Collaborators:  
- Michael M. Forbes (Seattle, now WSU)
- Yuan-Lung (Alan) Luo (Seattle, now at a start-up)
- Piotr Magierski (Warsaw/Seattle)
- Kenneth J. Roche (PNNL/Seattle)
- Rishi Sharma (Triumf, now Tata)
- Ionel Stetcu (UW, now LANL)
- Yongle Yu (Seattle, now Wuhan, PRC)
- Sukjin Yoon (Seattle, now at APCTP)
- Gabriel Wlazłowski (Seattle/Warsaw)

Funding: DOE, NSF

Computing:  
Athena UW Cluster, Hyak UW cluster, Franklin, Hopper, and Edison at NERSC and Jaguar and Titan at OLCF

Slides pptx with movies can be downloaded from http://www.phys.washington.edu/users/bulgac/Media/YITP_December_2014.pptx
Why should one study fermionic superfluidity?

Superconductivity (discovered on April 8th, 1911) and superfluidity in Fermi systems are manifestations of quantum coherence at a macroscopic level.

- **Dilute atomic Fermi gases**
  \[ T_c \approx 10^{-9} \text{ eV} \]
- **Liquid \(^3\text{He}\)**
  \[ T_c \approx 10^{-7} \text{ eV} \]
- **Metals, composite materials**
  \[ T_c \approx 10^{-3} - 10^{-2} \text{ eV} \]
- **Nuclei, neutron stars**
  \[ T_c \approx 10^5 - 10^6 \text{ eV} \]
- **QCD color superconductivity**
  \[ T_c \approx 10^7 - 10^8 \text{ eV} \]

*units (1 eV \(\approx 10^4 \text{ K}\))*
Physical systems and processes we are interested in:

- Dynamics of vortices, Anderson-Higgs Mode
- Vortex crossing and reconnection and the onset of quantum turbulence
- Domain wall solitons and shock waves in collision of fermionic superfluid atomic clouds

- Collective states in nuclei
- Nuclear large amplitude collective motion (LACM) (Induced) nuclear fission
- Excitation of nuclei with gamma rays and neutrons
- Coulomb excitation of nuclei with relativistic heavy-ions
- Nuclear reactions, fusion between colliding heavy-ions
- Neutron star crust and dynamics of vortices and their pinning mechanism
One option is the two-fluid hydrodynamics (here at T=0, only one fluid)

*N.B. There is no quantum statistics in two-fluid hydrodynamics*

\[
\begin{align*}
\frac{\partial n(\vec{r}, t)}{\partial t} + \vec{\nabla} \cdot \left[ \vec{v}(\vec{r}, t)n(\vec{r}, t) \right] &= 0 \\
m \frac{\partial \vec{v}(\vec{r}, t)}{\partial t} + \vec{\nabla} \left\{ \frac{m \vec{v}^2(\vec{r}, t)}{2} + \mu n(\vec{r}, t) + V_{ext}(\vec{r}, t) \right\} &= 0
\end{align*}
\]

Troubles:
- These are classical equations, **no Planck's constant**, thus no quantized vortices (unless one imposes by hand quantization)
- No physically clear physical mechanism to describe superfluid to normal transition (no role for the critical velocity)

*Two-fluid hydrodynamics + vortex quantization is equivalent to a ``Bohr model'' of a superfluid*
Another option is the phenomenological Ginzburg-Landau model or the Gross-Pitaevskii equation:

\[ i\hbar e^{i\gamma} \frac{\partial \Psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2 \Delta \Psi(\vec{r},t)}{2M} + U(|\Psi(\vec{r},t)|^2)\Psi(\vec{r},t) + V_{\text{ext}}(\vec{r},t)\Psi(\vec{r},t) + \text{fluct}. \]

Troubles:

- GLE valid only for temperatures near and below the critical temperature
- Even though is a quantum approach, it describes only the superfluid phase. There is no Cooper pair breaking mechanism
- GPE was the only microscopic equation available until recently, valid a superfluid of weakly interacting bosons at T=0
Other issues:

There are a number of modes, such as the Anderson-Higgs mode, which cannot be describes in either of these phenomenological approaches.
Energy of a Fermi system as a function of the pairing gap: Anderson-Higgs mode

\[ \dot{n} + \vec{V} \cdot [\vec{V} n] = 0 \]

\[ m \dot{\vec{V}} + \vec{V} \left\{ \frac{m \vec{V}^2}{2} + \mu [n] \right\} = 0 \]

Landau’s two-fluid hydrodynamics

Both fail

Ginzburg-Landau-like equation
Response of a unitary Fermi system to changing the scattering length with time

- All these modes have a very low frequency below the pairing gap, a very large amplitude and very large excitation energy.

- None of these modes can be described either within two-fluid hydrodynamics or Ginzburg-Landau like approaches.

Bulgac and Yoon, Phys. Rev. Lett. 102, 085302 (2009)
A new local extension of DFT to superfluid systems and time-dependent phenomena was developed.


DFT has been developed and used mainly to describe normal (non-superfluid) electron systems – *50 years old theory*, Kohn and Hohenberg, 1964.
Kohn-Sham theorem \((1965)\)

\[
H = \sum_{i}^{N} T(i) + \sum_{i<j}^{N} U(ij) + \sum_{i<j<k}^{N} U(ijk) + \ldots + \sum_{i}^{N} V_{\text{ext}}(i)
\]

\[
H \Psi_{0} (1,2,...N) = E_{0} \Psi_{0} (1,2,...N)
\]

\[
n(\vec{r}) = \langle \Psi_{0} \left| \sum_{i}^{N} \delta(\vec{r} - \vec{r}_{i}) \right| \Psi_{0} \rangle
\]

\[
\Psi_{0} (1,2,...N) \iff V_{\text{ext}}(\vec{r}) \iff n(\vec{r})
\]

\[
E_{0} = \min_{n(\vec{r})} \int d^{3}r \left\{ \frac{\hbar^{2}}{2m^{*}(\vec{r})} \tau(\vec{r}) + \varepsilon \left[ n(\vec{r}) \right] + V_{\text{ext}}(\vec{r})n(\vec{r}) \right\}
\]

\[
n(\vec{r}) = \sum_{i}^{N} \left| \varphi_{i}(\vec{r}) \right|^{2}, \quad \tau(\vec{r}) = \sum_{i}^{N} \left| \nabla \varphi_{i}(\vec{r}) \right|^{2}
\]

THEOREM: There exist an universal functional of particle density alone independent of the external potential

Normal Fermi systems only!
However, not everyone is normal!
I will illustrate the construction of the DFT functional for a superfluid unitary Fermi gas

What is a unitary Fermi gas and why would one want to study it?

One reason: (for the nerds, I mean the hard-core theorists, not for the phenomenologists)

What are the ground state properties of the many-body system composed of spin $\frac{1}{2}$ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Bertsch’s Many-Body X challenge, Seattle, 1999

\[
k \cotan \delta_0 = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + \cdots
\]

\[
\sigma = \frac{4\pi}{k^2} \sin^2 \delta_0 + \cdots = 4\pi a^2 + \cdots
\]
Let us consider a very old and simple example:

The hydrogen atom.

The ground state energy could only be a function of:

- Electron charge
- Electron mass
- Planck’s constant

and then trivial dimensional arguments lead to

\[ E_{gs} = \frac{e^4 m}{\hbar^2} \times \frac{1}{2} \]

Only the factor \( \frac{1}{2} \) requires some hard work.
Let us turn now to dilute fermion matter

The ground state energy is given by a function:

$$E_{gs} = f(N, V, \hbar, m, a, r_0)$$

Taking the scattering length to infinity and the range of the interaction to zero, we are left with:

$$E_{gs} = F(N, V, \hbar, m) = \frac{3}{5} \varepsilon_F N \times \xi$$

$$N = \frac{k_F^3}{3\pi^2}, \quad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

Pure number (dimensionless)
The SLDA (DFT) energy density functional for unitary Fermi gas

Dimensional arguments, renormalizability, Galilean invariance, and symmetries determine the functional (energy density)

\[
\varepsilon(\vec{r}) = \frac{\hbar^2}{m} \left\{ \alpha \frac{\tau_c(\vec{r})}{2} + \gamma \frac{|v_c(\vec{r})|^2}{n^{1/3}(\vec{r})} \right\} + \beta \frac{3(3\pi^2)^{2/3} n^{5/3}(\vec{r})}{5} \right) - \frac{\hbar^2}{m} (\alpha - 1) \frac{j^2(\vec{r})}{2n(\vec{r})}
\]

\[
\Delta(\vec{r}) = \frac{\hbar^2}{m} \tilde{\Delta}(\vec{r})
\]

\[n(\vec{r}) = 2 \sum_{0<E_k<E_c} \left| v_k(\vec{r}) \right|^2, \quad \tau_c(\vec{r}) = 2 \sum_{0<E_k<E_c} \left| \nabla v_k(\vec{r}) \right|^2,
\]

\[v_c(\vec{r}) = \sum_{0<E<E_c} u_k(\vec{r})v_k^*(\vec{r}) \quad \Leftarrow \quad \text{divergent without a cutoff, need RG}
\]

Three dimensionless constants \( \alpha, \beta, \) and \( \gamma \) determining the functional are extracted from QMC for homogeneous systems by fixing the total energy, the pairing gap and the effective mass

The unitary Fermi gas and the dilute Bose gas are the only superfluids for which a microscopic framework exist to describe both statics and dynamics
<table>
<thead>
<tr>
<th>((N_a, N_b))</th>
<th>(E_{\text{FNDMC}})</th>
<th>(E_{\text{ASLDA}})</th>
<th>(error)</th>
<th>((N_a, N_b))</th>
<th>(E_{\text{FNDMC}})</th>
<th>(E_{\text{ASLDA}})</th>
<th>(error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3, 1)</td>
<td>6.6 ± 0.01</td>
<td>6.687</td>
<td>1.3%</td>
<td>(1, 1)</td>
<td>2.002 ± 0</td>
<td>2.302</td>
<td>15%</td>
</tr>
<tr>
<td>(4, 1)</td>
<td>8.93 ± 0.01</td>
<td>8.962</td>
<td>0.36%</td>
<td>(2, 2)</td>
<td>5.051 ± 0.009</td>
<td>5.405</td>
<td>7%</td>
</tr>
<tr>
<td>(5, 1)</td>
<td>12.1 ± 0.1</td>
<td>12.22</td>
<td>0.97%</td>
<td>(3, 3)</td>
<td>8.639 ± 0.03</td>
<td>8.939</td>
<td>3.5%</td>
</tr>
<tr>
<td>(5, 2)</td>
<td>13.3 ± 0.1</td>
<td>13.54</td>
<td>1.8%</td>
<td>(4, 4)</td>
<td>12.573 ± 0.03</td>
<td>12.63</td>
<td>0.48%</td>
</tr>
<tr>
<td>(6, 1)</td>
<td>15.8 ± 0.1</td>
<td>15.65</td>
<td>0.93%</td>
<td>(5, 5)</td>
<td>16.806 ± 0.04</td>
<td>16.19</td>
<td>3.7%</td>
</tr>
<tr>
<td>(7, 2)</td>
<td>19.9 ± 0.1</td>
<td>20.11</td>
<td>1.1%</td>
<td>(6, 6)</td>
<td>21.278 ± 0.05</td>
<td>21.13</td>
<td>0.69%</td>
</tr>
<tr>
<td>(7, 3)</td>
<td>20.8 ± 0.1</td>
<td>21.23</td>
<td>2.1%</td>
<td>(7, 7)</td>
<td>25.923 ± 0.05</td>
<td>25.31</td>
<td>2.4%</td>
</tr>
<tr>
<td>(7, 4)</td>
<td>21.9 ± 0.1</td>
<td>22.42</td>
<td>2.4%</td>
<td>(8, 8)</td>
<td>30.876 ± 0.06</td>
<td>30.49</td>
<td>1.2%</td>
</tr>
<tr>
<td>(8, 1)</td>
<td>22.5 ± 0.1</td>
<td>22.53</td>
<td>0.14%</td>
<td>(9, 9)</td>
<td>35.971 ± 0.07</td>
<td>34.87</td>
<td>3.1%</td>
</tr>
<tr>
<td>(9, 1)</td>
<td>25.9 ± 0.1</td>
<td>25.97</td>
<td>0.27%</td>
<td>(10, 10)</td>
<td>41.302 ± 0.08</td>
<td>40.54</td>
<td>1.8%</td>
</tr>
<tr>
<td>(9, 2)</td>
<td>26.6 ± 0.1</td>
<td>26.73</td>
<td>0.5%</td>
<td>(11, 11)</td>
<td>46.889 ± 0.09</td>
<td>45</td>
<td>4%</td>
</tr>
<tr>
<td>(9, 3)</td>
<td>27.2 ± 0.1</td>
<td>27.55</td>
<td>1.3%</td>
<td>(12, 12)</td>
<td>52.624 ± 0.2</td>
<td>51.23</td>
<td>2.7%</td>
</tr>
<tr>
<td>(9, 5)</td>
<td>30 ± 0.1</td>
<td>30.77</td>
<td>2.6%</td>
<td>(13, 13)</td>
<td>58.545 ± 0.18</td>
<td>56.25</td>
<td>3.9%</td>
</tr>
<tr>
<td>(10, 1)</td>
<td>29.4 ± 0.1</td>
<td>29.41</td>
<td>0.034%</td>
<td>(14, 14)</td>
<td>64.388 ± 0.31</td>
<td>62.52</td>
<td>2.9%</td>
</tr>
<tr>
<td>(10, 2)</td>
<td>29.9 ± 0.1</td>
<td>30.05</td>
<td>0.52%</td>
<td>(15, 15)</td>
<td>70.927 ± 0.3</td>
<td>68.72</td>
<td>3.1%</td>
</tr>
<tr>
<td>(10, 6)</td>
<td>35 ± 0.1</td>
<td>35.93</td>
<td>2.7%</td>
<td>(1, 0)</td>
<td>1.5 ± 0.0</td>
<td>1.5</td>
<td>0%</td>
</tr>
<tr>
<td>(20, 1)</td>
<td>73.78 ± 0.01</td>
<td>73.83</td>
<td>0.061%</td>
<td>(2, 1)</td>
<td>4.281 ± 0.004</td>
<td>4.417</td>
<td>3.2%</td>
</tr>
<tr>
<td>(20, 4)</td>
<td>73.79 ± 0.01</td>
<td>74.01</td>
<td>0.3%</td>
<td>(3, 2)</td>
<td>7.61 ± 0.01</td>
<td>7.602</td>
<td>0.1%</td>
</tr>
<tr>
<td>(20, 10)</td>
<td>81.7 ± 0.1</td>
<td>82.57</td>
<td>1.1%</td>
<td>(4, 3)</td>
<td>11.362 ± 0.02</td>
<td>11.31</td>
<td>0.49%</td>
</tr>
<tr>
<td>(20, 20)</td>
<td>109.7 ± 0.1</td>
<td>113.8</td>
<td>3.7%</td>
<td>(7, 6)</td>
<td>24.787 ± 0.09</td>
<td>24.04</td>
<td>3%</td>
</tr>
<tr>
<td>(35, 4)</td>
<td>154 ± 0.1</td>
<td>154.1</td>
<td>0.078%</td>
<td>(11, 10)</td>
<td>45.474 ± 0.15</td>
<td>43.98</td>
<td>3.3%</td>
</tr>
<tr>
<td>(35, 10)</td>
<td>158.2 ± 0.1</td>
<td>158.6</td>
<td>0.27%</td>
<td>(15, 14)</td>
<td>69.126 ± 0.31</td>
<td>62.55</td>
<td>9.5%</td>
</tr>
<tr>
<td>(35, 20)</td>
<td>178.6 ± 0.1</td>
<td>180.4</td>
<td>1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bulgac, Forbes, and Magierski, Lecture Notes in Physics (2012)
EOS for spin polarized systems

\[ E(n_a, n_b) = \frac{3}{5} \frac{(6\pi^2)^{2/3}}{2m} \hbar^2 \left[ n_a g \left( \frac{n_b}{n_a} \right) \right]^{5/3} \]

**Red line:** Larkin-Ovchinnikov phase (unitary Fermi supersolid)

**Black line:** normal part of the energy density

**Blue points:** DMC calculations for normal state, Lobo et al, PRL 97, 200403 (2006)

**Gray crosses:** experimental EOS due to Shin, Phys. Rev. A 77, 041603(R) (2008)

Formalism for Time-Dependent Phenomena

“The time-dependent density functional theory is viewed in general as a reformulation of the exact quantum mechanical time evolution of a many-body system when only one-body properties are considered.”

V. Peuckert, J. Phys. C 11, 4945 (1978)

http://www.tddft.org

\[
E(t) = \int d^3r \left[ \mathcal{E}(n(\vec{r},t), \tau(\vec{r},t), \nu(\vec{r},t), \vec{j}(\vec{r},t)) + V_{\text{ext}}(\vec{r},t)n(\vec{r},t) + \ldots \right] \\
\left[ h(\vec{r},t) + V_{\text{ext}}(\vec{r},t) - \mu \right] u_i(\vec{r},t) + \left[ \Delta(\vec{r},t) + \Delta_{\text{ext}}(\vec{r},t) \right] v_i(\vec{r},t) = i\hbar \frac{\partial u_i(\vec{r},t)}{\partial t} \\
\left[ \Delta^*(\vec{r},t) + \Delta_{\text{ext}}^*(\vec{r},t) \right] u_i(\vec{r},t) - \left[ h(\vec{r},t) + V_{\text{ext}}(\vec{r},t) - \mu \right] v_i(\vec{r},t) = i\hbar \frac{\partial v_i(\vec{r},t)}{\partial t}
\]

For time-dependent phenomena one has to add currents. Galilean invariance determines the dependence on currents.
Below $T_c$ dissipation is included in TDSLDA formalism for the unitary Fermi gas!

- Bulk viscosity vanishes for the unitary Fermi gas

- Below $T_c$ shear viscosity is determined by phonons alone (yellow band in figure)
TDSLDA equations

\[
\begin{pmatrix}
u_{n\uparrow}(\vec{r},t) \\
u_{n\downarrow}(\vec{r},t) \\
v_{n\uparrow}(\vec{r},t) \\
v_{n\downarrow}(\vec{r},t)
\end{pmatrix}
\frac{i\hbar}{\partial t}
\begin{pmatrix}
h_{\uparrow\uparrow}(\vec{r},t) - \mu \\
h_{\downarrow\uparrow}(\vec{r},t) \\
h_{\downarrow\downarrow}(\vec{r},t) - \mu \\
\Delta^*(\vec{r},t)
\end{pmatrix}
\begin{pmatrix}
h_{\uparrow\uparrow}(\vec{r},t) \\
h_{\downarrow\uparrow}(\vec{r},t) \\
h_{\downarrow\downarrow}(\vec{r},t) \\
\Delta(\vec{r},t)
\end{pmatrix}
\begin{pmatrix}
u_{n\uparrow}(\vec{r},t) \\
u_{n\downarrow}(\vec{r},t) \\
v_{n\uparrow}(\vec{r},t) \\
v_{n\downarrow}(\vec{r},t)
\end{pmatrix}
\]

• The system is placed on a large 3D spatial lattice (adequate representation of continuum)
• Derivatives are computed with FFTW (this insures machine accuracy) and is very fast
• Fully self-consistent treatment with fundamental symmetries respected (isospin, gauge, Galilean, rotation, translation)
• Adams-Bashforth-Milne fifth order predictor-corrector-modifier integrator
  Effectively a sixth order method
• No symmetry restrictions
• Number of PDEs is of the order of the number of spatial lattice points
  – from 10,000s to 1-2,000,000

\[
\propto 4\left(\frac{2p_cL}{2\pi\hbar}\right)^3 = 4N_xN_yN_z
\]

• SLDA/TDSLDA (DFT) is formally by construction like meanfield HFB/BdG
• The code was implemented on Jaguar, Titan, Franklin, Hopper, Edison, Hyak, Athena
• Initially Fortran 90, 95, 2003 …, presently C, CUDA, and obviously MPI, threads, etc.
Sample Nuclear Code Comparisons (4-component qwfs)

<table>
<thead>
<tr>
<th>$N_xN_yN_z$</th>
<th>$N_{wf}$</th>
<th>memory</th>
<th>CPU comp. + comm.</th>
<th>CPU comp.</th>
<th>GPU comp. + comm.</th>
<th>GPU comp.</th>
<th># of GPUs</th>
<th>speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>$48^3$</td>
<td>110592</td>
<td>10 TB</td>
<td>3.9s</td>
<td>2.4s</td>
<td>0.39s</td>
<td>0.023s</td>
<td>6912</td>
<td>10</td>
</tr>
<tr>
<td>$64^3$</td>
<td>262144</td>
<td>56 TB</td>
<td>20s</td>
<td>9.1s</td>
<td>0.80s</td>
<td>0.48s</td>
<td>16384</td>
<td>25</td>
</tr>
</tbody>
</table>

Over 1 million time-dependent 3D nonlinear complex coupled PDEs
Several hours of videos

The Superfluid Local Density Approximation Applied to Unitary Fermi Gases - Supplementary Material

All simulations can be found here: [http://www.phys.washington.edu/groups/qm6nt/UPG](http://www.phys.washington.edu/groups/qm6nt/UPG). The simulations can be categorized by the excitations: ball and rod, centered ball, centered small ball, centered big ball, centered supersonic ball, off-centered ball, and twisted stirrer. The following table matches simulations with numerical experiments. In several studies, we present multiple perspectives of the event as well as different plotting schemes to reveal different features of the dynamics.

### 3D Simulations

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Link</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ball and Rod</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nt-ball-rod-dns.m4v</td>
<td>density volume plot of magnitude of pairing field; front facing with quarter segment slice; 5m28s duration (20.9 MB)</td>
<td></td>
</tr>
<tr>
<td>nt-ball-rod-dns-pin.m4v</td>
<td>density volume plot of magnitude of pairing field; 2D slice; 5m28s duration (9.8 MB)</td>
<td></td>
</tr>
<tr>
<td>nt-ball-rod-thin-angl.m4v</td>
<td>density contour plot of magnitude of pairing field focused on vortices; angled front-facing with quarter segment slice; 5m28s duration (12.8 MB)</td>
<td></td>
</tr>
<tr>
<td><strong>Centered Ball</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nt-ball-c.m4v</td>
<td>density contour plot of magnitude of pairing field focused on vortices; full geometry; 3m29s</td>
<td></td>
</tr>
</tbody>
</table>
Movie
Time = 0.0
Movie
Fig. 2. A spherical projectile flying along the symmetry axis leaves in its wake two vortex rings.

Fig. 3. (A to D) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.

A. Bulgac, Y.-L. Luo, P. Magierski, K.J. Roche, Y. Yu Science, 332, 1288 (2011)
Observation of shock waves in a strongly interacting Fermi gas
J. Joseph, J.E. Thomas, M. Kulkarni, and A.G. Abanov
PRL 106, 150401 (2011)

Number density of two colliding cold Fermi gases in TDSLDA
Bulgac, Luo, and Roche,
Collision of clouds with larger aspect ratio
Dark solitons/domain walls and shock waves in the collision of two UFG clouds

Phase of the pairing gap normalized to $\varepsilon_F$

Local velocity normalized to Fermi velocity
Heavy solitons in a fermionic superfluid

Tarik Yefsah¹, Ariel T. Sommer¹, Mark J. H. Ku¹, Lawrence W. Cheuk¹, Wenjie Ji¹, Waseem S. Bakr¹ & Martin W. Zwierlein¹
Construction of ground state (adiabatic switching with quantum friction), generation of a domain wall using an optical knife, followed by the spontaneous formation of a vortex ring. Approximately 1270 fermions on a 48x48x128 spatial lattice, ≈ 260,000 complex PDEs, ≈ 309,000 time-steps, 2048 GPUs on Titan, 27.25 hours of wall time (initial code) Wlazłowski et al, Phys. Rev. Lett. 112, 025301 (2014)
Imaging the vortex ring in experiment (movie)

Large ring  Small ring  Too large $B_{\text{min}}$
Large ring

Small ring

Insufficient ramping of magnetic field
Near harmonic motion close to T=0
(very small number of phonons)

Anti-damping of the motion in the presence of a considerable number of phonons

TDSLDA (movie)
Vortex Ring Motion

Buoyant force

Magnus effect

Vortex ring motion (here in the presence of “thermal” noise, hence the inverse decay)
What TDSLDA tells us in the case of an axially non-symmetric trap, similar to the 2014 MIT experiment? (movie)

In agreement with the new experiment, when axial symmetry is broken a domain wall, converts to a vortex ring, which shortly becomes a vortex line.
View along the long axis (y-axis vertical, movie)

In a slightly different geometry one can put directly in evidence in great detail the crossing and reconnection of vortex lines, the mechanism envisioned by Feynman in 1955 as the route to Quantum Turbulence (movie)
Quantum turbulence with no dissipation conjectured by Feynman (1955)
Exciting quantum turbulence in a unitary Fermi gas in a trap

Włazłowski et al, arXiv:1404.1038
Let us summarize some of the ingredients of the SLDA in nuclei

**Energy Density (ED) describing the normal system**

$$E_{gs} = \int d^3 r \left\{ \mathcal{E}_N[\rho_n(\vec{r}), \rho_p(\vec{r})] + \mathcal{E}_S[\rho_n(\vec{r}), \rho_p(\vec{r}), \nu_n(\vec{r}), \nu_p(\vec{r})] \right\}$$

$$\begin{align*}
\mathcal{E}_N[\rho_n(\vec{r}), \rho_p(\vec{r})] &= \mathcal{E}_N[\rho_p(\vec{r}), \rho_n(\vec{r})] \\
\mathcal{E}_S[\rho_n(\vec{r}), \rho_p(\vec{r}), \nu_n(\vec{r}), \nu_p(\vec{r})] &= \mathcal{E}_S[\rho_p(\vec{r}), \rho_n(\vec{r}), \nu_p(\vec{r}), \nu_n(\vec{r})]
\end{align*}$$

**ED contribution due to superfluid correlations**

**Isospin symmetry constraints**

(Coulomb energy and other relatively small terms not shown here.)

$$\mathcal{E}_S[\rho_n, \rho_p, \nu_p, \nu_n] = g(\rho_p, \rho_n)[|\nu_p|^2 + |\nu_n|^2]$$

$$+ f(\rho_p, \rho_n)[|\nu_p|^2 - |\nu_n|^2] \frac{\rho_p - \rho_n}{\rho_p + \rho_n}$$

where $g(\rho_p, \rho_n) = g(\rho_n, \rho_p)$

and $f(\rho_p, \rho_n) = f(\rho_n, \rho_p)$
Giant Dipole Resonance deformed and superfluid nuclei

Osmium is triaxial, and both protons and neutrons are superfluid.

Neutron scattering of $^{238}$U computed in TDSLDA

I. Stetcu et al.
Real-time induced fission of $^{280}\text{Cf}$ computed in TDSLDA

I. Stetcu et al.
How to compute the pinning energy of a vortex on nucleus in the neutron star crust