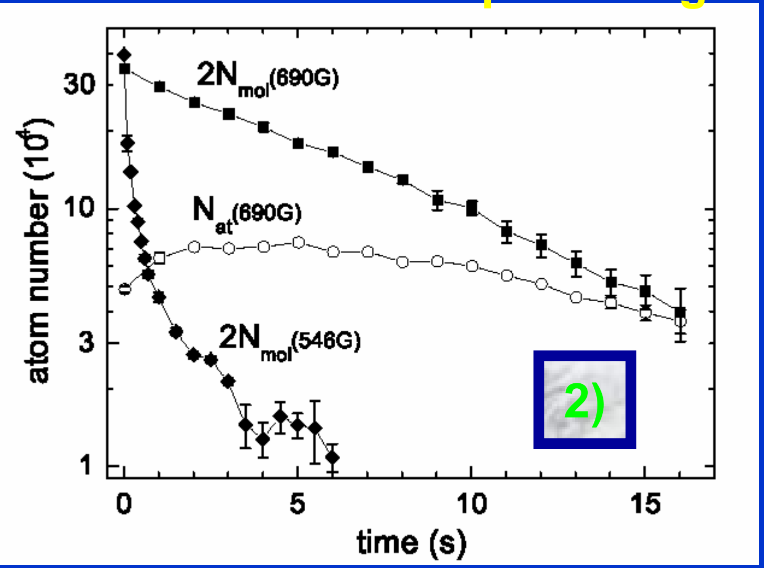
S. Jochim et al.
PRL 91, 240402 (2003)

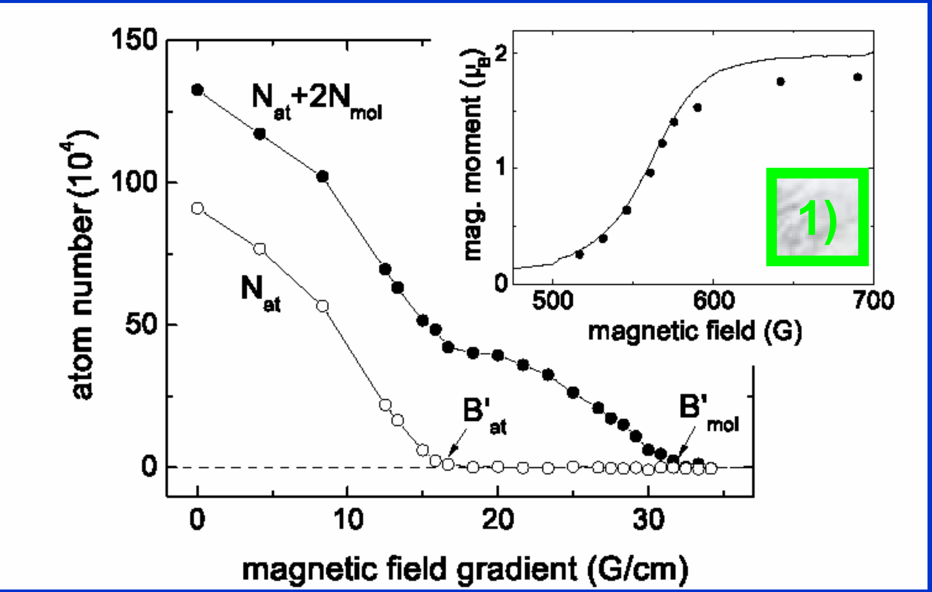
Electron spin opposite B





Electron spin along B

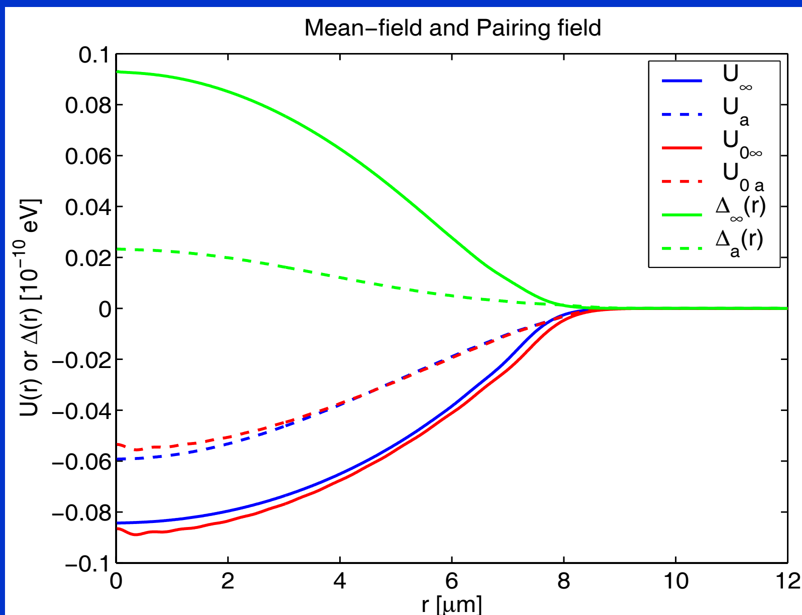


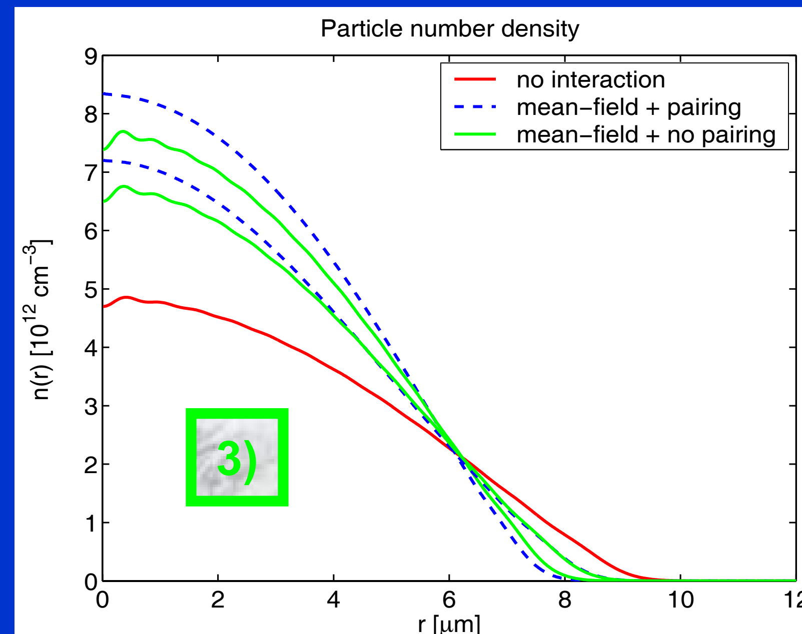


see also Petrov *et al.* cond-mat/0309010
Regal *et al.* PRL 92, 083201 (2004)

40K (Fermi) atoms in a spherical harmonic trap

Effect of interaction, with and without weak and strong pairing correlations with fixed particle number, N = 5200.

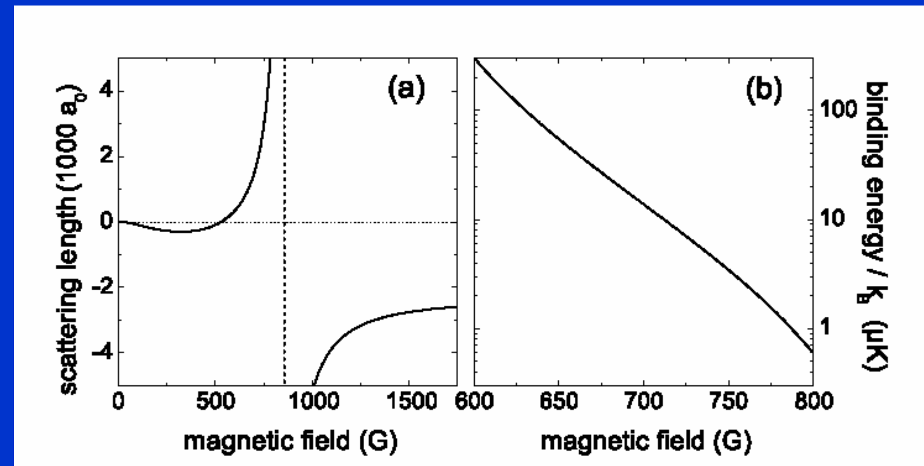


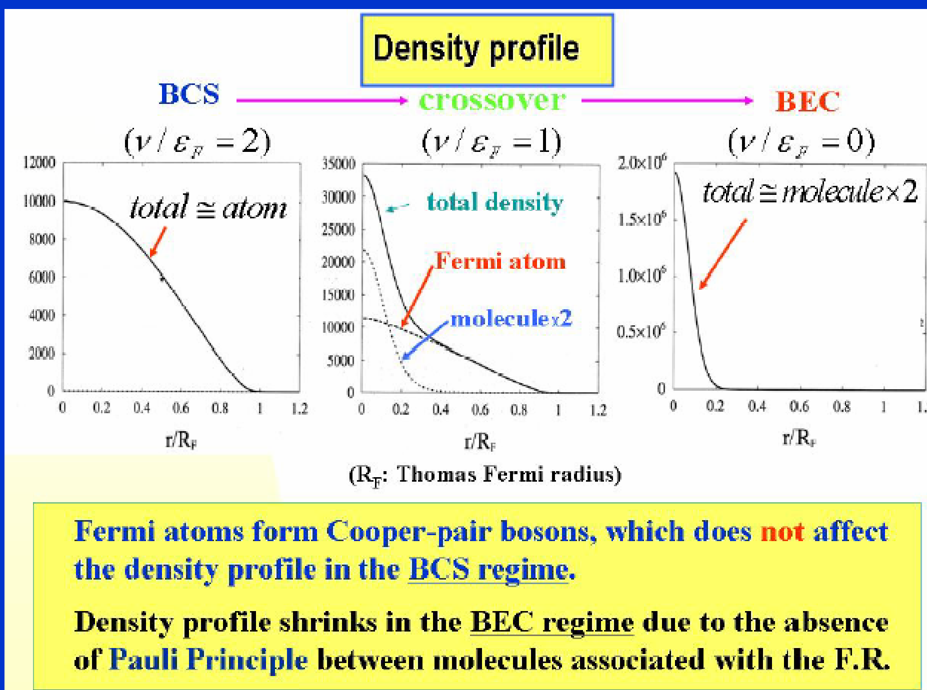


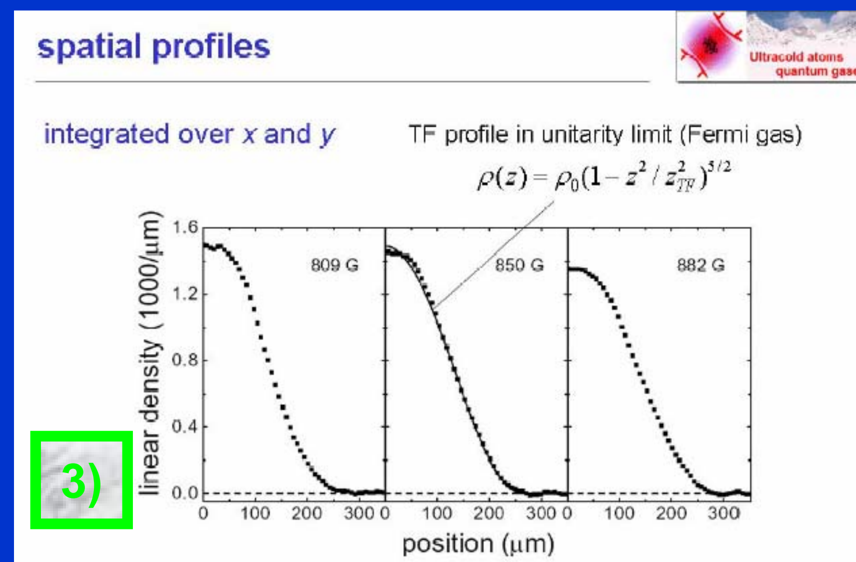
$\hbar w=0.568\ \mu 10^{-12}$eV, a = -12.63nm (when finite)

Unpublished, fully self-consistent SLDA (Kohn-Sham generalized to pairing) calculation performed by Yongle Yu in July 2003 and presented in a number of talks (ECT*, Trento, July 2003, and other talks).

The fermion-boson model predict that at resonance the size of the cloud is significantly smaller than the observed one.



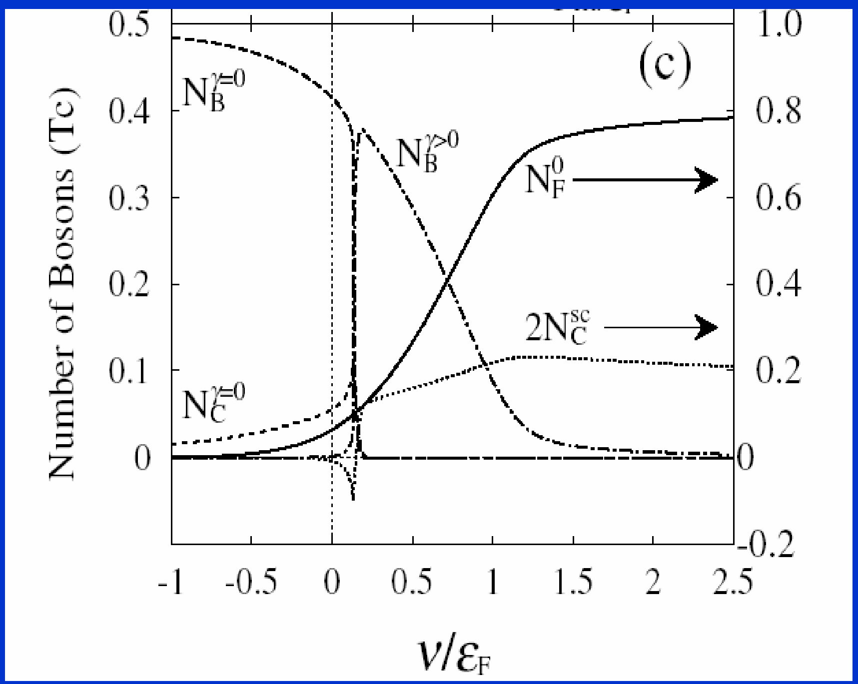




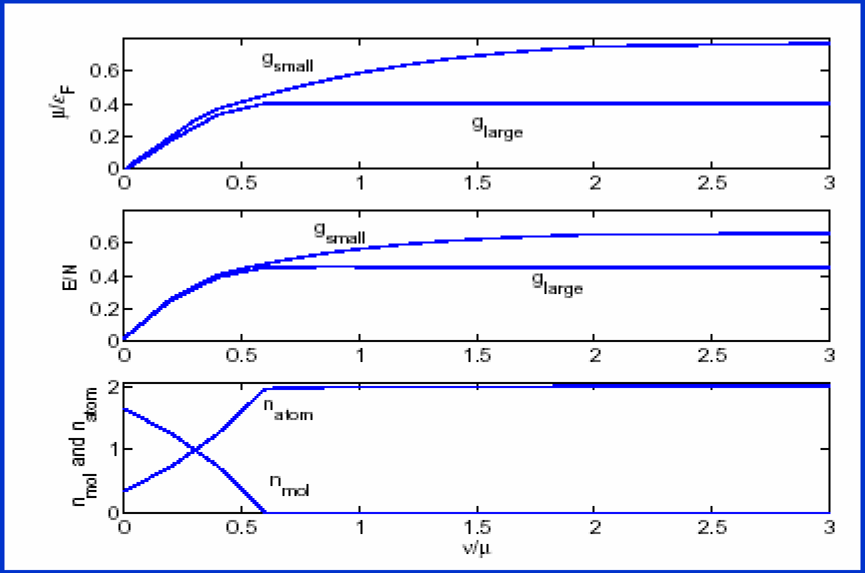
Ohashi, Levico 2004

Grimm, Levico 2004

The fermion-boson model predict that at resonance an atomic Fermi cloud consists predominantly of molecules in the closed channel (singlet), which thus have an almost vanishing magnetic moment.



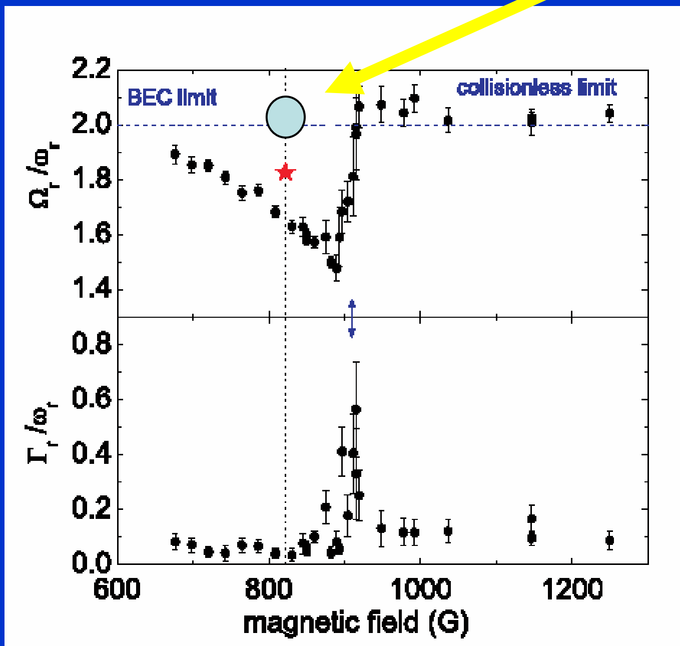
Ohashi and Griffin,
Phys. Rev. Lett. 89, 130402 (2002)

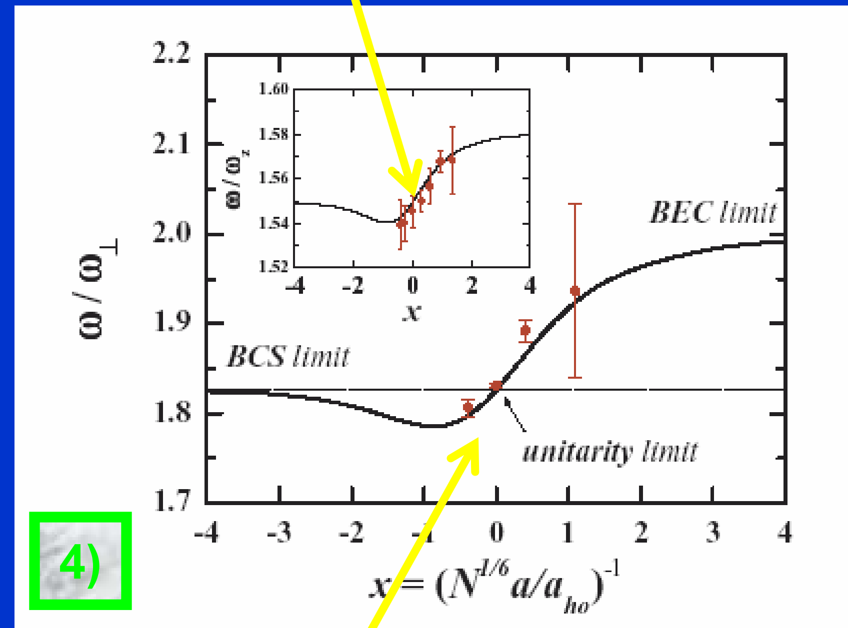


Bruun, cond-mat/0401497

Fermion-boson model would predict here

Grimm's group experiment

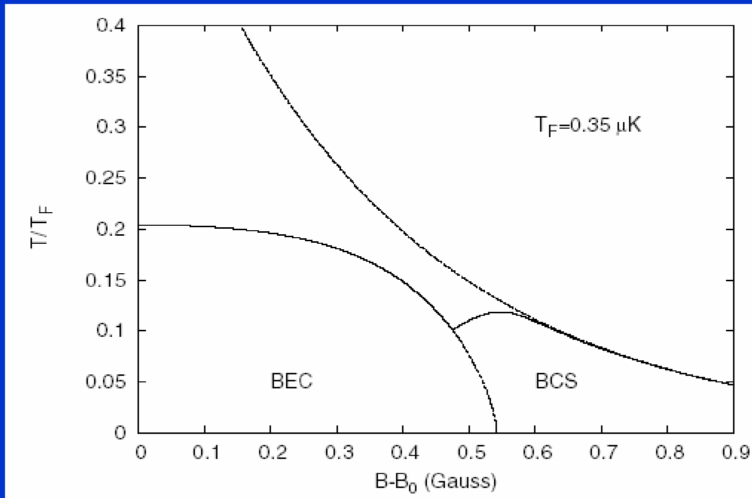


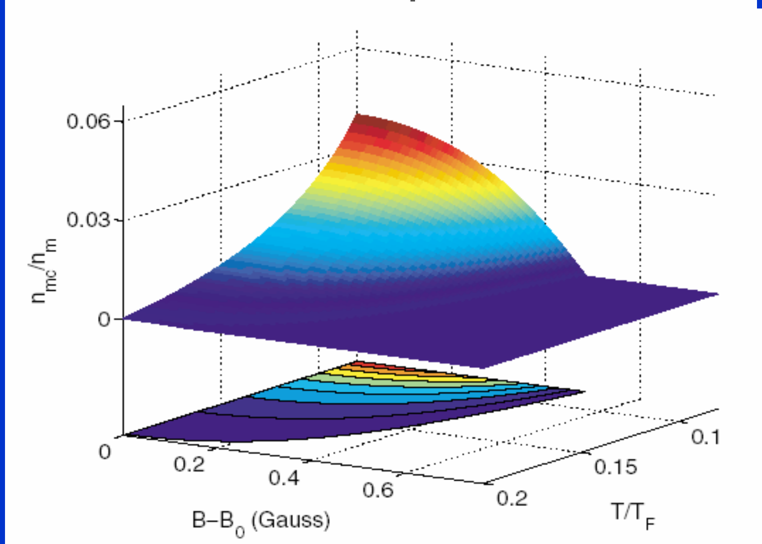


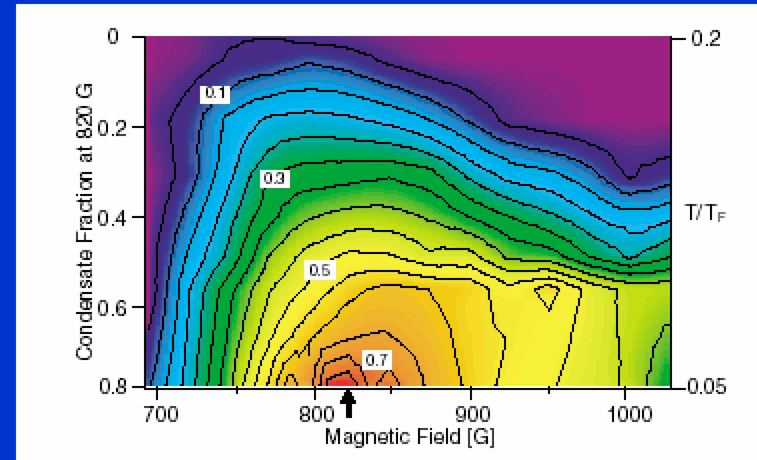
Thomas' group experiment

Hu *et al.* cond-mat/0404012, a semi-quantitative analysis (gap and chemical potential inaccurate) assuming a polytropic equation of state

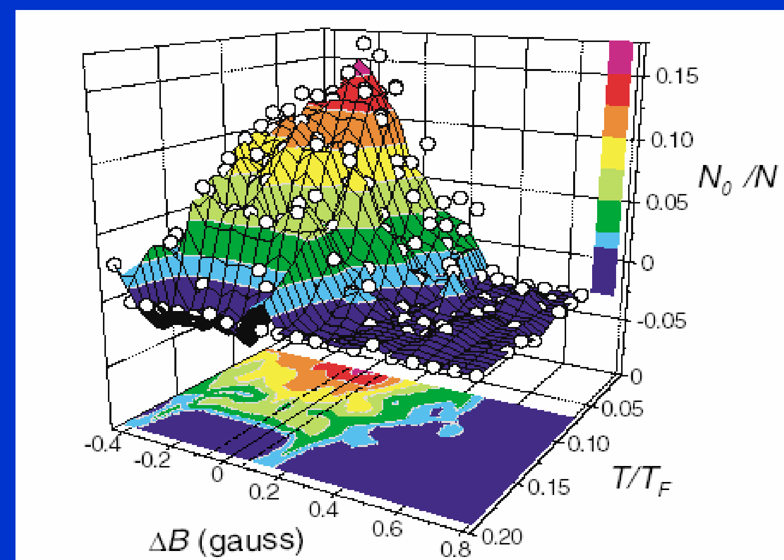
For a more careful analysis, using GFMC equation of state in a trap see Bulgac and Bertsch, cond-mat/0404687







Zwierlein *et al.*
Phys. Rev. Lett. 92, 120403 (2004)



Falco and Stoof,
Phys. Rev. Lett. 92, 130401 (2004)

$$n_m \simeq \frac{n}{2}\left[1-\frac{1}{(k_F|a|)^3}\right]$$

Regal, Greiner and Jin,
Phys. Rev. Lett. 92, 040403 (2004)

Theory declared full victory here!

Main conclusions

TABLE I: Character of the condensate as a function of the inverse scattering length a^{-1} in various in intervals, the approximate boundaries of these intervals being shown in the second row. The total electron spin and spin projection (S, S_Z) along the magnetic field for various pairs are shown in the last row.

$a^{-1} > 0$				$a^{-1} < 0$	
$+\infty$ r_0^{-1} k_F 0				0 k_F $-\infty$	
molecules	halo dimers (+ atoms ?[15])	?		BCS strong coupling	BCS weak coupling
(0,0)	(1,-1)	(1,-1)		(1,-1)	(1,-1)

- ✓ **Fermion superfluidity, more specificaly <u>superflow,</u> has not yet been demonstrated unambiguously experimentally. There is lots of circumstantial evidence and facts in agreement with theoretical models assuming its existence. <u>Vortices!?</u>**

- ✓ **Theory is able to make very precise predictions in this regime and the agreement with experiment can be check quantitatively.**