DILUTE QUANTUM DROPLETS

Aurel Bulgac

Phys. Rev. Lett. 89, 050402 (2002)

Quantum Fluids at T = 0

- **Liquid ^{4}He and ^{3}He**
- **Electrons (but only embedded in an ion background)**
- **BEC in alkali atoms and hydrogen and the Fermi-Dirac counterparts – gas (but only in traps)**
- **Nuclei (but Z < 115 and N < 186 so far)**
- **Neutron stars (gravitationally bound)**
  In the core of neutron stars color superconductivity (quarks) is likely

- **The most ethereal droplets ever: the boselets, the fermilets and the ferbolets**

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Number 599 #1, July 24, 2002 by Phil Schewe, James Riordon, and Ben Stein

An Ultra Low-Density Liquid

An ultra low-density liquid, some 10^{13} times thinner than water, might form inside Bose-Einstein condensates under the action of the "Efimov effect," a quantum phenomenon in which the atoms in the cloud attract each other when considered two at a time but repel each other when considered three at a time. In such an Efimov cloud the atoms would be some 20 times farther apart that in a BEC, which is itself pretty sparse---a million times thinner than air. And yet this new type of condensate would not be a gas but a liquid!

According to Aurel Bulgac of the University of Washington (bulgac@phys.washington.edu, 206-685-2988), the exquisite coordination of atoms in an Efimov condensation would allow it to be self-bound (the constraining magnetic fields used to keep a BEC from drifting apart would be unnecessary); moreover, it would be neither compressible nor dilutable. This extraordinary quantum liquid---the smallest density condensed matter system yet proposed---could probably only be formed at much colder temperatures than are now available in BEC experiments. Bulgac proposes that Efimov droplets made from boson atoms be called "boselets." The fermion version would be "fermilets." (Aurel Bulgac, *Physical Review Letters*, 29 July 2002.)

Energy density of a dilute bose gas

$$\mathcal{E}_2(\rho) = \frac{g_2 \rho^2}{2}\left\{1 + \frac{128}{15\sqrt{\pi}}\sqrt{\rho a^3} + \left[\frac{8(4\pi - 3\sqrt{3})}{3}\ln(\rho a^3) + C\right]\rho a^3 + \ldots\right\}$$

$$g_2 = \frac{4\pi\hbar^2 a}{m}$$

$a > 0$

$r a^3 \ll 1$

Bogoliubov	**1947**
Lee and Yang	**1957**
Wu	**1959**
Hugengoltz and Pines	**1959**
Sawada	**1959**
Braaten and Nieto	**1999**

BEC in alkali atoms

- Anderson, Ensher, Matthews, Wieman, Cornel
  Science, 269, 198 (1995)
  ^{87}Rb, $a>0$
- Bradley, Sackett, Tollett, Hulet
  PRL, 75, 1687 (1995)
  ^{7}Li, $a<0$!
- Davis, Mewes, Andrews, van Druten, Kurn, Ketterle
  PRL, 75, 3969 (1995)
  ^{23}Na, $a>0$

- $a > 0$, effective repulsion, need traps

– $r a^3 << 1$

- the state is stable with respect to long wave density fluctuations

- if there are bound two-particle states then BEC decays

$$A + A + A \longrightarrow A_2 + A$$

$$\Gamma \propto a^4 \rho^2$$

a < 0 ?

the system is unstable with respect to density fluctuations and collapses

$$\Gamma \propto \sqrt{r\,|a|}$$

- **Gerton, Strekalov, Prodan, Hulet, Nature 408, 692 (2000)**
  Direct Observation of Growth and Collapse of a Bose-Einstein Condensate with Attractive Interactions
- **Roberts, Claussen, Cornish, Donley, Cornell, Wieman, Phys. Rev. Lett. 86, 4211 (2001), Bosenova**
  Controlled Collapse of a Bose-Einstein Condensate
  Condensate with Attractive Interactions
  see also Nature, 412, 295 (2001).

Case a > 0

- r_0 (radius of interaction)

  $a = O(r_0)$

- only s-wave interaction is relevant, since $r a^3 \ll 1$

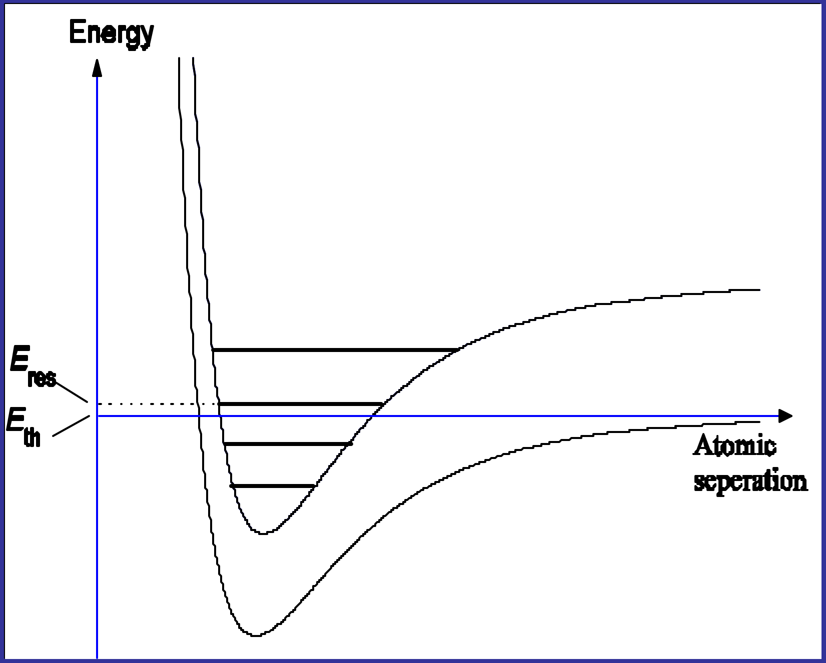
- However, the Feshbach resonance changes the physics

Feshbach resonance

$$H = \frac{\vec{p}^{\,2}}{2m_r} + \sum_{i=1}^{2} (V_i^{hf} + V_i^{Z}) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + V^d$$

$$V^{hf} = \frac{a_{hf}}{\hbar^2} \vec{S}^e \cdot \vec{S}^n, \quad V^Z = (g_e S_z^e - g_n S_z^n) B$$

Tiesinga, Verhaar, Stoof
Phys. Rev. A47, 4114 (1993)



Tiesinga, Verhaar, Stoof
Phys. Rev. A47, 4114 (1993)

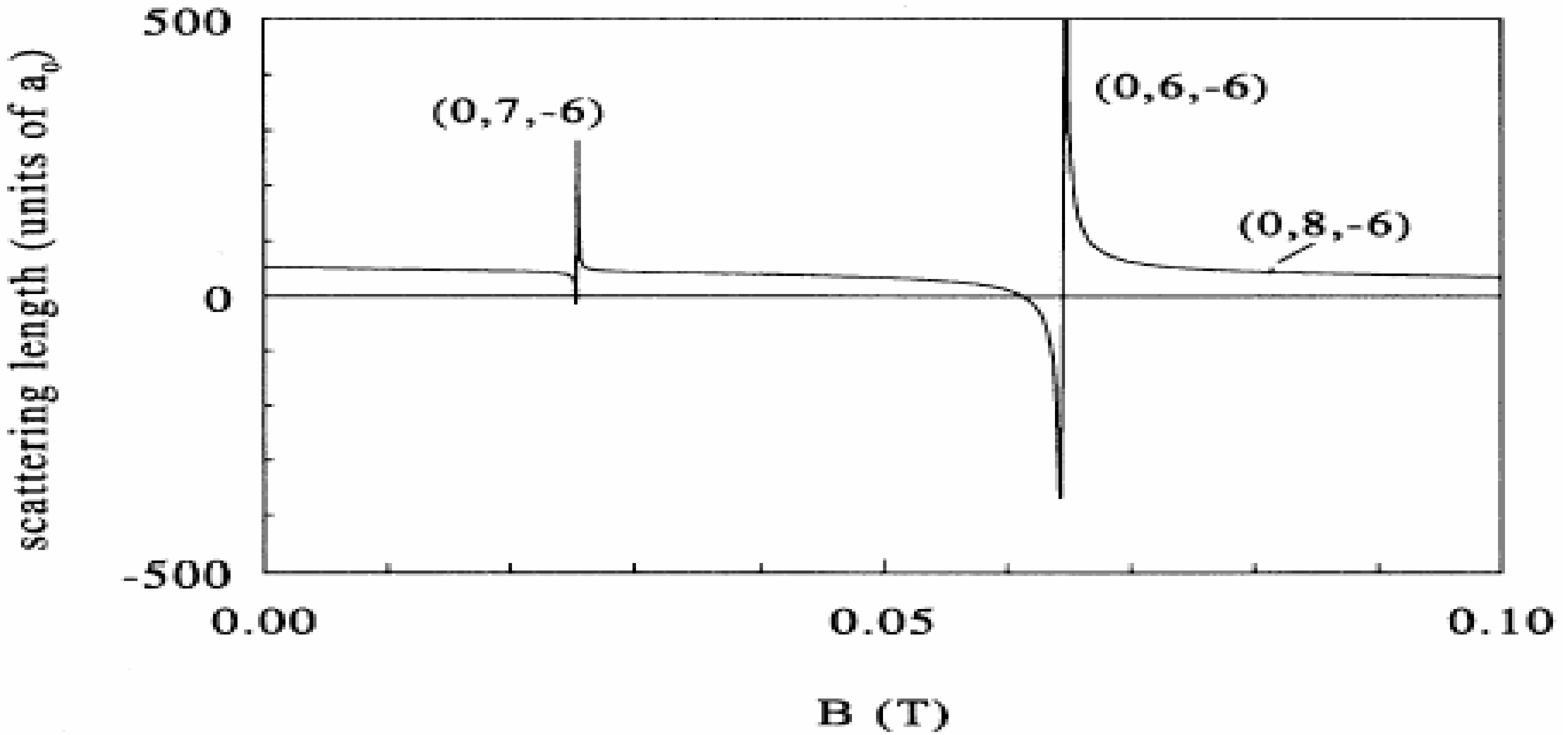


FIG. 2. Scattering length for elastic $|3,-3\rangle + |3,-3\rangle$ scattering as a function of magnetic field. Labels denote quantum numbers (l, F, M_F).

Vogels, Tsai, Freeland, Kokkelmans, Verhaar, Heinzen
Phys. Rev. A56, R1067 (1997)

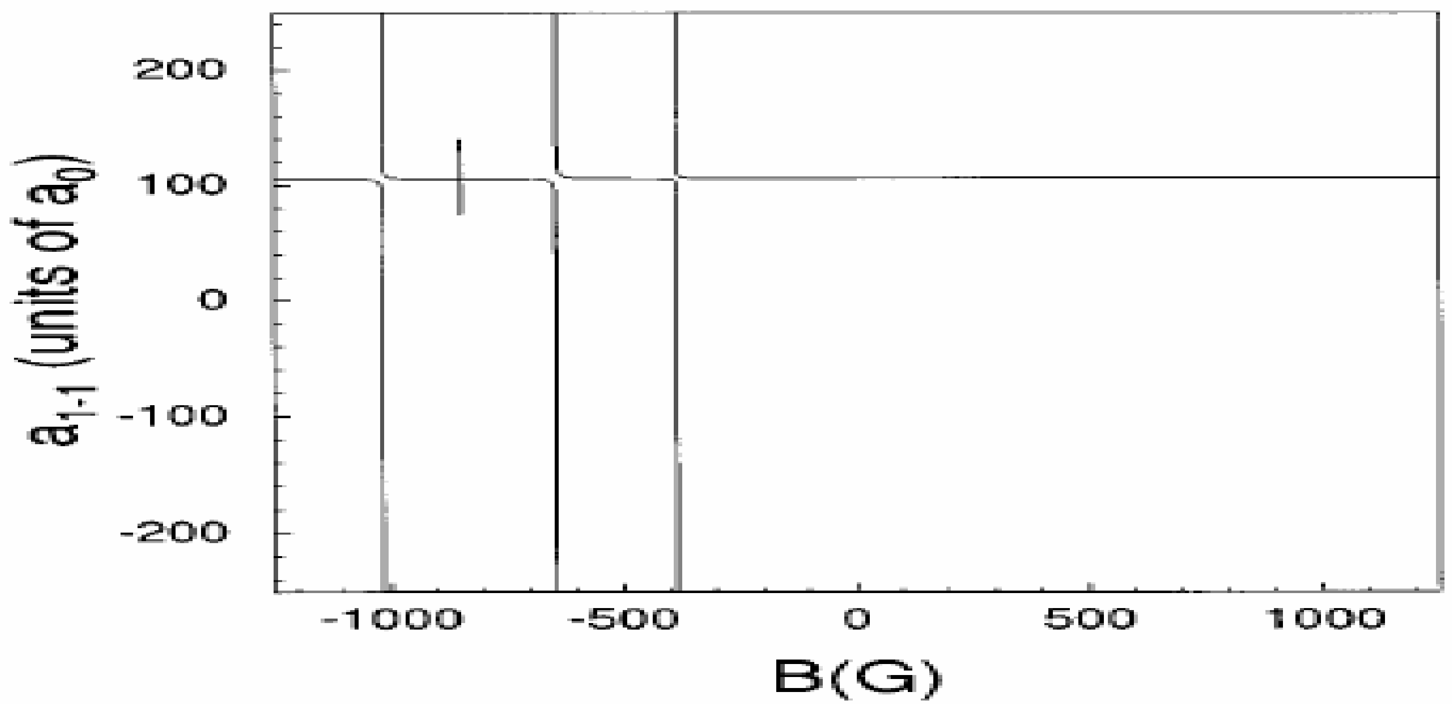


FIG. 3. Predicted field-dependent scattering length for collisions of ^{87}Rb atoms in $|1,-1\rangle$ state. Three broad Feshbach resonances occur for negative fields at 383, 643, and 1018 G. A narrow resonance occurs at 850 G.

Vogels, Tsai, Freeland, Kokkelmans, Verhaar, Heinzen
Phys. Rev. A56, R1067 (1997)

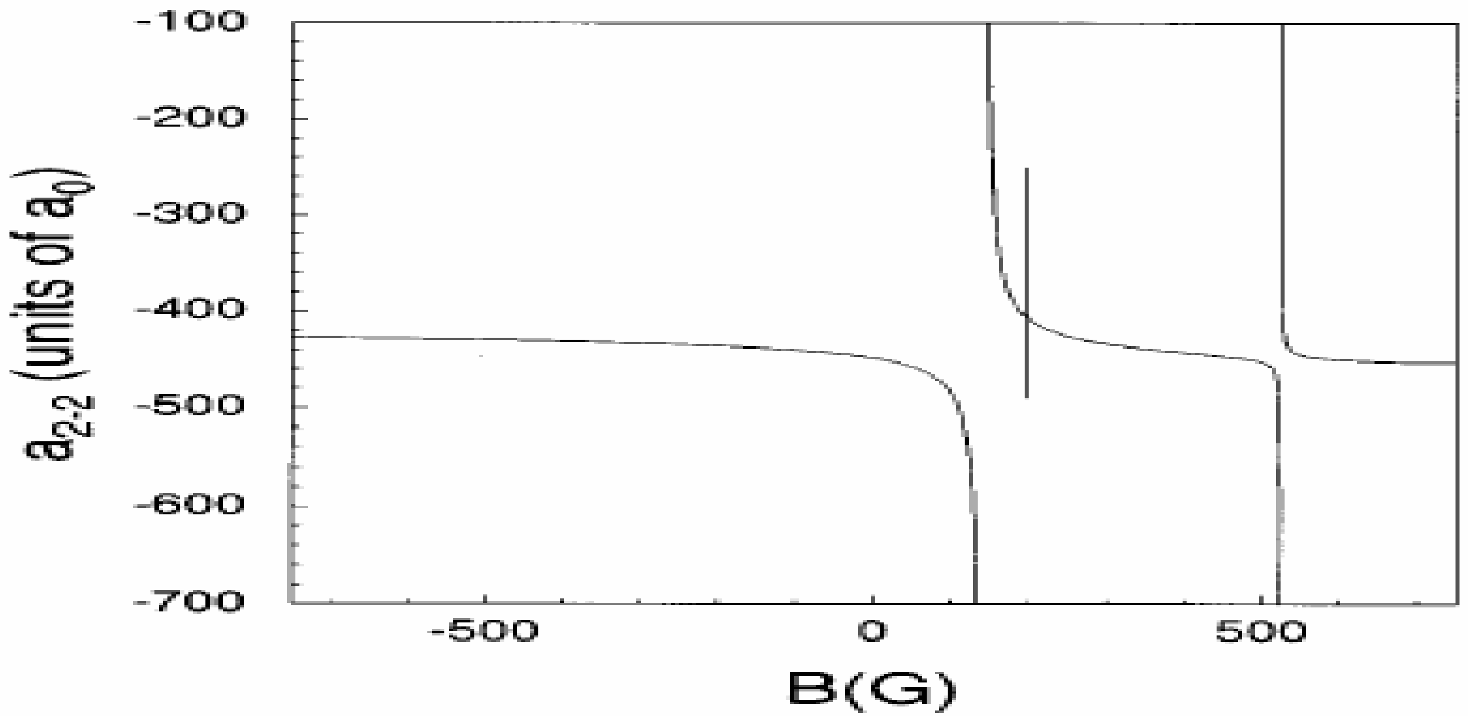


FIG. 4. Predicted field-dependent scattering length for collisions of ^{85}Rb atoms in the $|2,-2\rangle$ state. Two broad Feshbach resonances occur in the weak-field seeking range at 142 and 524 G, and a narrow one at 198 G.

Courteille, Freeland, Heinzen, van Abeelen,Verhaar, Phys. Rev. Lett. 81, 69 (1998)

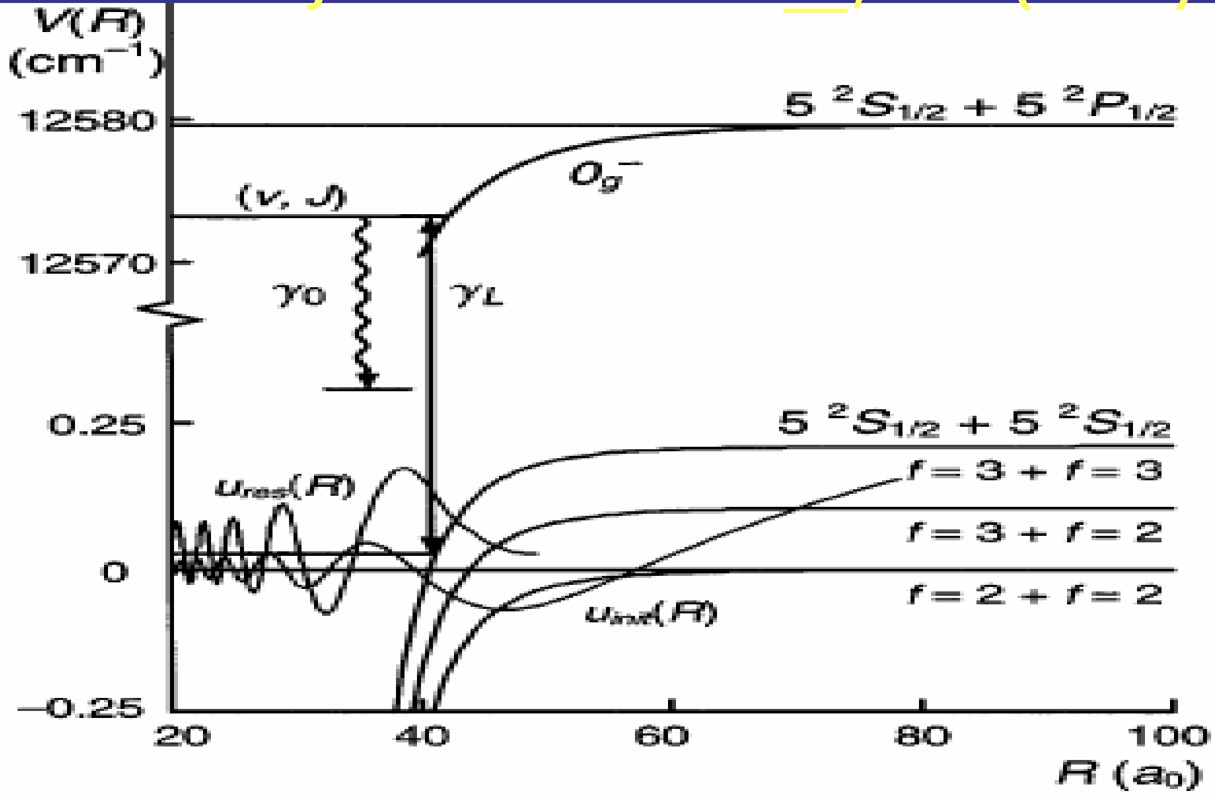


FIG. 1. Photoassociation method for detecting a Feshbach resonance in collisions of ultracold ^{85}Rb($f = 2, m_f = -2$) atoms. The entrance channel wave function $u_{init}(R)$ couples to a quasibound state with wave function $u_{res}(R)$. A laser field induces photoassociation of this state to an excited, bound $0_g^-(v, J)$ molecular state at a rate γ_L, which then decays back to free atoms at a rate γ_0. As a magnetic field is varied, the quasibound state tunes through zero energy, producing a Feshbach resonance for ultracold collisions. The resulting enhancement of $u_{res}(R)$ produces an enhancement of γ_L that we detect with a trap loss method.

Inouye, Andrews, Stenger, Miesner, Stamper-Kurn, Ketterle, Nature, 392, 151 (1998)

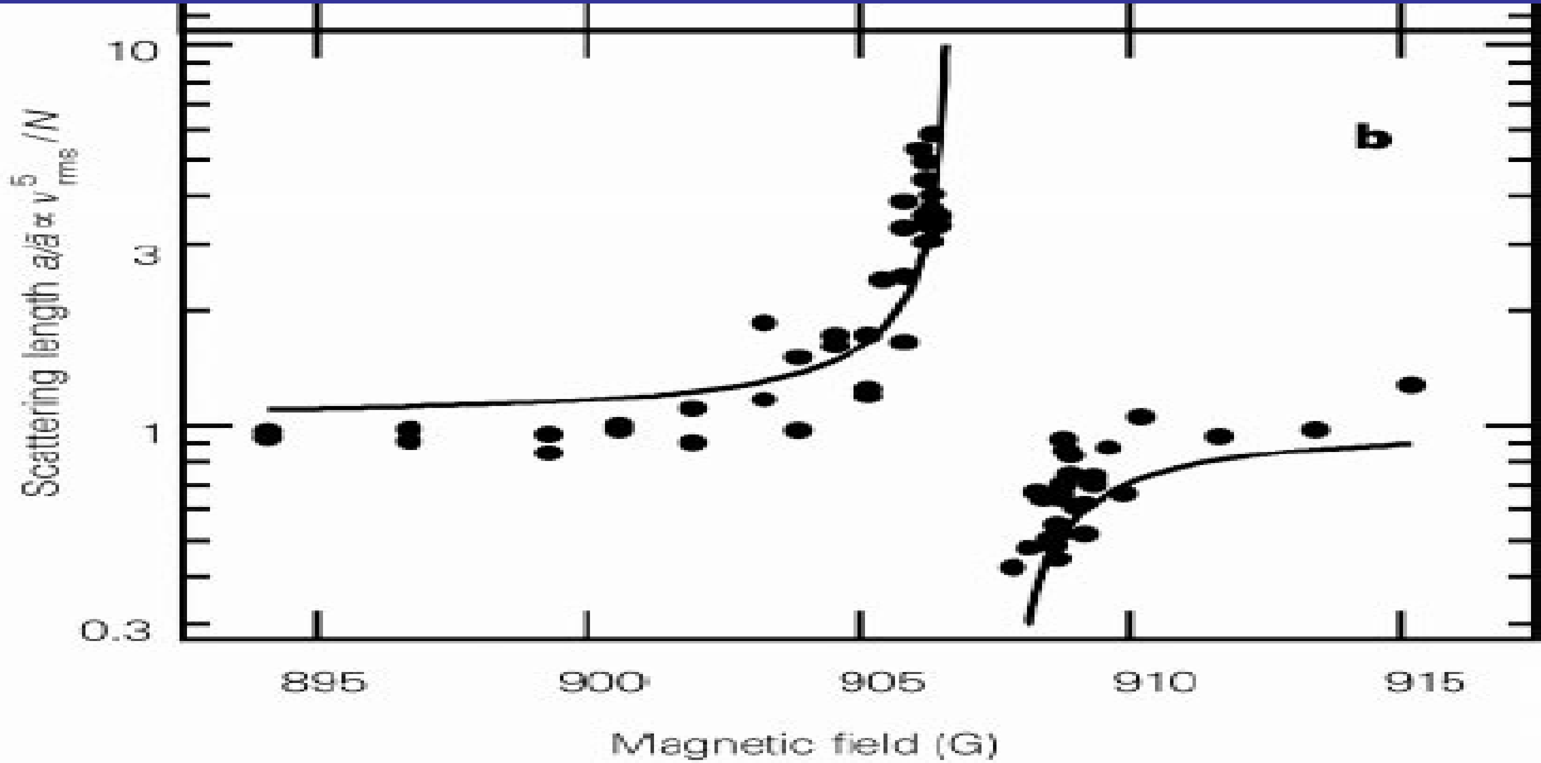


Figure 2 Observation of the Feshbach resonance at 907 G using time-of-flight absorption imaging. **a**, Number of atoms in the condensate versus magnetic field. Field values above the resonance were reached by quickly crossing the resonance from below and then slowly approaching from above. **b**, The normalized scattering length $a/\tilde{a} \propto v^5_{rms}/N$ calculated from the released energy, together with the predicted shape (equation (1), solid line). The values of the magnetic field in the upper scan relative to the lower one have an uncertainty of <0.5 G.

Cornish, Clausen, Roberts, Cornel, Weinman,
Phys. Rev. Lett. 85, 1795 (2000)

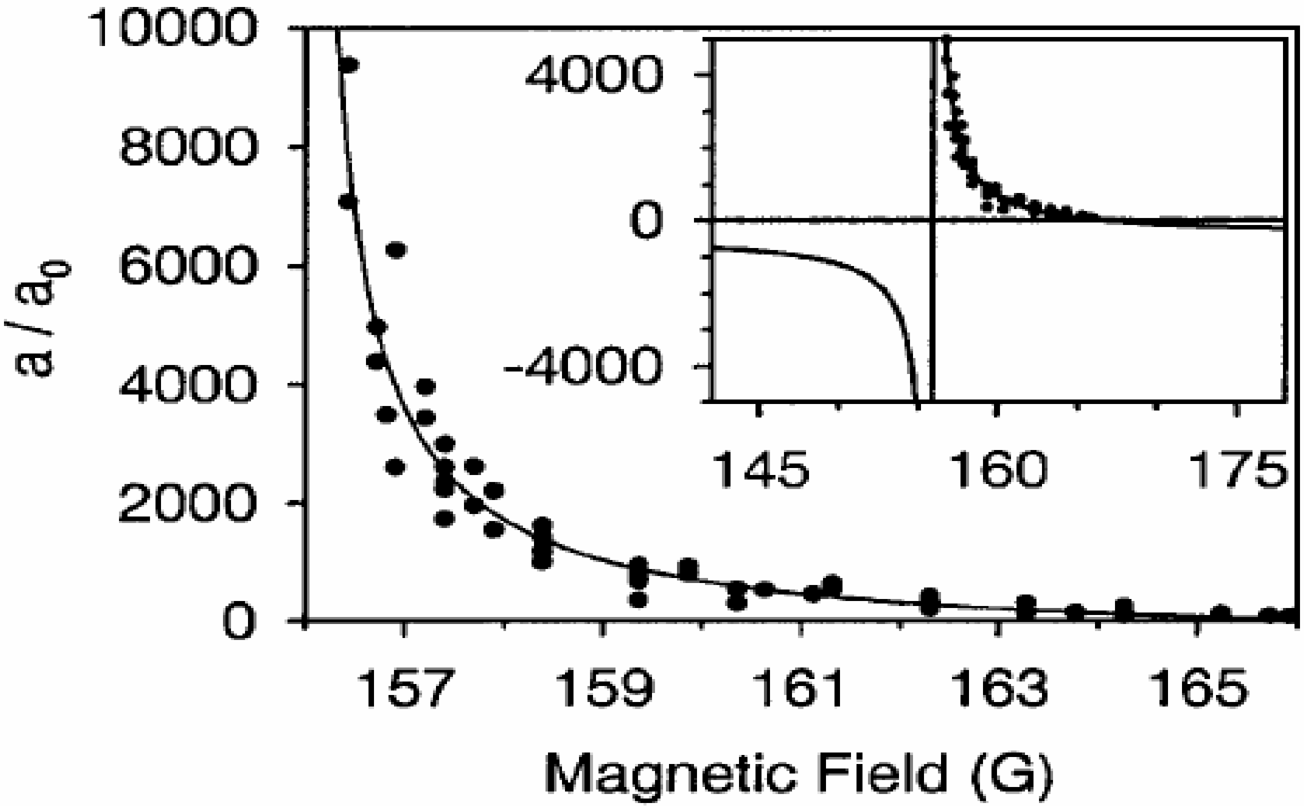


FIG. 1. Scattering length in units of the Bohr radius (a_0) as a function of the magnetic field. The data are derived from the condensate widths. The solid line illustrates the expected shape of the Feshbach resonance using a peak position and resonance width consistent with our previous measurements [4,18]. For reference, the shape of the full resonance has been included in the inset.

Efimov effect - 1970

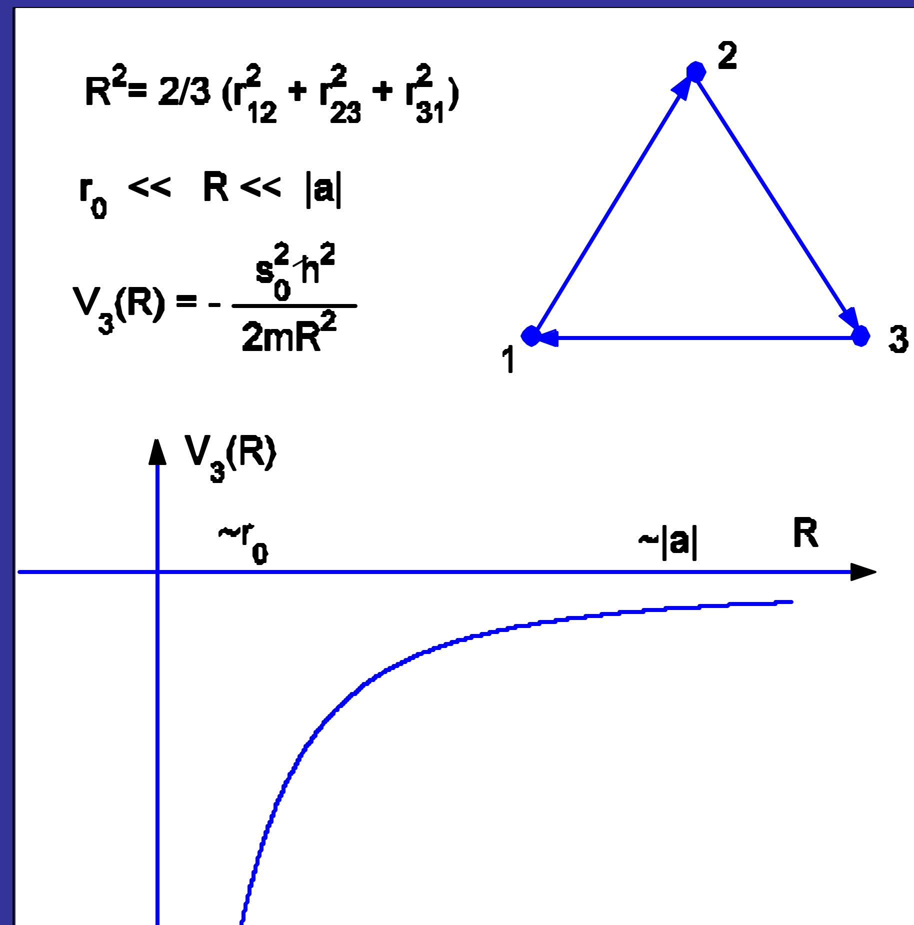
$|a| >> r_0$

$$\psi(R) \propto \sin\left(s_0 \ln k_n R + \delta\right)$$

$$N \approx \frac{s_0}{p} \ln\frac{|a|}{r_0}, \quad s_0 = 1.00624...$$

$$E_n = E_0 \exp\left(-\frac{2pn}{s_0}\right) = -\frac{\hbar^2 k_n^2}{2m}, \quad n = 0,1,2,...$$

$$\exp\left(\frac{p}{s_0}\right) = 22.694..., \quad \exp\left(\frac{2p}{s_0}\right) = 515.028...$$



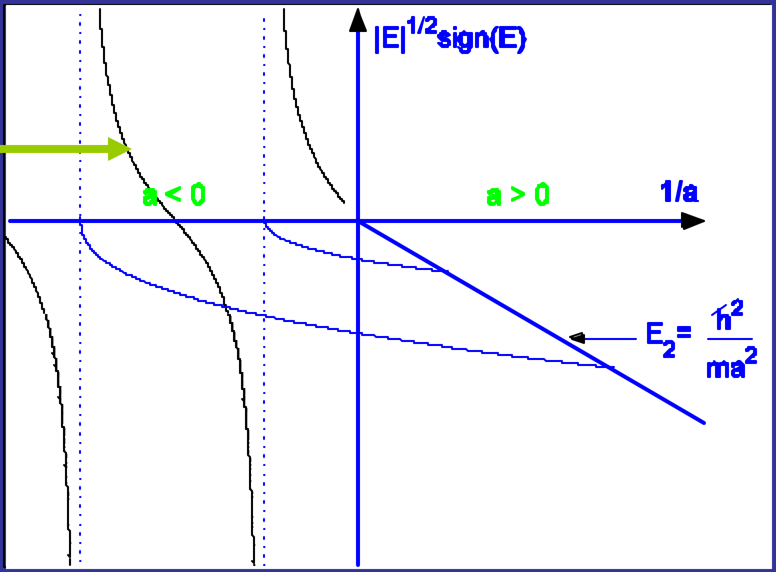
-Scaling
-All states have the same $J^P=0+$

Case a <0

- Now there are two dimensionless parameters
  $r|a|^3$ and $r|r_0|^3$
- g_3 - the zero energy three-particle amplitude
  d_1, d_2<0 are universal constants and a_0 is a genuine system dependent three-body parameter, Efimov 1979, numerical values computed by Braaten, Hammer and Mehen, 2002

$$g_3 = \frac{12p\hbar^2 a^4}{m}\left[d_1 + d_2 \tan\left(s_0 \ln\frac{|a|}{|a_0|} + \frac{p}{2}\right)\right]$$

$$E_3 = \frac{1}{6} g_3 r^3$$



E_3 – the contribution to the energy density of the three-body collisions

Quantum fluctuations

- **Even though boselets are formed when $a<0$ the next order corrections to the gs energy are still suppressed by the factor $(r|a|^3)^{1/2}$, as in the case of a dilute Bose gas with $a>0$.**
- **Higher-order loops, including both g_2 and g_3, are controlled by the same parameter $(r|a|^3)^{1/2}$ - *preliminary***

***Paulo Bedaque*, private communication**

The ground state energy of an ensemble of N bosons

$$E(N) = \int d^3r \varepsilon(\boldsymbol{r}) = \int d^3r \left[\frac{\hbar^2}{2m} |\boldsymbol{\nabla}\psi(\boldsymbol{r}))|^2 + \frac{1}{2} g_2 \rho(\boldsymbol{r})^2 + \frac{1}{6} g_3 \rho(\boldsymbol{r})^3\right],$$

$$\rho(\boldsymbol{r}) = |\psi(\boldsymbol{r})|^2$$

Gross-Pitaevski equation

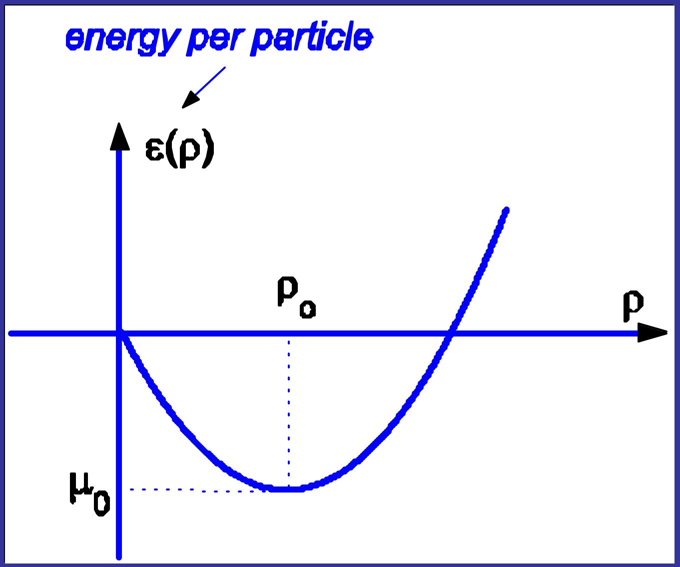
$$-\frac{\hbar^2 \Delta}{2m} y(\vec{r}) + \left[g_2 r(\vec{r}) + \frac{1}{2} g_3 r(\vec{r})^2 \right] y(\vec{r}) = m y(\vec{r})$$

$$\int d^3\vec{r}\, |y(\vec{r})|^2 = \int d^3\vec{r}\, r(\vec{r}) = N$$

One can rescale the density, the energy and the unit of length

$$-\frac{\Delta}{2} y(\vec{r}) + \left[- r(\vec{r}) + \frac{1}{2} r(\vec{r})^2 \right] y(\vec{r}) = m y(\vec{r})$$

Infinite homogeneous matter



$$r_0 = -\frac{3g_2}{2g_3},\quad m_0 = -\frac{3g_2^2}{8g_3}$$

$$s = \sqrt{\frac{3g_2^2}{4mg_3}}$$

Sound velocity

Semi-infinite mater

$$r(z) = \frac{r_0}{1+\exp(2k_0 z)}$$

$$k_0 = \sqrt{\frac{2m\,|\,m_0\,|}{\hbar}}$$

$$s = \int dz\,\left[e(z) - m_0\, r(z)\right] = \frac{g_3\, r_0^3}{12 k_0}$$

Surface tension

$$w_s^2 = \frac{s\, k^3}{m\, r_0}$$

Dispersion law for surface waves

Slab of finite width

$$r(z) = r_0 \frac{m}{m_0} \frac{1}{1+\sqrt{1-\frac{m}{m_0}\cosh(2kz)}}$$

$$m = -\frac{\hbar^2 k^2}{2m} \geq m_0$$

$$r(z) \leq r_0$$

Spherical droplets – boselets

$$r(r) \cong r_0\left(1+\frac{1}{2kR}\right)\frac{1}{1+\dfrac{\cosh(2kr)}{\cosh(2kR)}}$$

$$m = -\frac{\hbar^2 k^2}{2m}, \quad r(0) \geq r_0$$

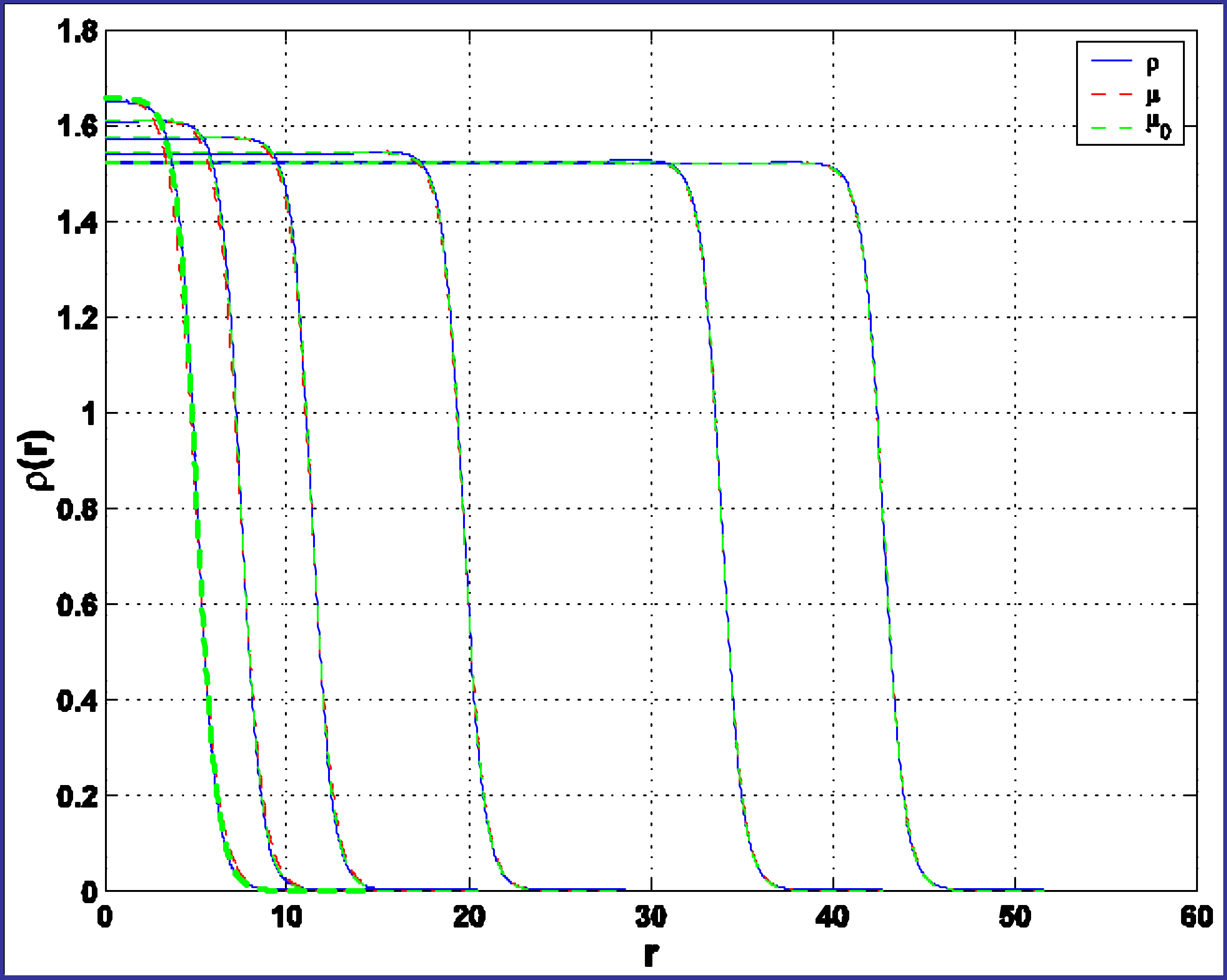
$$R = r_0 N^{1/3} + r_1 + \ldots$$

$$r_0 = \left(\frac{3}{4p r_0}\right)^{1/3}, \quad \frac{4p}{3} R^3 r(0) \cong N$$

$$E(N) = m_0 N + 4p r_0^2 s N^{2/3} + \ldots$$

$$w_l^2 = \frac{s\, l(l-1)(l+2)}{m r_0 R^3} + \ldots, \; l = 2,3,\ldots$$

Density profiles of boselets with N = 1000, 3000, 10000, 50000, 250000 and 500000 particles



Fermilets

Efimov effect occurs only if three fermions are simultaneously in a relative s-state, thus:

- Need fermions with spin s> 1/2 or
- At least two different species with s=1/2

Bulgac and Efimov - 1975

Energy of a fermilet:

$$E(N_\pi, N_\nu) = \int d^3 r \left\{ \frac{\hbar^2}{2m}[\tau_\pi(\boldsymbol{r}) + \tau_\nu(\boldsymbol{r})] + \frac{g_2}{2}[\rho_\pi(\boldsymbol{r}) + \rho_\nu(\boldsymbol{r})]^2 - \frac{g_2}{4}[\rho_\pi(\boldsymbol{r})^2 + \rho_\nu(\boldsymbol{r})^2] \right.$$
$$\left. + \frac{g_3}{4}\rho_\pi(\boldsymbol{r})\rho_\nu(\boldsymbol{r})[\rho_\pi(\boldsymbol{r}) + \rho_\nu(\boldsymbol{r})] \right\},$$

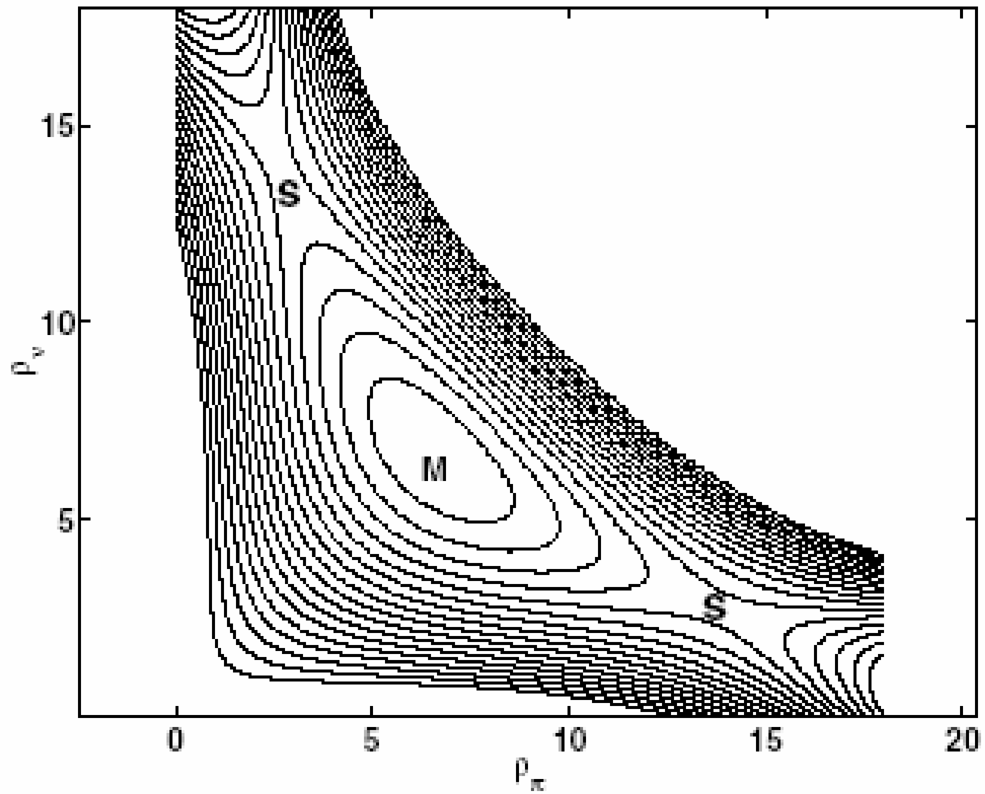
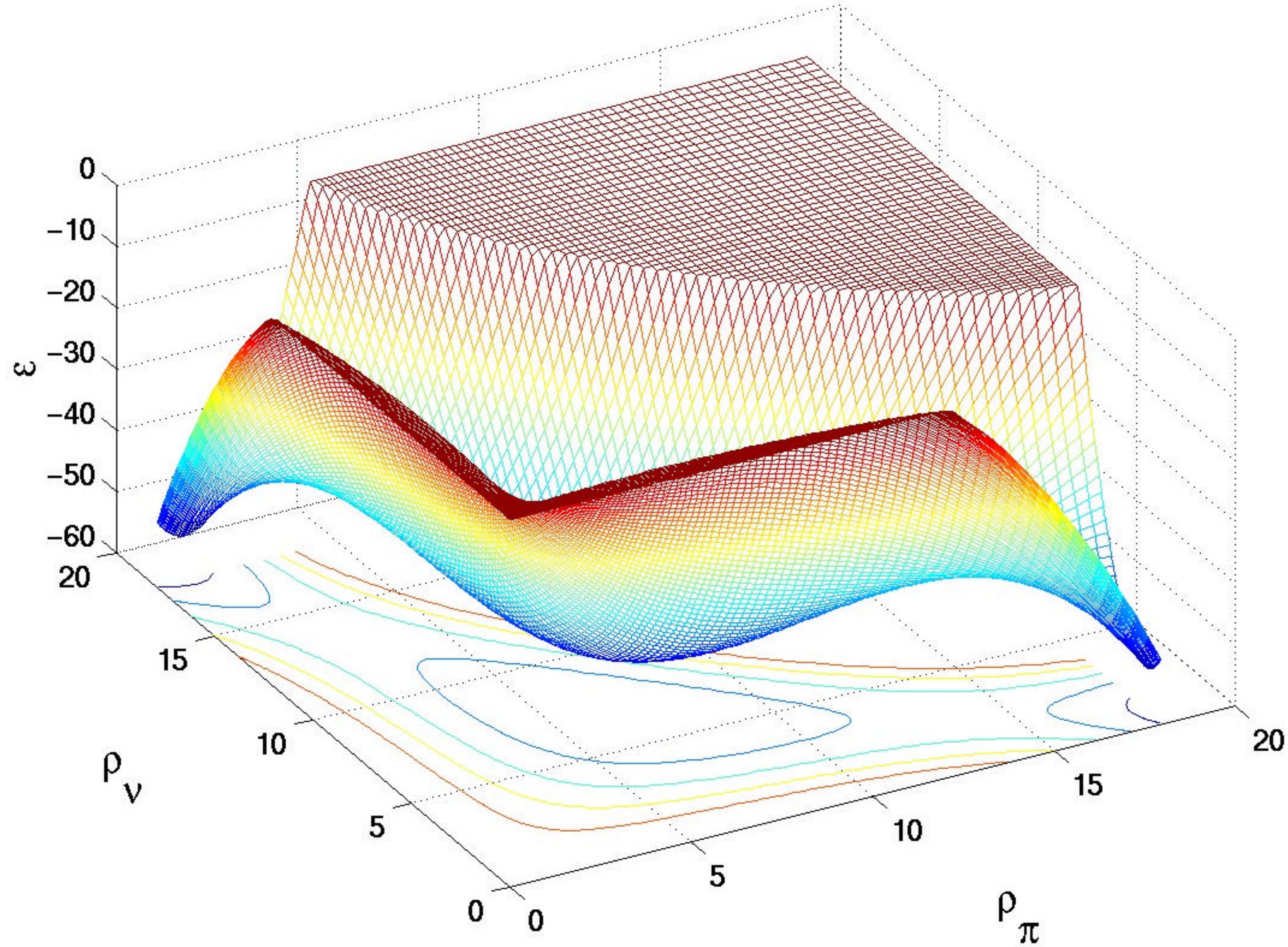


FIG. 1. A typical contour plot of the energy density for an homogeneous Fermi system consisting of two fermion species π and ν [see Eq. (16)] for the case $\hbar = m = g_3 = 1$ and $g_2 = -5$. Only the negative part of the energy density surface is plotted. The local minimum and the two saddle points are labeled by M and S, respectively. For other sets of parameters there is either only one saddle point or none at all.



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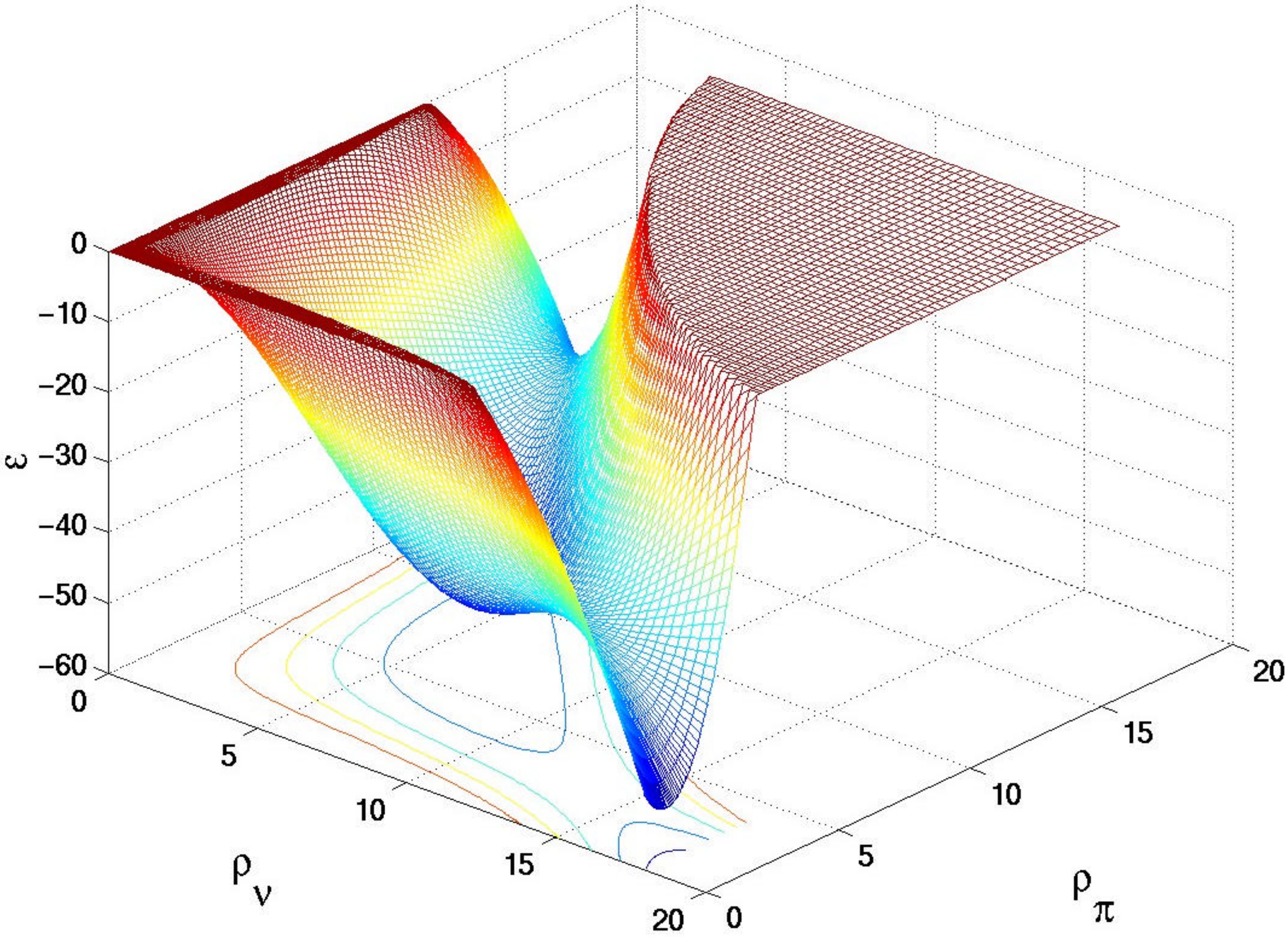
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Trimer phase

- Trimers can form and either a pure trimer BEC can form or a atom-trimer coherence can occur. Since the trimer size << |a| the density drops by a factor of 3.

  NB Atom-molecule coherence in BEC has been observed: Donley, Claussen, Thompson, Wieman, Nature, 417, 529 (2002).

. In Fermilets trimers (fermions) can form as well and compete with Cooper pairs (bosons).

  NB The size of Cooper pairs is expected to be >> |a| while trimers' size is << |a|.

. Since Efimov effect occurs only in a three-body system, these trimer phases are perhaps unique in a sense if they take place.

Conclusions

- A whole new class of self-bound quantum liquids, superfluids.
- At T=0 the boselets are almost 100% BEC and liquid (not gaseous) and extremely dilute
- Widely tunable properties, one can manufacture “designer nuclei” with given density, arbitrary number of particles and “isotopic” composition
- Mixtures of bosons and fermions – ferbolets
- Unusual new phases, in particular the trimer phase
- Particularly simple properties, since essentially only s-wave interactions are relevant