I shall present a many body problem which is:

of fundamental theoretical interest,

of an extreme richness and a host of a large number of unanswered questions,

of extreme computational complexity,

of relevance to several fields,

a compelling testing ground for many existing theoretical methods.

See webpage of the INT Pairing Workshop November 14-17, 2005

Homework problems

In the presentations and the discussions of the Fall INT program and workshops, the idea came up that it would be useful to have a simple model Hamiltonian that could be used as a testing ground for theories and calculational approximations. The two component trapped Fermi (TCFG) gas in the unitary limit seems ideal for this purpose. Not only is it well-defined and challenging, but it is very relevant to experimental atomic trap physics, besides been of interest as well to the case of low density neutron matter. The Hamiltonian includes, besides the kinetic energy for the two Fermion species, a contact interaction, with a strength chosen so as to reproduce a specified value of the scattering length.

$$\hat{H} = \hat{T} + \hat{V} = \int d^3x \left[\psi_\uparrow^\dagger(\vec{x}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \psi_\uparrow(\vec{x}) + \psi_\downarrow^\dagger(\vec{x}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \psi_\downarrow(\vec{x}) \right] - g \int d^3x \; \hat{n}_\uparrow(\vec{x}) \hat{n}_\downarrow(\vec{x})$$

$$\hat{N} = \int d^3x \; \left[\hat{n}_\uparrow(\vec{x}) + \hat{n}_\downarrow(\vec{x}) \right], \qquad \hat{n}_s(\vec{x}) = \psi_s^\dagger(\vec{x}) \psi_s(\vec{x}), \qquad s = \uparrow, \downarrow$$

Bertsch's regime introduced in 1999 as a many-body Challenge for the MBX in Seattle is nowadays called *the unitary regime*

The system is very dilute, but strongly interacting!

$$n\, r_0^3 \ll 1$$

$$n\, |a|^3 \gg 1$$

n - number density

$$r_0 \ll n^{-1/3} \approx \lambda_F/2 \ll |a|$$

r_0 - range of interaction

a - scattering length

1) Consider at the beginning two cases in a harmonic trap, N=8 and N=20. Calculate the ground state energy in units of the trap frequency, the spectrum of the excited states, and the density distribution at the unitary point (where the scattering length is infinite) and in a region around the unitary point, when the scattering length is still much larger than the interparticle separation. Having exact results for these systems one can then compare them with DFT results, assess the role of derivative corrections, and also assess the range of validity of other theoretical schemes.

2) A number of ground state and excited states properties of the homogeneous TCFG in the unitary regime have been established in rather accurate fully non-perturbative calculations (Green Function Monte Carlo, Diffusion Monte Carlo, Auxiliary Field Monte Carlo calculations, respectively). One should test various approximate theoretical methods used so far to compute in particular pairing properties of neutron matter against these accurate numerical results.

The two component Fermi gas with contact interaction

- So far lattices 10^3 times 3000 with up to 100 fermions with a contact interaction have been considered within auxiliary PIMC and GFMC/DMC studies at T=0 for up to 66 fermions .

- One should aim for spatial lattices up to 32^3 and maybe even higher. The computational size of the problem scales roughly as the <u>fifth</u> power of the spatial lattice size.

- One can extend the calculations to include a finite range interaction in the Effective Field Theory spirit. At least a lattice perturbative approach to finite range effects is clearly feasible.

- Calculations should be extended to spin unsaturated systems. There exist (incomplete) arguments that the sign problem is not a major hurdle for spin unsaturated system as one would naively expect.

<u>Why shall we look at this problem?</u>

- Is a pretty good model for dilute neutron matter and one can thus extract the EOS and paring properties of such a system

- It is a very clean many-body problem where one can obtain a numerically very accurate description

- This problem allows us to understand very well the many body physics of strongly interacting systems – <u>textbook</u>

- This is directly observable in Fermionic atomic clouds, where the interaction strength can be varied at will, and very likely it will be also directly relevant to high T_c superconductivity

- One can extract the "exact" energy density functional from these calculations for both normal and superfluid phases and at both zero and finite temperatures

- This case should serve as a standard testing ground of essentially all other approximate many body techniques, <u>hopefully finally settle the pairing properties of neutron matter in particular</u>

Things become even better!

Consider the No Core Shell Model for two Fermion species with a contact interaction, and at the beginning N=8 and N=20 only in a harmonic potential (not necessarily spherical). This problem can be treated in PIMC method as well.

One can obtain a complete converged solution for the ground state and hopefully a large number of excited states, including the matter distribution.

One can try to compare that with a DFT approach. In particular one can start a serious study of nonlocal and gradient corrections.

Such systems are very likely to be created in optical lattices as well, thus of interest to another field

One can extend such studies in EFT spirit and include finite range effects

One can then consider the extension of these studies to four Fermion species

To summarize

It is possible to perform an essentially complete study of a highly nontrivial strongly interacting many body system, which displays many phase transitions and crossover physics.

The system has universal properties and as such is relevant to several fields in physics: nuclear physics and neutron stars, atomic clouds, condensed matter physics

One can extend the range of applicability of the model to more complex interactions

One can test and (in)validate a large number of theoretical approaches

One can test the limits of our understanding and implementation of DFT to normal, superfluid fermion system at both zero and finite temperatures