Unexpected goings-on in the structure of a neutron star crust

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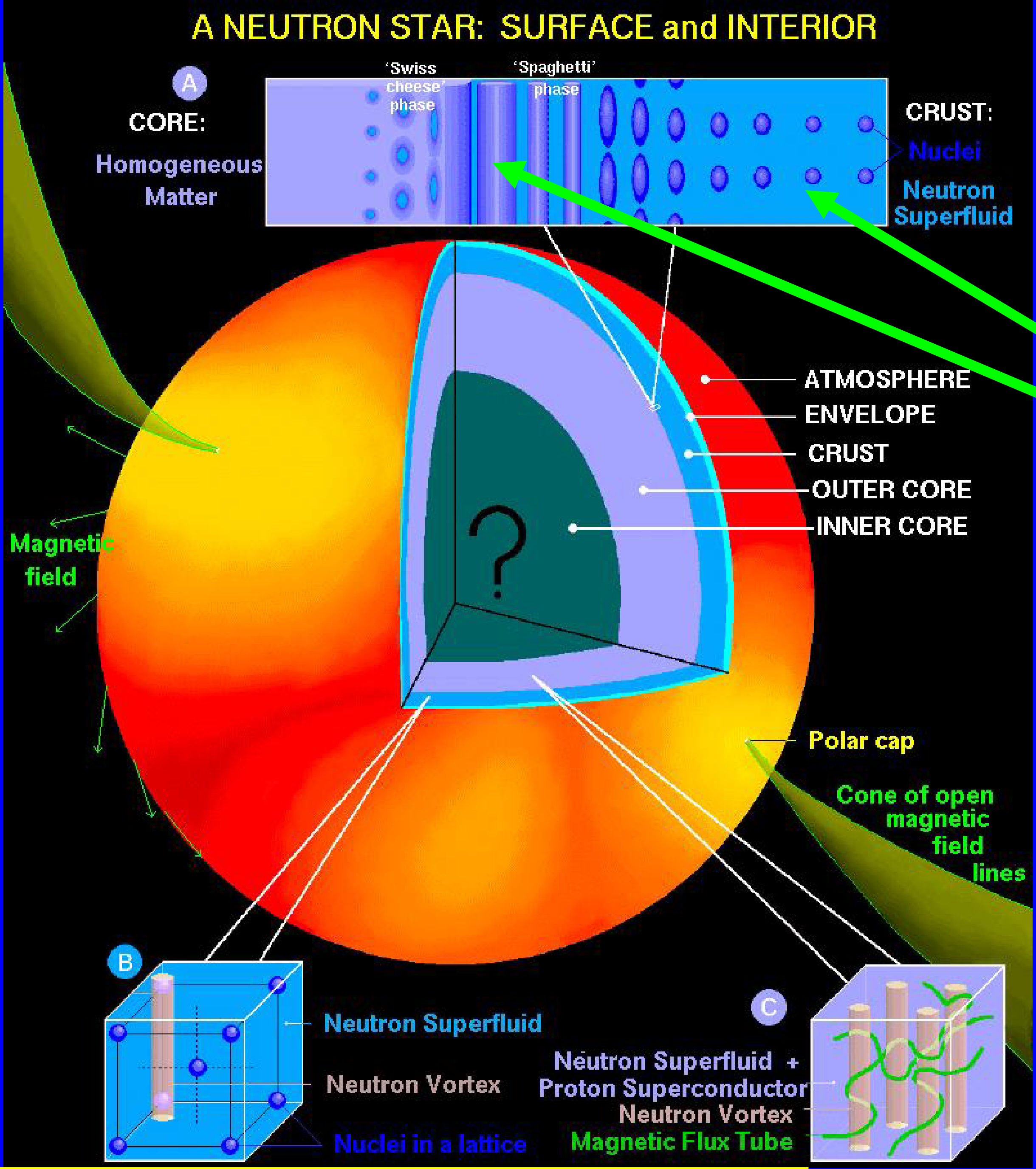
Transparencies (both in ppt and pdf format) shall become shortly available at http://www.phys.washington.edu/~bulgac

What shall I talk about?

✓ I shall describe how a rather subtle quantum phenomenon, the Fermion counterpart of the Casimir effect, is affecting quite drastically the crystalline structure of the neutron star crust, leading likely to a more complex phase, with a richer structure.

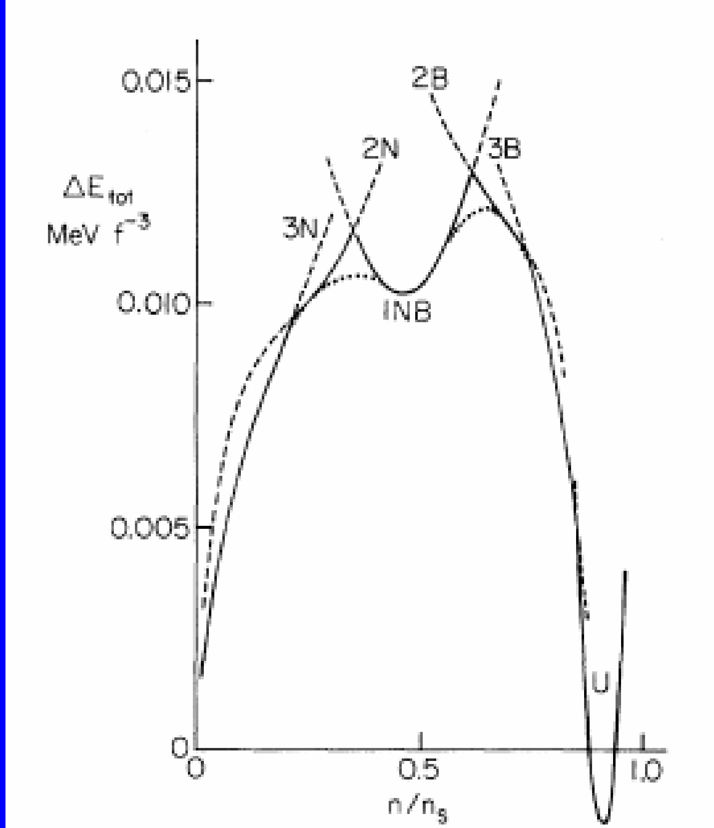
✓ I shall also show that in low density neutron matter, when neutron matter becomes superfluid and vortices can form, the spatial profile of a vortex resembles more its Bose counterpart, and develops a strong density depletion along its axis.

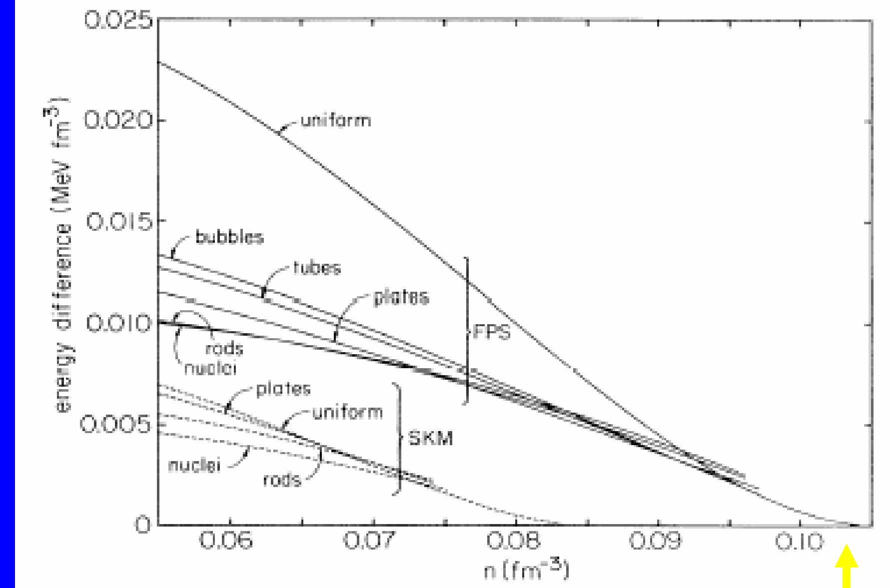


"meat balls"

"lasagna"

Borrowed from www.lsw.uni-heidelberg.de/~mcamenzi/NS_Mass.html





Lorenz, Ravenhall and Pethick
Phys. Rev. Lett. 70, 379 (1993)

Ravenhall, Pethick and Wilson
Phys. Rev. Lett. 50, 2066 (1983)

$$\frac{\mathrm{E_s}}{\mathrm{V}} = \frac{u\,\sigma\,d}{r}$$ surface energy

$$\frac{\mathrm{E_C}}{\mathrm{V}} = 2\pi n'^2\, x^2 e^2 r^2 u\,\frac{1}{d+2}\left\{\frac{2-du^{1-\frac{2}{d}}}{d-2}+u\right\}$$ Coulomb energy

$$\widetilde{\mathrm{E}}(n') = n'\left\{\mathrm{E}_0 + \frac{K_s}{18}\left(1-\frac{n'}{n_s}\right)^2\right\}$$ bulk energy of dense phase

$$\mathrm{E_{tot}} = \mathrm{E_e} + u\widetilde{\mathrm{E}}(\mathrm{n'}) + \mathrm{E_s} + \mathrm{E_c}$$

Figure of merit to remember

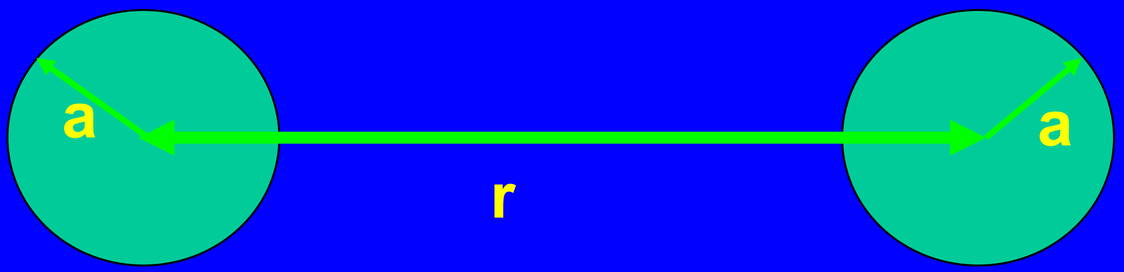
0.005 MeV/fm^3 = 5 keV/fm^3

Let me create a caricature of a the “pasta phase” in the crust of a neutron star.

Imagine that the entire plane is a cross section through a space filled with non-interacting fermions at some finite density and zero temperature.

Let me create two empty spherical or cylindrical regions of radius a and separated by r.

Question: What is the most favorable arrangement of these two spheres?



Answer: The energy of the system does not depend on r as long as r > 2a.
NB Assuming that a liquid drop model for the fermions is accurate!

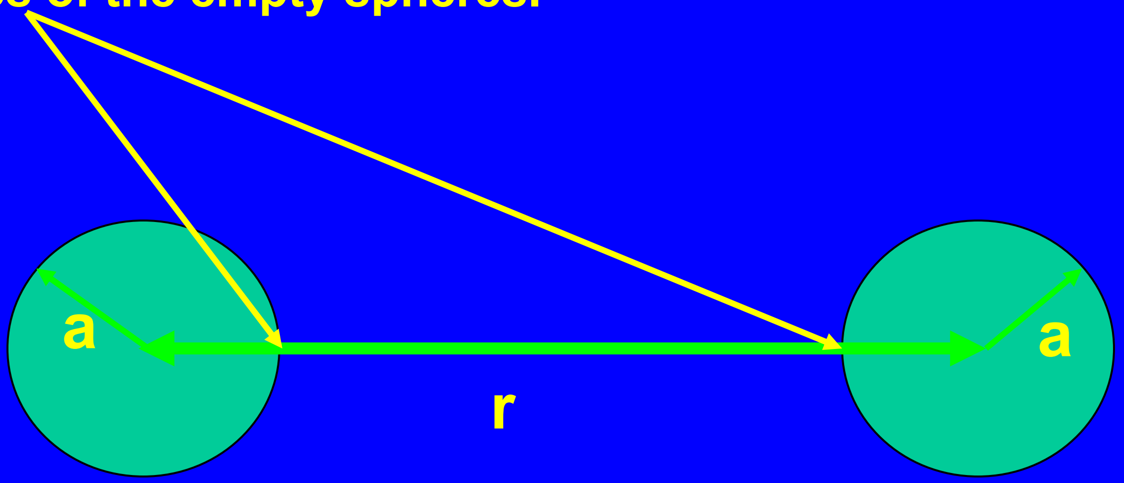
This is a very strange answer! Isn’t it? Something is amiss here.

Let us try to think of this situation now in quantum mechanical terms.

The dark blue region is really full of de Broglie's waves, which is in the absence of homogeneities are simple plane waves.

When inhomogeneities are present, there are a lot of scattered waves.

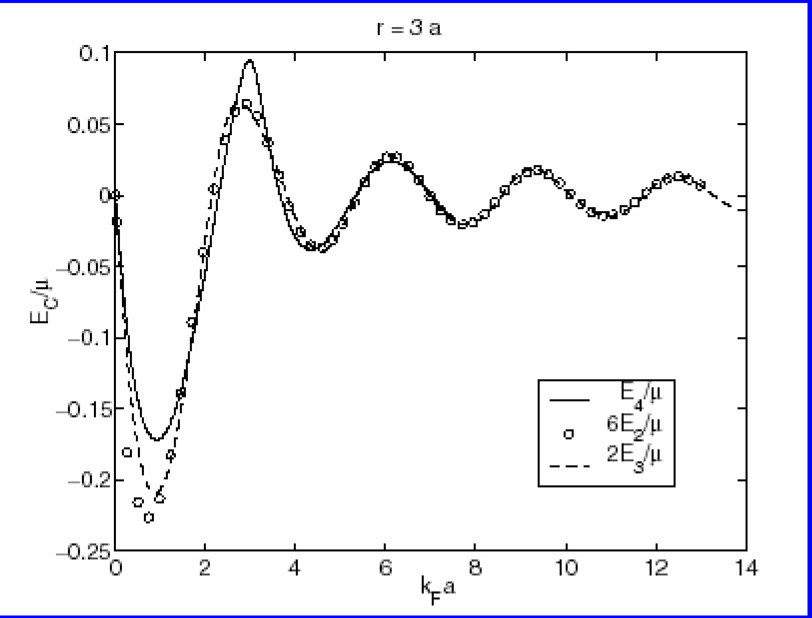
Also, there are some almost stationary waves, which reflect back and forth from the two tips of the empty spheres.

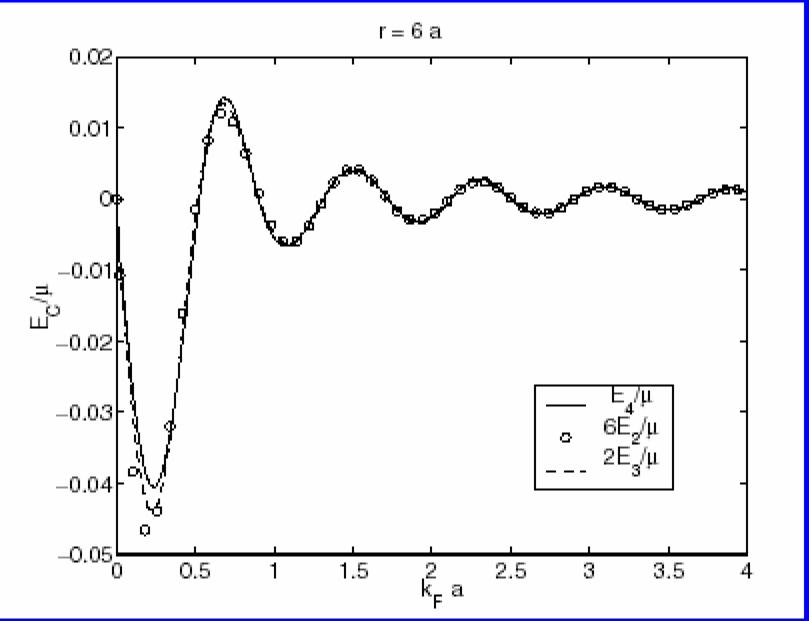


As in the case of a musical instrument, in the absence of damping, the stable "musical notes" correspond to stationary modes.

Problems: 1) There is a large number of such modes.
2) The tip-to-tip modes cannot be absolutely stable, as the reflected wave disperses in the rest of the space.

Casimir Interaction among Objects Immersed in a Fermionic Environment





The ratio of the exact Casimir energy and the chemical potential for four equidistant spheres of radius a separated by r forming a tetrahedron and also the same ratio computed as a sum of interactions between pairs or triplets for two different separations.

$$E_C \approx -\mu \frac{a^2}{\pi r (r-2a)} j_1[2k_F(r-2a)] \xrightarrow[r>>a]{} \frac{\hbar^2 k_F a^2}{8\pi m} \frac{\cos(2k_F(r-2a))}{r^3}$$

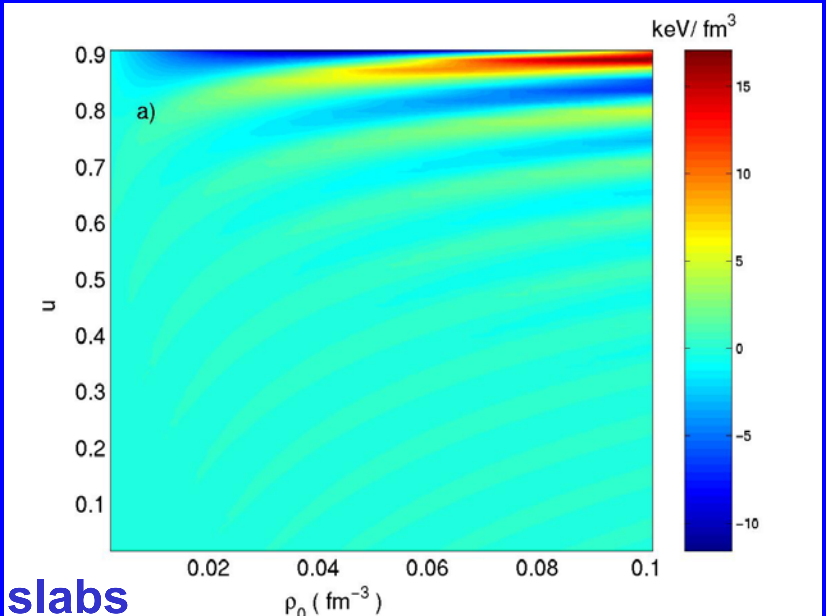
two spheres

$$E_C \approx -\mu \frac{a}{\pi (r-a)} j_1[2k_F(r-a)]$$

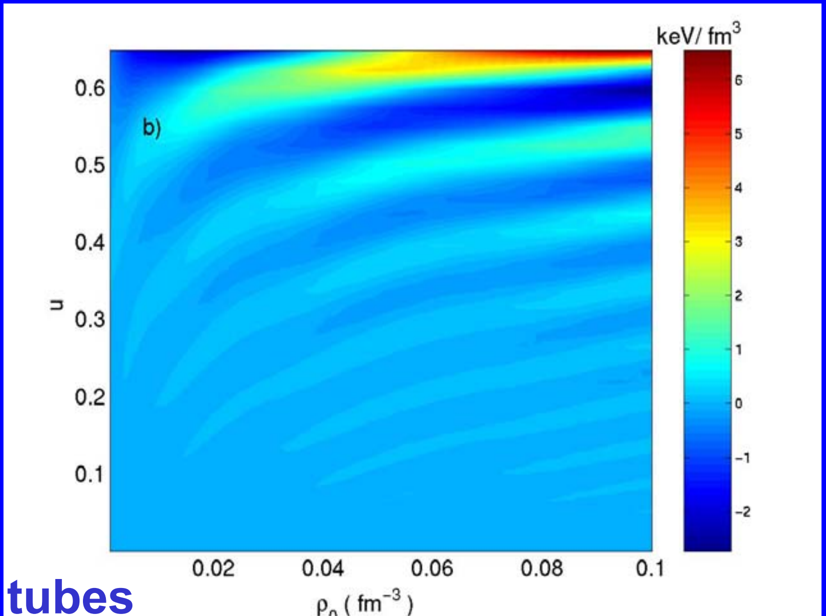
sphere next to a plane

A. Bulgac and A. Wirzba, Phys. Rev. Lett. 87, 120404 (2001).

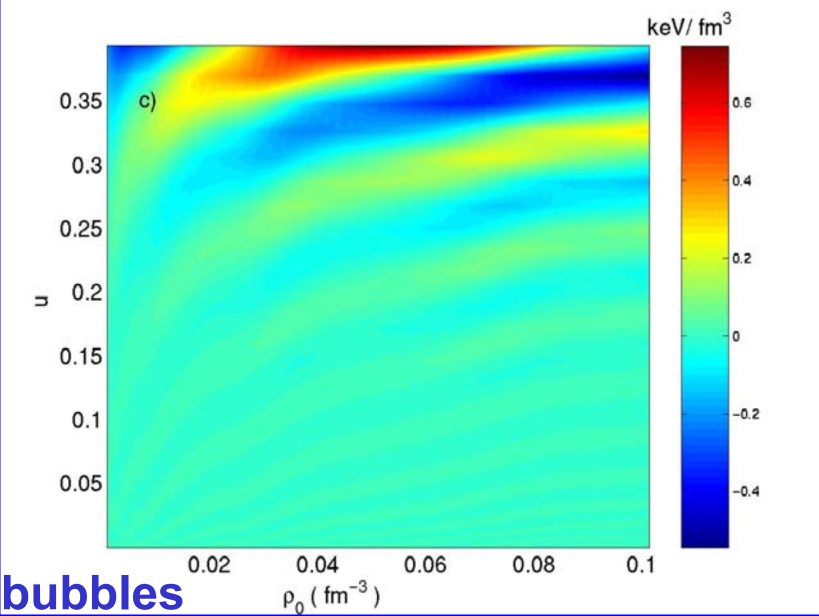
Quantum Corrections to the GS Energy of Inhomogeneous NM



slabs



tubes

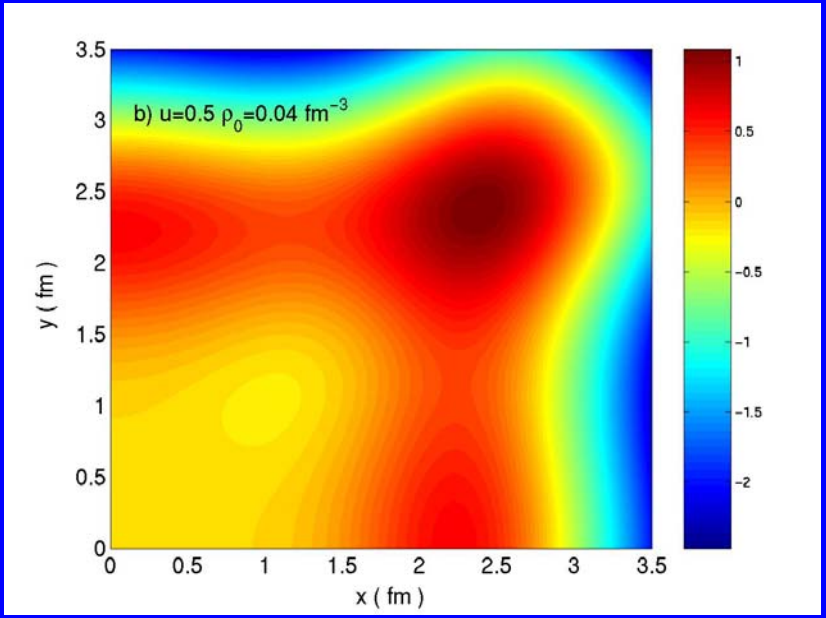


bubbles

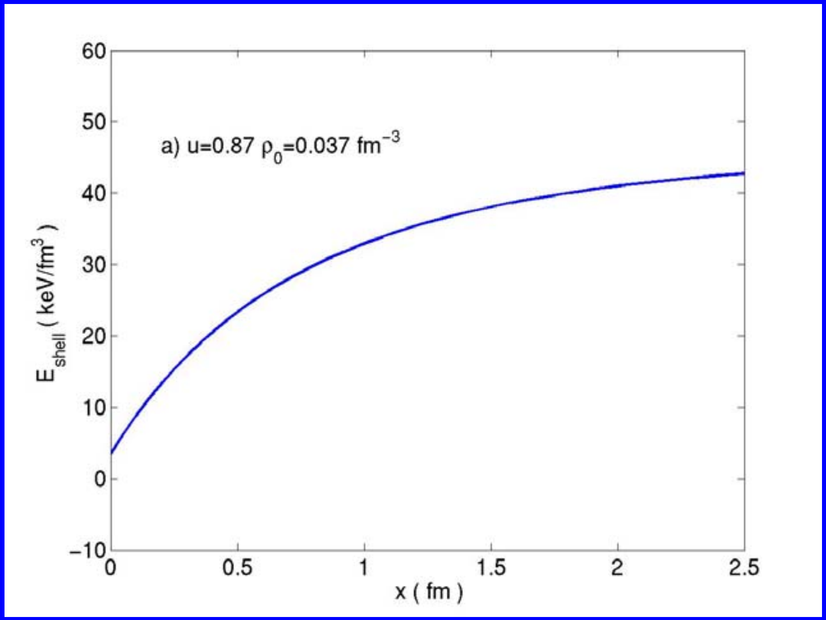
The Casimir energy for various phases.
The lattice constants are:
L = 23, 25 and 28 fm respectively.
u — anti-filling factor
$\mathbf{r}_0$ — average density

A. Bulgac and P. Magierski
Nucl. Phys. 683, 695 (2001)
Nucl. Phys, 703, 892 (2002) (E)

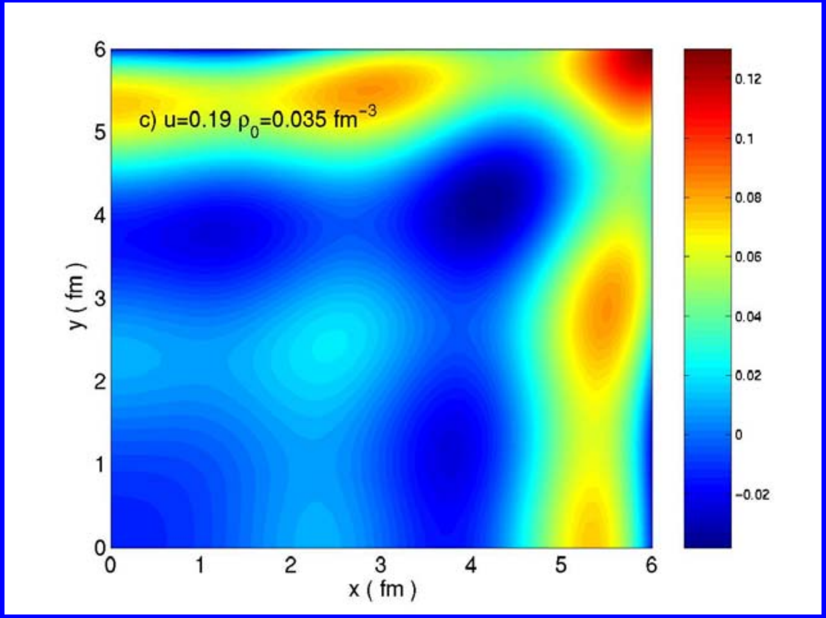
The Casimir energy for the displacement of a single void in the lattice



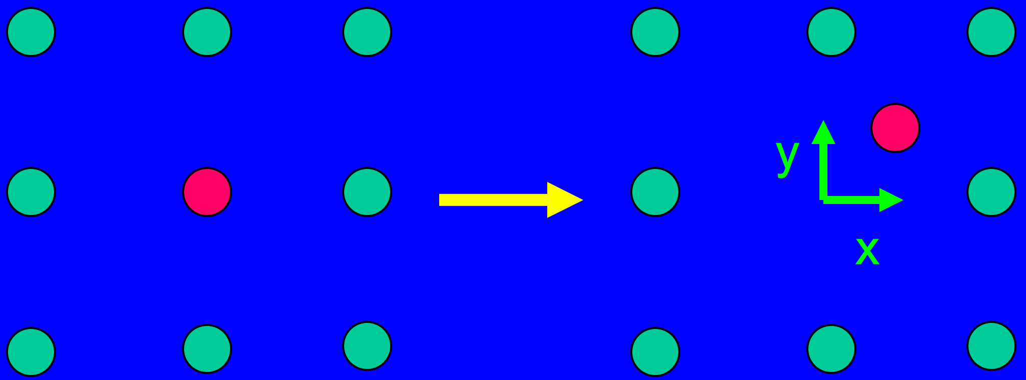
Rod phase



Slab phase



Bubble phase



A. Bulgac and P. Magierski
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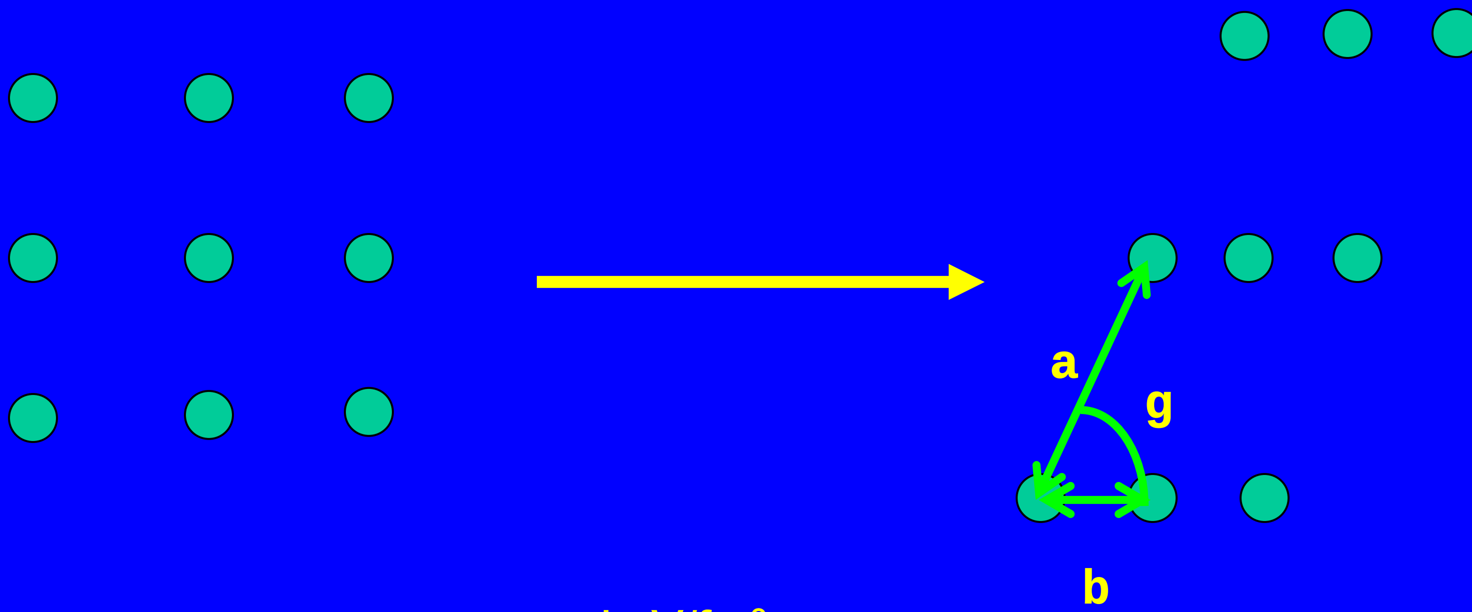
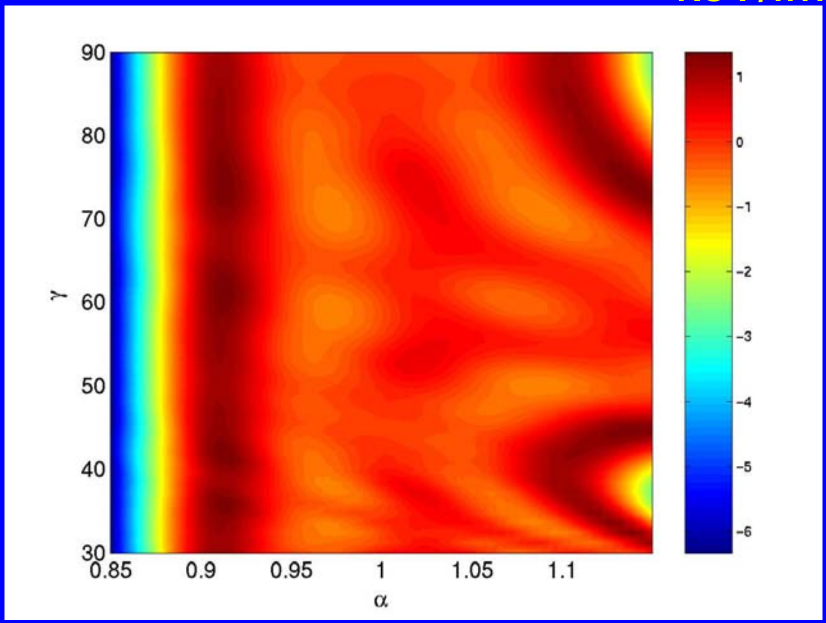
Deformation of the rod-like phase lattice



keV/fm^3



$$\alpha\beta\sin\gamma = 1$$

volume conservation

A. Bulgac and P. Magierski
Nucl. Phys. 683, 695 (2001)
Nucl. Phys, 703, 892 (2002) (E)

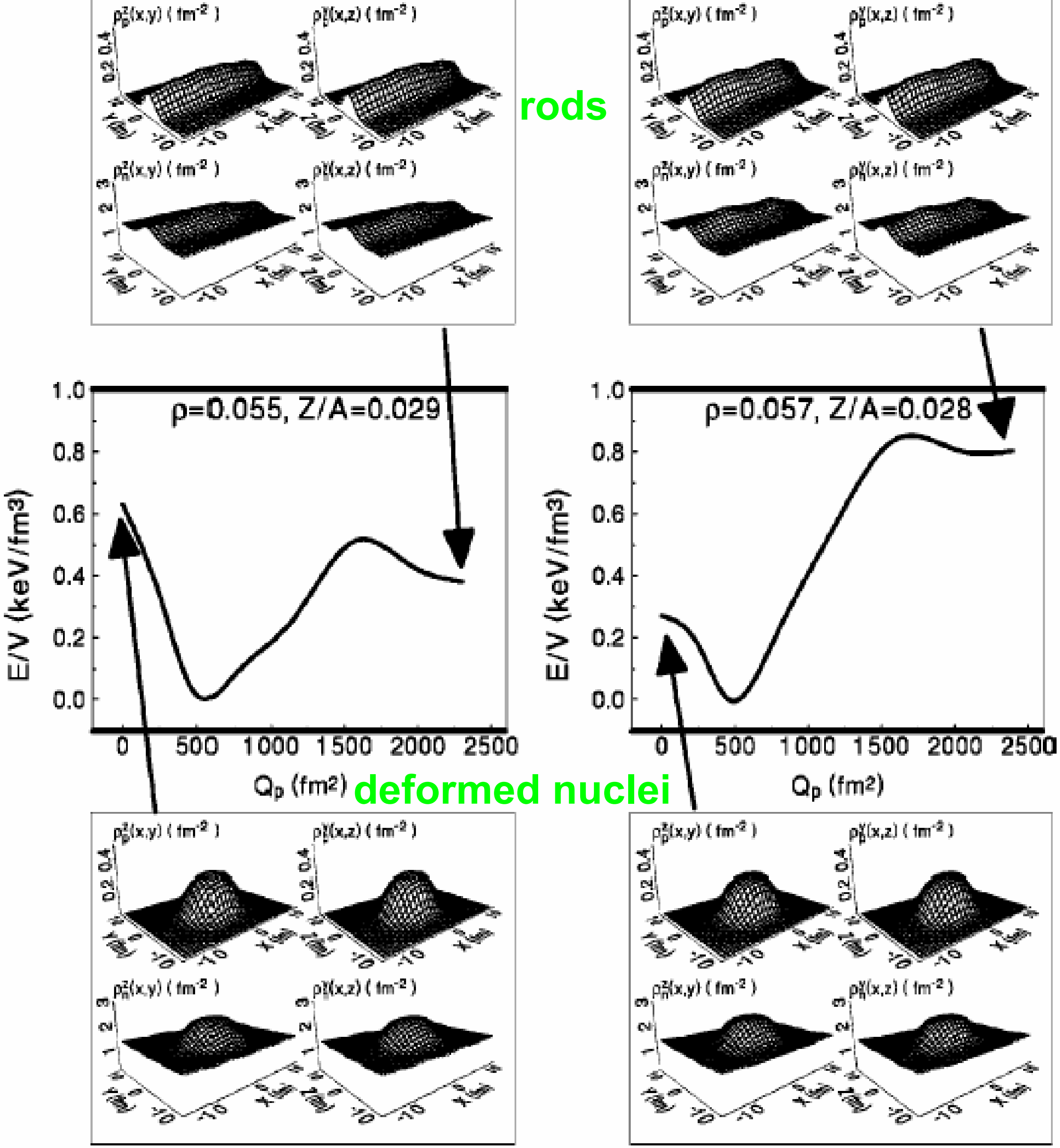


FIG. 2. The total energy density (1) of npe matter as a function of the proton quadrupole moment $Q_p = Q_{20}^p$ (middle subfigures). The integrated proton and neutron densities (see text for definition) corresponding to nuclear configurations indicated by arrows are shown in the lower and upper subfigures.

Skyrme HF with SLy4, Magierski and Heenen, Phys. Rev. C 65, 045804 (2002)

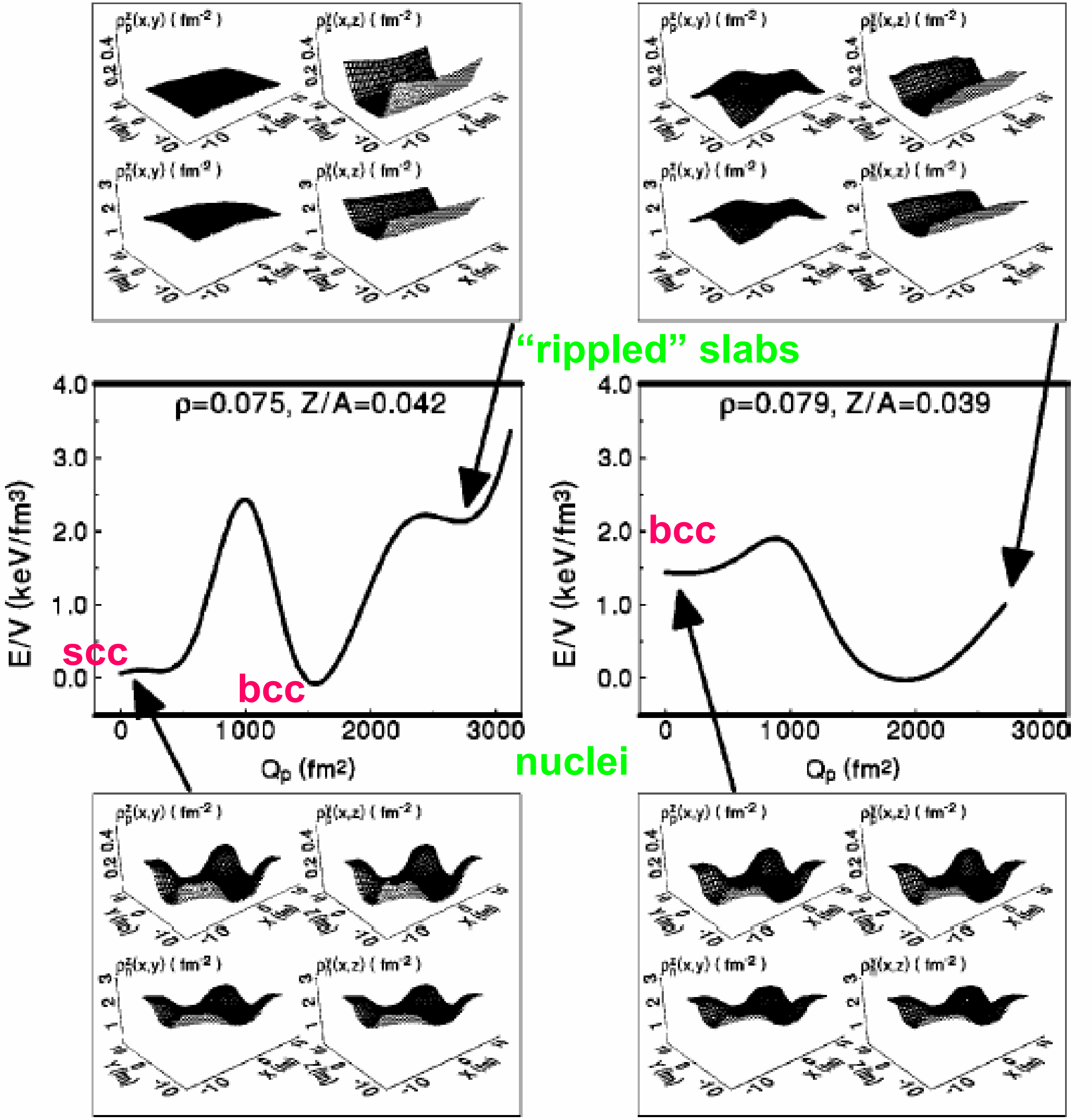


FIG. 3. The same as in the Fig. 2 but for different densities.

Skyrme HF with SLy4, Magierski and Heenen, Phys. Rev. C 65, 045804 (2002)

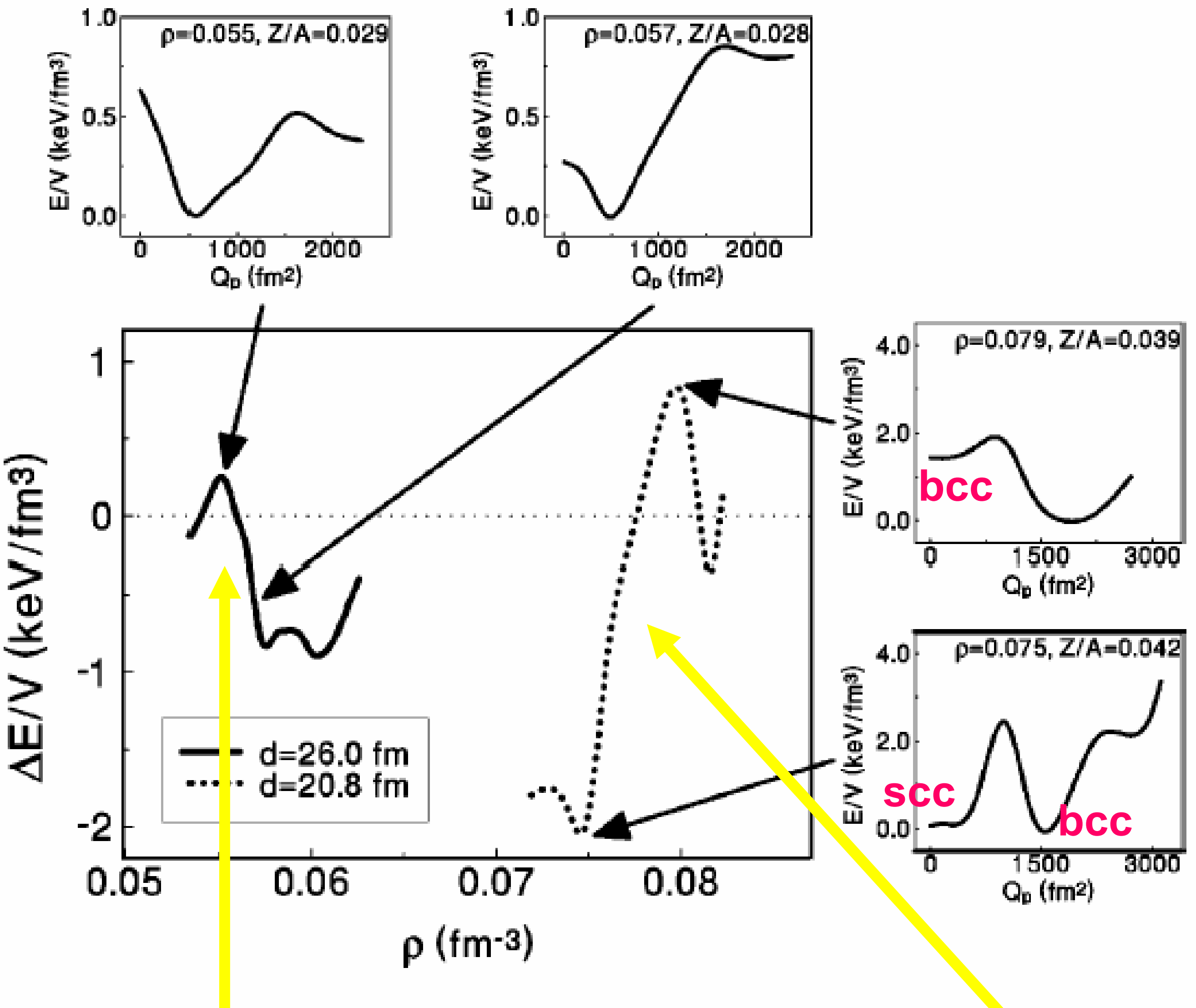


FIG. 4. The energy density difference $\Delta E/V$ between nuclear phases as a function of the total density. Solid curve denotes the difference between the spherical and rodlike phase. Dotted curve denotes the difference between the spherical and slablike phase. Smaller subfigures show the energy density of *npe* matter as a function of the proton quadrupole moment for four different densities. Parameter *d* denotes the length of the cubic box.

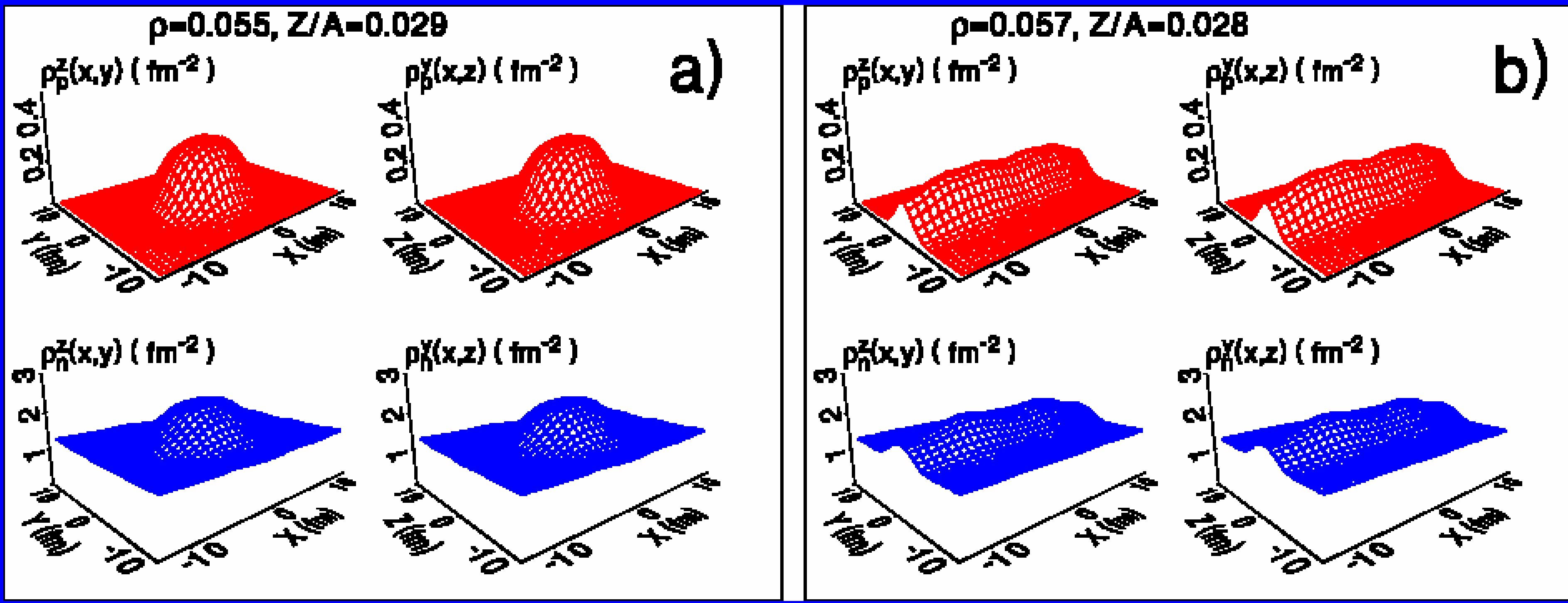
ΔE between spherical and rod-like phases

ΔE between spherical and slab-like phases

Skyrme HF with SLy4, Magierski and Heenen, Phys. Rev. C 65, 045804 (2002)

"Spherical" phase (scc)

Rod-like phase

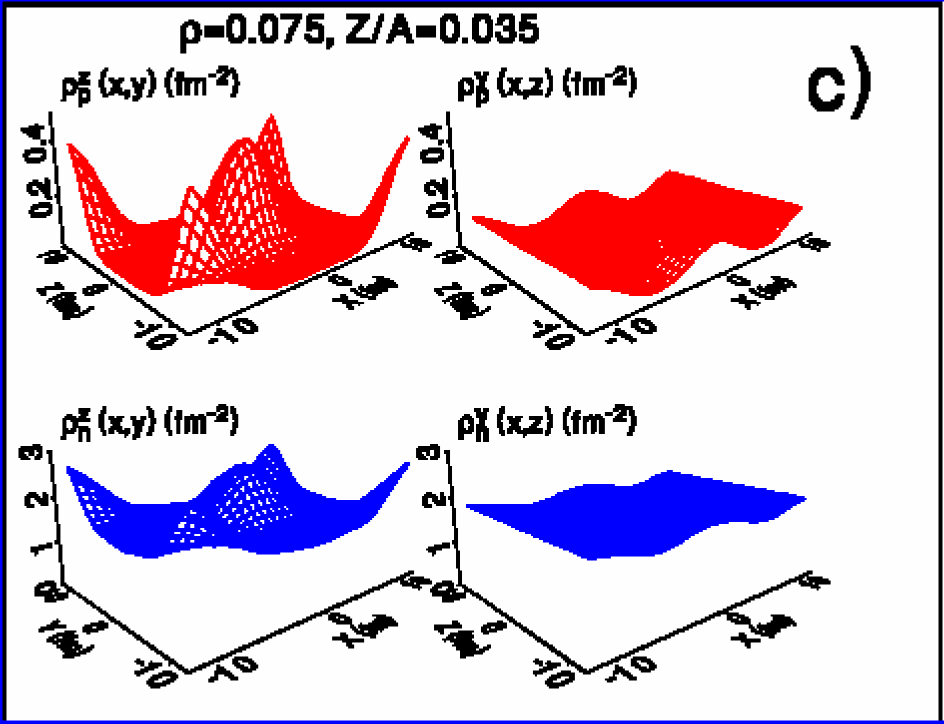


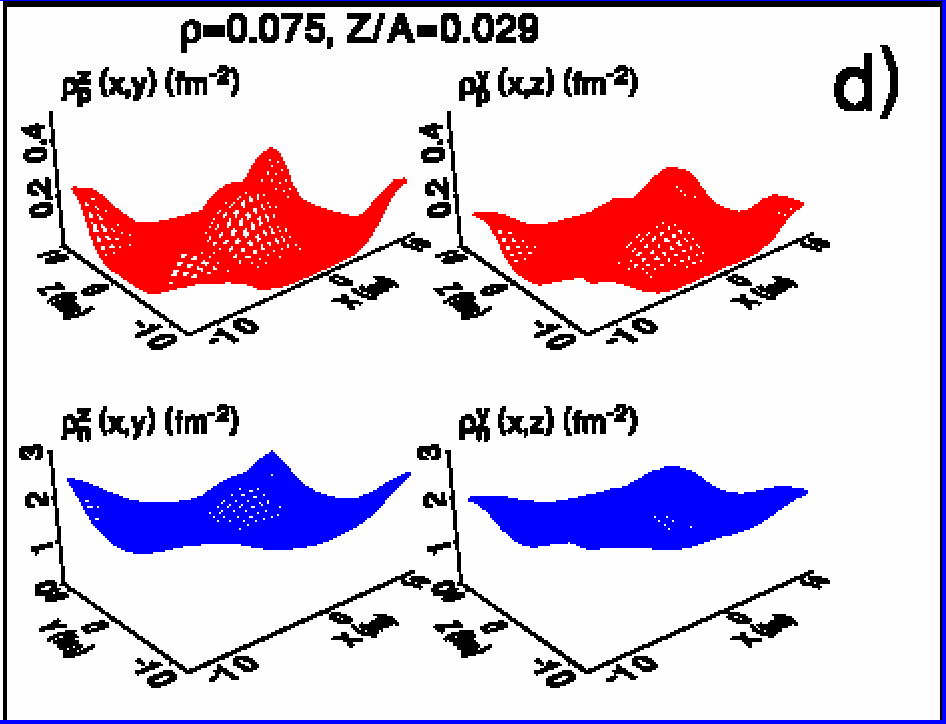
Size of box d = 26 fm

P. Magierski, A. Bulgac and P.-H. Heenen, nucl-ph/0112003

Rod-like phase

"Spherical" phase (bcc)



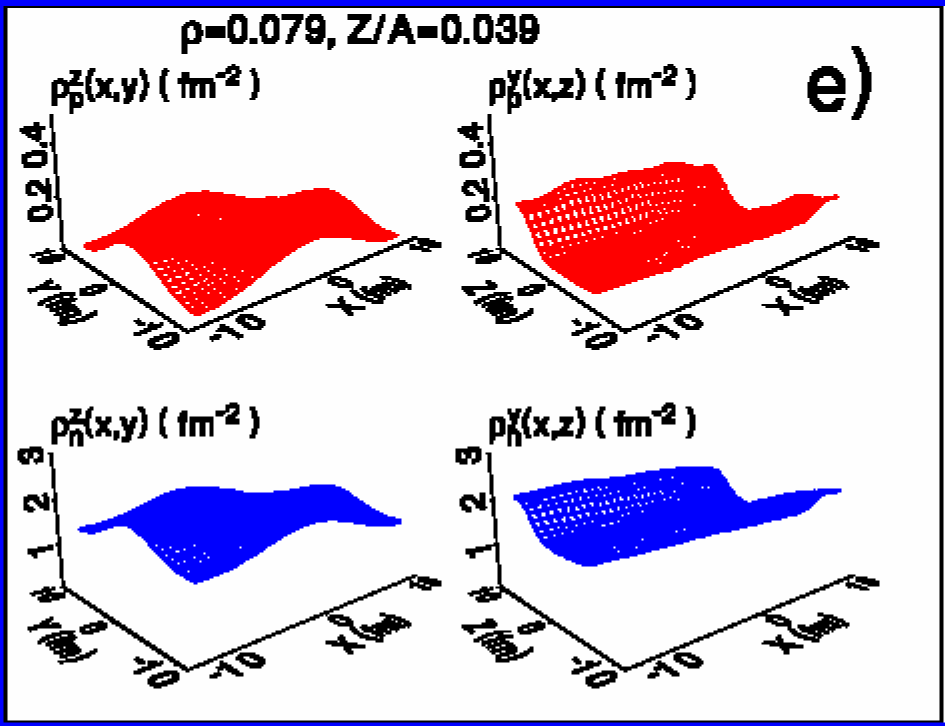


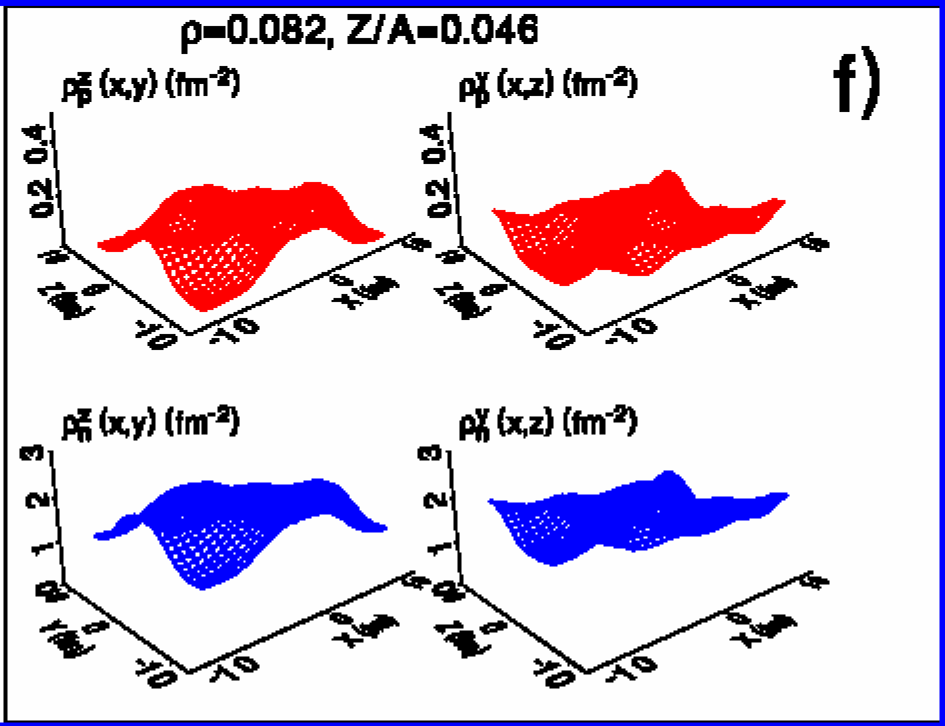
Size of box d = 23.4 fm

P. Magierski, A. Bulgac and P.-H. Heenen, nucl-ph/0112003

Slab-like phase

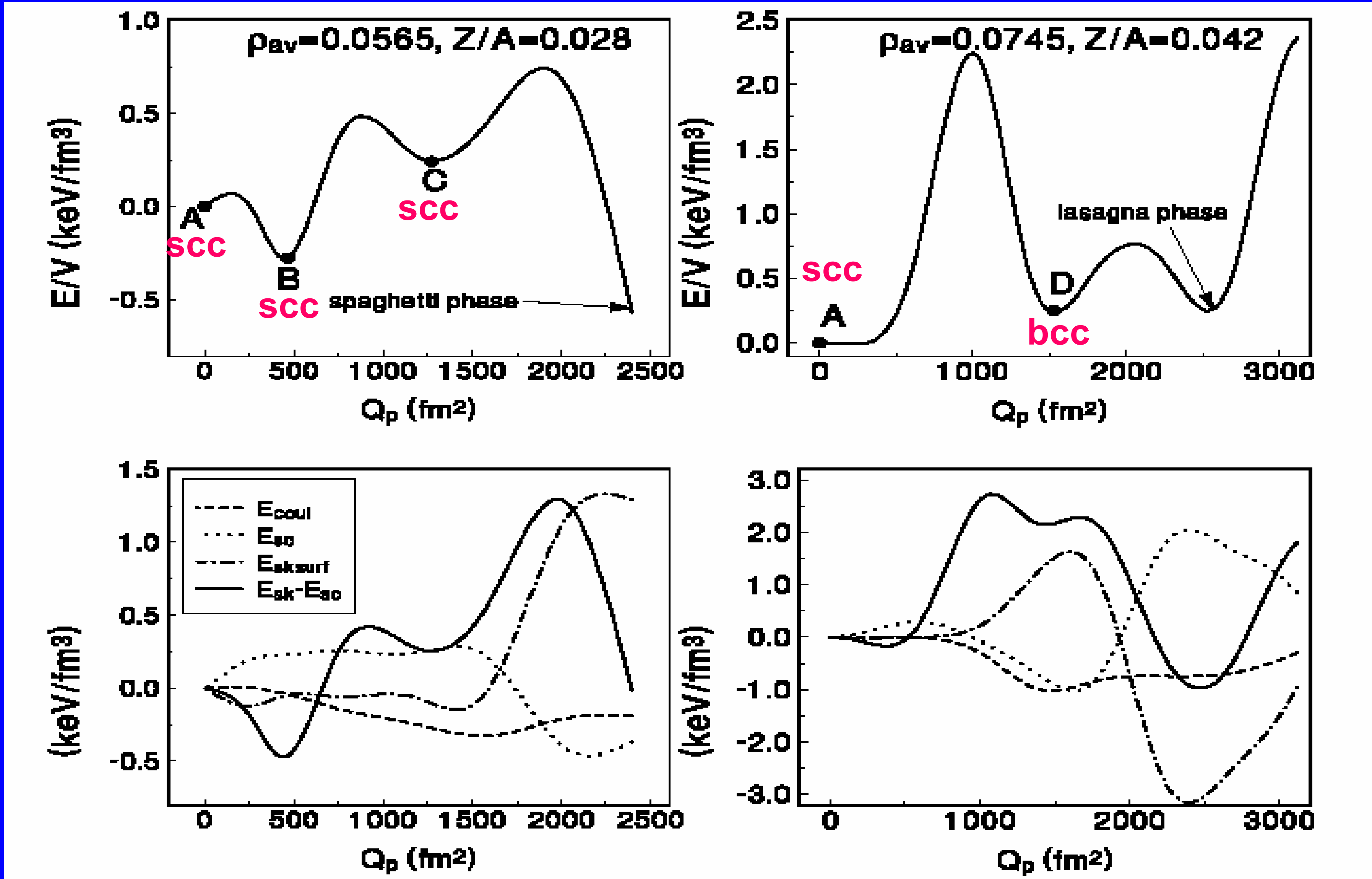
Bubble-like phase





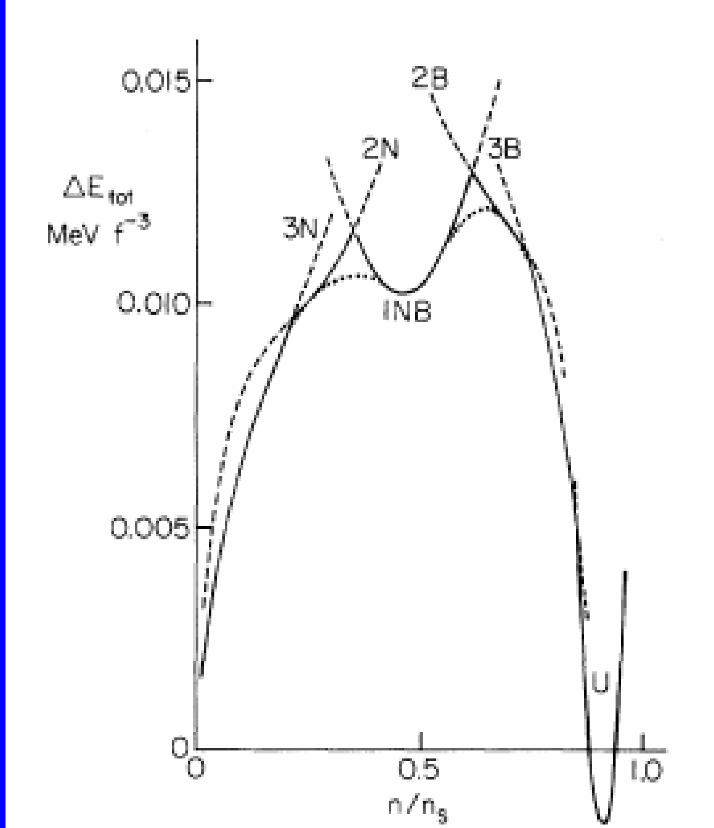
Size of box d = 20.8 fm

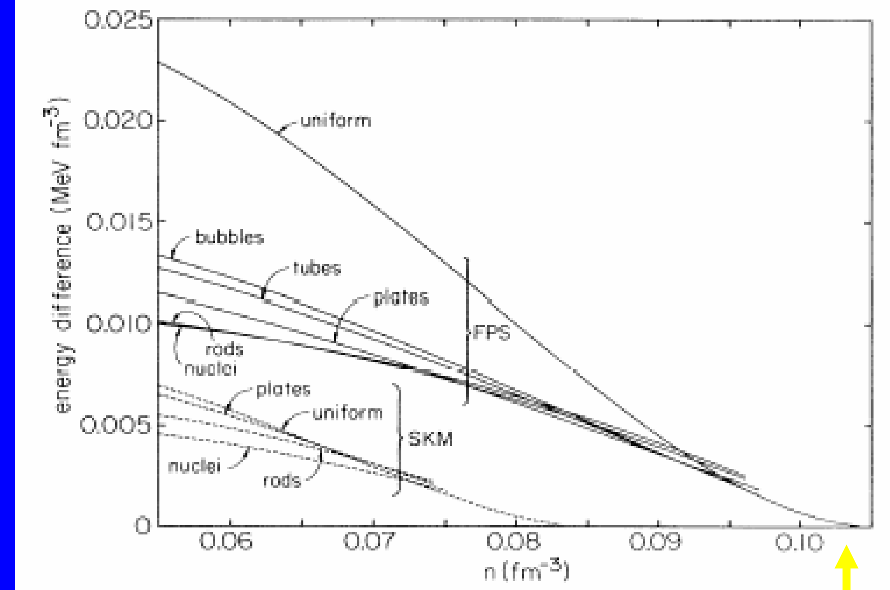
P. Magierski, A. Bulgac and P.-H. Heenen, nucl-ph/0112003



Various contributions to the energy density as a function of the proton quadrupole moment.

Magierski, Bulgac and Heenen, Nucl.Phys. A719, 217c (2003)





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Phys. Rev. Lett. 70, 379 (1993)

Ravenhall, Pethick and Wilson
Phys. Rev. Lett. 50, 2066 (1983)

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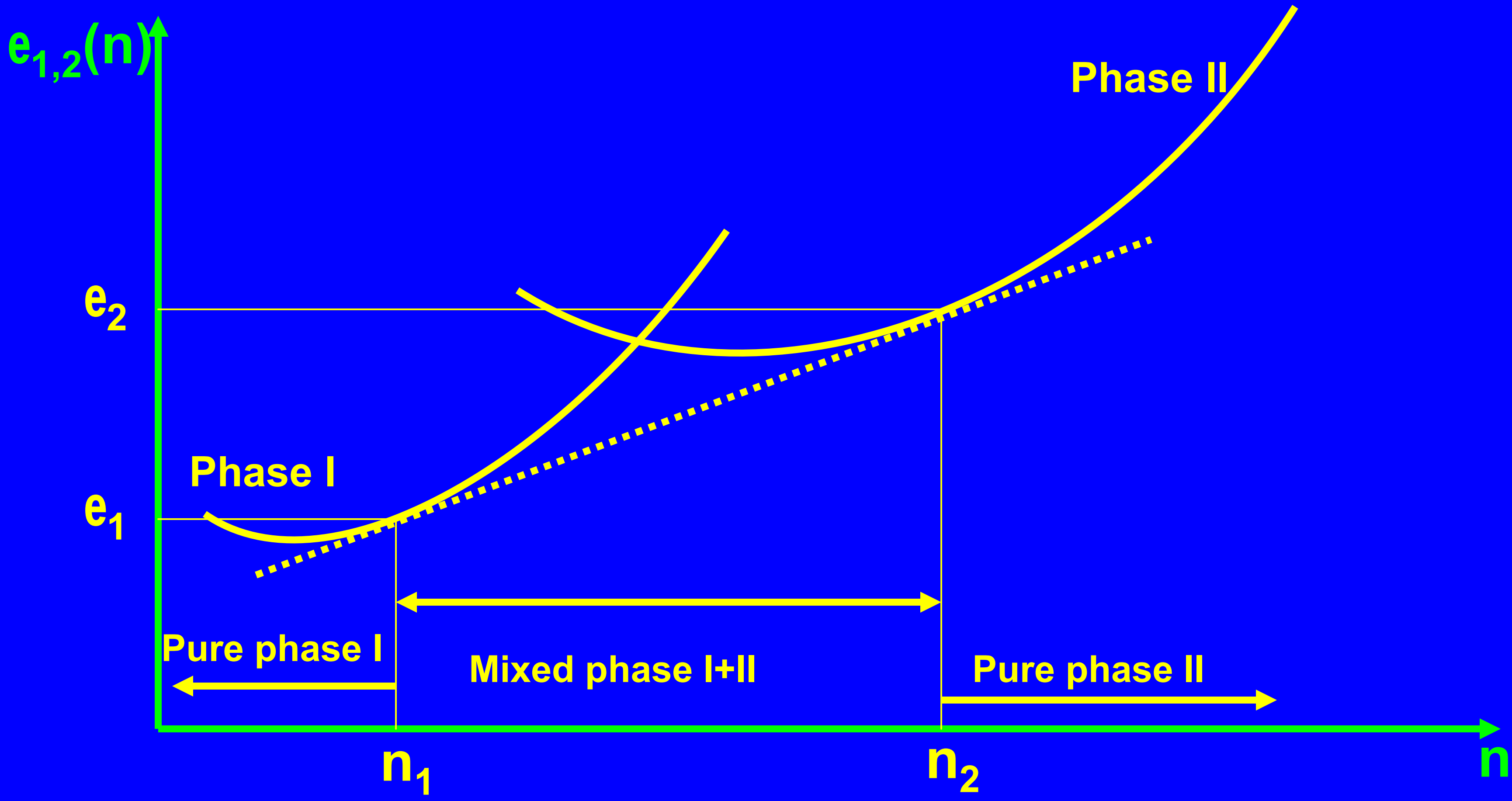
$$\frac{E_C}{V} = 2\pi n'^2 x^2 e^2 r^2 u \frac{1}{d+2}\left\{\frac{2 - d u^{1-\frac{2}{d}}}{d-2} + u\right\}$$ Coulomb energy

$$\tilde{E}(n') = n'\left\{E_0 + \frac{K_s}{18}\left(1 - \frac{n'}{n_s}\right)^2\right\}$$ bulk energy of dense phase

$$E_{tot} = E_e + u\tilde{E}(n') + E_s + E_c$$



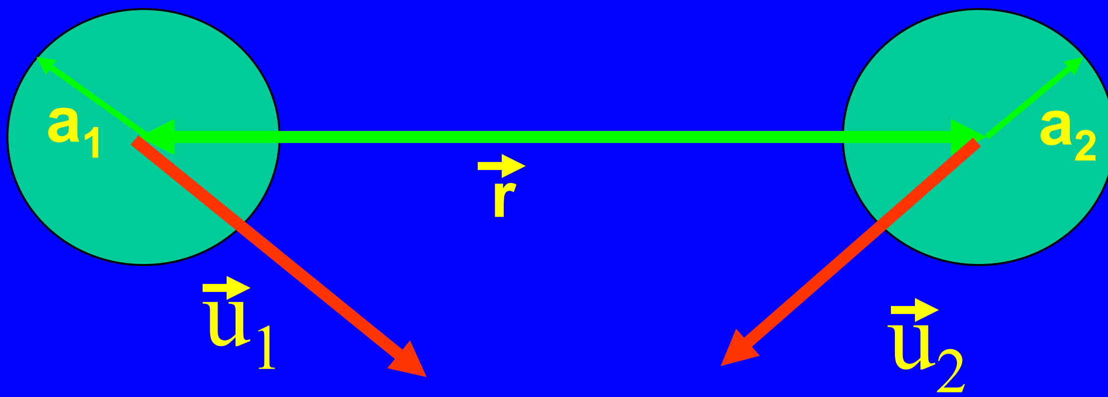
$$E = \left[x\,\varepsilon_1(n_1) + (1-x)\,\varepsilon_2(n_2)\right]V \qquad \frac{d(E-\mu N)}{dn_1} = 0$$

$$N = \left[x n_1 + (1-x)\,n_2\right]V = nV \qquad \frac{d(E-\mu N)}{dn_2} = 0$$

$$x = \frac{n - n_1}{n_2 - n_1} \qquad \frac{d(E-\mu N)}{dx} = 0$$

$$\mu = \frac{d\varepsilon_1(n_1)}{dn_1} = \frac{d\varepsilon_2(n_2)}{dn_2} = \frac{\varepsilon_2(n_2) - \varepsilon_1(n_1)}{n_2 - n_1}$$

Let us consider now two moving spheres in the superfluid medium at velocities below the critical velocity for the loss of superfluidity.



$$T_{kin} = \frac{M_1^{ren}\vec{u}_1^2 + M_2^{ren}\vec{u}_2^2}{2} + 4\pi\rho_{out}\left(\frac{1-\gamma}{2+\gamma}\right)^2\left(\frac{a_1 a_2}{r}\right)^3\left[\vec{u}_1\cdot\vec{u}_2 - \frac{3}{r^2}(\vec{u}_1\cdot\vec{r})(\vec{u}_2\cdot\vec{r})\right]$$

$$M_i^{ren} = \frac{4\pi}{3}a_i^3\rho_{in}m\frac{(1-\gamma)^2}{2\gamma+1} = M_i\frac{(1-\gamma)^2}{2\gamma+1}, \qquad i = 1,2$$

$$\gamma = \frac{\rho_{out}}{\rho_{in}}$$

Kinetic energy has a similar $1/r^3$ dependence as the Casimir energy!

What have we established so far?

✓ Quantum corrections (Casimir energy) to the ground state energy of inhomogeneous neutron matter are of the same magnitude or larger then the energy differences between various simple phases.

✓ Lattice defects and lattice distortions have characteristic energy changes of the same order of magnitude.

✓ Only relatively large temperatures (of order of 10 MeV) lead to the disappearance of these quantum energy corrections.

✓ Fully self-consistent calculations confirm the fact that the "pasta phase" might have a rather complex structure, various shapes can coexist, at the same time significant lattice distortions are likely and the neutron star crust could be on the verge of a disordered phase.

• Pethick and Potekhin, Phys. Lett. B 427, 7 (1998) present argument in favor of a liquid crystal structure of the "pasta phase."

• Jones, Phys. Rev. Lett. 83, 3589 (1999) claims that the thermal fluctuations are so large that the system likely cools down to an amorphous and heterogeneous phase.

✓ Dynamics of these structures is important (not really covered in detail here)

Now I shall switch gears and discuss some aspects of the physics of vortices in low density neutron matter .

A vortex is just about the only phenomenon in which a true stable superflow is created in a neutral system

- ✓ **I shall describe briefly the DFT-LDA to superfluid Fermi systems:**

  SLDA (Superfluid LDA)

- ✓ **I shall apply this theory to describe the basic properties of a vortex in low density neutron matter.**

SLDA equations for superfluid Fermi systems:

Energy Density (ED) describing the normal phase

Additional contribution to ED due to superfluid correlations

$$E_{gs} = \int d^3r [\mathcal{E}_N(\boldsymbol{r}) + \mathcal{E}_S(\boldsymbol{r})],$$

$$\mathcal{E}_S(\boldsymbol{r}) := -\Delta(\boldsymbol{r})\nu_c(\boldsymbol{r}) = g_{\text{eff}}(\boldsymbol{r})|\nu_c(\boldsymbol{r})|^2,$$

$$\begin{cases} [h(\boldsymbol{r}) - \mu]u_i(\boldsymbol{r}) + \Delta(\boldsymbol{r})v_i(\boldsymbol{r}) = E_i u_i(\boldsymbol{r}), \\ \Delta^*(\boldsymbol{r})u_i(\boldsymbol{r}) - [h(\boldsymbol{r}) - \mu]v_i(\boldsymbol{r}) = E_i v_i(\boldsymbol{r}), \end{cases}$$

$$h(\boldsymbol{r}) = -\boldsymbol{\nabla}\frac{\hbar^2}{2m(\boldsymbol{r})}\boldsymbol{\nabla} + U(\boldsymbol{r}), \quad \Delta(\boldsymbol{r}) := -g_{\text{eff}}(\boldsymbol{r})\nu_c(\boldsymbol{r}),$$

$$\frac{1}{g_{\text{eff}}(\boldsymbol{r})} = \frac{1}{g(\boldsymbol{r})} - \frac{mk_c(\boldsymbol{r})}{2\pi^2\hbar^2}\left[1 - \frac{k_F(\boldsymbol{r})}{2k_c(\boldsymbol{r})}\ln\frac{k_c(\boldsymbol{r}) + k_F(\boldsymbol{r})}{k_c(\boldsymbol{r}) - k_F(\boldsymbol{r})}\right]$$

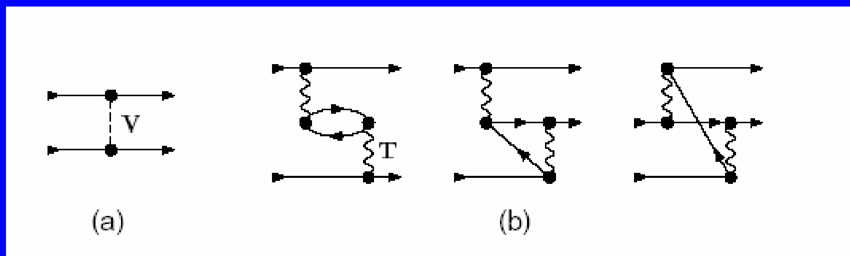
$$\rho_c(\boldsymbol{r}) = \sum_{E_i \geq 0}^{E_c} 2|v_i(\boldsymbol{r})|^2, \quad \nu_c(\boldsymbol{r}) = \sum_{E_i \geq 0}^{E_c} v_i^*(\boldsymbol{r})u_i(\boldsymbol{r}),$$

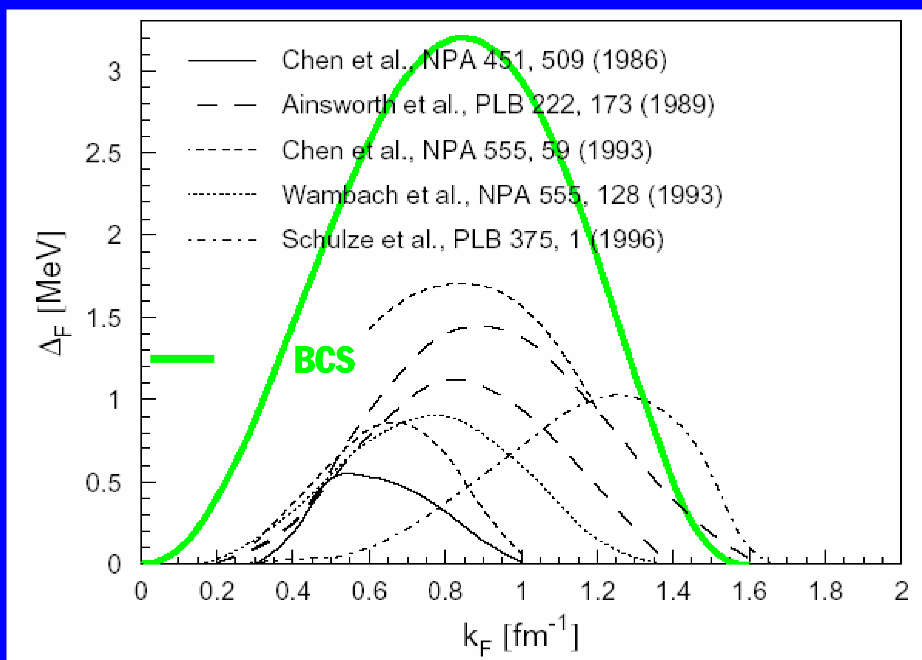
$$E_c + \mu = \frac{\hbar^2 k_c^2(\boldsymbol{r})}{2m(\boldsymbol{r})} + U(\boldsymbol{r}), \ \mu = \frac{\hbar^2 k_F^2(\boldsymbol{r})}{2m(\boldsymbol{r})} + U(\boldsymbol{r}).$$

Typo: replace m by m(r)

Y.Yu and A. Bulgac, PRL 90, 222501 (2003)

“Screening effects” are significant!



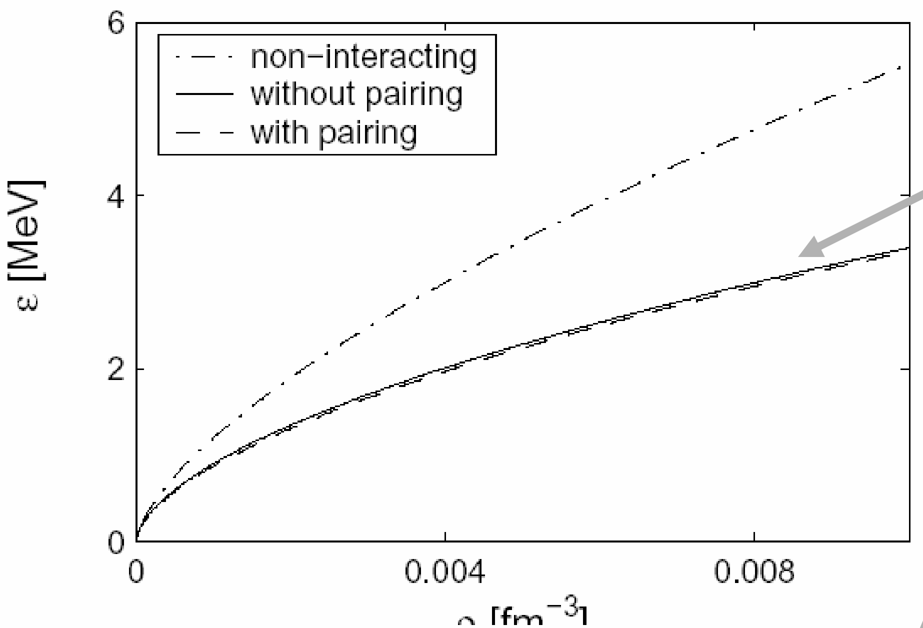


s-wave pairing gap in infinite neutron matter with realistic NN-interactions

from Lombardo and Schulze astro-ph/0012209

These are major effects beyond the naïve HFB

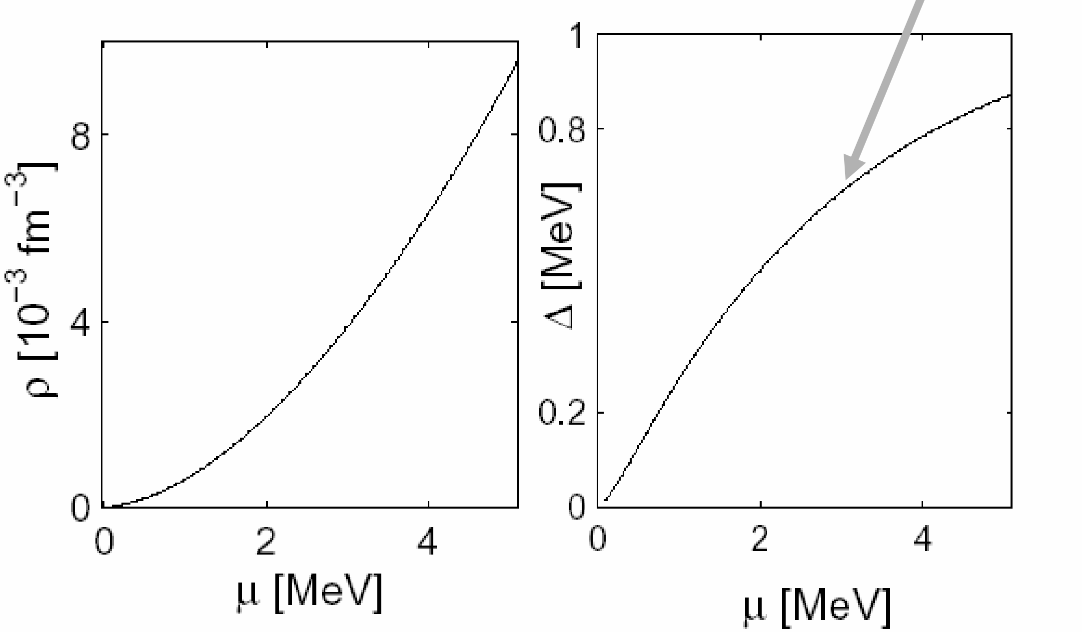
Fayans's FaNDF^0



$$\Delta = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(-\frac{\pi}{2\tan\delta(k_F)}\right)$$

An additional factor of **1/(4e)$^{1/3}$** is due to induced interactions

Naïve HFB/BCS not valid.



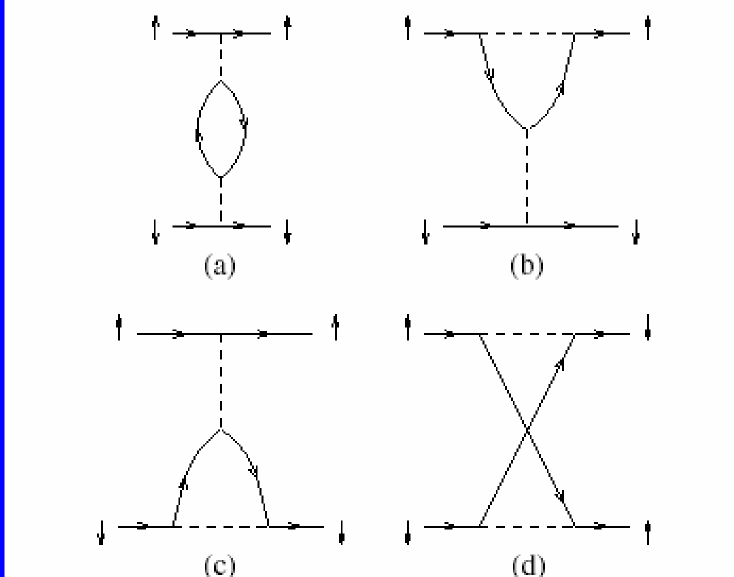


FIG. 1. Diagrams for the induced interactions between two fermions in different internal states to second order in the effective interaction.

from Heiselberg et al Phys. Rev. Lett. 85, 2418, (2000)

Y. Yu and A. Bulgac, PRL 90, 161101 (2003)

Landau criterion for superflow stability

(flow without dissipation)

Consider a superfluid flowing in a pipe with velocity v_{s}:

$$E_0 + \frac{Nm\mathrm{v}_s^2}{2} < E_0 + \varepsilon_{\vec{p}} + \vec{\mathrm{v}}_S \cdot \vec{p} + \frac{Nm\mathrm{v}_s^2}{2} \Rightarrow \mathrm{v}_s < \frac{\varepsilon_{\vec{p}}}{p}$$

no internal excitations

One single quasi-particle excitation with momentum p

In the case of a Fermi superfluid this condition becomes

$$\frac{\mathrm{v}_S}{\mathrm{v}_F} < \frac{\Delta}{2\varepsilon_F}$$

Vortex in neutron matter

$$\begin{pmatrix} u_{\alpha\,\mathrm{kn}}(\vec{r}) \\ v_{\alpha\,\mathrm{kn}}(\vec{r}) \end{pmatrix} = \begin{pmatrix} u_{\alpha}(r)\exp[i(n+1/2)\phi - ikz] \\ v_{\alpha}(r)\exp[i(n-1/2)\phi - ikz] \end{pmatrix}, \quad \text{n - half-integer}$$

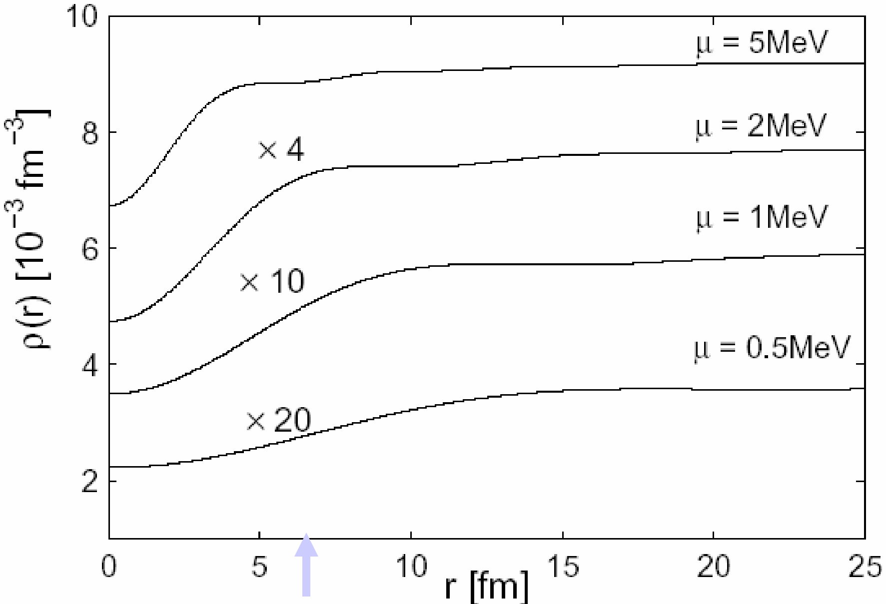
$\Delta(\vec{r}) = \Delta(r)\exp(i\phi),$ $\quad \vec{r} = (r,\phi,z)$ [cyllindrical coordinates]

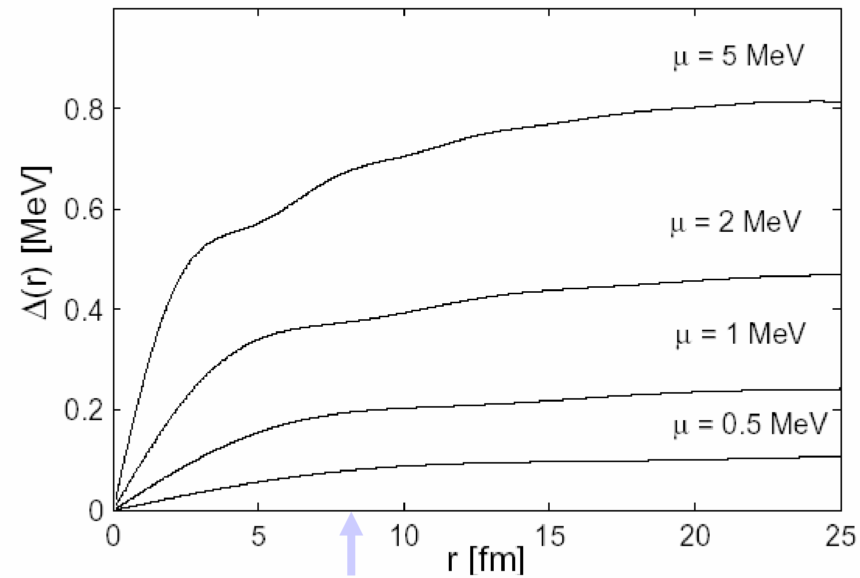
Oz - vortex symmetry axis

Ideal vortex, Onsager's quantization (one $\hbar$ per Cooper pair)

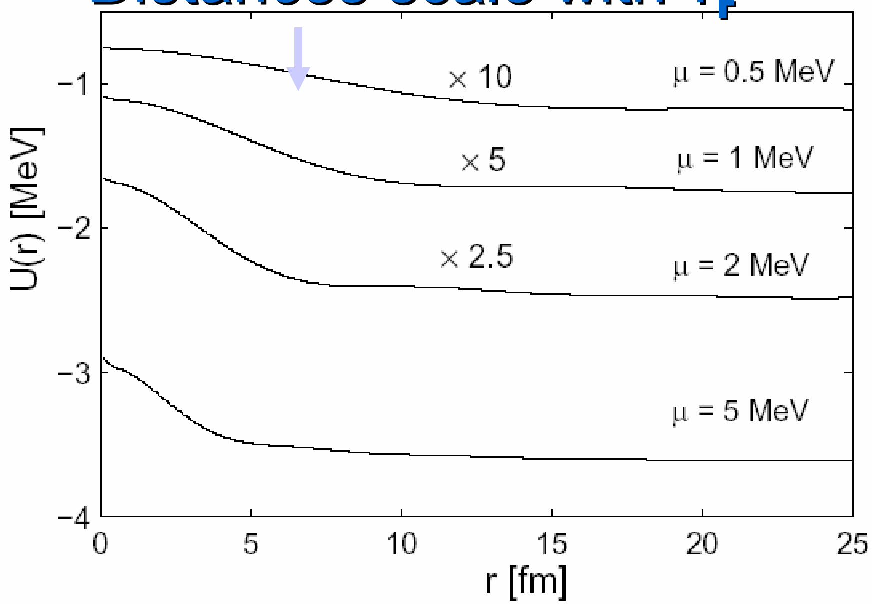
$$\vec{V}_{\mathrm{v}}(\vec{r}) = \frac{\hbar}{2mr^2}(y,-x,0) \quad \Leftarrow \quad \frac{1}{2\pi}\oint_C \vec{V}_{\mathrm{v}}(\vec{r})\cdot d\vec{r} = \frac{\hbar}{2m}$$

Y. Yu and A. Bulgac, PRL 90, 161101 (2003)

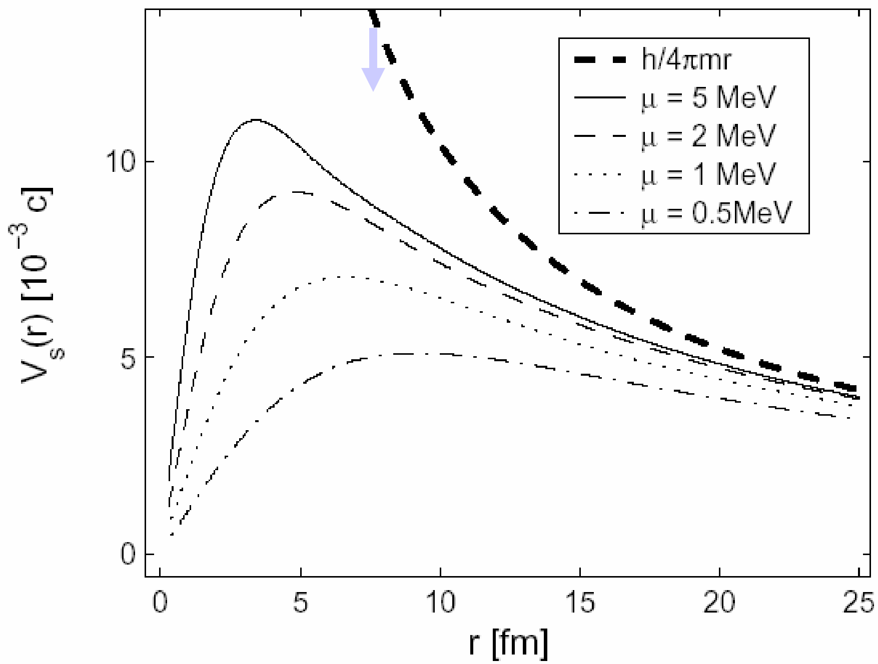




Distances scale with l_F



Distances scale with $x >> l_F$



Y. Yu and A. Bulgac, PRL 90, 161101 (2003)

Dramatic structural changes of the vortex state naturally lead to significant changes in the energy balance of a neutron star

- $\frac{v_S}{v_F} \leq \left.\frac{\Delta}{2\varepsilon_F}\right|_{max} \approx 0.12,$ extremely fast vortical motion,

  $\frac{\lambda_F}{\xi} \propto \frac{\Delta}{\varepsilon_F}$

- In low density region $\varepsilon(\rho_{out})\rho_{out} > \varepsilon(\rho_{in})\rho_{in}$

which thus leads to a large anti-pinning energy $E^V_{pin} > 0$:

$$E^V_{pin} = [\varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in}]V$$

- The energy per unit length is going to be changed dramatically

when compared to previous estimates, by

$$\frac{\Delta E_{vortex}}{L} \approx [\varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in}]\pi R^2$$

- Specific heat, transport properties are expected to significantly

affected as well.

Some similar conclusions have been reached recently also by Donati and Pizzochero, Phys. Rev. Lett. 90, 211101 (2003).

Main conclusions of this presentation:

- ✓ The crust of a neutron star has most likely a rather complex structure, among candidates: regular solid lattice, liquid crystal, significant number of defects and lattice distortions, disordered phase, amorphous and heterogeneous phase. The elastic properties of such structures vary, naturally, a lot from one structure to another.

- ✓ At very low neutron densities vortices are expected to have a very unusual spatial profile, with a prominent density depletion along the axis of the vortex. The energetics of a star is thus affected in a major way and the pinning mechanism of the vortex to impurities is changed as well.