What have we learned so far about dilute Fermi gases?

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These slides will be posted shortly at
http://www.phys.washington.edu/~bulgac/
What is the *Holy Grail* of this field?

Fermionic superfluidity!
Superconductivity and superfluidity in Fermi systems

- Dilute atomic Fermi gases
  \[ T_c \approx 10^{-12} - 10^{-9} \text{ eV} \]

- Liquid $^3$He
  \[ T_c \approx 10^{-7} \text{ eV} \]

- Metals, composite materials
  \[ T_c \approx 10^{-3} - 10^{-2} \text{ eV} \]

- Nuclei, neutron stars
  \[ T_c \approx 10^5 - 10^6 \text{ eV} \]

- QCD color superconductivity
  \[ T_c \approx 10^7 - 10^8 \text{ eV} \]

*units (1 eV $\approx 10^4$ K)*
Memorable years in the history of superfluidity and superconductivity of Fermi systems

- 1913  Kamerlingh Onnes
- 1972  Bardeen, Cooper and Schrieffer
- 1973  Esaki, Giaever and Josephson
- 1987  Bednorz and Muller
- 1996  Lee, Osheroff and Richardson
- 2003  Abrikosov, Ginzburg and Leggett
One of my favorite times in the academic year occurs in early spring when I give my class of extremely bright graduate students, who have mastered quantum mechanics but are otherwise unsuspecting and innocent, a take-home exam in which they are asked to deduce superfluidity from first principles. There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is impossible. Superfluidity, like the fractional quantum Hall effect, is an emergent phenomenon—a low-energy collective effect of huge numbers of particles that cannot be deduced from the microscopic equations of motion in a rigorous way and that disappears completely when the system is taken apart. There are prototypes for superfluids, of course, and students who memorize them have taken the first step down the long road to understanding the phenomenon, but these are all approximate and in the end not deductive at all, but fits to experiment. The students feel betrayed and hurt by this experience because they have been trained to think in reductionist terms and thus to believe that everything not amenable to such thinking is unimportant. But nature is much more heartless than I am, and those students who stay in physics long enough to seriously confront the experimental record eventually come to understand that the reductionist idea is wrong a great deal of the time, and perhaps always.
Topics to be covered (not necessarily in this order):

- A single atom in magnetic field
- Two atoms in magnetic field
- Atomic traps
- Basic parameters of fermionic dilute atomic clouds
- A review of a number of key experimental results
- What theory tells us so far?
HISTORY

✓ 1995 BEC was observed.
✓ 2000 vortices in BEC were created thus BEC confirmed un-ambiguously.

✓ In 1999 DeMarco and Jin created a degenerate atomic Fermi gas.
✓ 2002 O’Hara, Hammer, Gehm, Granada and Thomas observed expansion of a Fermi cloud compatible with the existence of a superfluid fermionic phase.
✓ 2003 Jin’s, Grimm’s, Ketterle’s groups and others ultracold molecules, mBEC from Fermi gas
✓ 2004 Jin’s group (and bit later Ketterle’s group too) announces the observation of the resonance condensation of fermionic atomic pairs?
✓ Grimm’s group reports measurements of the gap
✓ Thomas’ group reports measurements of the specific heat
One fermionic atom in magnetic field

\[ \text{Li ground state in a magnetic field} \]
Feshbach resonance

\[ H = \frac{\vec{p}^2}{2\mu_r} + \sum_{i=1}^{2} (V_i^{hf} + V_i^Z) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \mathcal{K}^d \]

\[ V^{hf} = \frac{a_{hf}}{\hbar^2} \vec{S}^e \cdot \vec{S}^n, \quad V^Z = (\gamma_e S^e_z - \gamma_n S^n_z)B \]

Tiesinga, Verhaar, Stoof

Regal and Jin

Atomic separation
Energy
Channel coupling
Atomic separation
Köhler, Gasenzer, Jullienne and Burnett
PRL 91, 230401 (2003), inspired by Braaten, Hammer and Kusunoki
cond-mat/0301489

The size of the “Feshbach molecule” (closed channel state) is largely B-independent and smaller than the interparticle separation.

\[ \frac{P(r > r_0)}{P(r < r_0)} \propto \frac{a}{r_0} \gg 1 \]
\[
\begin{bmatrix}
H_{11} & V_{12} \\
V_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= k^2
\begin{bmatrix}
u \\
v
\end{bmatrix}
\]  

Closed channel

Open channel

\[
H_{11} u = k^2 u
\]

\[
H_{22} \phi_0 = \kappa_0^2 \phi_0
\]

\[
\phi_0(r) = \sqrt{\frac{2}{r_0}} \sin \left( \frac{\pi r}{r_0} \right), \quad u_0(r) = \sin(kr), \quad v_0(r) \propto \phi_0(r)
\]

\[
V_{12}(r) = g \delta(r - r_1), \quad r_1 = \frac{r_0}{2}
\]

\[
u(r) = \sin(kr) + \frac{g^2 \exp(ikr_>) \sin(kr_<)}{k^2 - \kappa_0^2}, \quad \begin{cases}
 r_> = \max(r, r_1) \\
 r_< = \min(r, r_1)
\end{cases}
\]

\[
\begin{align*}
-\frac{1}{a} &= \frac{\kappa_0^2 - k^2}{g^2 r_1} \\
\int_0^r |v(r)|^2 &= \frac{k^2 a^2}{(k^2 a^2 + 1) g^2 r_1} \leq \frac{1}{g^2 r_1} = O(r_0) \quad \text{Open channel } \text{wf (triplet)}
\end{align*}
\]

\[
\int_{r_0}^R |u(r)|^2 \approx \int_{r_0}^R dr \sin^2(kr + \delta) = O(R) \gg O(r_0) \quad \text{Open channel } \text{wf (triplet)}
\]

\[
\frac{d \ln u(r)}{dr} \bigg|_{r=r_0} = -\frac{1}{a}
\]

\[
r_0 \approx \left( \frac{C_6 m}{\hbar^2} \right)^{1/4}
\]

\[
r_0 r^3 \ll 1, \quad a \gg r_0, \quad O(k_F r_0) \ll 1
\]

\[
O(\kappa_0^2) = O(g^2) = \frac{2 \mu_B B_0 m}{\hbar^2}
\]

\[
r_0 \ll R \approx \frac{\lambda_F}{2}
\]

Particles in a pair spend most of the time outside the interaction zone, in the triplet state.

Halo dimers
Optical trap for evaporative cooling

borrowed from R. Grimm

special feature #1
precise control of laser power
10 W $\rightarrow$ few 100µW

special feature #2: axial magnetic confinement
- spatial compression at very weak optical traps
- perfectly harmonic !!
- precisely known trap frequency for weak optical trap

$\nu_z = 24.5 \text{ Hz @ 1kG}$
What is in a trap?

• Typically about $10^5$-$10^6$ atoms divided 50-50 among the lowest two hyperfine states

• Due to the high diluteness atoms in the same hyperfine state do not interact with one another

• Atoms in different hyperfine states experience interactions only in s-wave. *The strength of this interaction is fully tunable!*

Who does experiments?

• Jin’s group at Boulder
• Grimm’s group in Innsbruck
• Thomas’ group at Duke
• Ketterle’s group at MIT
• Salomon’s group in Paris
• Hulet’s group at Rice
Typical parameters

hyperfine splitting $\approx 80$ MHz
Fermi energy $\varepsilon_F = 20\cdots50$ kHz

Temperature $T \approx 0\cdots2\varepsilon_F$

trap frequencies
\[
\begin{align*}
\nu_z &\approx 25 \text{ Hz} \\
\nu_r &\approx 750 \text{ Hz}
\end{align*}
\]

cloud has cigar shape

number density $n \approx 10^{13}$ atoms/cm$^3$
typical atom separation $\approx 5000\ a_0$

Number of atoms $N \approx 10^5\cdots10^6$

radius of interaction $r_0 \approx 100a_0$
diluteness $nr_0^3 \ll 1$
In dilute Fermi systems only very few characteristics are relevant.

- These systems are typically very cold

\[
T \ll T_F = \frac{\epsilon_F}{k_B}, \quad \epsilon_F = \frac{p_F^2}{2m}
\]

- A dilute Fermi system is degenerate and the fastest particle has a momentum of the order of the Fermi momentum

\[
p_F = (6\pi^2 n_\uparrow)^{1/3} \hbar = (6\pi^2 n_\downarrow)^{1/3} \hbar
\]

\[
n_\uparrow = n_\downarrow = \frac{n}{2}
\]

- The wave functions are basically constant over the interaction volume of two particles and thus they cannot “see” any details, except the scattering length typically.
Expected phases of a two species dilute Fermi system
BCS-BEC crossover

High T, normal atomic (plus a few molecules) phase

Strong interaction

weak interaction

BCS Superfluid

\[ \frac{\pi}{k_F} \leq |a| \]

weak interactions

Molecular BEC and/or Atomic + Molecular Superfluids

\[ a < 0 \]

no 2-body bound state

\[ a > 0 \]

shallow 2-body bound state

halo dimers

\[ \frac{1}{a} \]
BCS → BEC crossover


If \( a < 0 \) at \( T=0 \) a Fermi system is a BCS superfluid

\[
\Delta \approx \left( \frac{2}{e} \right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp \left( \frac{\pi}{2k_F a} \right) \ll \varepsilon_F, \quad \text{iff} \quad k_F |a| \ll 1 \quad \text{and} \quad \xi = \frac{1}{k_F} \frac{\varepsilon_F}{\Delta} \gg \frac{1}{k_F}
\]

If \(|a|=\infty\) and \(nr_0^3 \ll 1\) a Fermi system is strongly coupled and its properties are universal. Carlson et al. PRL 91, 050401 (2003)

\[
\frac{E_{\text{normal}}}{N} \approx 0.54 \frac{3}{5} \varepsilon_F, \quad \frac{E_{\text{superfluid}}}{N} \approx 0.44 \frac{3}{5} \varepsilon_F \quad \text{and} \quad \xi = O(\lambda_F), \quad \Delta = O(\varepsilon_F)
\]

If \(a>0 \) (\(a \gg r_0\)) and \(na^3 \ll 1\) the system is a dilute BEC of tightly bound dimers

\[
\varepsilon_2 = -\frac{\hbar^2}{ma^2} \quad \text{and} \quad n_b a^3 \ll 1, \quad \text{where} \quad n_b = \frac{n_f}{2} \quad \text{and} \quad a_{bb} = 0.6a > 0
\]
What are the ground state properties of the many-body system composed of spin $\frac{1}{2}$ fermions interacting via a zero-range, infinite scattering-length contact interaction.

In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not.

- systems of bosons are unstable (Efimov effect)
- systems of three or more fermion species are unstable (Efimov effect)

• Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)

• Carlson et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided best the theoretical estimates for the ground state energy of such systems.

• Thomas’ Duke group (2002) demonstrated experimentally that such systems are (meta)stable.
Experimental signatures/predictions of the large size pairs/ halo dimer model versus fermion-boson model:

1) Near the Feshbach resonance the pair is in triplet state
   \textit{in agreement with Grimm’s group experiments}
   (fermion-boson model predicts a singlet state)

2) Particle loss is consistent with large spatial size pairs
   \textit{in agreement with Grimm’s and Jin’s groups experiments}
   (fermion-boson model is consistent with small spatial size pairs)

3) Spatial size of the cloud
   \textit{in agreement with Grimm’s group experiment}
   (fermion-boson model disagrees with experiment)

4) Frequencies and frequency shift of the frequencies of the collective oscillations
   (if at the Feshbach resonance the system would be made of small size pairs the frequency
   would be higher and the frequency shift would be much smaller than observed in experiments, see
   Pitaevskii and Stringari, PRL 81, 4541 (1998), Braaten and Pearson, PRL 82, 255 (1999))

\[
\frac{\delta \omega}{\omega} \approx \frac{63\sqrt{\pi}}{128} \sqrt{n(0)r_0^3} \ll 1
\]
Coupled Fermion-Boson model in a trap

Superconductivity: Lee, Ranninger,
Fermi atom gas: Timmermans, Holland, Ohashi, and their collaborators

two hyperfine states = ⬇️ ⬆️

$$H = -\sum_\sigma \int dr \Psi^\dagger_\sigma(r) \left[ \frac{p^2}{2m} + V_{\text{Fermion}}(r) \right] \Psi_\sigma(r) - U \int dr \Psi^\dagger_\uparrow \Psi^\dagger_\downarrow \Psi_\downarrow \Psi_\uparrow$$

$$+ \Phi^\dagger(r) \left[ \frac{p^2}{2M} + 2\nu + V_{\text{Boson}}(r) \right] \Phi(r)$$

$$+ g \sum_\sigma \int dr [\Phi^\dagger(r) \Psi^\dagger_\downarrow(r) \Psi_\uparrow(r) + h.c.]$$

Feshbach resonance

Open (triplet) channel
Closed (singlet) channel

Ohashi, Levico 2004
Even though two atoms can bind, there is no binding among dimers!

Fixed node GFMC results, J. Carlson et al. (2003)
\[
\Delta_{Gorkov} = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)
\]

\[
\Delta_{BCS} = \frac{8}{e^2} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)
\]

Fixed node GFMC results, J. Carlson et al. (2003)
S. Jochim et al.
PRL 91, 240402 (2003)

Electron spin along $B$

Electron spin opposite $B$

see also Petrov et al. cond-mat/0309010
Regal et al. PRL 92, 083201 (2004)
The fermion-boson model predict that at resonance an atomic Fermi cloud consists predominantly of molecules in the closed channel (singlet), which thus have an almost vanishing magnetic moment.

Ohashi and Griffin,

Bruun, cond-mat/0401497
40K (Fermi) atoms in a spherical harmonic trap

Effect of interaction, with and without weak and strong pairing correlations with fixed particle number, $N = 5200$.

$\hbar\omega = 0.568 \times 10^{-12}\text{eV}$, $a = -12.63\text{nm}$ (when finite)

Unpublished, fully self-consistent SLDA (Kohn-Sham generalized to pairing) calculation performed by Yongle Yu in July 2003.
The fermion-boson model predicts that at resonance the size of the cloud is significantly smaller than the observed one.

Fermi atoms form Cooper-pair bosons, which does not affect the density profile in the BCS regime. Density profile shrinks in the BEC regime due to the absence of Pauli Principle between molecules associated with the F.R.

Ohashi, Levico 2004

Grimm, Levico 2004
<table>
<thead>
<tr>
<th>Local shape of Fermi surface</th>
<th>Sound velocity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td>$v_s \approx \frac{V_F}{\sqrt{3}}$</td>
<td>First sound</td>
</tr>
<tr>
<td>Spherical</td>
<td>$v_s \approx \frac{V_F}{\sqrt{3}}$</td>
<td>Anderson-Bogoliubov sound</td>
</tr>
<tr>
<td>Elongated along propagation direction</td>
<td>$v_s = SV_F$ \quad $S &gt; 1$</td>
<td>Landau’s zero sound</td>
</tr>
</tbody>
</table>
\[ \varepsilon(n) = \frac{3}{5} \frac{\hbar^2 k_F^2}{2m} \left[ \xi - \frac{\zeta}{k_F a} \left( 3 \left( \frac{k_F a}{r} \right)^2 + O \left( \frac{1}{(k_F a)^3} \right) \right) \right] \]

\[ \xi \approx 0.44, \quad \zeta \approx 1, \quad \ell \approx 1 \]

\[ U = \frac{m \omega_0^2 \left( x^2 + y^2 + \lambda^2 z^2 \right)}{2} \]

\[ \frac{\delta \omega^2}{\omega^2} = \frac{\zeta}{\xi k_F(0)} K \]

**TABLE II: Results for \( K \).**

<table>
<thead>
<tr>
<th>trap type</th>
<th>mode</th>
<th>( f_1 )</th>
<th>( \omega )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>spherical</td>
<td>dipole</td>
<td>( z )</td>
<td>( \omega_0 )</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>monopole</td>
<td>( 1 - 2r^2 )</td>
<td>( 2\omega_0 )</td>
<td>( \frac{256}{525\pi} )</td>
</tr>
<tr>
<td></td>
<td>quadrupole</td>
<td>( xy )</td>
<td>( \sqrt{2\omega_0} )</td>
<td>0</td>
</tr>
<tr>
<td>axial</td>
<td>( M = \pm 2 )</td>
<td>( xy, x^2 - y^2 )</td>
<td>( \sqrt{2\omega_0} )</td>
<td>0</td>
</tr>
<tr>
<td>( \lambda \ll 1 )</td>
<td>( M = \pm 1 )</td>
<td>( xz, yz )</td>
<td>( \omega_0 )</td>
<td>0</td>
</tr>
<tr>
<td>radial</td>
<td>( x^2 + y^2 + \frac{2}{5} \lambda^2 z^2 - \frac{2}{5} )</td>
<td>( \sqrt{\frac{10}{3} \omega_0} )</td>
<td>( \frac{1024}{2625\pi} )</td>
<td>0</td>
</tr>
<tr>
<td>axial</td>
<td>( 1 - 6\lambda^2 z^2 )</td>
<td>( \sqrt{\frac{12}{5} \lambda \omega_0} )</td>
<td>( \frac{256}{2625\pi} )</td>
<td>0</td>
</tr>
</tbody>
</table>

**Adiabatic regime**

**Spherical Fermi surface**

**Perturbation theory result using GFMC equation of state in a trap**

**Frequency shifts in these modes might carry information about possible atom-halo dimer mixture**
Hu et al. cond-mat/0404012, a semi-quantitative analysis (gap and chemical potential inaccurate) assuming a polytropic equation of state

For a more careful analysis, using GFMC equation of state in a trap see Bulgac and Bertsch, cond-mat/0404687
FIG. 1. Measurement of the Feshbach resonance position $B_0$. Shown in the inset is a schematic of the magnetic field as a function of time $t$ measured with respect to the optical trap turn off at $t = 0$. Molecules are first created by a slow magnetic-field sweep across the resonance (dotted line) and then dissociated if $B_{\text{probe}}$ (indicated by the arrow in the inset) is beyond the magnetic field where the two-body physics supports a new bound state. The number of atoms, measured at $t = 17$ ms, is shown as a function of $B_{\text{probe}}$. The two error bars indicate the spread in repeated points at these values of $B$. A fit of the data to an error function reveals $B_0 = 202.10 \pm 0.07$ G, where the uncertainty is given conservatively by the 10%-90% width.

\[
a(B) = a_{bg} \left(1 - \frac{w}{B - B_0}\right)
\]

$a_{bg} = 174a_0$

$w = 7.8 \pm 0.6$ G

$B_0 = 202.10 \pm 0.07$ G

FIG. 2. Measured condensate fraction as a function of detuning from the Feshbach resonance $\Delta B = B_{\text{hold}} - B_0$. Data here were taken for $t_{\text{hold}} = 2$ ms (●) and $t_{\text{hold}} = 30$ ms (△) with an initial cloud at $T/T_F = 0.08$ and $T_F = 0.35 \mu$K. The area between the dashed lines around $\Delta B = 0$ reflects the uncertainty in the Feshbach resonance position based on the 10%-90% width of the feature in Fig. 1. Condensation of fermionic atom pairs is seen near and on either side of the Feshbach resonance. Comparison of the data taken with the different hold times indicates that the pair condensed state has a significantly longer lifetime near the Feshbach resonance and on the BCS ($\Delta B > 0$) side. The inset shows a schematic of a typical magnetic-field sweep used to measure the fermionic condensate fraction. The system is first prepared by a slow magnetic-field sweep towards the resonance (dotted line) to a variable position $B_{\text{hold}}$, indicated by the two-sided arrow. After a time $t_{\text{hold}}$ the optical trap is turned off and the magnetic field is quickly lowered by $\sim 10$ G to project the atom gas onto a molecular gas. After free expansion, the molecules are imaged on the BEC side of the resonance (○).
Falco and Stoof,

\[ n_m \approx \frac{n}{2} \left[ 1 - \frac{1}{(k_F a)^3} \right] \]

Theory declared full victory here! 😞

Regal, Greiner and Jin,

Zwierlein et al.
Fermion superfluidity, more specifically superflow, has not yet been demonstrated unambiguously experimentally. There is lots of circumstantial evidence and facts in agreement with theoretical models assuming its existence.

Theory is able to make very precise predictions in this regime and the agreement with experiment can be check quantitatively.
radio-frequency spectroscopy

meas. of mol. bind. energy in $^{40}$K

rf spectroscopy of $^6$Li:

~80MHz

breaking pairs costs energy
→
pair signal up-shifted

borrowed from R. Grimm
rf spectra in crossover regime

rf offset

borrowed from R. Grimm
temperature dependence of pairing

\begin{align*}
T \lessapprox 0.1 \, T_F & \approx 0.5 \, T_F \\
& \approx 0.4 \, T_F \\
& \approx 0.2 \, T_F \\
T \lessapprox 0.1 \, T_F & \quad \text{(b)} \\
& \quad \text{(c)} \\
& \quad \text{(d)} \\
\end{align*}

borrowed from R. Grimm
J. Thomas’ group at Duke

Consistent with $E \approx aT^{5/2}$

Consistent with $E \approx bT^2$
Superfluid LDA (SLDA)

\[ n(\vec{r}) = 2 \sum_{0<E_k<E_c} \left| v_k(\vec{r}) \right|^2, \quad \tau(\vec{r}) = 2 \sum_{0<E_k<E_c} \left| \nabla v_k(\vec{r}) \right|^2 \]

\[ v(\vec{r}) = \sum_{0<E_k<E_c} v_k^*(\vec{r}) u_k(\vec{r}) \]

\[ E = \int d^3r \left\{ \frac{\hbar^2}{2m} \tau(\vec{r}) + \frac{\hbar^2}{m} \beta \left[ x(\vec{r}) \right] n(\vec{r})^{5/3} - \Delta \left[ x(\vec{r}) \right] v^*(\vec{r}) \right\} \]

\[ \Delta \left[ x(\vec{r}) \right] = -\frac{\hbar^2}{m} \gamma_{\text{eff}} \frac{x(\vec{r})}{n(\vec{r})^{1/3}} v(\vec{r}), \quad x(\vec{r}) = \frac{1}{k_F(\vec{r})a} \]

\[ \begin{pmatrix} T + U(\vec{r}) - \mu & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -(T + U(\vec{r}) - \mu) \end{pmatrix} \begin{pmatrix} u_k(\vec{r}) \\ v_k(\vec{r}) \end{pmatrix} = E_i \begin{pmatrix} u_k(\vec{r}) \\ v_k(\vec{r}) \end{pmatrix} \]

Bogoliubov-de Gennes like equations. Correlations are however included by default!

number and kinetic densities

anomalous density

Cutoff and position running coupling constant!

Divergent!
Vortex in fermion matter

\[
\begin{pmatrix}
  u_{\alpha k n} (\vec{r}) \\
  v_{\alpha k n} (\vec{r})
\end{pmatrix}
= \begin{pmatrix}
  u_\alpha (\rho) \exp[i(n+1/2)\phi - ikz] \\
  v_\alpha (\rho) \exp[i(n-1/2)\phi - ikz]
\end{pmatrix}, \quad n - \text{half-integer}
\]

\[\Delta(\vec{r}) = \Delta(\rho) \exp(i\phi), \quad \vec{r} = (\rho, \phi, z) \quad [\text{cylindrical coordinates}]\]

Oz - vortex symmetry axis

Ideal vortex, Onsager's quantization (one \(\hbar\) per Cooper pair)

\[
\vec{V}_v (\vec{r}) = \frac{\hbar}{2m\rho^2} (y, -x, 0) \iff \frac{1}{2\pi} \oint_C \vec{V}_v (\vec{r}) \cdot d\vec{r} = \frac{\hbar}{2m}
\]
How can one put in evidence a vortex in a Fermi superfluid?

Hard to see, since density changes are not expected, unlike the case of a Bose superfluid.

However, if the gap is not small, one can expect a noticeable density depletion along the vortex core, and the bigger the gap the bigger the depletion, due to an extremely fast vortical motion.

\[
\frac{V_s}{V_F} < \frac{\Delta}{2\varepsilon_F} \propto \frac{T_c}{T_F}
\]

NB $T_c$ unknown in the strong coupling limit!
The depletion along the vortex core is reminiscent of the corresponding density depletion in the case of a vortex in a Bose superfluid, when the density vanishes exactly along the axis for 100% BEC.

### Fermions with $1/k_F a = 0.3, 0.1, 0, -0.1, -0.5$

### Bosons with $n a^3 = 10^{-3}$ and $10^{-5}$

**Number density and pairing field profiles**

**Local vortical speed as fraction of Fermi speed**

From Ketterle’s group.
Phases of a two species dilute Fermi system BCS-BEC crossover

“Before”- Life
Atoms form rather unstable couples, spread over large distances, with many other couples occupying the same space

LIFE!
Atoms form strong couples, living as in an apartment building, in close proximity of one another, and obviously annoying each other quite a bit

After – Life
Atoms form very strong couples, mostly inert and at rest, **halo dimers**, widely separated from one another and weakly interacting

---

**Weak interaction**

\( a < 0 \)

no 2-body bound state

**Strong interaction**

**Weak interaction**

\( a > 0 \)

shallow 2-body bound state

**halo dimers**
Conclusions:

✓ The field of dilute atomic systems is going to be for many years to come one of the most exciting fields in physics, with lots of surprises at every corner.