What have we learned so far about dilute Fermi gases?

Aurel Bulgac
University of Washington, Seattle

These slides will be posted shortly at
http://www.phys.washington.edu/~bulgac/

What is the *Holy Grail* of this field?

Fermionic superfluidity!

Superconductivity and superfluidity in Fermi systems

- Dilute atomic Fermi gases $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ Liquid ^3He $T_c \approx 10^{-7}$ eV
- ✓ Metals, composite materials $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ Nuclei, neutron stars $T_c \approx 10^{5} - 10^{6}$ eV
- QCD color superconductivity $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

Memorable years in the history of superfluidity and superconductivity of Fermi systems

- **1913 Kamerlingh Onnes**
- **1972 Bardeen, Cooper and Schrieffer**
- **1973 Esaki, Giaever and Josephson**
- **1987 Bednorz and Muller**
- **1996 Lee, Osheroff and Richardson**
- **2003 Abrikosov, Ginzburg and Leggett**

One of my favorite times in the academic year occurs in early spring when I give my class of extremely bright graduate students, who have mastered quantum mechanics but are otherwise unsuspecting and innocent, a take-home exam in which they are asked to deduce superfluidity from first principles. There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is impossible. Superfluidity, like the fractional quantum Hall effect, is an emergent phenomenon – a low-energy collective effect of huge numbers of particles that cannot be deduced from the microscopic equations of motion in a rigorous way and that disappears completely when the system is taken apart[A)]. There are prototypes for superfluids, of course, and students who memorize them have taken the first step down the long road to understanding the phenomenon, but these are all approximate and in the end not deductive at all, but fits to experiment. The students feel betrayed and hurt by this experience because they have been trained to think in reductionist terms and thus to believe that everything not amenable to such thinking is unimportant. But nature is much more heartless than I am, and those students who stay in physics long enough to seriously confront the experimental record eventually come to understand that the reductionist idea is wrong a great deal of the time, and perhaps always.

Robert B. Laughlin, Nobel Lecture, December 8, 1998

Topics to be covered (not necessarily in this order):

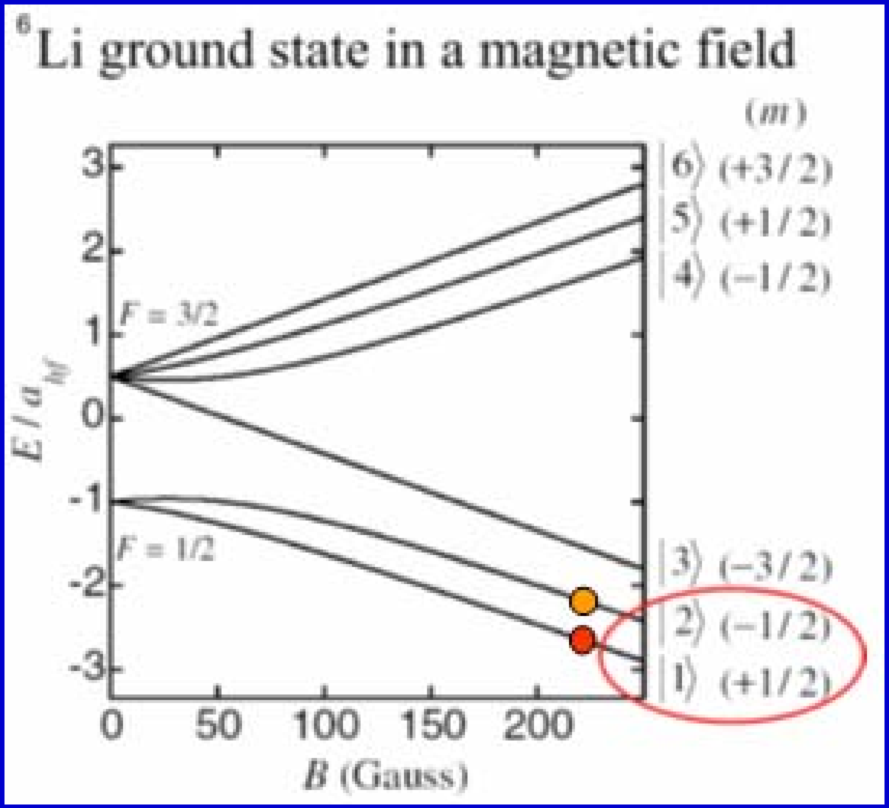
- **A single atom in magnetic field**
- **Two atoms in magnetic field**
- **Atomic traps**
- **Basic parameters of fermionic dilute atomic clouds**
- **A review of a number of key experimental results**
- **What theory tells us so far?**

HISTORY

- ✓ 1995 BEC was observed.
- ✓ 2000 vortices in BEC were created
  thus BEC confirmed un-ambiguously.

- ✓ In 1999 DeMarco and Jin created a degenerate atomic Fermi gas.
- ✓ 2002 O'Hara, Hammer, Gehm, Granada and Thomas observed expansion of a Fermi cloud compatible with the existence of a superfluid fermionic phase.
- ✓ 2003 Jin's, Grimm's, Ketterle's groups and others
  ultracold molecules, mBEC from Fermi gas
- ✓ 2004 Jin's group (and bit later Ketterle's group too) announces the observation of the resonance condensation of fermionic atomic pairs ?
- ✓ Grimm's group reports measurements of the gap
- ✓ Thomas' group reports measurements of the specific heat

One fermionic atom in magnetic field



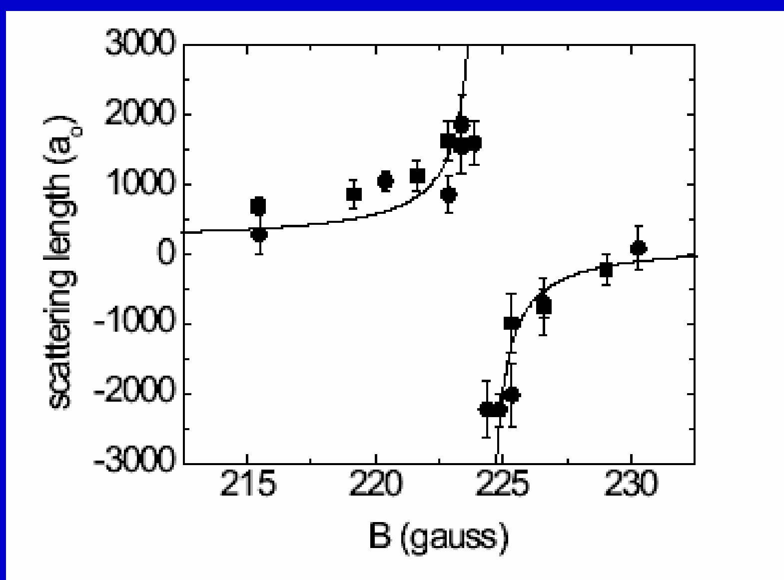
Feshbach resonance

$$H = \frac{\vec{p}^{\,2}}{2\mu_r} + \sum_{i=1}^{2}\left(V_i^{hf} + V_i^{Z}\right) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \cancel{V^d}$$

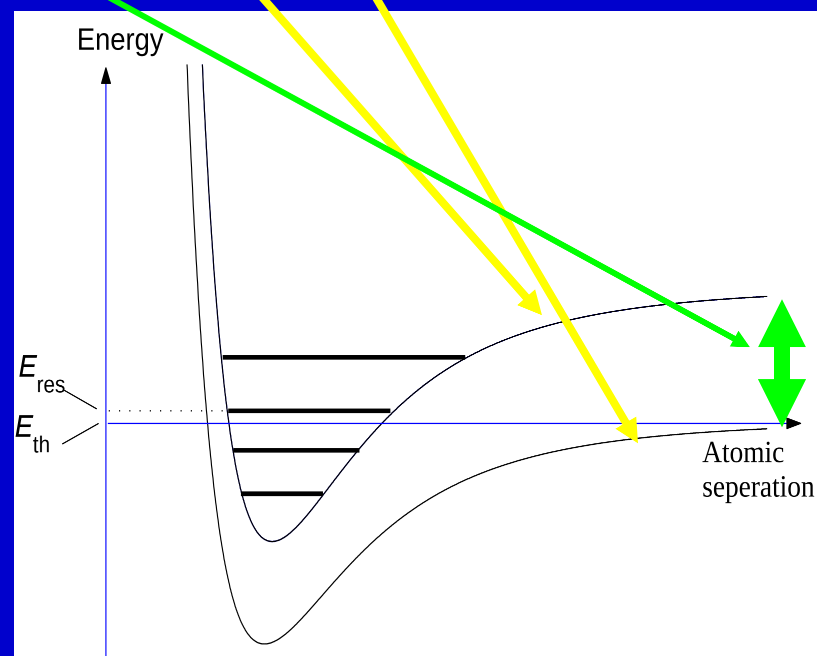
$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{S}^e\cdot\vec{S}^n,\quad V^Z = (\gamma_e S_z^e - \gamma_n S_z^n)B$$

Channel coupling

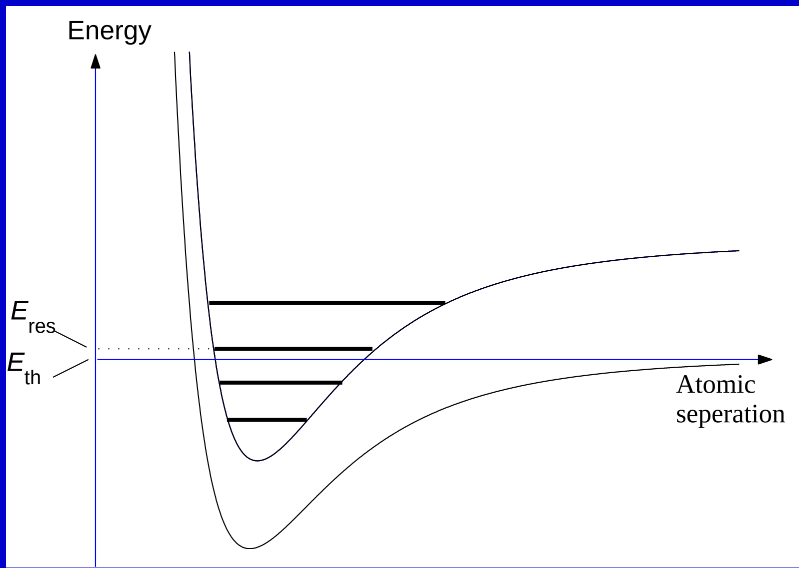
Tiesinga, Verhaar, Stoof
Phys. Rev. A**47**, 4114 (1993)

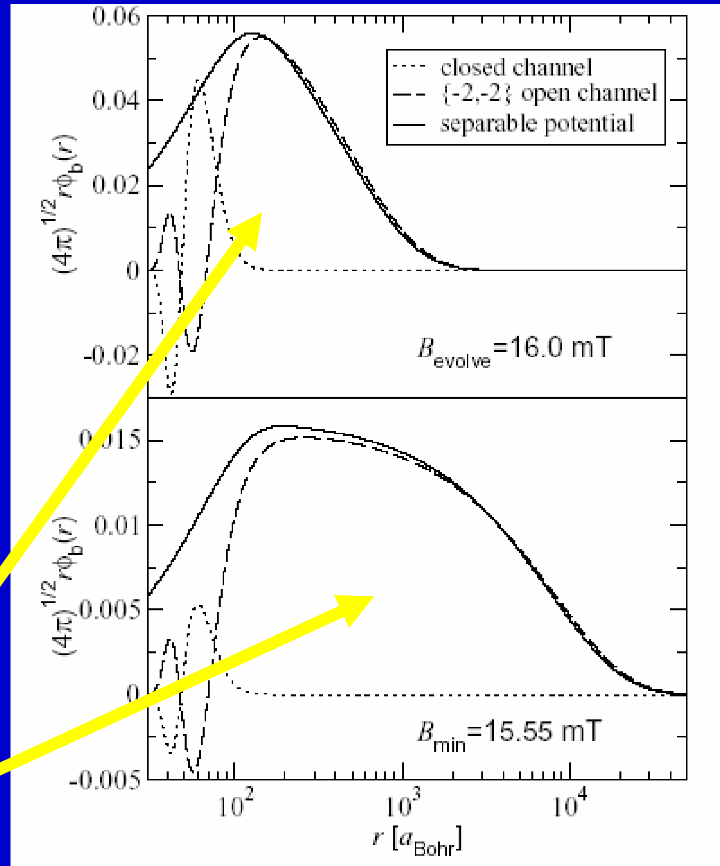


Regal and Jin
Phys. Rev. Lett. **90**, 230404 (2003)

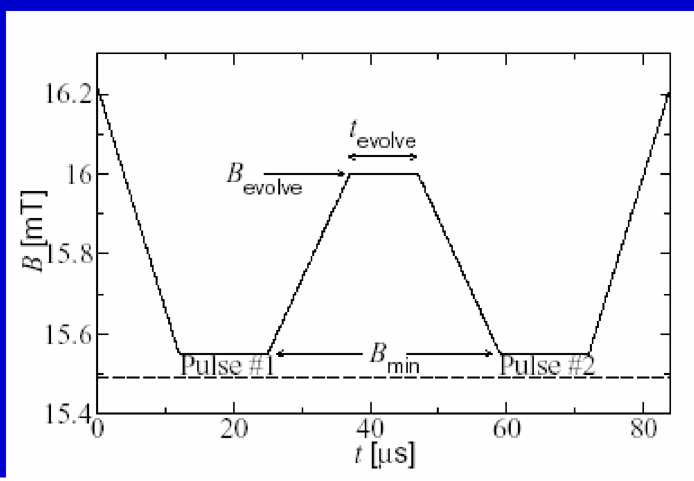


Köhler, Gasenzer, Jullienne and Burnett PRL 91, 230401 (2003), inspired by Braaten, Hammer and Kusunoki cond-mat/0301489





Halo dimer



$$\frac{P(r > r_0)}{P(r < r_0)} \propto \frac{a}{r_0} \quad >> \quad 1$$

NB The size of the "Feshbach molecule" (closed channel state) is largely B-independent and smaller than the interparticle separation.

$$\begin{bmatrix} H_{11} & V_{12} \\ V_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \mathrm{u} \\ \mathrm{v} \end{bmatrix} = k^2 \begin{bmatrix} \mathrm{u} \\ \mathrm{v} \end{bmatrix}$$

Open channel

Closed channel

$$\begin{cases} H_{11}\mathrm{u} = k^2 \mathrm{u} \\ H_{22}\phi_0 = \kappa_0^2 \phi_0 \end{cases}$$

$$\phi_0(r) = \sqrt{\frac{2}{r_0}} \sin\frac{\pi r}{r_0}, \qquad \mathrm{u}_0(r) = \sin(kr), \qquad \mathrm{v}_0(r) \propto \phi_0(r)$$

$$V_{12}(r) = g\delta(r - r_1), \qquad r_1 = \frac{r_0}{2}$$

$$\mathrm{u}(r) = \sin(kr) + \frac{g^2 \exp(ikr_>)\sin(kr_<)}{k^2 - \kappa_0^2}, \qquad \begin{cases} r_> = \max(r, r_1) \\ r_< = \min(r, r_1) \end{cases}$$

$$\begin{cases} -\dfrac{1}{a} = \dfrac{\kappa_0^2 - k^2}{g^2 r_1} \\ \mathrm{u}(r) = \exp(i\delta)\sin k(r - a), \qquad r > r_1 \end{cases}$$

$$\begin{cases} \int \mathrm{d}r \left|\mathrm{v}(r)\right|^2 = \dfrac{k^2 a^2}{(k^2 a^2 + 1) g^2 r_1} \le \dfrac{1}{g^2 r_1} = \mathrm{O}(r_0) \\ \int_0^{\mathrm{R}} \mathrm{d}r \left|\mathrm{u}(r)\right|^2 \approx \int_{r_0}^{R} \mathrm{d}r \sin^2(kr + \delta) = \mathrm{O}(R) \gg \mathrm{O}(r_0) \end{cases}$$

Closed channel wf (singlet)

Open channel wf (triplet)

$$\left.\frac{d \ln \mathrm{u}(r)}{dr}\right|_{r = r_0} = -\frac{1}{a}$$

$$r_0 \approx \left(\frac{C_6 m}{\hbar^2}\right)^{1/4}$$

$$n r_0^3 \ll 1, \quad a \gg r_0, \qquad \mathrm{O}(k_F r_0) \ll 1$$

$$\mathrm{O}(\kappa_0^2) = \mathrm{O}(g^2) = \frac{2\mu_B B_0 m}{\hbar^2}$$

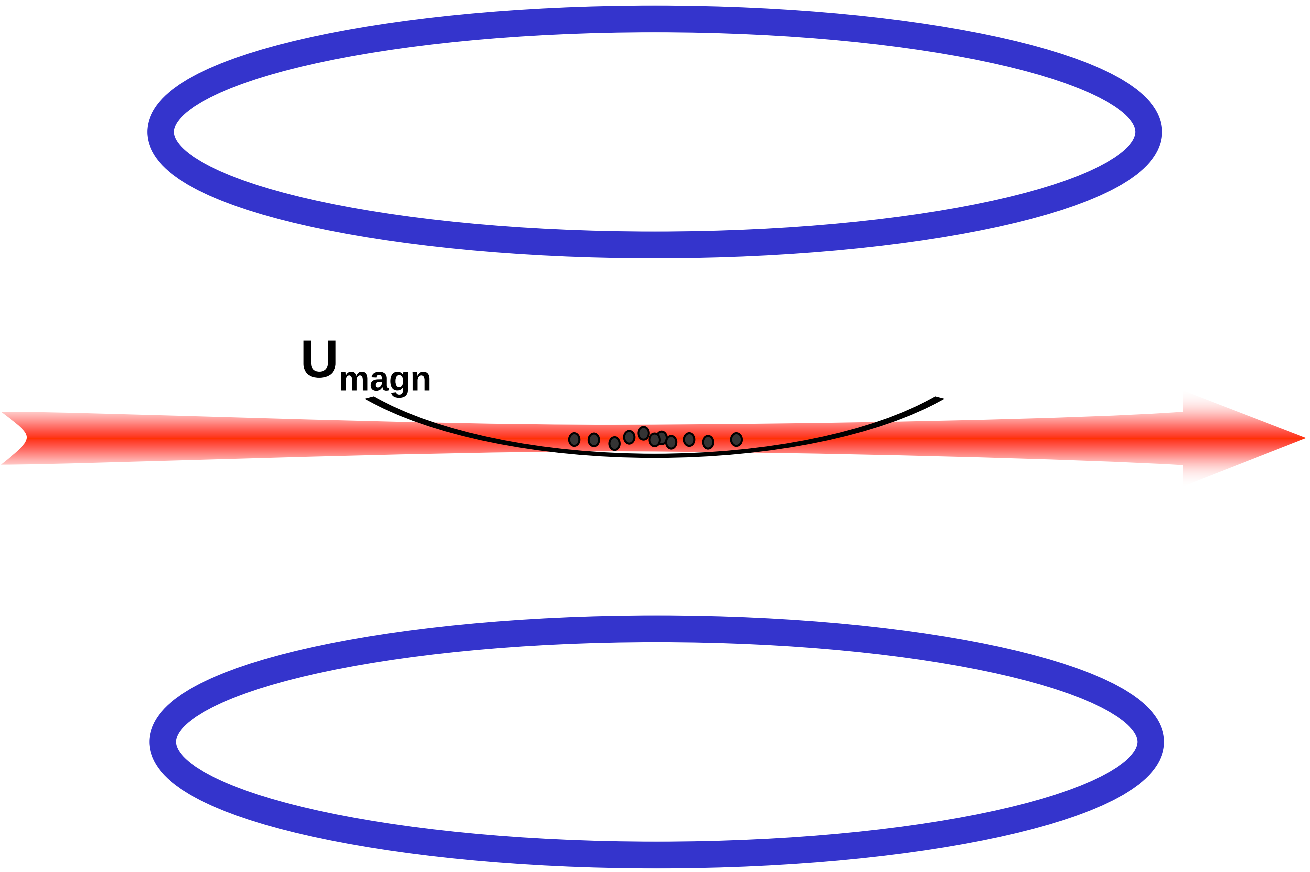
$$r_0 \ll R \approx \frac{\lambda_F}{2}$$

Particles in a pair spend most of the time outside the interaction zone, in the triplet state.

Halo dimers

Optical trap for evaporative cooling

borrowed from R. Grimm



special feature #1
precise control
of laser power
10 W → few 100μW

special feature #2: axial magnetic confinement
- spatial compression at very weak optical traps
- perfectly harmonic !!
- precisely known trap frequency for weak optical trap

ν_z =
24.5 Hz @ 1kG

What is in a trap?

- Typically about 10^5-10^6 atoms divided 50-50 among the lowest two hyperfine states

- Due to the high diluteness atoms in the same hyperfine state do not interact with one another

- Atoms in different hyperfine states experience interactions only in s-wave. <u>The strength of this interaction is fully tunable!</u>

Who does experiments?

- Jin's group at Boulder
- Grimm's group in Innsbruck
- Thomas' group at Duke
- Ketterle's group at MIT
- Salomon's group in Paris
- Hulet's group at Rice

Typical parameters

hyperfine splitting $\approx$ 80 MHz

Fermi energy $\varepsilon_{\mathrm{F}} \simeq 20 \cdots 50$ kHz

Temperature $T \simeq 0 \cdots 2\, \varepsilon_F$

trap frequencies $\begin{cases} \nu_z \simeq 25 \text{ Hz} \\ \nu_r \simeq 750 \text{ Hz} \end{cases}$

cloud has cigar shape

number density $n \simeq 10^{13}$ atoms/cm^3

typical atom separation $\simeq 5000\, a_0$

Number of atoms $N \simeq 10^5 \cdots 10^6$

radius of interaction $r_0 \simeq 100 a_0$

diluteness $n r_0^3 \ll 1$

In dilute Fermi systems only very few characteristics are relevant.

- These systems are typically very cold

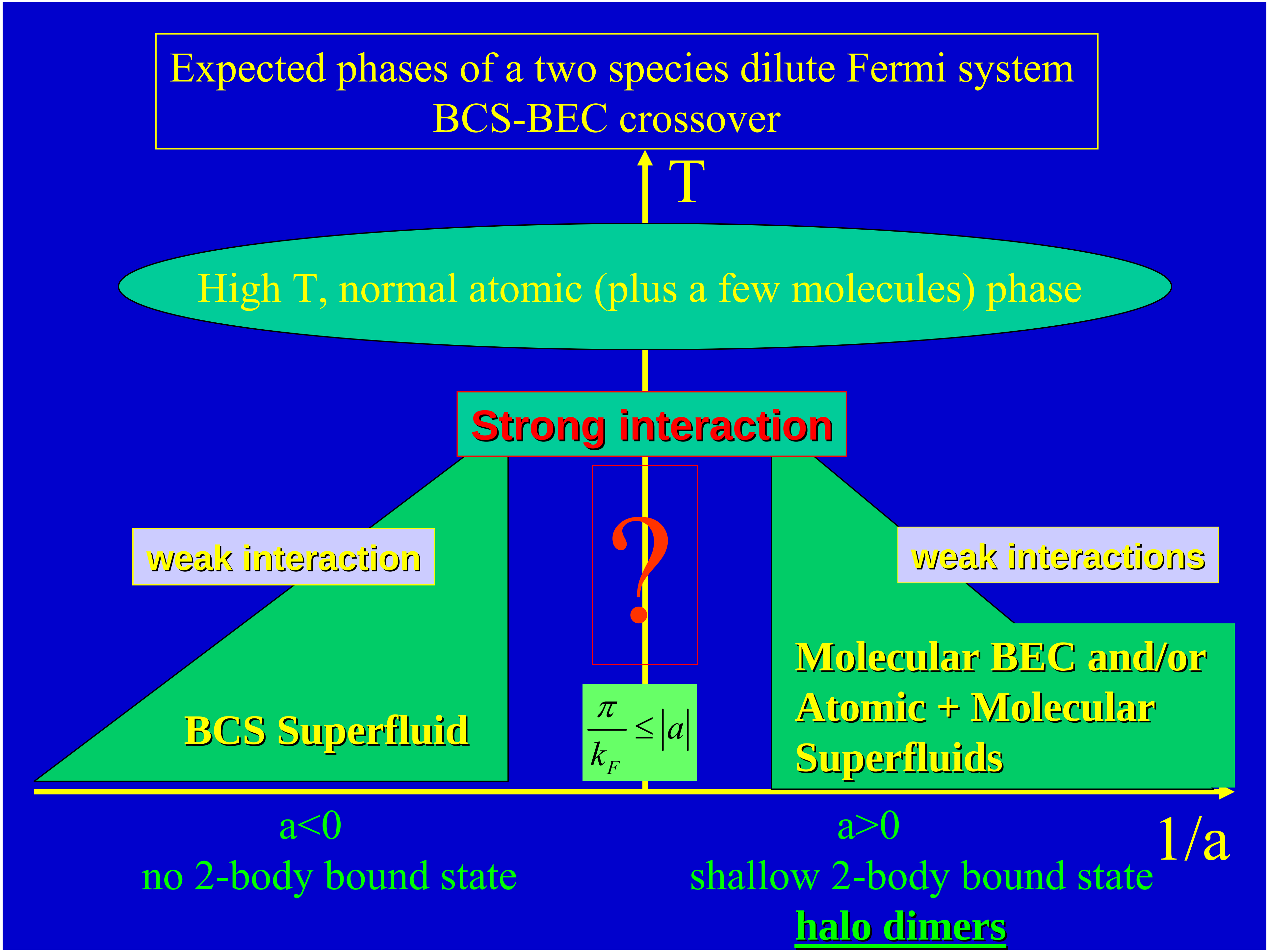
$$T \ll T_F = \frac{\varepsilon_F}{k_B}, \qquad \varepsilon_F = \frac{p_F^2}{2m}$$

- A dilute Fermi system is degenerate and the fastest particle has a momentum of the order of the Fermi momentum

$$p_F = (6\pi^2 n_\uparrow)^{1/3}\hbar = (6\pi^2 n_\downarrow)^{1/3}\hbar$$

$$n_\uparrow = n_\downarrow = \frac{n}{2}$$

- The wave functions are basically constant over the interaction volume of two particles and thus they cannot “see” any details, except the <u>scattering length</u> typically.


Expected phases of a two species dilute Fermi system
BCS-BEC crossover
T
High T, normal atomic (plus a few molecules) phase
Strong interaction
weak interaction
weak interactions
?
BCS Superfluid
$\frac{\pi}{k_F} \leq |a|$
Molecular BEC and/or
Atomic + Molecular
Superfluids
a<0
no 2-body bound state
a>0
shallow 2-body bound state
halo dimers
1/a

BCS $\rightarrow$ BEC crossover

Eagles (1969), Leggett (1980), Nozieres and Schmitt-Rink (1985), Randeria *et al.* (1993),…

If a<0 at T=0 a Fermi system is a BCS superfluid

$$\Delta \approx \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right) << \varepsilon_F, \quad \text{iff} \quad k_F |a| << 1 \text{ and } \xi = \frac{1}{k_F}\frac{\varepsilon_F}{\Delta} >> \frac{1}{k_F}$$

If $|a|=\infty$ and $nr_0^3 \ll 1$ a Fermi system is strongly coupled and its properties are universal. Carlson *et al*. PRL <u>91</u>, 050401 (2003)

$$\frac{E_{\text{normal}}}{N} \approx 0.54 \frac{3}{5}\varepsilon_F, \qquad \frac{E_{superfluid}}{N} \approx 0.44 \frac{3}{5}\varepsilon_F \quad \text{and } \xi = O(\lambda_F),\ \Delta = O(\varepsilon_F)$$

If $a>0$ ($a \gg r_0$) and $na^3 \ll 1$ the system is a dilute BEC of tightly bound dimers

$$\varepsilon_2 = -\frac{\hbar^2}{ma^2} \quad \text{and} \quad n_b a^3 << 1, \quad \text{where} \quad n_b = \frac{n_f}{2} \quad \text{and} \quad a_{bb} = 0.6a > 0$$

Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not.

- *systems of bosons are unstable (Efimov effect)*
- *systems of three or more fermion species are unstable (Efimov effect)*

- Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)

- Carlson et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided best the theoretical estimates for the ground state energy of such systems.

- Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.

Theory is now in such a state that it can make verifiable or falsifiable predictions.

Experimental signatures/predictions of the large size pairs/ halo dimer model versus fermion-boson model:

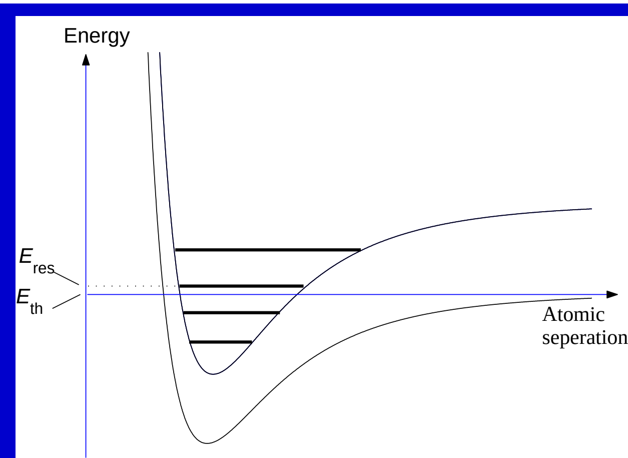
1) Near the Feshbach resonance the pair is in triplet state
in agreement with Grimm's group experiments
(fermion-boson model predicts a singlet state)

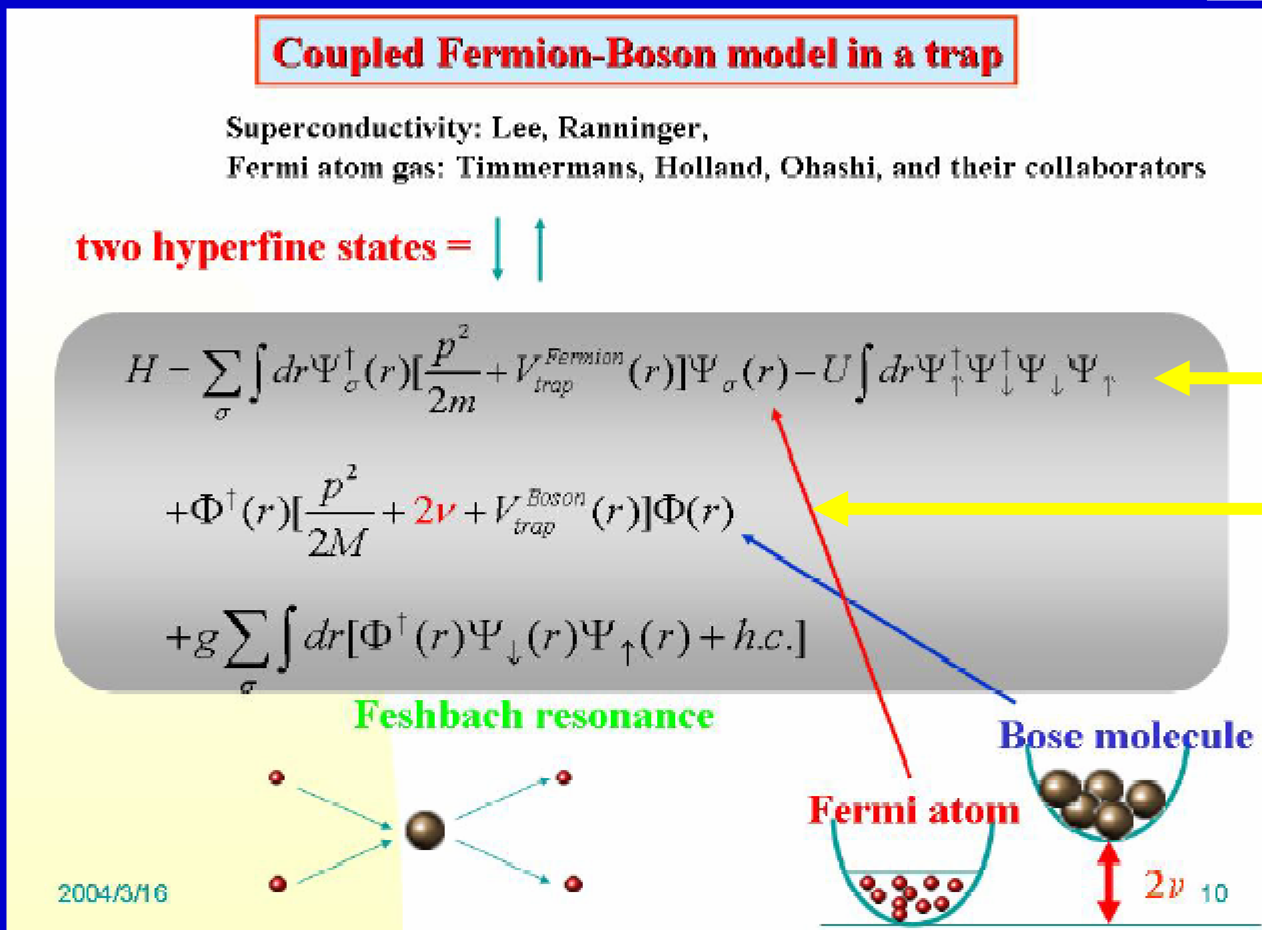
2) Particle loss is consistent with large spatial size pairs
in agreement with Grimm's and Jin's groups experiments
(fermion-boson model is consistent with small spatial size pairs)

3) Spatial size of the cloud
in agreement with Grimm's group experiment
(fermion-boson model disagrees with experiment)

4) Frequencies and frequency shift of the frequencies of the collective oscillations
(if at the Feshbach resonance the system would be made of small size pairs the frequency would be higher and the frequency shift would be much smaller than observed in experiments, see Pitaevskii and Stringari, PRL 81, 4541 (1998), Braaten and Pearson, PRL 82, 255 (1999))

$$\frac{\delta\omega_M}{\omega_M} \approx \frac{63\sqrt{\pi}}{128}\sqrt{n(0)r_0^3} \ll 1$$

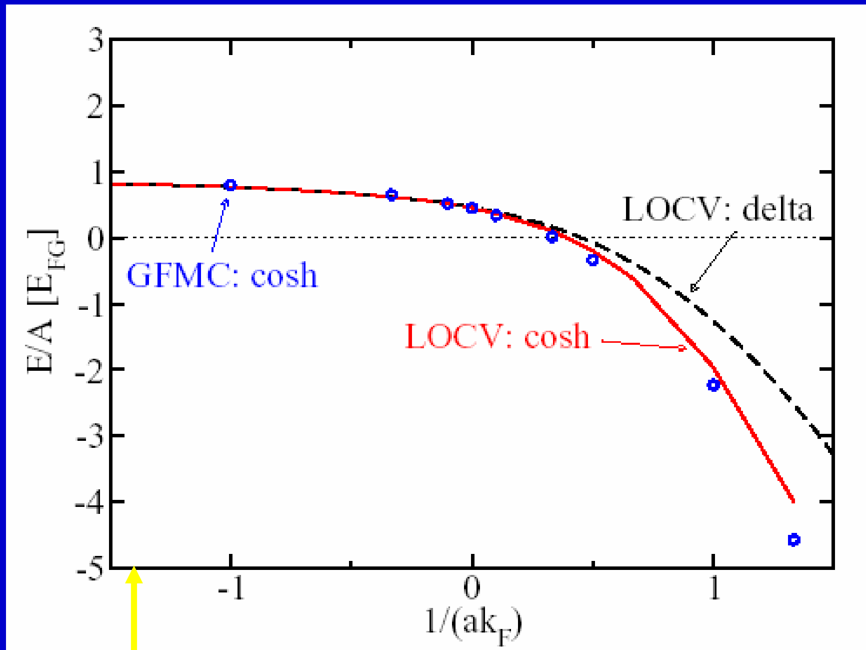


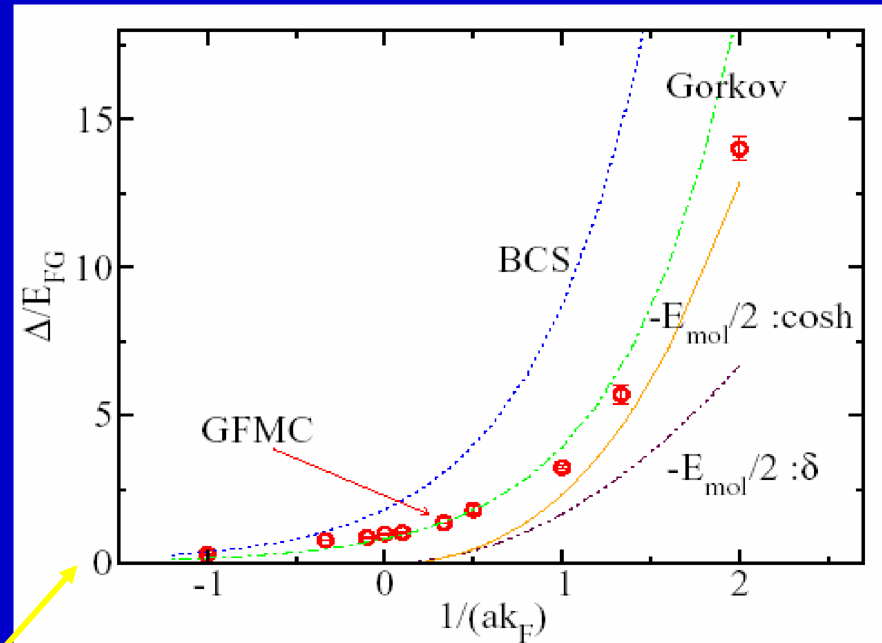


Open (triplet) channel

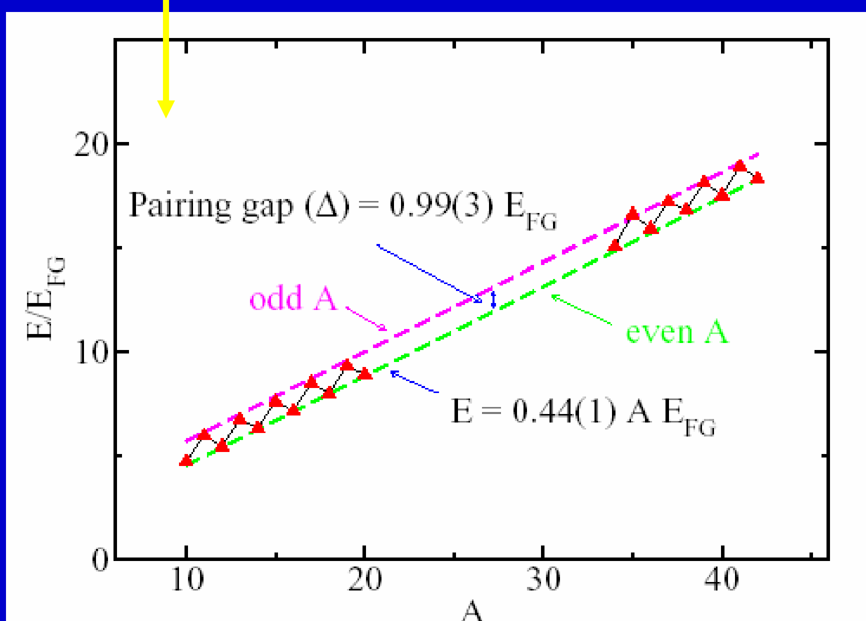
Closed (singlet) channel

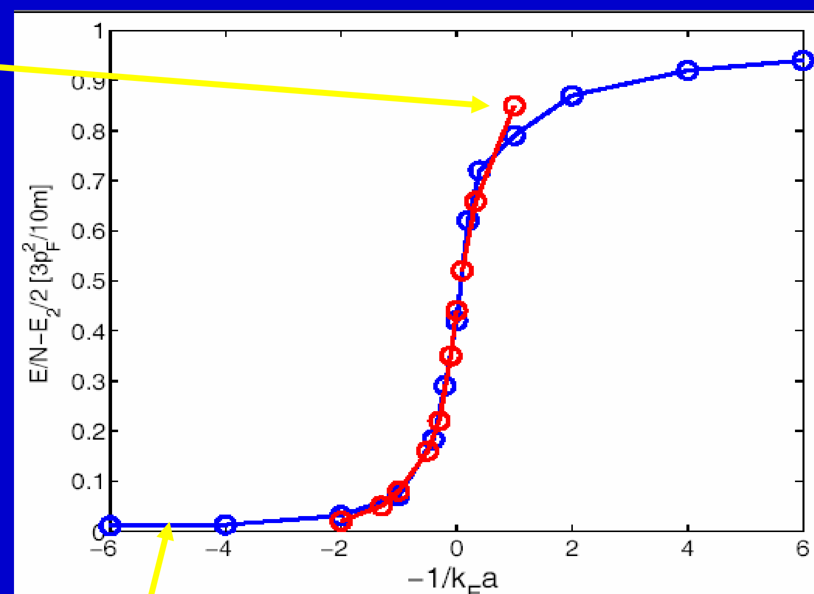
Ohashi, Levico 2004



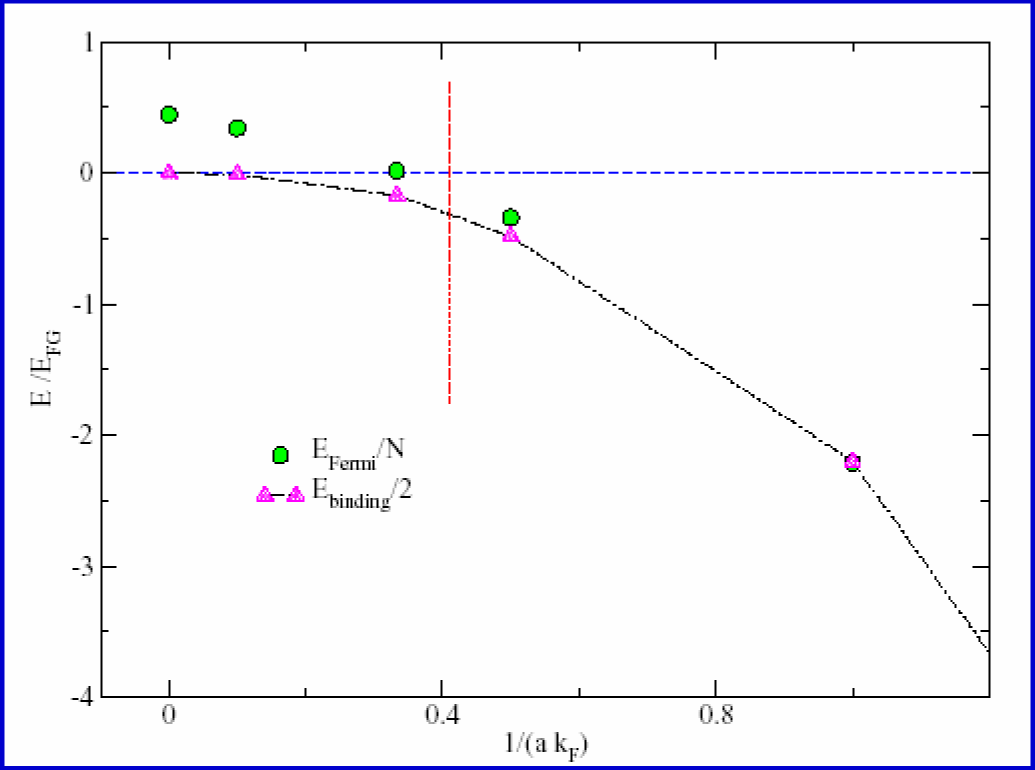


Chang *et al.* physics/0404115



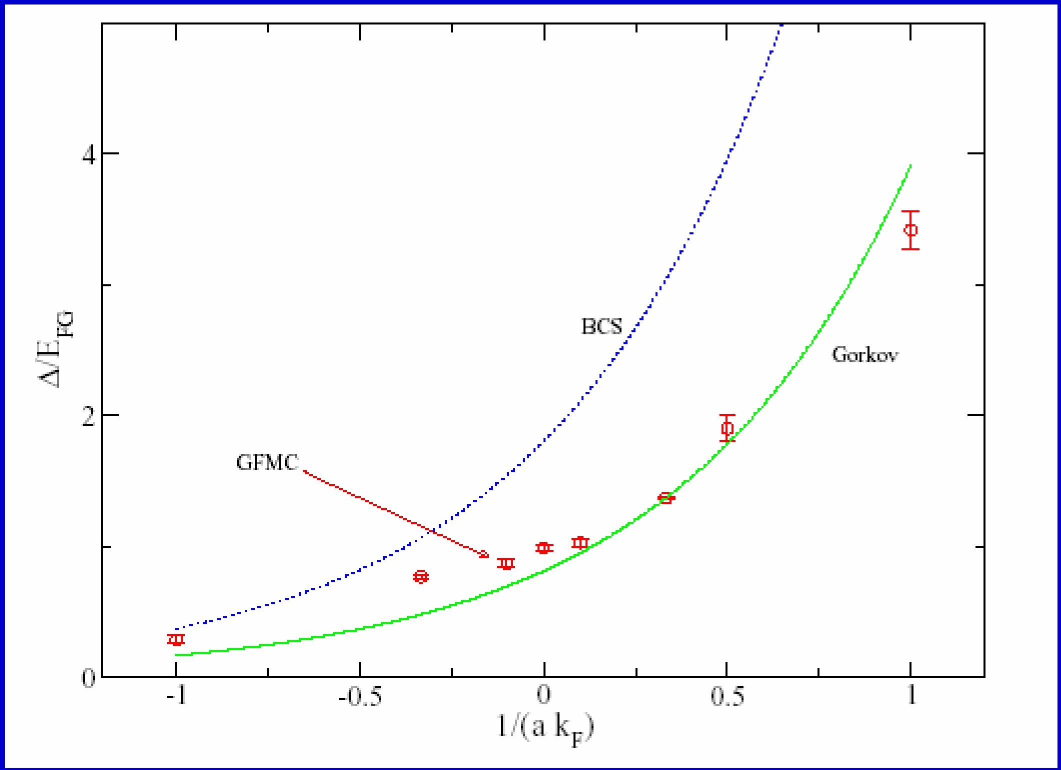


Astrakharchik et al, cond-mat/0406113



Even though two atoms can bind,
there is no binding among dimers!

Fixed node GFMC results, J. Carlson *et al.* (2003)

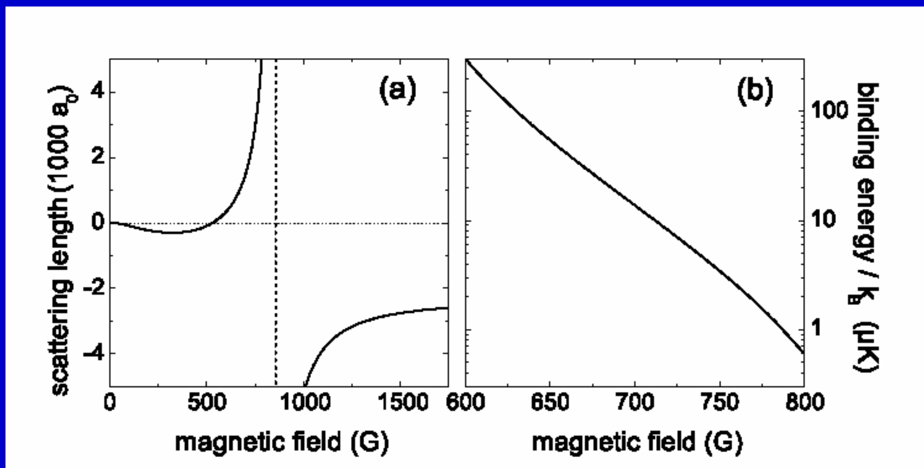


$$\Delta_{Gorkov} = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)$$

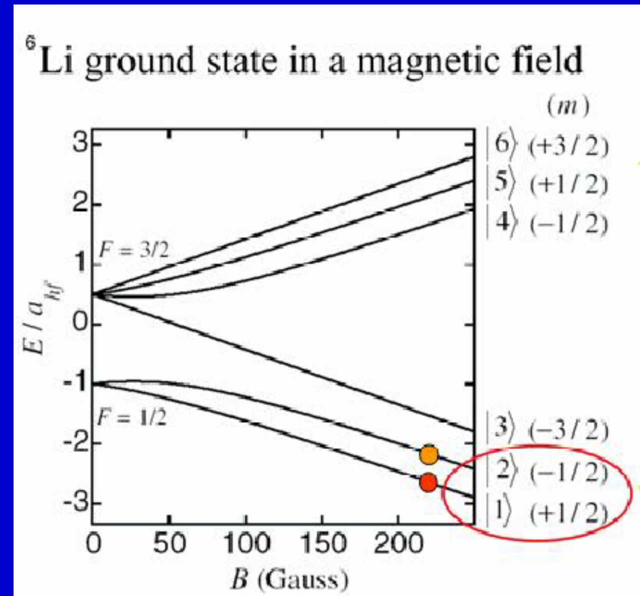
$$\Delta_{BCS} = \frac{8}{e^2} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)$$

Fixed node GFMC results, J. Carlson *et al.* (2003)

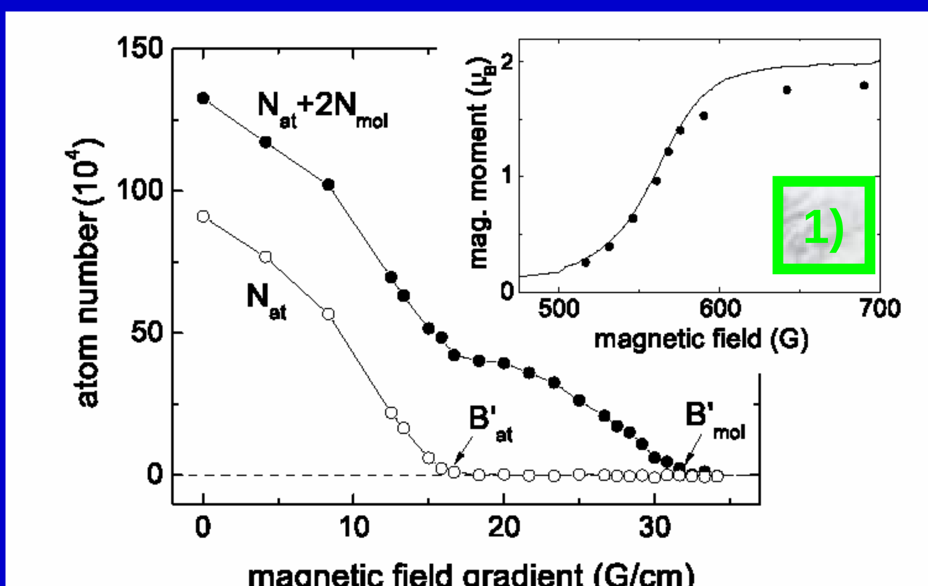
S. Jochim et al.
PRL 91, 240402 (2003)

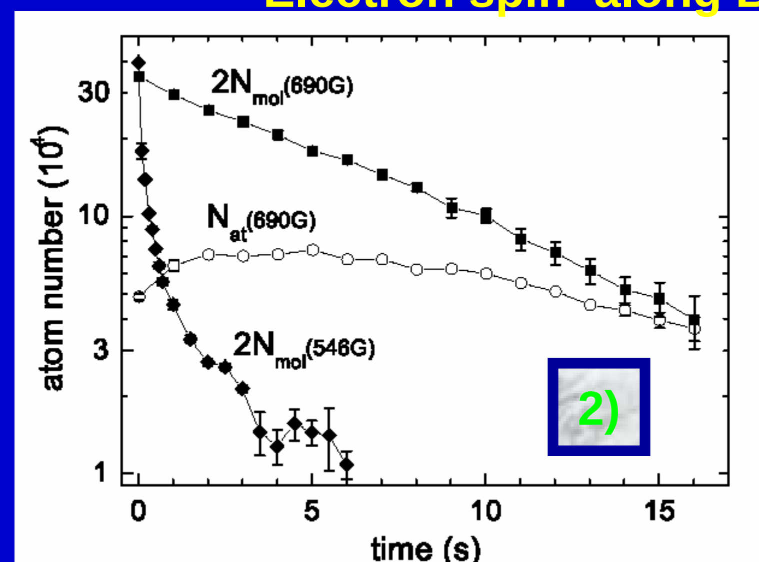


Electron spin opposite B



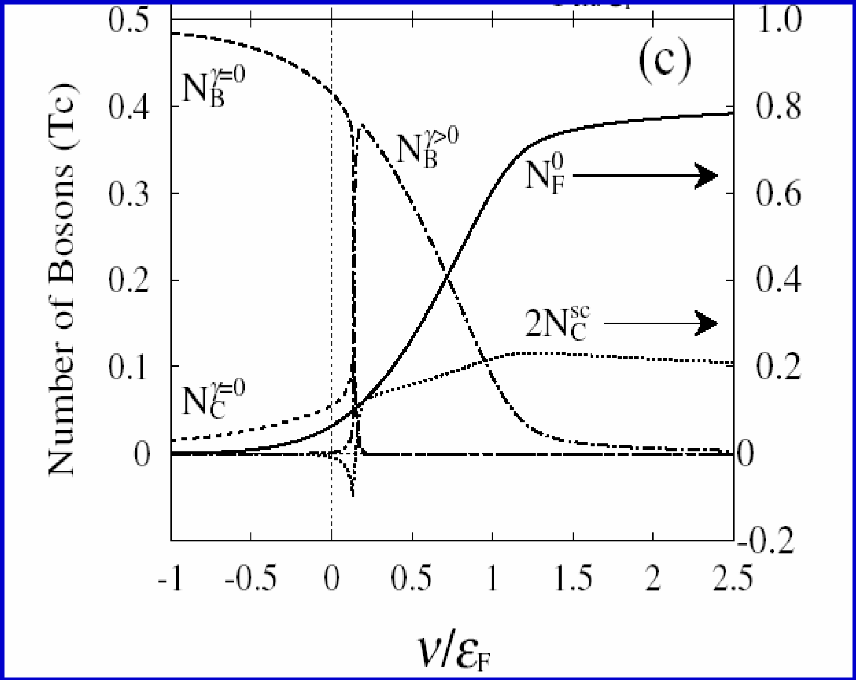
Electron spin along B



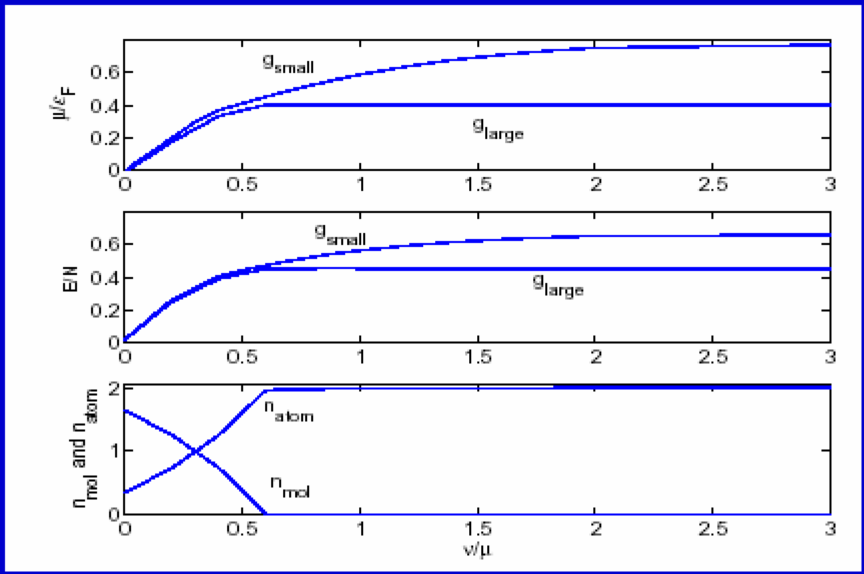


see also Petrov *et al.* cond-mat/0309010
Regal *et al.* PRL 92, 083201 (2004)

The fermion-boson model predict that at resonance an atomic Fermi cloud consists predominantly of molecules in the closed channel (singlet), which thus have an almost vanishing magnetic moment.



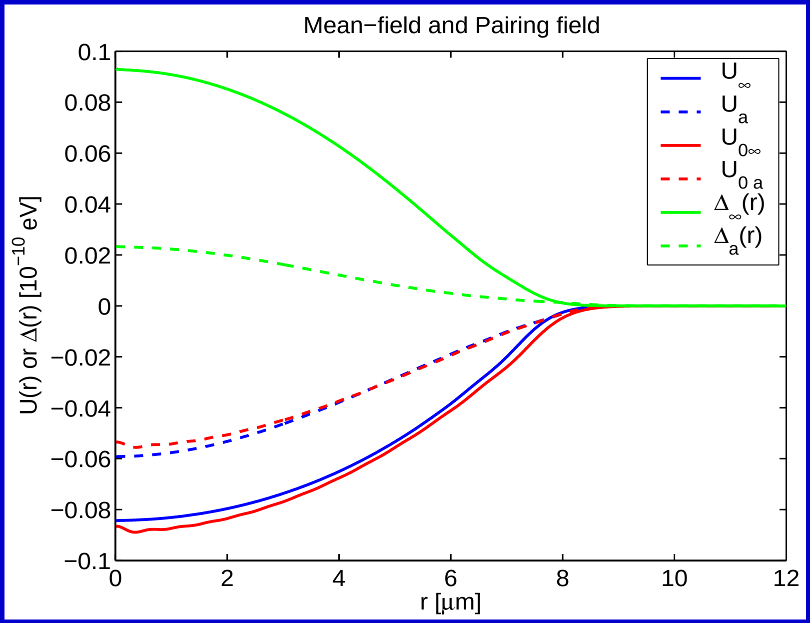
Ohashi and Griffin,
Phys. Rev. Lett. 89, 130402 (2002)

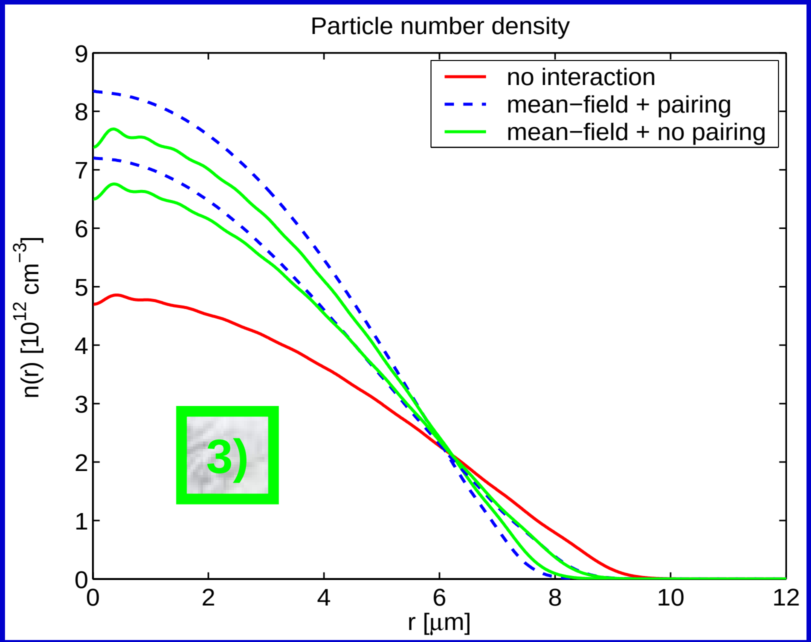


Bruun, cond-mat/0401497

40K (Fermi) atoms in a spherical harmonic trap

Effect of interaction, with and without weak and strong pairing correlations with fixed particle number, N = 5200.

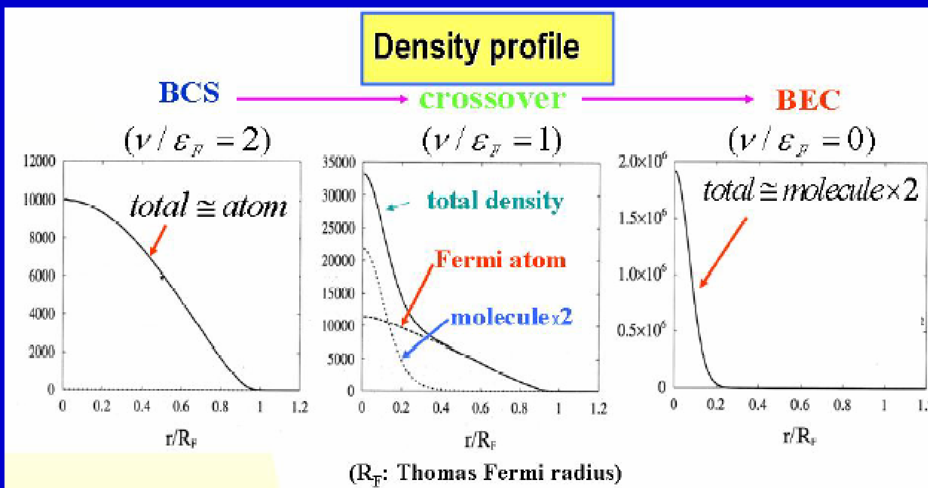




$\hbar\omega = 0.568 \times 10^{-12}$eV, a = -12.63nm (when finite)

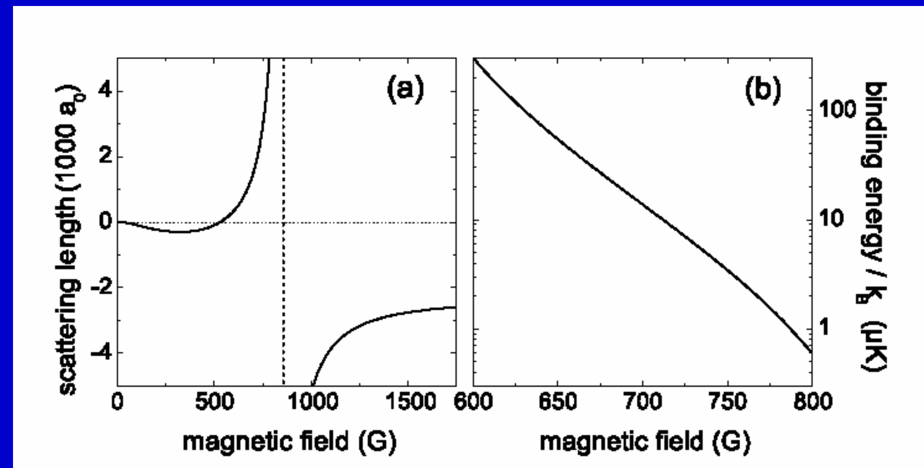
Unpublished, fully self-consistent SLDA (Kohn-Sham generalized to pairing) calculation performed by Yongle Yu in July 2003.

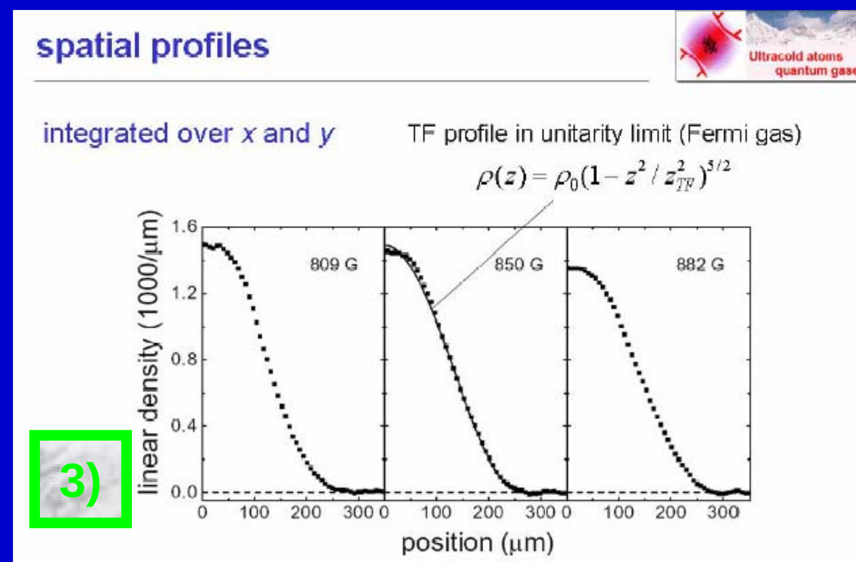
The fermion-boson model predict that at resonance the size of the cloud is significantly smaller than the observed one.



Fermi atoms form Cooper-pair bosons, which does **not** affect the density profile in the BCS regime.

Density profile shrinks in the BEC regime due to the absence of Pauli Principle between molecules associated with the F.R.



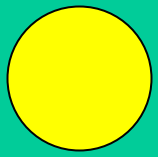
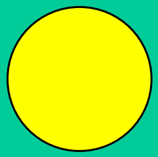
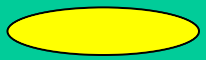


Ohashi, Levico 2004

Grimm, Levico 2004

Sound in infinite fermionic matter

$$\omega = \mathrm{v}_s k$$

	Local shape of Fermi surface	Sound velocity	
Collisional regime Compressional mode	Spherical 	$\mathrm{v}_s \approx \frac{\mathrm{v}_F}{\sqrt{3}}$	First sound
Superfluid collisionless Compressional mode	Spherical 	$\mathrm{v}_s \approx \frac{\mathrm{v}_F}{\sqrt{3}}$	Anderson-Bogoliubov sound
Normal Fermi fluid collisionless Incompressional mode	 Elongated along propagation direction	$\mathrm{v}_\mathrm{s} = s\mathrm{v}_\mathrm{F}$ $s > 1$	Landau's zero sound

$$\varepsilon(n) = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}\left[\xi - \frac{\varsigma}{k_F a} - \frac{5\iota}{3(k_F a)^2} + O\left(\frac{1}{(k_F a)^3}\right)\right]$$

$$\xi \approx 0.44, \qquad \varsigma \approx 1, \qquad \iota \approx 1$$

$$U = \frac{m\omega_0^2\left(x^2 + y^2 + \lambda^2 z^2\right)}{2}$$

$$\frac{\delta\omega^2}{\omega^2} = \frac{\varsigma}{\xi}\frac{1}{k_F(0)a}K$$

Adiabatic regime
Spherical Fermi surface

Perturbation theory result using GFMC equation of state in a trap

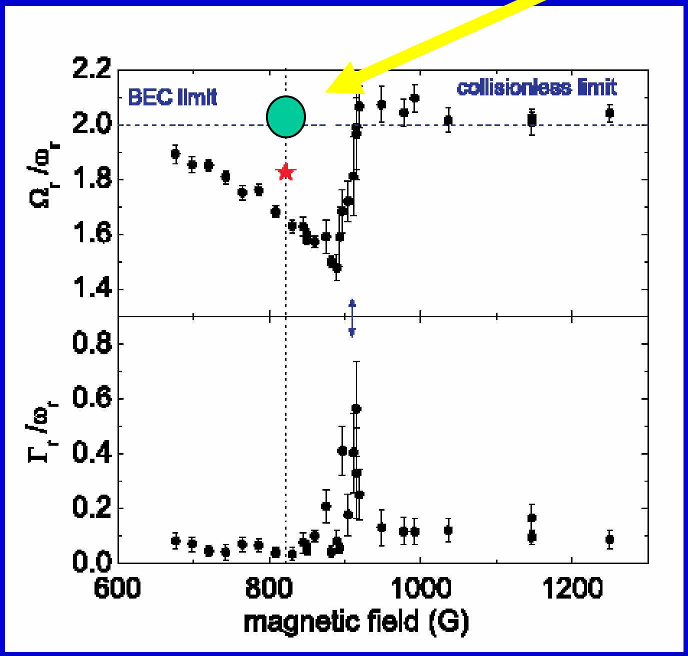
TABLE II: Results for K.

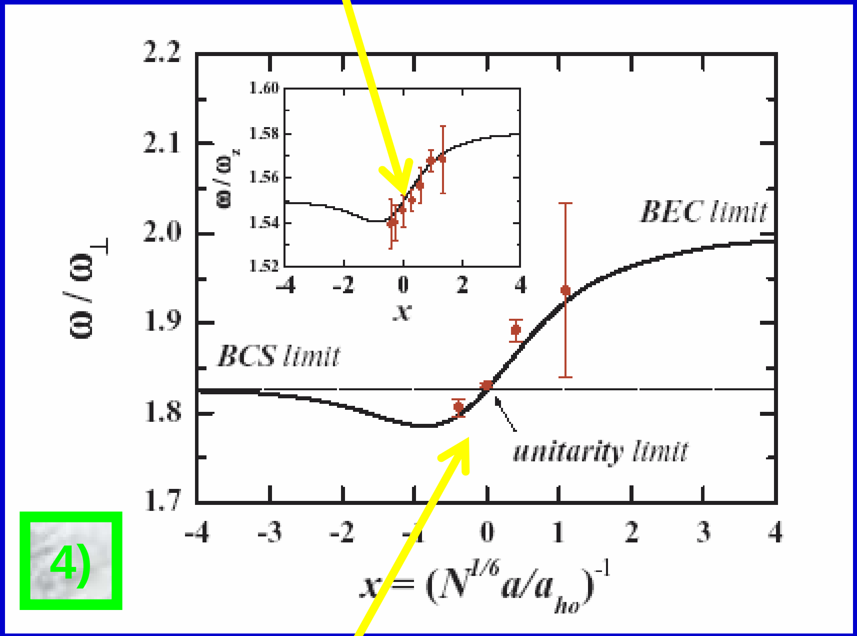
trap type	mode	f_1	ω	K
spherical	dipole	z	ω_0	0
$\lambda = 1$	monopole	$1 - 2r^2$	$2\omega_0$	$\frac{256}{525\pi}$
	quadrupole	xy	$\sqrt{2}\omega_0$	0
axial	$M = \pm 2$	$xy,\ x^2 - y^2$	$\sqrt{2}\omega_0$	0
$\lambda \ll 1$	$M = \pm 1$	$xz,\ yz$	ω_0	0
	radial	$x^2 + y^2 + \frac{2}{5}\lambda^2 z^2 - \frac{2}{5}$	$\sqrt{\frac{10}{3}}\omega_0$	$\frac{1024}{2625\pi}$
	axial	$1 - 6\lambda^2 z^2$	$\sqrt{\frac{12}{5}}\lambda\omega_0$	$\frac{256}{2625\pi}$

Frequency shifts in these modes might carry information about possible atom-halo dimer mixture

Fermion-boson model would predict here

Grimm's group experiment





Thomas' group experiment

Hu *et al.* cond-mat/0404012, a semi-quantitative analysis (gap and chemical potential inaccurate) assuming a polytropic equation of state

For a more careful analysis, using GFMC equation of state in a trap see Bulgac and Bertsch, cond-mat/0404687

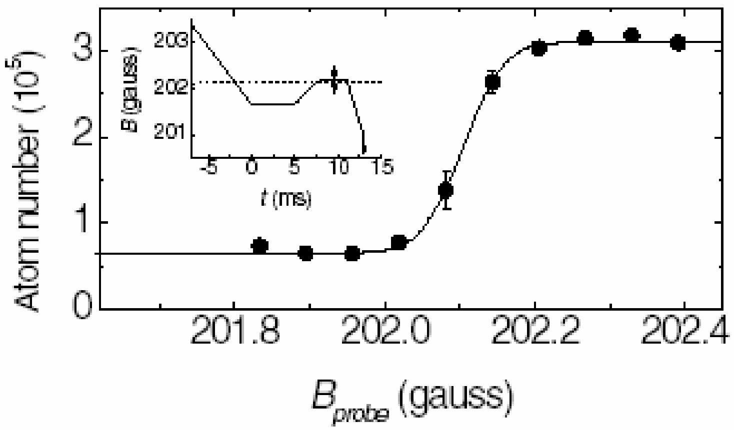


FIG. 1. Measurement of the Feshbach resonance position B_0. Shown in the inset is a schematic of the magnetic field as a function of time t measured with respect to the optical trap turn off at $t = 0$. Molecules are first created by a slow magnetic-field sweep across the resonance (dotted line) and then dissociated if B_{probe} (indicated by the arrow in the inset) is beyond the magnetic field where the two-body physics supports a new bound state. The number of atoms, measured at $t = 17$ ms, is shown as a function of B_{probe}. The two error bars indicate the spread in repeated points at these values of B. A fit of the data to an error function reveals $B_0 = 202.10 \pm 0.07$ G, where the uncertainty is given conservatively by the 10%–90% width.

$$a(B) = a_{bg}\left(1 - \frac{w}{B - B_0}\right)$$

$$a_{bg} = 174 a_0$$

$$w = 7.8 \pm 0.6 \text{ G}$$

$$B_0 = 202.10 \pm 0.07 \text{ G}$$

a>0 a<0

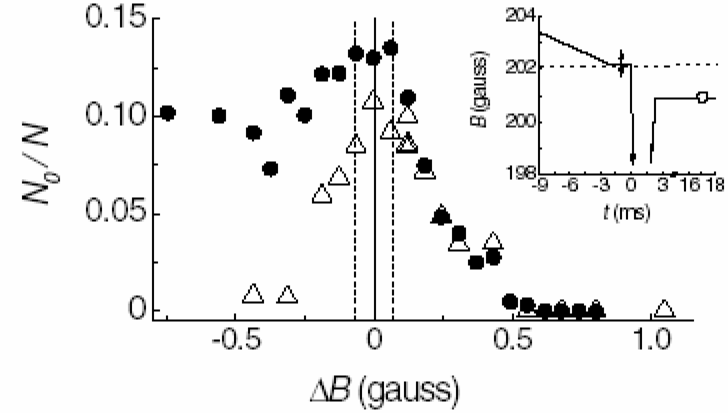
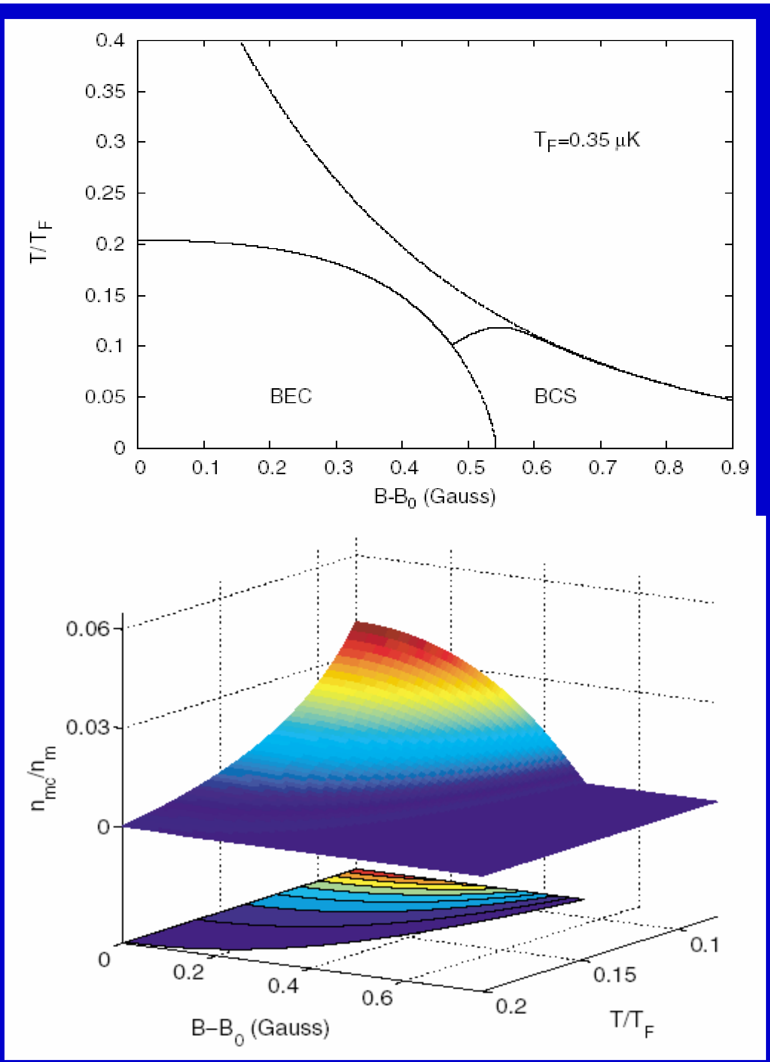
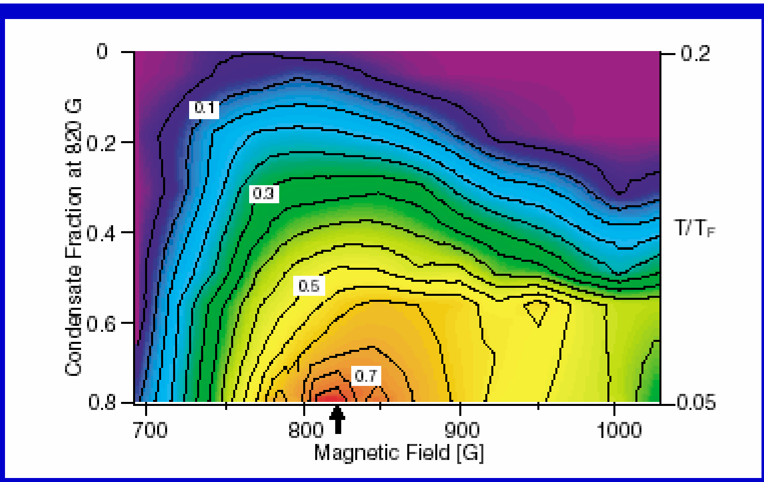


FIG. 2. Measured condensate fraction as a function of detuning from the Feshbach resonance $\Delta B = B_{\mathrm{hold}} - B_0$. Data here were taken for $t_{\mathrm{hold}} = 2$ ms (●) and $t_{\mathrm{hold}} = 30$ ms (△) with an initial cloud at $T/T_F = 0.08$ and $T_F = 0.35\ \mu$K. The area between the dashed lines around $\Delta B = 0$ reflects the uncertainty in the Feshbach resonance position based on the 10%–90% width of the feature in Fig. 1. Condensation of fermionic atom pairs is seen near and on either side of the Feshbach resonance. Comparison of the data taken with the different hold times indicates that the pair condensed state has a significantly longer lifetime near the Feshbach resonance and on the BCS ($\Delta B > 0$) side. The inset shows a schematic of a typical magnetic-field sweep used to measure the fermionic condensate fraction. The system is first prepared by a slow magnetic-field sweep towards the resonance (dotted line) to a variable position B_{hold}, indicated by the two-sided arrow. After a time t_{hold} the optical trap is turned off and the magnetic field is quickly lowered by ~10 G to project the atom gas onto a molecular gas. After free expansion, the molecules are imaged on the BEC side of the resonance (○).

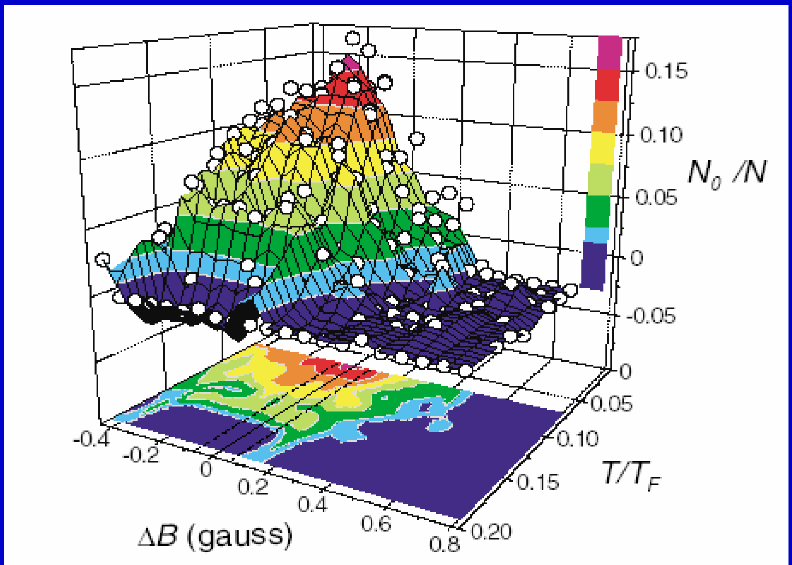


Falco and Stoof,
Phys. Rev. Lett. <u>92</u>, 130401 (2004)

$$n_m \simeq \frac{n}{2}\left[1-\frac{1}{(k_F|a|)^3}\right]$$



Zwierlein *et al.*
Phys. Rev. Lett. <u>92</u>, 120403 (2004)



Regal, Greiner and Jin,
Phys. Rev. Lett. <u>92</u>, 040403 (2004)

<u>Theory declared full victory here!</u>

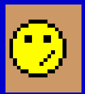


TABLE I: Character of the condensate as a function of the inverse scattering length a^{-1} in various in intervals, the approximate boundaries of these intervals being shown in the second row. The total electron spin and spin projection (S, S_Z) along the magnetic field for various pairs are shown in the last row.

$a^{-1} > 0$			$a^{-1} < 0$	
$+\infty$	r_0^{-1}	k_F 0	0 k_F	$-\infty$
molecules	halo dimers (+ atoms ?[15])	?	BCS strong coupling	BCS weak coupling
(0,0)	(1,-1)	(1,-1)	(1,-1)	(1,-1)

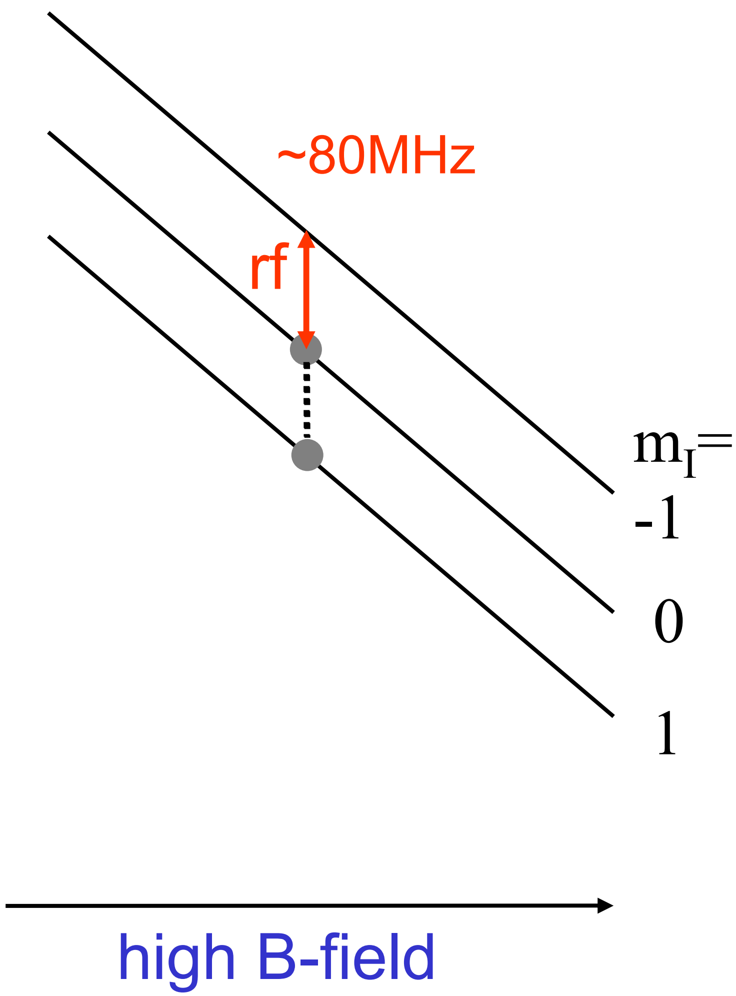
✓ **Fermion superfluidity, more specificaly <u>superflow,</u> has not yet been demonstrated unambiguously experimentally. There is lots of circumstantial evidence and facts in agreement with theoretical models assuming its existence.**

✓ **Theory is able to make very precise predictions in this regime and the agreement with experiment can be check quantitatively.**

radio-frequency spectroscopy

meas. of mol. bind. energy in ^{40}K
Regal *et al.*, Nature **424**, 47 (2003)

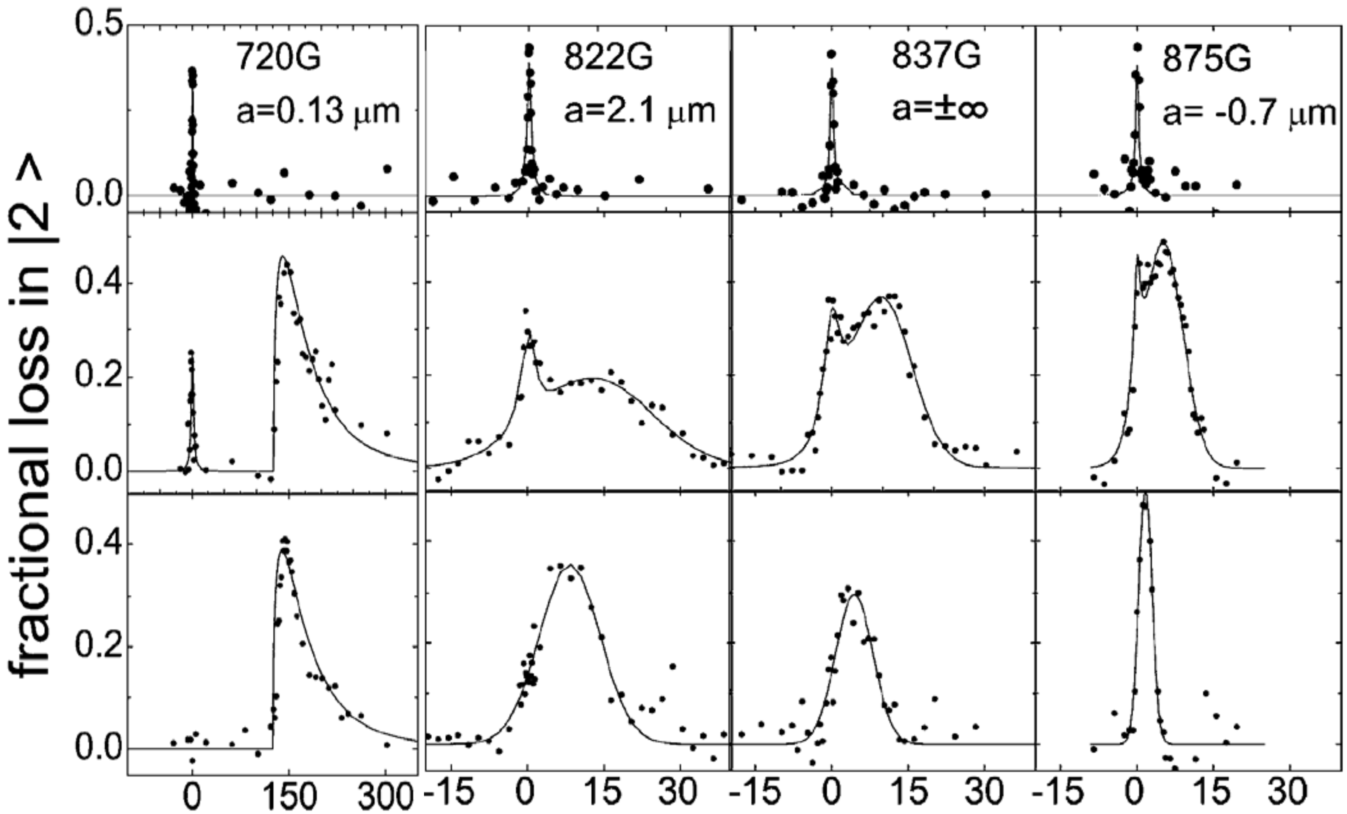
rf spectroscopy of ^{6}Li:
Gupta *et al.*, Science **300**, 1723 (2003)



breaking ***pairs***
costs energy
→
pair signal
up-shifted

borrowed from R. Grimm

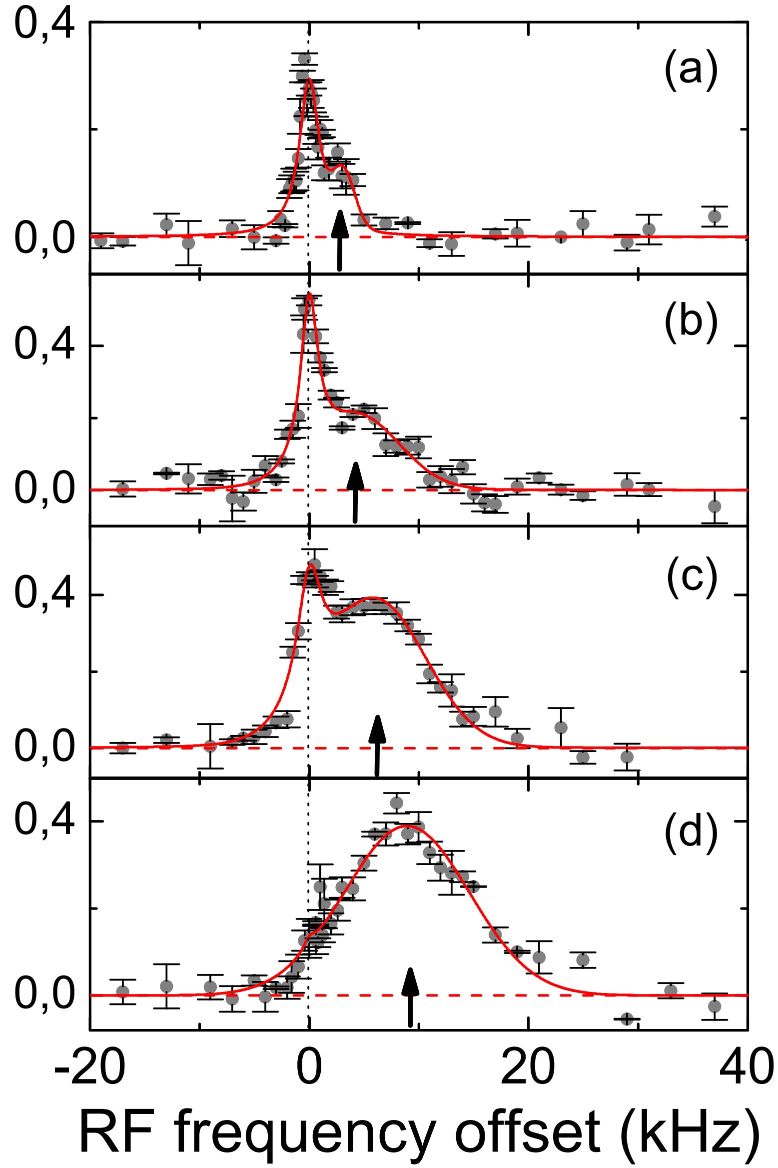
rf spectra in crossover regime



rf offset

borrowed from R. Grimm

temperature dependence of pairing



$\approx 0.5\ T_F$

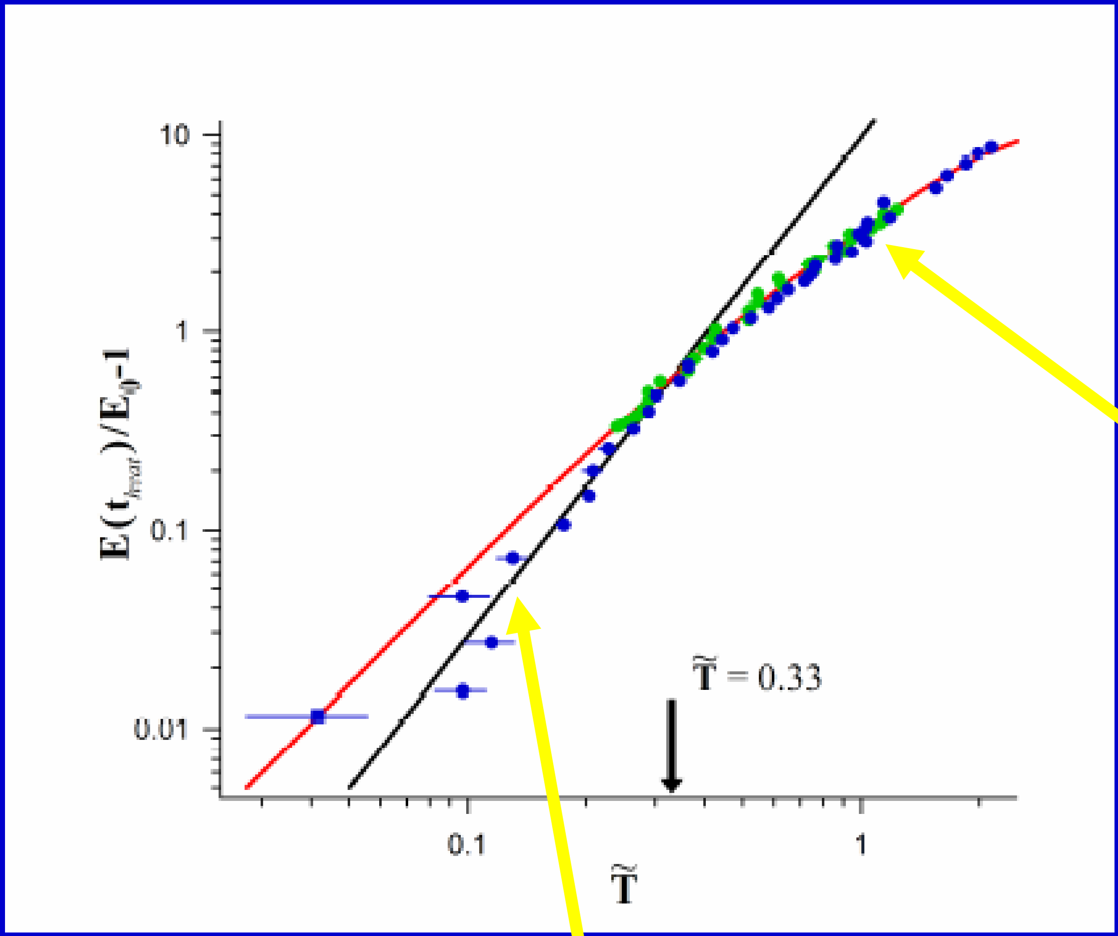
$\approx 0.4\ T_F$

$\approx 0.2\ T_F$

$T < 0.1\ T_F$

borrowed from R. Grimm

J. Thomas' group at Duke



Consistent with $E \approx bT^2$

Consistent with $E \approx aT^{5/2}$

Superfluid LDA (SLDA)

number and kinetic densities

$$n(\vec{r}) = 2 \sum_{0<E_k<E_c} \left|\mathrm{v}_k(\vec{r})\right|^2, \qquad \tau(\vec{r}) = 2 \sum_{0<E_k<E_c} \left|\vec{\nabla}\mathrm{v}_k(\vec{r})\right|^2$$

anomalous density

$$\nu(\vec{r}) = \sum_{0<E_k<E_c} \mathrm{v}_k^*(\vec{r})\mathrm{u}_k(\vec{r})$$

Divergent!

$$\mathrm{E} = \int d^3r \left\{ \frac{\hbar^2}{2m}\tau(\vec{r}) + \frac{\hbar^2}{m}\beta\left[x(\vec{r})\right]n(\vec{r})^{5/3} - \Delta\left[x(\vec{r})\right]\nu^*(\vec{r}) \right\}$$

Cutoff and position running coupling constant!

$$\Delta\left[x(\vec{r})\right] = -\frac{\hbar^2}{m}\frac{\gamma_{eff}\left[x(\vec{r})\right]}{n(\vec{r})^{1/3}}\nu(\vec{r}), \qquad x(\vec{r}) = \frac{1}{k_F(\vec{r})a}$$

$$\begin{pmatrix} T+U(\vec{r})-\mu & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -(T+U(\vec{r})-\mu) \end{pmatrix}\begin{pmatrix} \mathrm{u}_k(\vec{r}) \\ \mathrm{v}_k(\vec{r}) \end{pmatrix} = E_i \begin{pmatrix} \mathrm{u}_k(\vec{r}) \\ \mathrm{v}_k(\vec{r}) \end{pmatrix}$$

Bogoliubov-de Gennes like equations.
Correlations are however included by default!

Vortex in fermion matter

$$\begin{pmatrix} \mathrm{u}_{\alpha\,\mathrm{kn}}(\vec{r}) \\ \mathrm{v}_{\alpha\,\mathrm{kn}}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \mathrm{u}_{\alpha}(\rho)\exp[i(n+1/2)\phi - ikz] \\ \mathrm{v}_{\alpha}(\rho)\exp[i(n-1/2)\phi - ikz] \end{pmatrix},\quad \text{n - half-integer}$$

$$\Delta(\vec{r}) = \Delta(\rho)\exp(i\phi), \qquad \vec{r} = (\rho, \phi, z) \ \text{[cyllindrical coordinates]}$$

Oz - vortex symmetry axis

Ideal vortex, Onsager's quantization (one $\hbar$ per Cooper pair)

$$\vec{\mathrm{V}}_{\mathrm{v}}(\vec{r}) = \frac{\hbar}{2m\rho^2}(y, -x, 0) \quad \Leftarrow \quad \frac{1}{2\pi}\oint_C \vec{\mathrm{V}}_{\mathrm{v}}(\vec{r})\cdot d\vec{r} = \frac{\hbar}{2m}$$

How can one put in evidence a vortex in a Fermi superfluid?

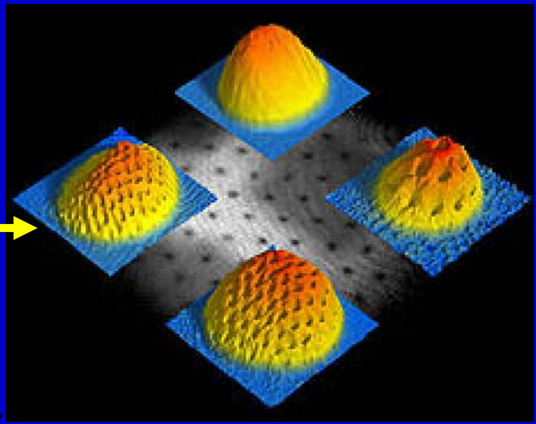
Hard to see, since density changes are not expected, unlike the case of a Bose superfluid.

However, if the gap is not small, one can expect a noticeable density depletion along the vortex core, and the bigger the gap the bigger the depletion, due to an <u>extremely fast vortical motion</u>.

$$\frac{v_s}{v_F} < \frac{\Delta}{2\varepsilon_F} \propto \frac{T_c}{T_F}$$

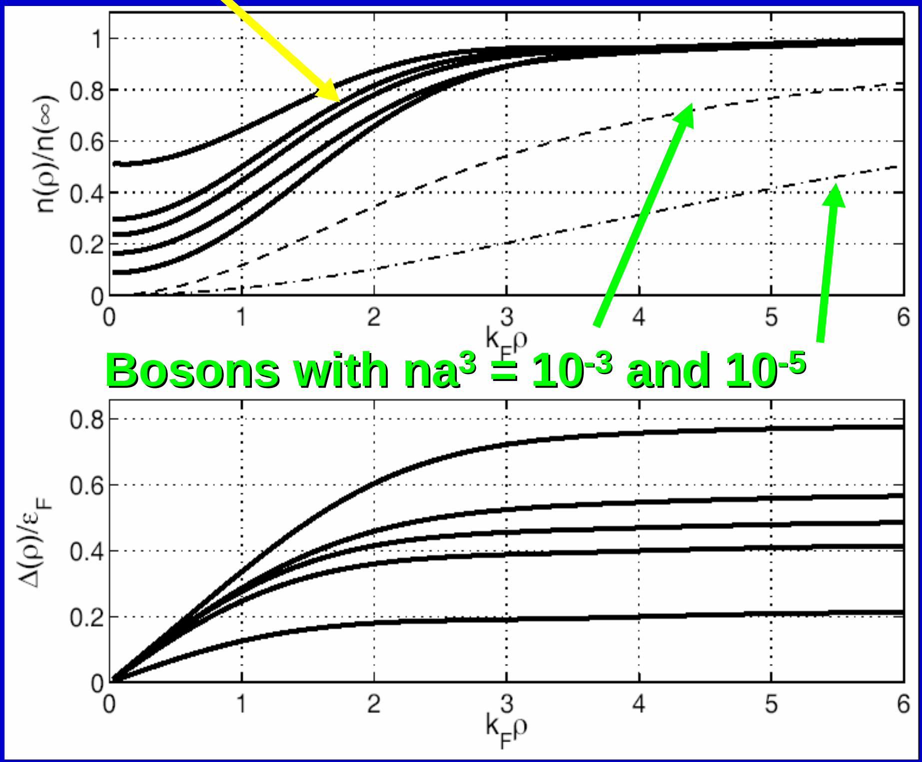
NB T_c unknown in the strong coupling limit!

The depletion along the vortex core is reminiscent of the corresponding density depletion in the case of a vortex in a Bose superfluid, when the density vanishes exactly along the axis for 100% BEC.

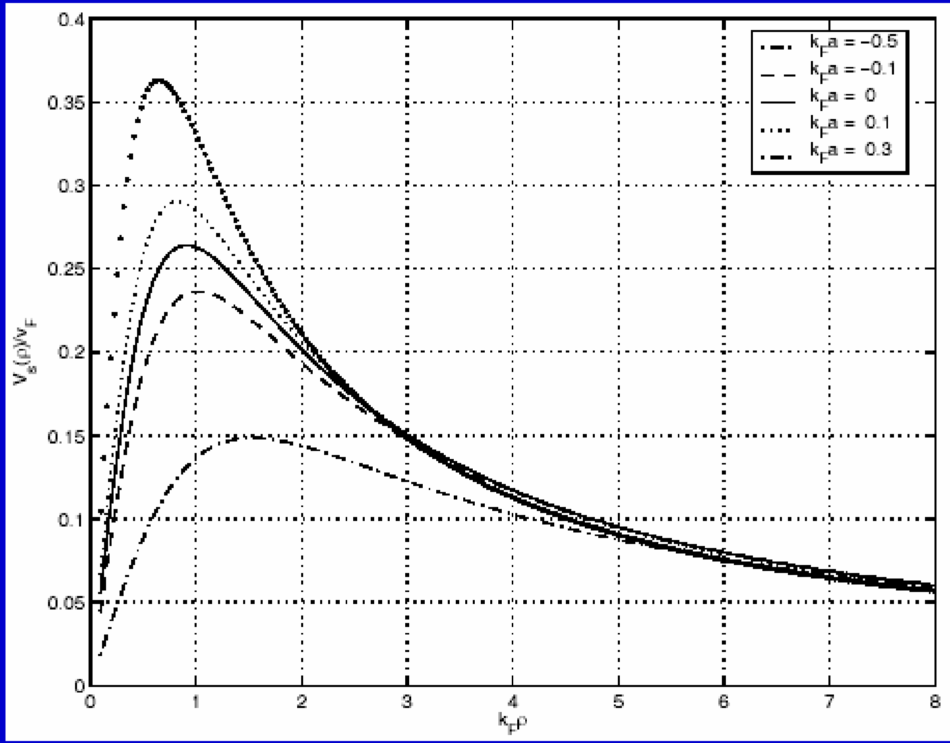


From Ketterle's group

Fermions with $1/k_F a$ = 0.3, 0.1, 0, -0.1, -0.5



Number density and pairing field profiles



Extremely fast quantum vortical motion!

Local vortical speed as fraction of Fermi speed

Phases of a two species dilute Fermi system BCS-BEC crossover

T

"Before"- Life	**LIFE!**	After – Life
Atoms form rather unstable couples, spread over large distances, with many other couples occupying the same space	Atoms form strong couples, living as in an apartment building, in close proximity of one another, and obviously anoying each other quite a bit	Atoms form very strong couples, mostly inert and at rest, ***halo dimers***, widely separated from one another and weakly interacting
weak interaction	**Strong interaction**	**weak interactions**

1/a

a<0
no 2-body bound state

a>0
shallow 2-body bound state
halo dimers

Conclusions:

✓ The field of dilute atomic systems is going to be for many years to come one of the most exciting fields in physics, with lots surprises at every corner.