Empty regions of space interact with one another because of quantum pressure

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In 1948 the Dutch physicist H.B.G. Casimir predicted a peculiar effect following from the laws of quantum mechanics: the attraction between two metallic parallel plates in vacuum. While early experiments in the 1950's gave qualitative support for the Casimir effect, the existence of such an attraction has been confirmed with high accuracy only recently (Lamoreaux 1997, Mohideen and Roy 1998, Chan *et al.* 2001, Buks and Roukes 2001). According to Heisenberg's uncertainty relations, the vacuum is never really empty, but always contains fluctuating fields. The fluctuations of the electromagnetic fields are modified by the introduction of the metallic surfaces, and thus the energy of the vacuum changes. The Casimir effect is therefore a macroscopic manifestation of quantum mechanics. It is not limited to modifications of electromagnetic fields, but is expected to exist for various other fields – even matter fields. Since the pioneering work of Louis de Broglie in 1924, matter particles such as electrons in metals, neutrons in neutron stars, or even atoms of the recently experimentally realized dilute atomic Bose–Einstein and Fermi condensates, can be described as waves or, in modern terms, as excitations of the corresponding quantum fields. Furthermore, as first calculated by Timothy Boyer in 1968, the introduction of metallic obstacles does not always lead to an attractive force, but can also induce a repulsive force, depending on the shape and geometry of the obstacle(s) and the nature of the fields. Thus, the Casimir effect can be interpreted, in general terms, as the geometry–dependent part of the vacuum energy in the presence of fluctuating quantum fields.

The present work has the goal of reaching a qualitative understanding of the interaction between cavities (literal empty regions of space) immersed in the background of non–interacting fermion matter fields. This topic is relevant to the physics of neutron stars, to inhomogeneous quark–gluon plasma, and also to buckyballs (carbon–60 molecules) in liquid mercury, for example. Surprisingly the new form of Casimir energy is neither attractive nor repulsive in character, but it changes depending on the relative arrangement of various scatterers. Qualitatively this effect can be explained by the formation of standing matter waves between scatterers. When such a wave can be formed there is attraction, but repulsion otherwise. For more than two scatterers the pattern of possible standing waves becomes rather complicated. The simplest standing wave forms between two scatterers only. However a wave can scatter first off one scatterer, then off a second one and a third time off a third scatterer before coming back to the first scatterer and closing the cycle. The actual Casimir energy is given by a coherent sum over the effects of all possible matter waves among all scatterers. In this way one arrives at an extremely curious and somewhat unexpected picture. The matter filling up the space between the voids – even when it is nothing else but an ideal quantum gas and thus with no interactions of any kind – leads to an effective interaction between the empty regions of space.

While the evaluation of the Casimir energy for complicated geometries can be rather involved, the calculation simplifies for a system of non–overlapping cavities, since it can be mapped onto the quantum mechanical problem of the scattering of a point–particle of a fixed, but tunable initial energy from the very same cavity–geometry. The Casimir energy can then be extracted from the delay time of the scattered point particle, summed over all relevant initial energies. Although the cavity–systems can be very complicated, such that the scattering pattern for a classical particle would be totally chaotic, the calculations of this work show that the Casimir interaction energy in the case of more than two scatterers can be evaluated quite accurately as a sum of pairwise interactions between these scatterers. As is known from Bohr's correspondence principle, for large initial energies the quantum mechanical scattering calculation can be very well approximated by the interference of periodic orbits running between and being reflected off the cavities. The effective interaction of the empty regions of space thus decreases with distance roughly for the same reason why in a game of billiards it is very difficult to make a long shot: long trajectories are very unstable. The increased instability of relatively long periodic orbits bouncing off more than two scatterers is what makes the three or more body interactions rather short–ranged.