Local Density Approximation for Systems with Pairing Correlations:

Nuclei, Neutron Stars and Dilute Atomic Systems

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A few introductory clarifications:

✓ LDA is understood here as the Kohn-Sham extension of the Hohenberg-Kohn extension of the Energy Density Functional (EDF).

✓ We shall not discuss here the existence and/or the uniqueness of the LDA (EDF) to systems with pairing correlations.

✓ However, we shall discuss the feasibility, the implementation, how LDA for systems with pairing correlations works and the accuracy of this approach.

✓ Since the Hartree-Fock-Bogoliubov approximation is not accurate for the description of the pairing correlations (explicit examples will be given during the presentation) in either the weak or strong coupling limits, one is left with no option but to use an EDF or some other approach (still to be formulated, but it is really unclear what else could be feasible).

Contents

- Rather lengthy introduction, motivating the LDA approach
- Description of the main technical stumbling block in formulating a LDA for systems with pairing correlations and how this difficulty is overcome.
- Results of application of this new LDA approach to a rather large number of spherical nuclei in a fully self-consistent approach with continuum correctly accounted for.
- Description of the new features of the vortex state in low density neutron matter (neutron stars)
- Application of this new LDA approach in the limit of strong coupling, (when the pairing gap is of the order of the Fermi energy) and the description of the vortex state in a dilute atomic Fermi gas
- The role of paring correlations on the particle number density profiles in cases when paring correlations are in the weak and in the strong coupling limits respectively
- Summary

References

A. Bulgac and Y. Yu, Phys. Rev. Lett. **88**, 0402504 (2002)
A. Bulgac, Phys. Rev. C **65**, 051305(R) (2002)
A. Bulgac and Y. Yu, nucl-th/0109083 (Lectures)
Y. Yu and A. Bulgac, Phys. Rev. Lett. **90**, 222501 (2003)
Y. Yu and A. Bulgac, nucl-th/0302007 (Appendix to PRL)
Y. Yu and A. Bulgac, Phys. Rev. Lett. 90, 161101 (2003)
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Y. Yu, PhD thesis (2003), almost done.
A. Bulgac and Y. Yu, in preparation

Superconductivity and superfluidity in Fermi systems

- ✓ Dilute atomic Fermi gases — $T_c > 10^{-12}$ eV
- Liquid ^3He — $T_c > 10^{-7}$ eV
- Metals, composite materials — $T_c > 10^{-3} – 10^{-2}$ eV
- ✓ Nuclei, neutron stars — $T_c > 10^5 – 10^6$ eV
- QCD color superconductivity — $T_c > 10^7 – 10^8$ eV

units (1 eV > 10^4 K)

Why is the nuclear pairing problem still an open problem ?

- We do not know yet whether pairing has a volume or/and surface character.
- The isospin properties of the pairing field are largely unknown (more about this later).
- Coupling of the pairing field to vibrations is a field still in its infancy. Even though nuclear pairing coupling occurs in the so called weak coupling limit, screening effects are large and effects beyond mean-field (HFB) are substantial.
- Pairing is a key feature of neutron stars and we understand this part still rather badly: proton and neutron pairing in *s* and *p* partial waves, properties of vortices, mixed phases, etc.
- Prediction of nucleon drip lines and basic nuclear properties for exotic nuclei - RIA physics and astrophysics (nucleo-synthesis).

Density Functional Theory (DFT)

Hohenberg and Kohn, 1964

$$E_{gs} = \int d^3 r \varepsilon[\rho(\vec{r})]$$

particle density

Local Density Approximation (LDA)

Kohn and Sham, 1965

$$E_{gs} = \int d^3 r \varepsilon[\rho(\vec{r}), \tau(\vec{r})]$$

$$\rho(\vec{r}) = \sum_{i=1}^{N} |\, \mathrm{v}_i(\vec{r}) \,|^2$$

$$\tau(\vec{r}) = \sum_{i=1}^{N} |\, \vec{\nabla} \mathrm{v}_i(\vec{r}) \,|^2$$

Normal Fermi systems only!

$$\Psi_A(\vec{r}_1,\ldots,\vec{r}_N) \Rightarrow \rho(\vec{r})$$

$$\Psi_B(\vec{r}_1,\ldots,\vec{r}_N) \Rightarrow \rho(\vec{r})$$

$$\Psi_A(\vec{r}_1,\ldots,\vec{r}_N) \neq \Psi_B(\vec{r}_1,\ldots,\vec{r}_N)$$

Assume that there are two different many-body wave functions, corresponding to the same number particle density!

$$H = \sum_i T_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

$$E_A = \left\langle \Psi_A \,|\, H + \sum_i V_i \,|\, \Psi_A \right\rangle = \left\langle \Psi_A \,|\, H \,|\, \Psi_A \right\rangle + \mathrm{Tr}(V\rho)$$

$$E_B = \left\langle \Psi_B \,|\, H + \sum_i U_i \,|\, \Psi_B \right\rangle = \left\langle \Psi_B \,|\, H \,|\, \Psi_B \right\rangle + \mathrm{Tr}(U\rho)$$

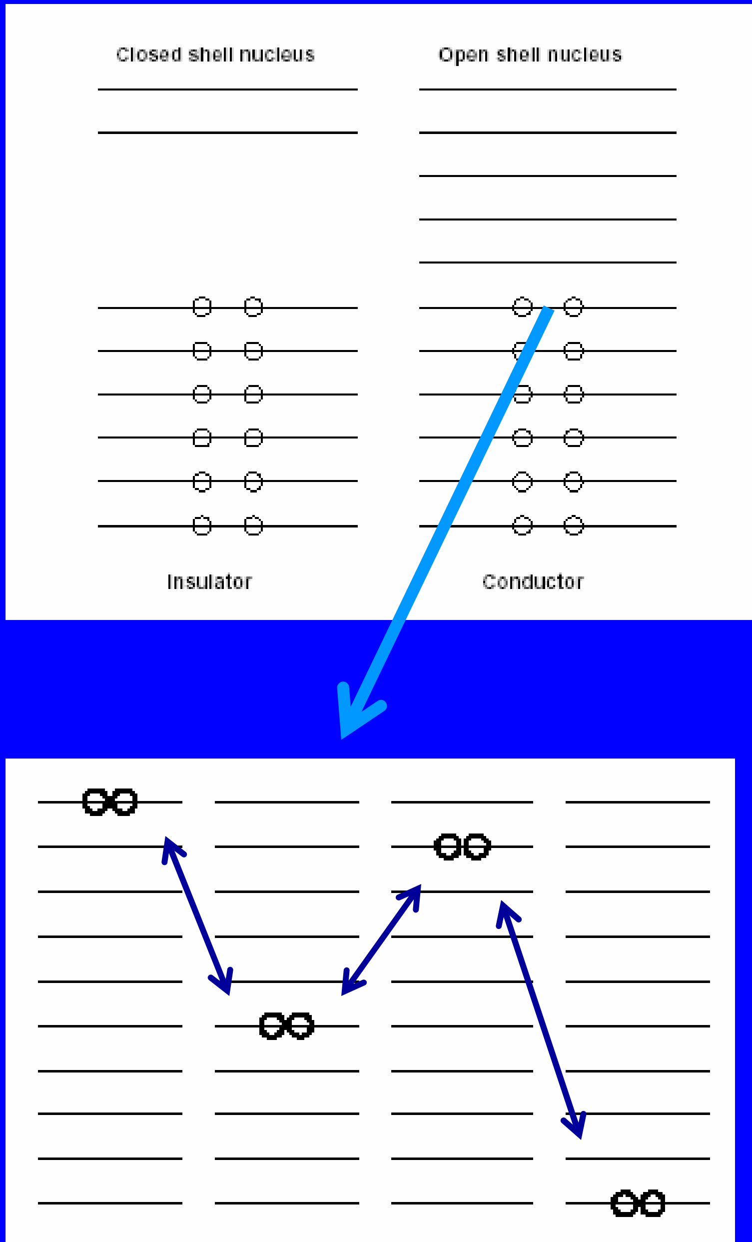
$$E_A < \left\langle \Psi_B \,|\, H \,|\, \Psi_B \right\rangle + \mathrm{Tr}(V\rho)$$

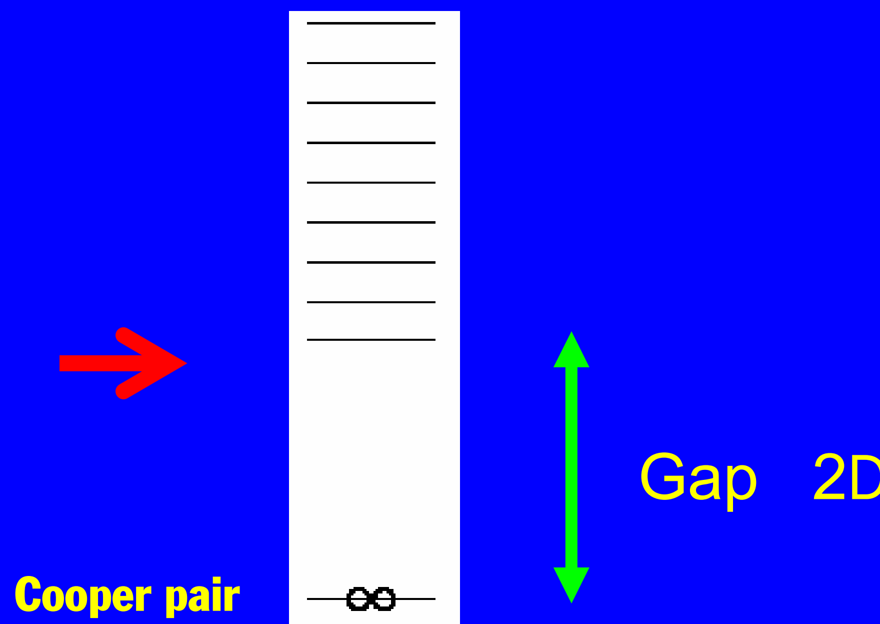
$$E_B < \left\langle \Psi_A \,|\, H \,|\, \Psi_A \right\rangle + \mathrm{Tr}(U\rho)$$

$$E_A + E_B < E_A + E_B$$

Nonsense!

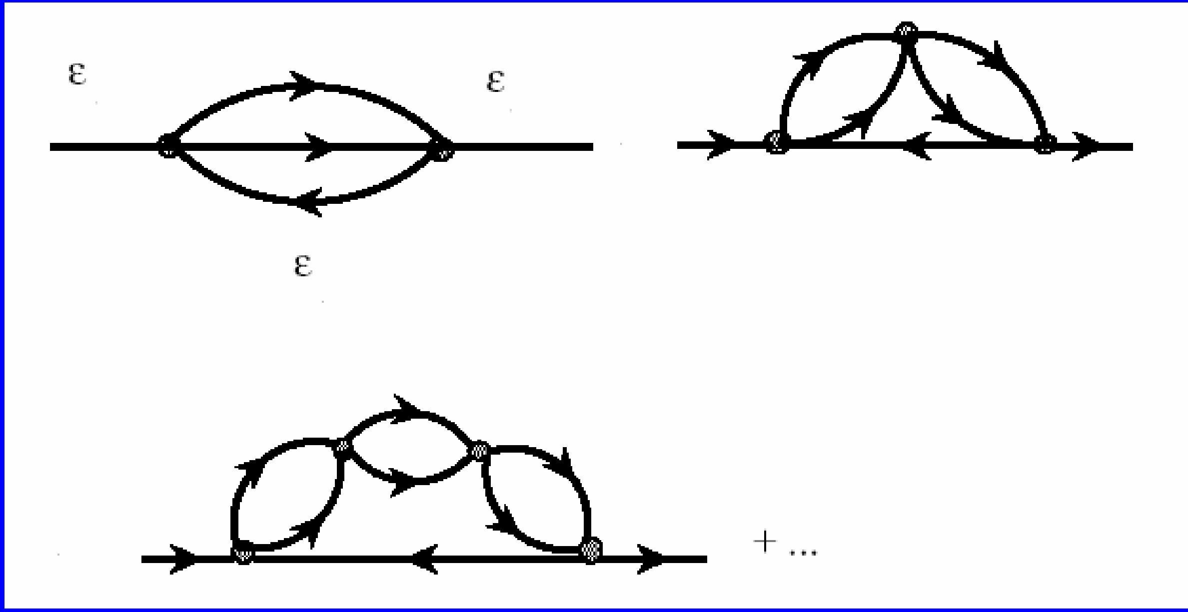
Cooper's argument





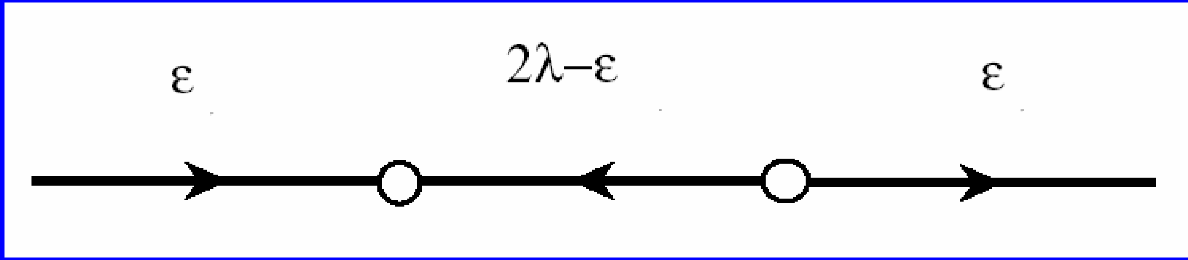
Following Migdal and using a half-century old language:
Feynman diagrams (sum over histories)

Improve the particle propagator beyond the usual mean-field



After retaining only the pole (bound state) contribution in the 2-particle channel

A new field, the pairing field, is appearing, which mixes particle and hole states.



Hartree-Fock-Bogoliubov approximation

$$\alpha_k \left| gs \right\rangle = 0$$

$$\begin{cases} \alpha_k = \int dx \left[\mathrm{u}_\mathrm{k}^*(\mathrm{x})\psi(\mathrm{x}) + \mathrm{v}_\mathrm{k}(x)\psi^+(x) \right] \\ \alpha_k^+ = \int dx \left[\mathrm{v}_\mathrm{k}^*(x)\psi(\mathrm{x}) + \mathrm{u}_\mathrm{k}(x)\psi^+(x) \right] \end{cases}$$

$$E_{gs} - \lambda N = \left\langle gs \left| \hat{H} - \lambda \hat{N} \right| gs \right\rangle$$

$$\rho(x,y) = \sum_{E_k > 0} \mathrm{v}_\mathrm{k}(x)\mathrm{v}_\mathrm{k}^*(y)$$

$$\nu(x,y) = \sum_{E_k > 0} \mathrm{u}_\mathrm{k}(x)\mathrm{v}_\mathrm{k}^*(y)$$

$$\begin{pmatrix} h - \lambda & \Delta \\ \Delta^+ & -(h^* - \lambda) \end{pmatrix} \begin{pmatrix} \mathrm{u}_\mathrm{k} \\ \mathrm{v}_\mathrm{k} \end{pmatrix} = E_k \begin{pmatrix} \mathrm{u}_\mathrm{k} \\ \mathrm{v}_k \end{pmatrix}$$

LDA for superfluid fermi systems
(Bogoliubov-de Gennes equations)

$$E_{gs} = \int d^3 r \varepsilon(\rho(\vec{r}), \tau(\vec{r}), \nu(\vec{r}))$$

$$\rho(\vec{r}) = 2\sum_k |\mathrm{v}_\mathrm{k}(\vec{r})|^2, \quad \tau(\vec{r}) = 2\sum_k |\vec{\nabla}\mathrm{v}_\mathrm{k}(\vec{r})|^2$$

$$\nu(\vec{r}) = \sum_k \mathrm{u}_\mathrm{k}(\vec{r})\mathrm{v}_\mathrm{k}^*(\vec{r})$$

$$\begin{pmatrix} T + U(\vec{r}) - \lambda & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -(T + U(\vec{r}) - \lambda) \end{pmatrix} \begin{pmatrix} \mathrm{u}_k(\vec{r}) \\ \mathrm{v}_k(\vec{r}) \end{pmatrix} = E_k \begin{pmatrix} \mathrm{u}_k(\vec{r}) \\ \mathrm{v}_k(\vec{r}) \end{pmatrix}$$

Mean-field and pairing field are both local fields!

(for sake of simplicity spin degrees of freedom are not shown)

There is a little problem! The pairing field D diverges.

Nature of the problem

$$\nu(\vec{r}_1,\vec{r}_2) = \sum_{E_k>0} \mathrm{v}_\mathrm{k}^*(\vec{r}_1)\mathrm{u}_\mathrm{k}(\vec{r}_2) \propto \frac{1}{|\vec{r}_1 - \vec{r}_2|}$$

at small separations

$$\Delta(\vec{r}_1,\vec{r}_2) = \frac{1}{2} V(\vec{r}_1,\vec{r}_2)\nu(\vec{r}_1,\vec{r}_2)$$

It is easier to show how this singularity appears in infinite homogeneous matter (BCS model)

$$\mathrm{v}_\mathrm{k}(\vec{r}) = \mathrm{v}_\mathrm{k} \exp(i\vec{k}\cdot\vec{r}), \qquad \mathrm{u}_\mathrm{k}(\vec{r}) = \mathrm{u}_\mathrm{k} \exp(i\vec{k}\cdot\vec{r})$$

$$\mathrm{v}_\mathrm{k}^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_\mathrm{k} - \lambda}{\sqrt{(\varepsilon_\mathrm{k} - \lambda)^2 + \Delta^2}}\right), \quad \mathrm{u}_\mathrm{k}^2 + \mathrm{v}_\mathrm{k}^2 = 1, \quad \varepsilon_\mathrm{k} = \frac{\hbar^2 \vec{k}^2}{2m} + U, \quad \Delta = \frac{\hbar^2 \delta}{2m}$$

$$\nu(r) = \frac{\Delta m}{2\pi^2\hbar^2} \int_0^\infty dk \frac{\sin(kr)}{kr} \frac{k^2}{\sqrt{(k^2 - k_F^2)^2 + \delta^2}}$$

A (too) simple case

$$k_F \to 0, \delta \to 0$$

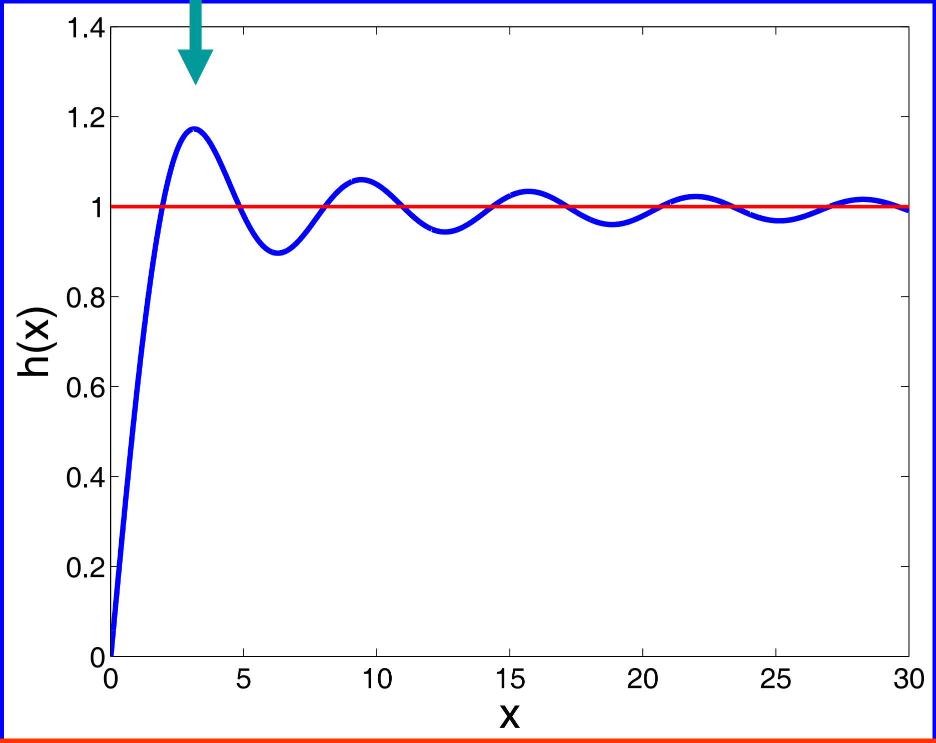
$$\nu(|\vec{r}_1 - \vec{r}_2|) \to \frac{\Delta m}{2\pi^2\hbar^2}\int_0^\infty dk \frac{\sin kr}{kr} = \frac{\Delta m}{2\pi^2\hbar^2}\frac{\pi}{2|\vec{r}_1 - \vec{r}_2|}$$

The integral converges (conditionally) at k > 1/r (iff r>0)

The divergence is due to high momenta and thus its nature is independent of whether the system is finite or infinite

If one introduces an explicit momentum cut-off one has deal with this integral iff r>0.

$$h(x) = \frac{2}{\pi}\int_0^x dy \frac{\sin y}{y}$$



If r=0 then the integral is simply:

$$h(x) = \frac{2}{\pi}\int_0^x dy = \frac{2x}{\pi}$$

In the final analysis all is an issue of the order of taking various limits: r $\rightarrow$ 0 versus cut-off x $\rightarrow \infty$

Solution of the problem in the case of the homogeneous matter (Lee, Huang and Yang and others)

Gap equation

$$V(\vec{r}_1 - \vec{r}_2) = g\delta(\vec{r}_1 - \vec{r}_2)$$

$$1 = -\frac{g}{2}\int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}}$$

Lippmann-Schwinger equation
(zero energy collision)
T = V + VGT

$$-\frac{mg}{4\pi\hbar^2 a} + 1 = -\frac{g}{2}\int \frac{d^3k}{(2\pi)^3} \frac{1}{\varepsilon_k}$$

Now combine the two equations and the divergence is (magically) removed!

$$\frac{m}{4\pi\hbar^2 a} = -\frac{1}{2}\int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{\sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}} - \frac{1}{\varepsilon_k} \right\}$$

How people deal with this problem in finite systems?

- ✓ Introduce an explicit energy cut-off, which can vary from 5 MeV to 100 MeV (sometimes significantly higher) from the Fermi energy.
- ✓ Use a particle-particle interaction with a finite range, the most popular one being Gogny's interaction.

Both approaches are in the final analysis equivalent in principle, as a potential with a finite range $\mathbf{r_0}$ provides a (smooth) cut-off at an energy $\mathbf{E_c} = \hbar^2/mr_0^2$

➢ The argument that nuclear forces have a finite range is superfluous, because nuclear pairing is manifest at small energies and distances of the order of the coherence length, which is much smaller than nuclear radii.

➢ Moreover, LDA works pretty well for the regular mean-field.

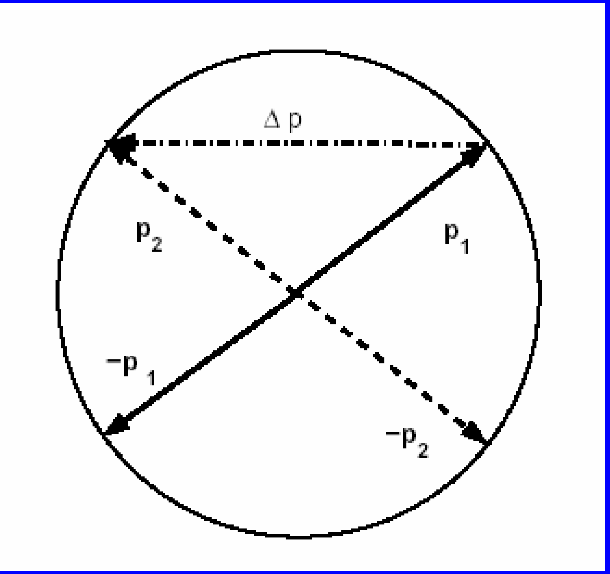
➢ A similar argument fails as well in case of electrons, where the radius of the interaction is infinite and LDA is fine.

Why would one consider a local pairing field?

✓Because it makes sense physically!

✓The treatment is so much simpler!

✓Our intuition is so much better also.



$$r_0 \cong \frac{\hbar}{p_F} = k_F^{-1}$$

radius of interaction   interparticle separation

$$\Delta = \omega_D Exp\left(-\frac{1}{|V|N}\right) << \varepsilon_F$$

$$\xi \approx \frac{1}{k_F}\frac{\varepsilon_F}{\Delta} >> r_0$$

coherence length

size of the Cooper pair

Pseudo-potential approach

(appropriate for very slow particles, very transparent but somewhat difficult to improve)

Lenz (1927), Fermi (1931), Blatt and Weiskopf (1952)
Lee, Huang and Yang (1957)

$$-\frac{\hbar^2 \Delta_{\vec{r}}}{m}\psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = E\,\psi(\vec{r}), \quad V(\vec{r}) \approx 0 \text{ if } r > R$$

$$\psi(\vec{r}) = \exp(i\vec{k}\cdot\vec{r}) + \frac{f}{r}\exp(ikr) \approx 1 + \frac{f}{r} + \ldots \approx 1 - \frac{a}{r} + O(kr)$$

$$f^{-1} = -\frac{1}{a} + \frac{1}{2}r_0 k^2 - ik, \qquad g = \frac{4\pi\hbar^2 a}{m(1+ika)} + \ldots$$

$$\text{if} \quad kr_0 << 1 \quad \text{then} \qquad V(\vec{r})\psi(\vec{r}) \Rightarrow g\,\delta(\vec{r})\frac{\partial}{\partial r}\left[r\,\psi(\vec{r})\right]$$

$$\text{Example}: \quad \psi(\vec{r}) = \frac{A}{r} + B + \ldots \Rightarrow \delta(\vec{r})\frac{\partial}{\partial r}\left[r\,\psi(\vec{r})\right] = \delta(\vec{r})B$$

How to deal with an inhomogeneous/finite system?

$$\nu_{reg}(\vec{r}) \stackrel{def}{=} \sum_i \left[\mathrm{v}_i^*(\vec{r})\mathrm{u}_i(\vec{r}) + \frac{\Delta(\vec{r})\psi_i^*(\vec{r})\psi_i(\vec{r})}{2(\lambda - \varepsilon_i)} \right] - \frac{\Delta(\vec{r})}{2} G_{reg}(\lambda, \vec{r})$$

$$G_{reg}(\lambda, \vec{r}) \stackrel{def}{=} \lim_{\vec{r}\,' \to \vec{r}} \left[G(\vec{r}, \vec{r}\,', \lambda) + \frac{m}{2\pi\hbar^2 |\vec{r} - \vec{r}\,'|} \right]$$

$$[h(\vec{r}) - \varepsilon_i]\psi_i(\vec{r}) = 0$$

$$[\lambda - h(\vec{r})]G(\vec{r}, \vec{r}\,', \lambda) = \delta(\vec{r} - \vec{r}\,')$$

There is complete freedom in choosing the Hamiltonian h and we are going to take advantage of this!

We shall use a "Thomas-Fermi" approximation for the propagator G.

$$G(\vec{r},\vec{r}',\lambda) = -\frac{m\exp(ik_F(\vec{r})|\vec{r}-\vec{r}'|)}{2\pi\hbar^2|\vec{r}-\vec{r}'|}$$

$$\approx -\frac{m}{2\pi\hbar^2|\vec{r}-\vec{r}'|} - \frac{ik_F(\vec{r})m}{2\pi\hbar^2} + \mathrm{O}(|\vec{r}-\vec{r}'|)$$

$$\frac{\hbar^2 k_F^2(\vec{r})}{2m} + U(\vec{r}) = \lambda, \quad \frac{\hbar^2 k_c^2(\vec{r})}{2m} + U(\vec{r}) = \lambda + E_c$$

$$\nu_{\mathrm{reg}}(\vec{r}) \overset{def}{=} \left\{ \sum_{E_i \le E_c} \mathrm{v}_i^*(\vec{r})\mathrm{u}_i(\vec{r}) + \frac{\Delta(\vec{r})}{4\pi^2} \int_0^{k_c(\vec{r})} \frac{1}{\lambda - \frac{\hbar^2 k^2}{2m} - U(\vec{r}) + i\gamma} k^2 dk \right\} + \frac{i\Delta(\vec{r})k_F(\vec{r})m}{4\pi\hbar^2}$$

Regularized anomalous density

Regular part of G

The renormalized equations:

$$E_{gs} = \int d^3r[\mathcal{E}_N(\boldsymbol{r}) + \mathcal{E}_S(\boldsymbol{r})],$$

$$\mathcal{E}_S(\boldsymbol{r}) := -\Delta(\boldsymbol{r})\nu_c(\boldsymbol{r}) = g_{\text{eff}}(\boldsymbol{r})|\nu_c(\boldsymbol{r})|^2,$$

$$\begin{cases} [h(\boldsymbol{r}) - \mu]u_i(\boldsymbol{r}) + \Delta(\boldsymbol{r})v_i(\boldsymbol{r}) = E_i u_i(\boldsymbol{r}), \\ \Delta^*(\boldsymbol{r})u_i(\boldsymbol{r}) - [h(\boldsymbol{r}) - \mu]v_i(\boldsymbol{r}) = E_i v_i(\boldsymbol{r}), \end{cases}$$

$$h(\boldsymbol{r}) = -\boldsymbol{\nabla}\frac{\hbar^2}{2m(\boldsymbol{r})}\boldsymbol{\nabla} + U(\boldsymbol{r}), \quad \Delta(\boldsymbol{r}) := -g_{\text{eff}}(\boldsymbol{r})\nu_c(\boldsymbol{r}),$$

$$\frac{1}{g_{\text{eff}}(\boldsymbol{r})} = \frac{1}{g(\boldsymbol{r})} - \frac{mk_c(\boldsymbol{r})}{2\pi^2\hbar^2}\left[1 - \frac{k_F(\boldsymbol{r})}{2k_c(\boldsymbol{r})}\ln\frac{k_c(\boldsymbol{r}) + k_F(\boldsymbol{r})}{k_c(\boldsymbol{r}) - k_F(\boldsymbol{r})}\right]$$

$$\rho_c(\boldsymbol{r}) = \sum_{E_i \geq 0}^{E_c} 2|v_i(\boldsymbol{r})|^2, \quad \nu_c(\boldsymbol{r}) = \sum_{E_i \geq 0}^{E_c} v_i^*(\boldsymbol{r})u_i(\boldsymbol{r}),$$

$$E_c + \mu = \frac{\hbar^2 k_c^2(\boldsymbol{r})}{2m(\boldsymbol{r})} + U(\boldsymbol{r}), \ \mu = \frac{\hbar^2 k_F^2(\boldsymbol{r})}{2m(\boldsymbol{r})} + U(\boldsymbol{r}).$$

Typo: replace m by m(r)

New renormalization scheme

Vacuum renormalization

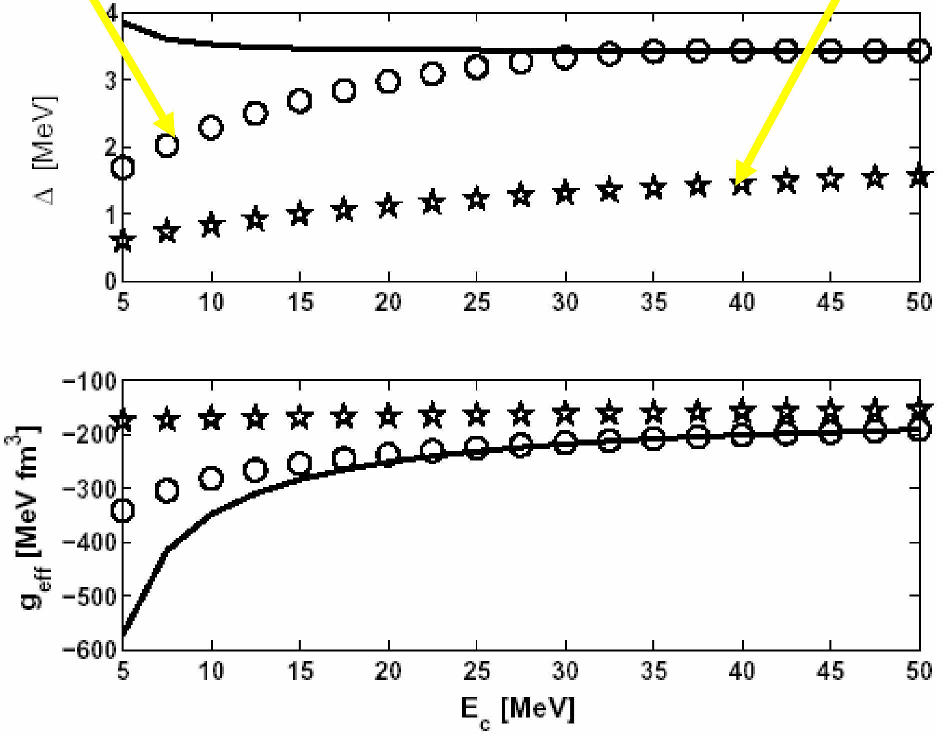


FIG. 2. The gap Δ and the effective coupling constant g_{eff} as a function of the cut-off energy E_c for three regularization schemes. The full lines correspond to calculations using Eqs. (15 – 17). Circles correspond to the regularization scheme presented in Ref. [5] (when only terms with k_c are present). The pentagrams correspond to the vacuum regularization scheme [16]. The calculation was performed for homogeneous neutron matter with $\rho = 0.08$ fm^{-3} and $g = -250$ MeV $\cdot$ fm^3.

A. Bulgac, Phys. Rev. C 65, 051305 (2002)

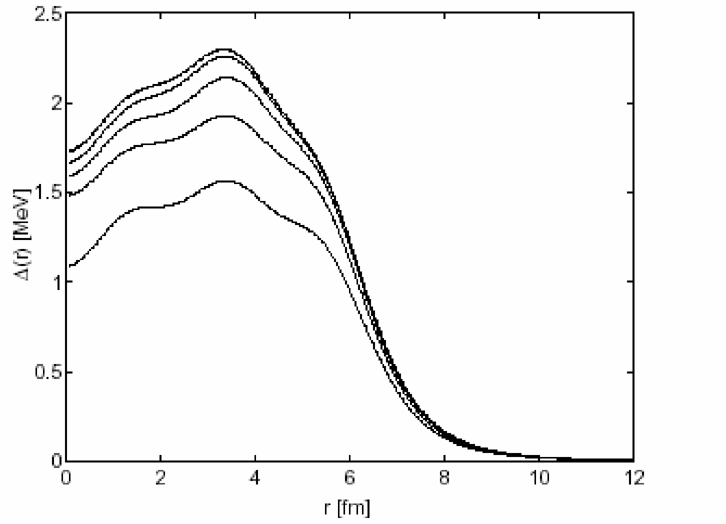


FIG. 1. The neutron pairing field (17) as a function of the radial coordinate and of the cut–off energy E_c. Upward various curves correspond to E_c = 20, 30, 35, 40, 45 and 50 MeV respectively. On the scale of the figure the last two curves are indistinguishable.

A. Bulgac and Y. Yu,
Phys. Rev. Lett. 88, 042504 (2002)

A few notes:

- The cut-off energy E_c should be larger than the Fermi energy.
- It is possible to introduce an even faster converging scheme for the pairing field with E_c of a few **D**'s only.
- Even though the pairing field was renormalized, the total energy should be computed with care, as the "pairing" and "kinetic" energies separately diverge.

$$E_{gs} = \int d^3r [\mathcal{E}_N(\boldsymbol{r}) + \mathcal{E}_S(\boldsymbol{r})],$$

$$\mathcal{E}_S(\boldsymbol{r}) := -\Delta(\boldsymbol{r})\nu_c(\boldsymbol{r}) = g_{\text{eff}}(\boldsymbol{r})|\nu_c(\boldsymbol{r})|^2,$$

Still diverges!

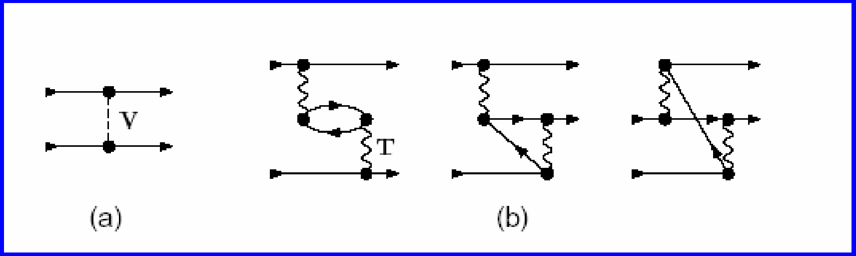
- One should now introduce the normal and the superfluid contributions to the Energy Density Functional (EDF).

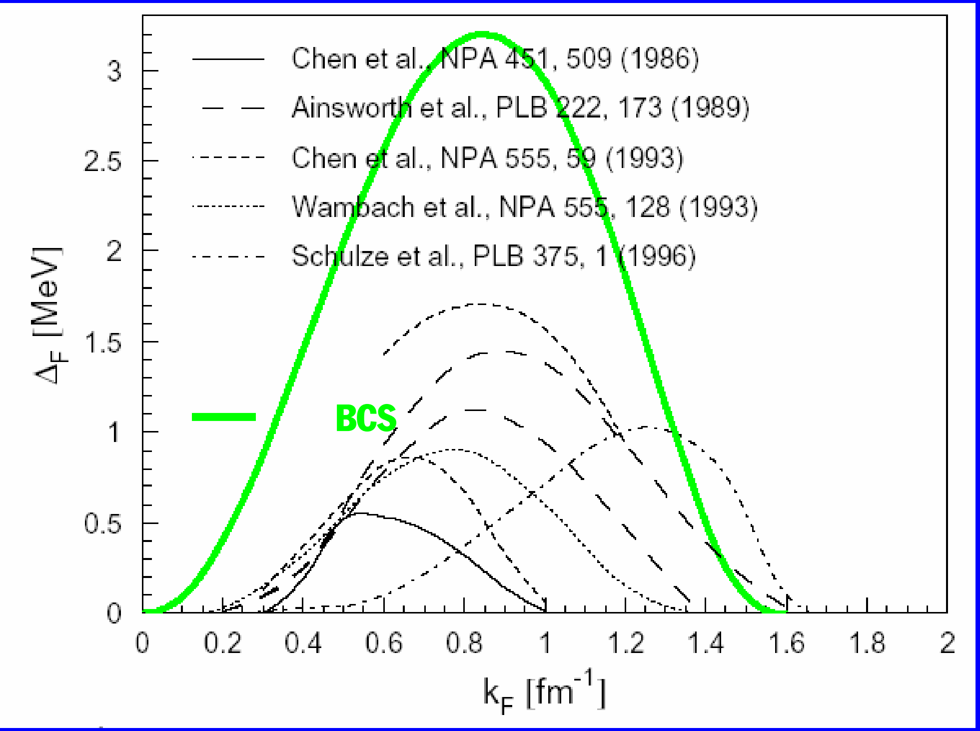
Isospin symmetry

$$\mathcal{E}_S(\boldsymbol{r}) = g_0(\boldsymbol{r})|\nu_p(\boldsymbol{r}) + \nu_n(\boldsymbol{r})|^2 + g_1(\boldsymbol{r})|\nu_p(\boldsymbol{r}) - \nu_n(\boldsymbol{r})|^2$$

We considered so far only the case $g_0 = g_1$.

“Screening effects” are significant!





s-wave pairing gap in infinite neutron matter with realistic NN-interactions

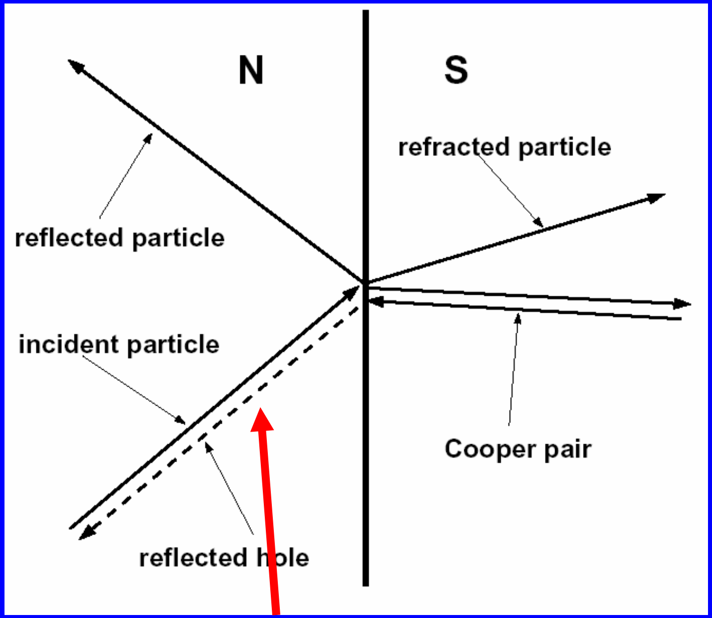
from Lombardo and Schulze astro-ph/0012209

These are major effects beyond the naïve HFB

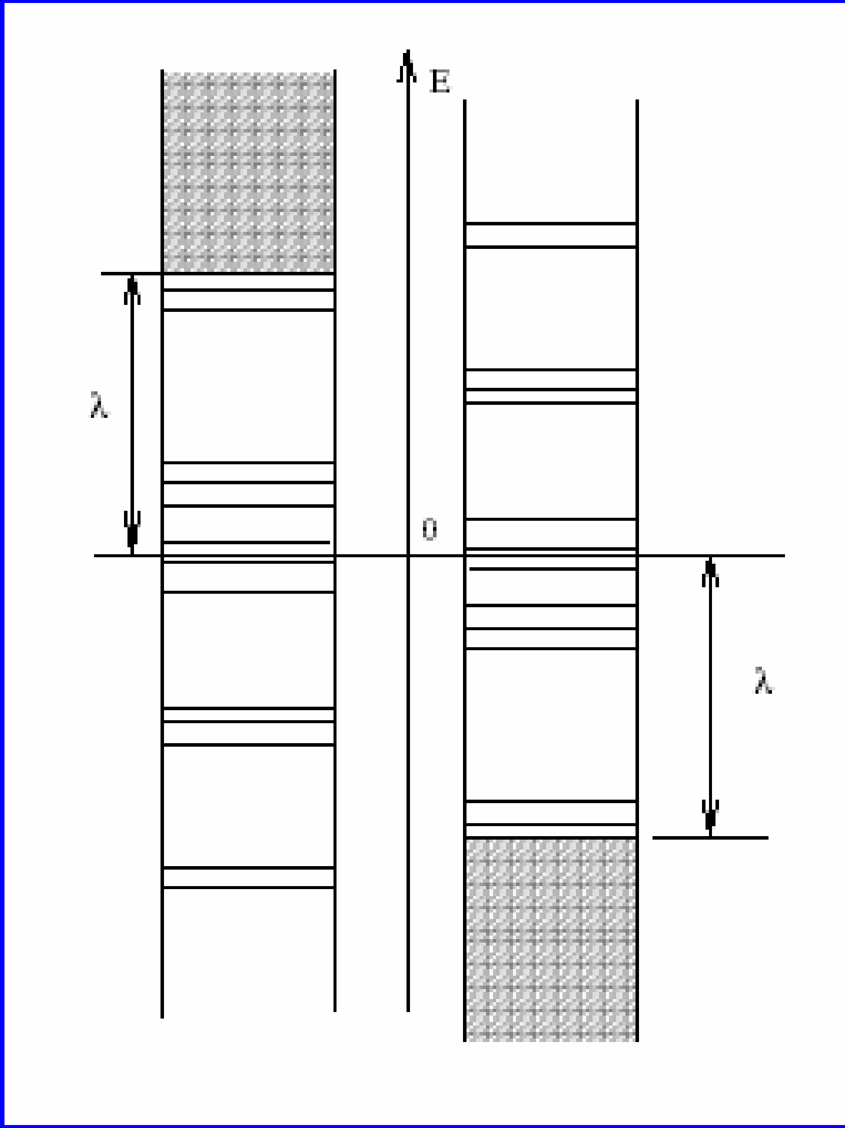
Peculiarity of the finite systems:
deep hole states are continuum states.

$$\begin{pmatrix} h-\lambda & \Delta \\ \Delta^{+} & -(h^{*}-\lambda) \end{pmatrix}\begin{pmatrix} u_k \\ v_k \end{pmatrix} = E_k \begin{pmatrix} u_k \\ v_k \end{pmatrix}$$

outside inside

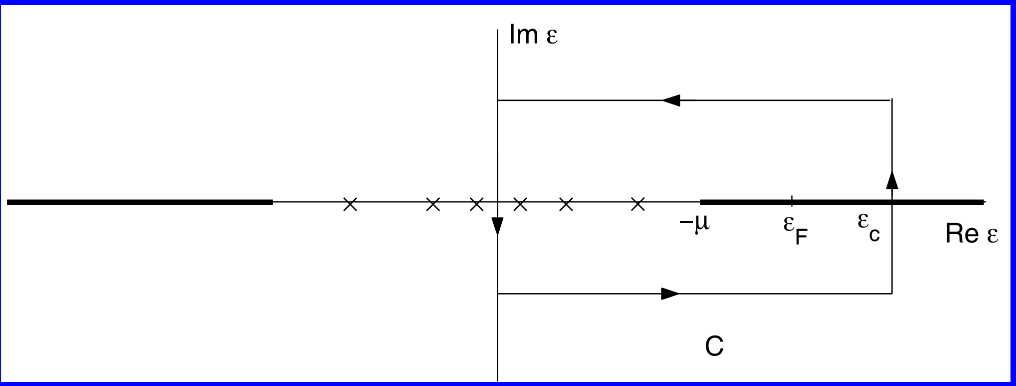


Andreev reflection

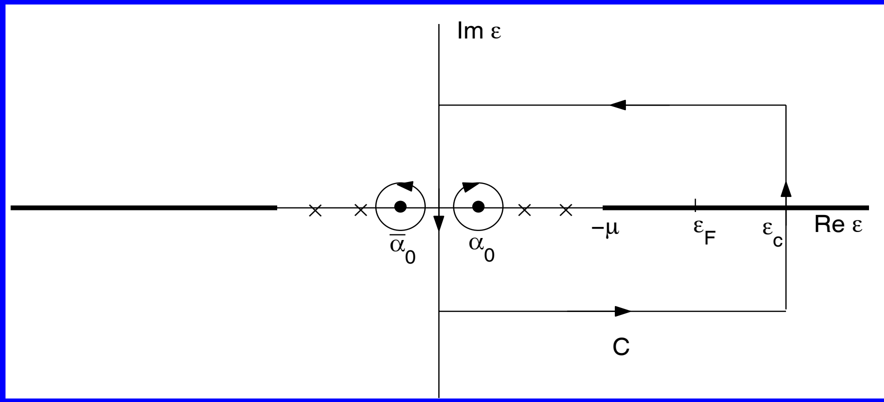


Integration contours used to construct the normal and anomalous densities from the Gorkov Green functions Belyaev *et al*, Sov. J. Nucl. Phys. 45, 783 (1987).

$$\begin{pmatrix} E-(h-\lambda) & -\Delta \\ -\Delta^{*} & E+(h^{*}-\lambda) \end{pmatrix} \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

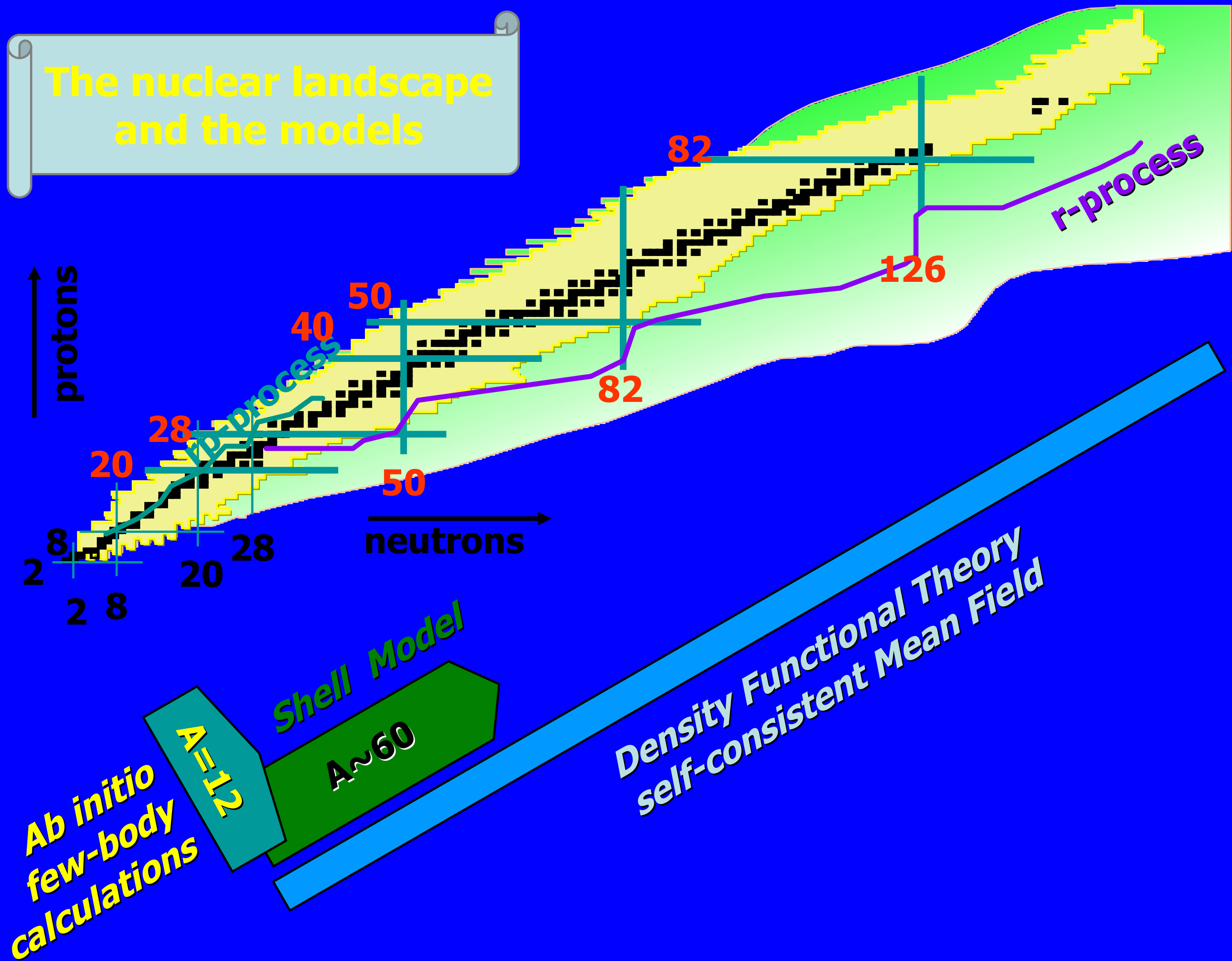


Even systems



Odd systems

The nuclear landscape and the models



Courtesy of Mario Stoitsov

The isotope and isotone chains treated by us are indicated with red numbers.

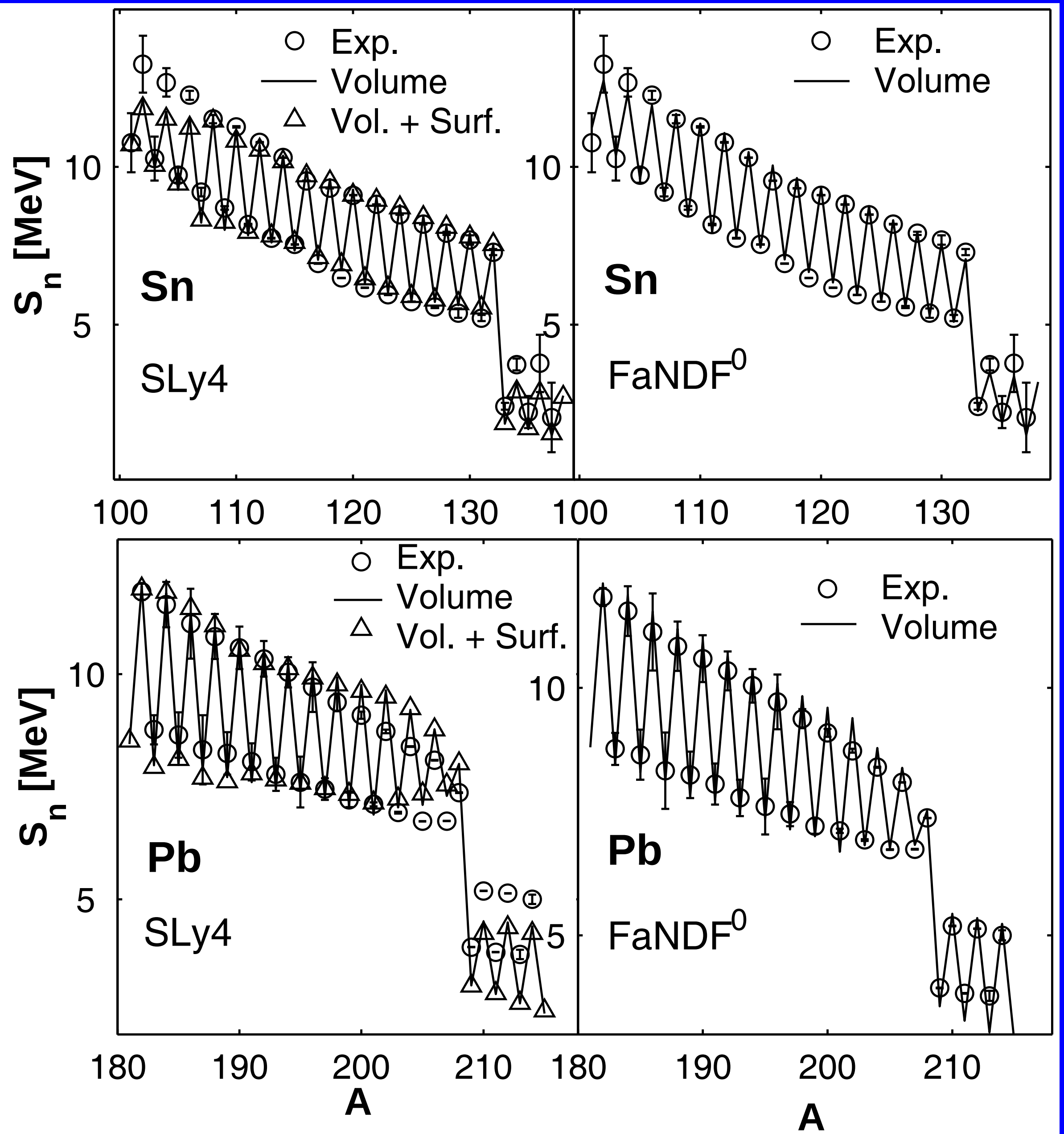
How well does the new approach work?

TABLE I. The rms of S_{2N} and S_N deviations, respectively, from experiment [21] (in MeV's) for several isotope and isotone chains.

Z or N chain	S_{2N}/S_N present	S_{2N}/S_N Ref. [11]	S_{2N} Ref. [23]
$Z = 20$	0.82/0.76	1.02/0.92	0.96
$Z = 28$	0.67/0.50	0.66/0.55	1.30
$Z = 40$	0.93/0.63	0.66/0.63	2.21
$Z = 50$	0.29/0.21	0.43/0.35	0.95
$Z = 82$	0.23/0.37	0.58/0.53	0.74
$N = 50$	0.37/0.26	0.41/0.23	NA
$N = 82$	0.43/0.31	0.50/0.56	NA
$N = 126$	0.42/0.23	0.88/0.52	NA

Ref. 21, Audi and Wapstra, Nucl. Phys. A595, 409 (1995).
Ref. 11, S. Goriely *et al.* Phys. Rev. C 66, 024326 (2002) - HFB
Ref. 23, S.Q. Zhang *et al.* nucl-th/0302032. - RMF

One-neutron separation energies



Volume pairing

$$g(\vec{r}) = g$$

Volume + Surface pairing

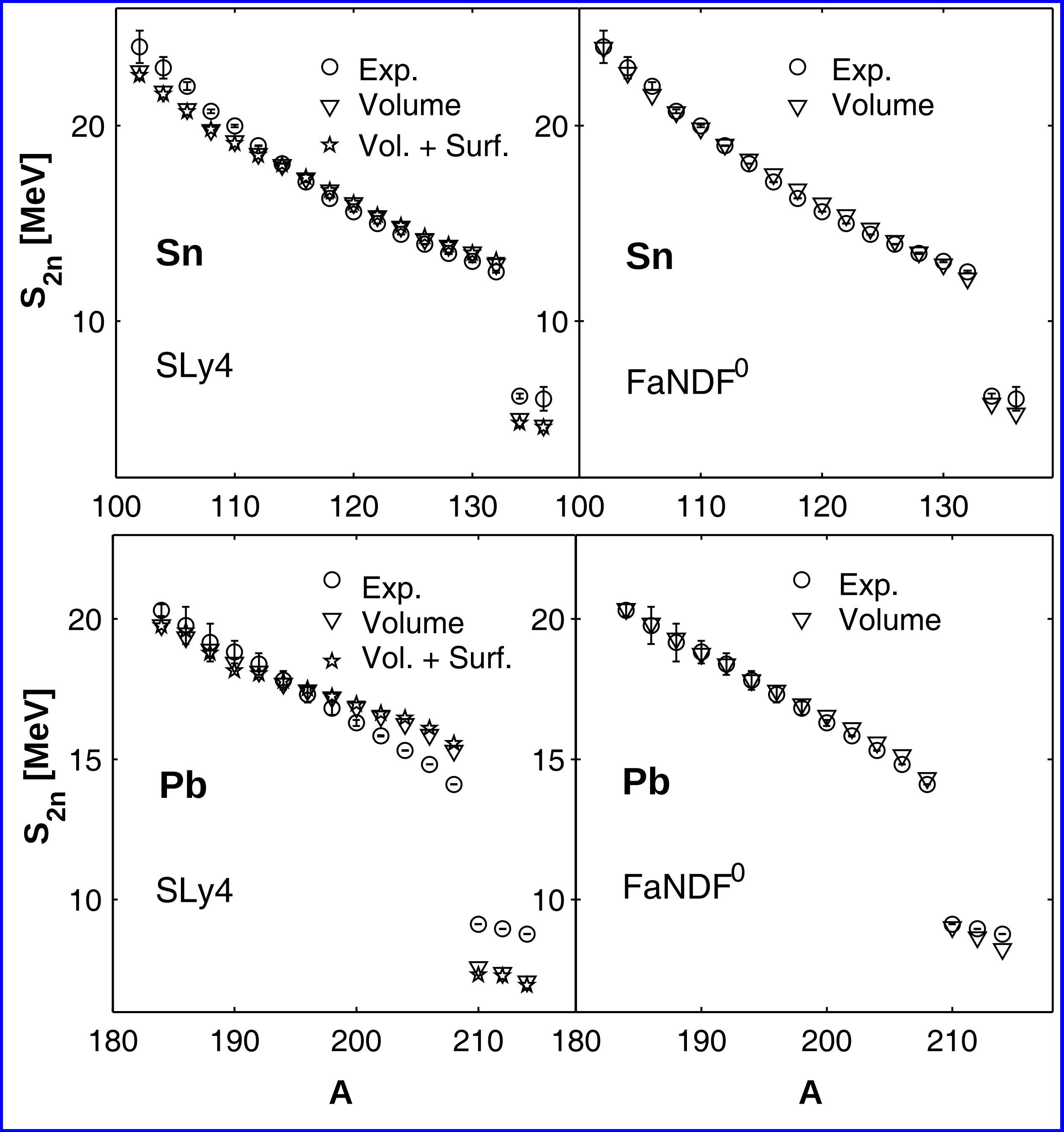
$$g(\vec{r}) = V_0\left(1 - \frac{\rho(\vec{r})}{\rho_c}\right)$$

Normal EDF:

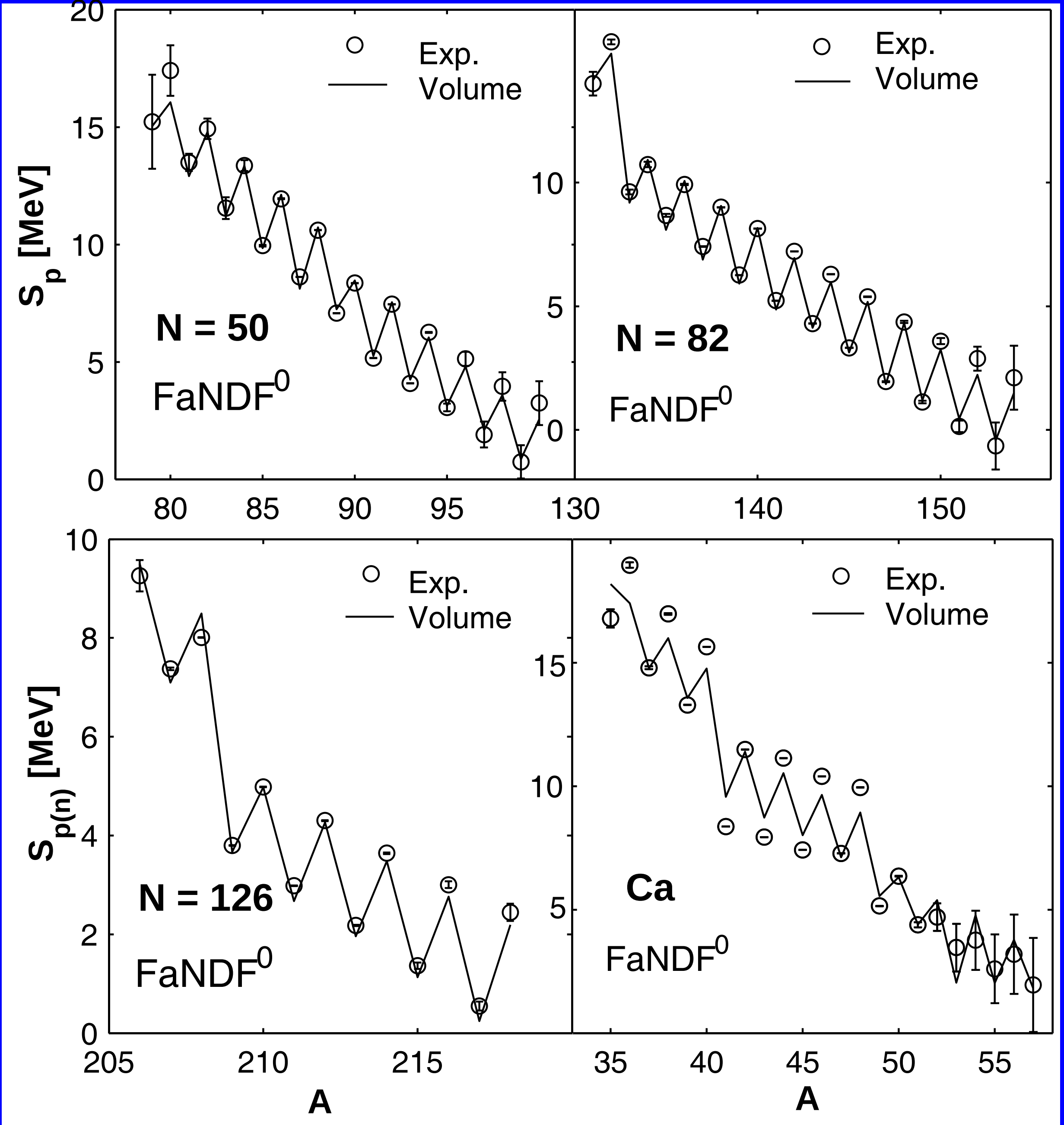
- SLy4 - Chabanat et al.
Nucl. Phys. A627, 710 (1997)
Nucl. Phys. A635, 231 (1998)
Nucl. Phys. A643, 441(E)(1998)

- FaNDF^0 – Fayans
JETP Lett. 68, 169 (1998)

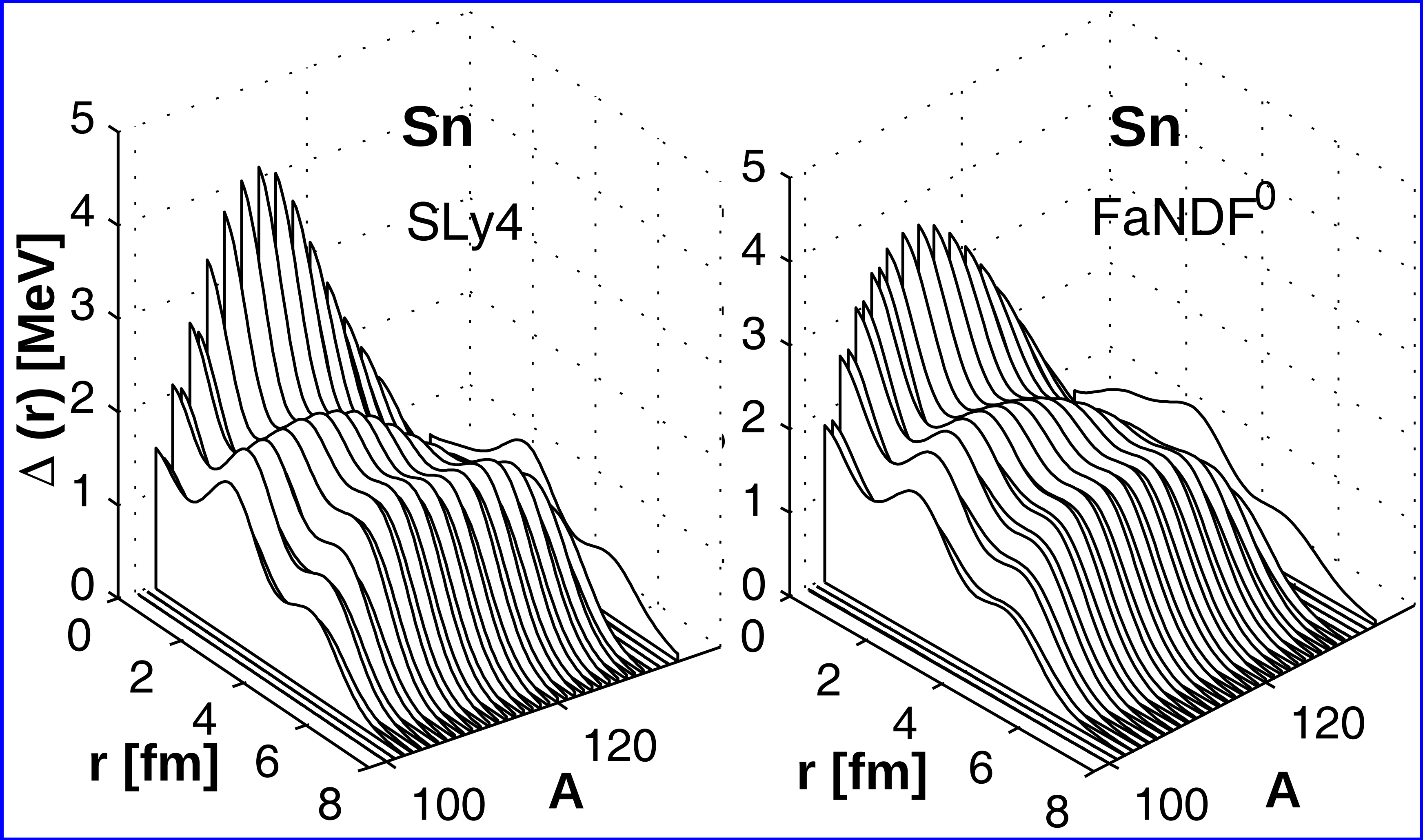
Two-neutron separation energies



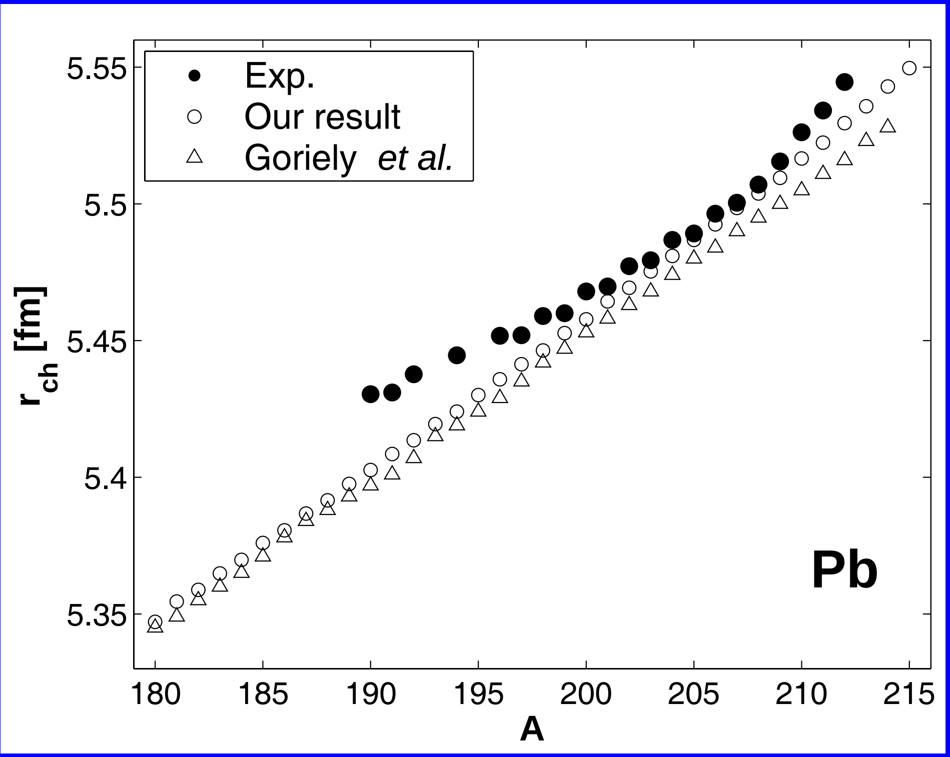
One-nucleon separation energies

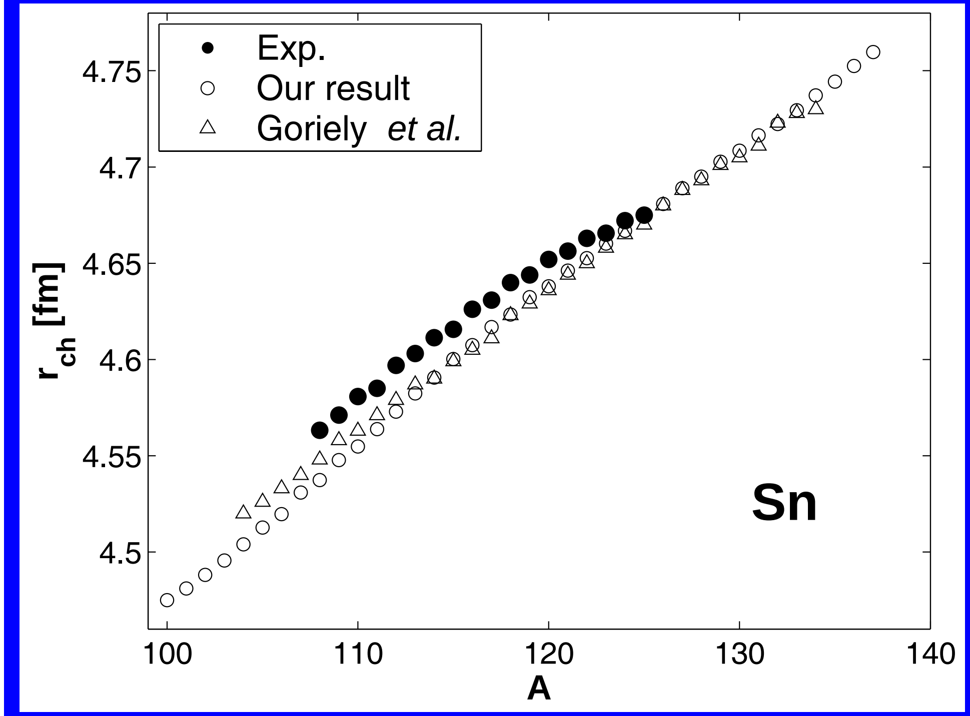


Spatial profiles of the pairing field for tin isotopes and two different (normal) energy density functionals

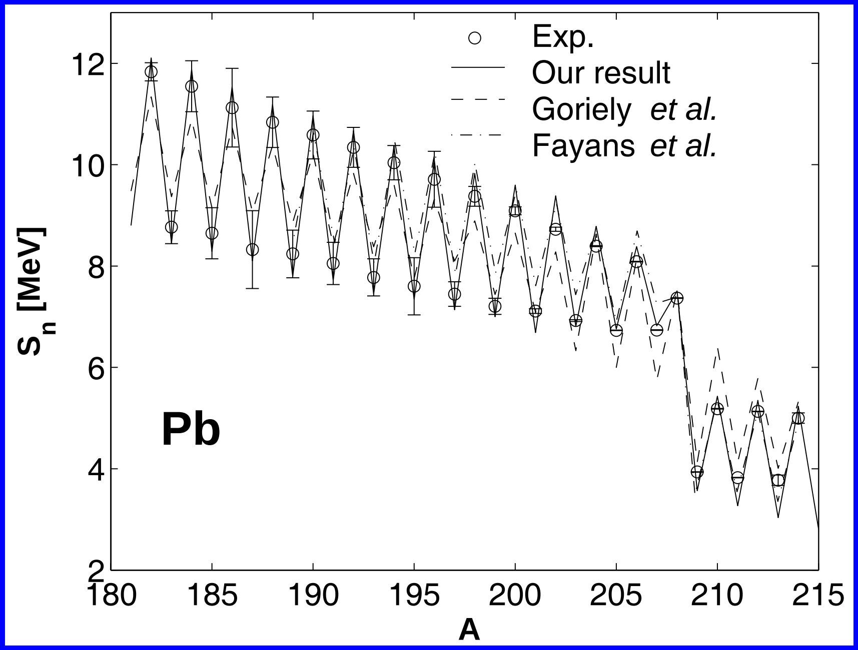


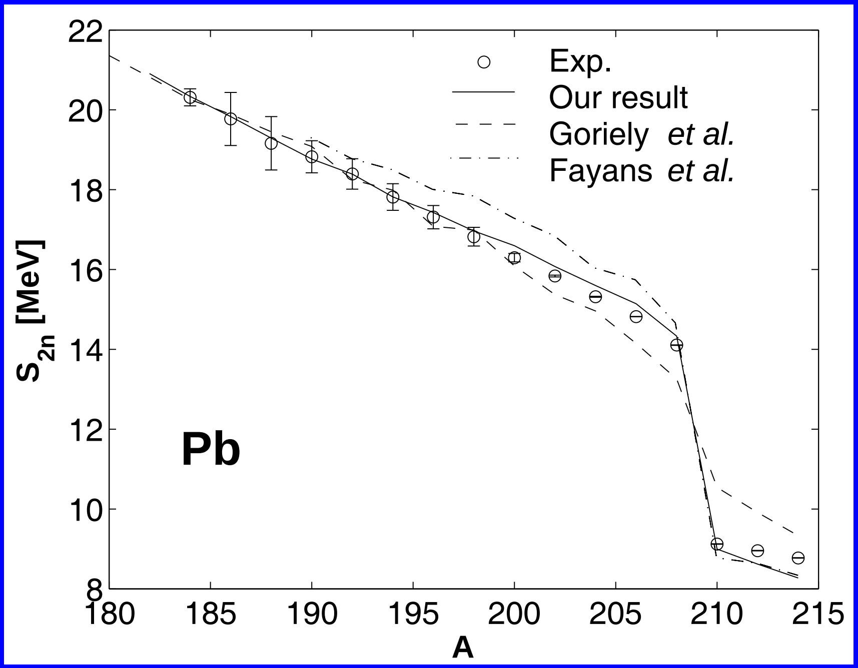
Charge radii



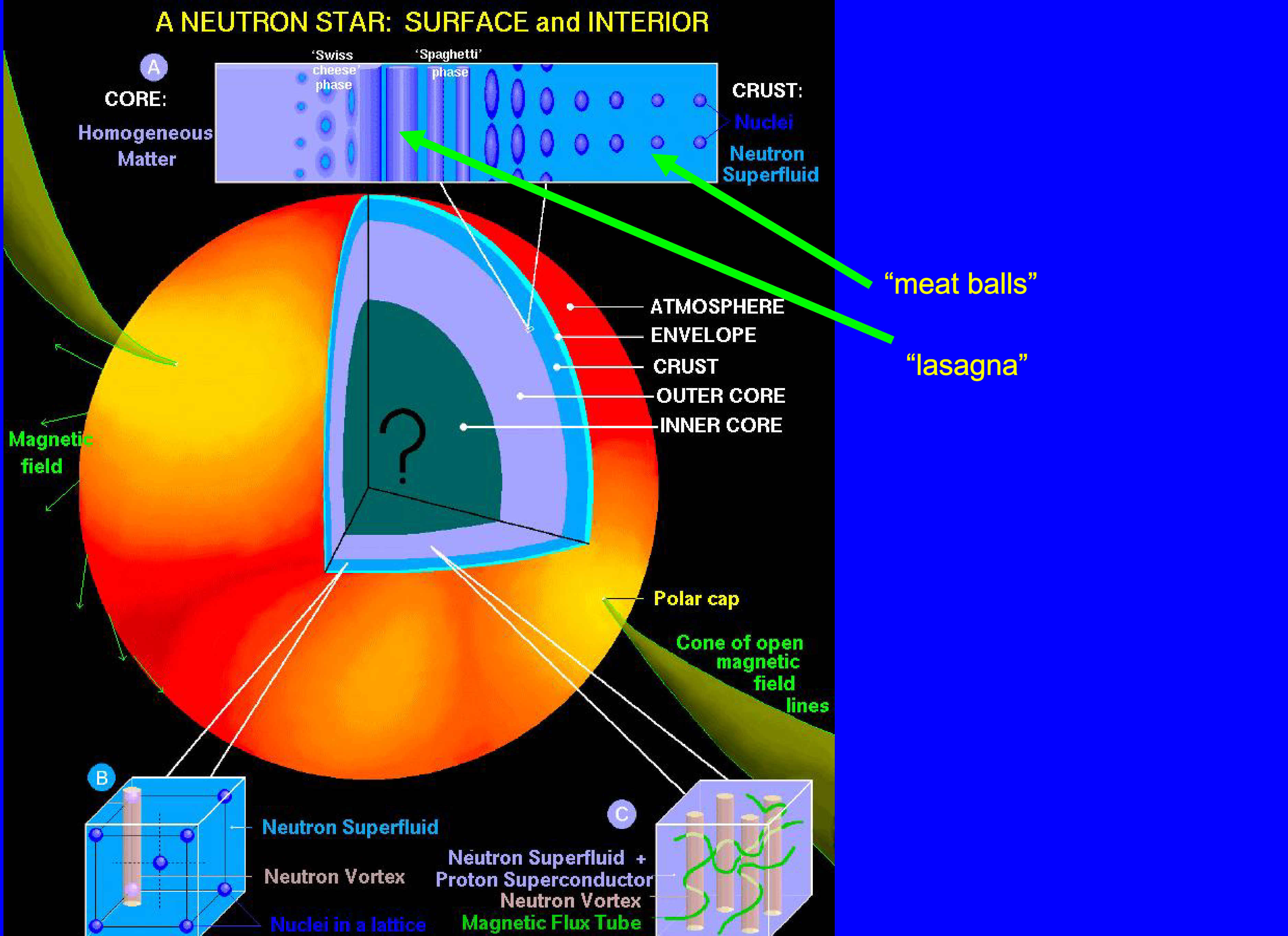


Exp. - Nadjakov et al. At. Data and Nucl. Data Tables, 56, 133 (1994)





- We use the same normal EDF as Fayans *et al.*
  volume pairing only with one universal constant
- Fayans *et al.* Nucl. Phys. A676, 49 (2000)
  5 parameters for pairing (density dependence with gradient terms (neutrons only).
- Goriely *et al.* Phys. Rev. C 66, 024326 (2002)
  volume pairing, 5 parameters for pairing, isospin symmetry broken
- Exp. - Audi and Wapstra, Nucl. Phys. A595, 409 (1995)



Borrowed from http://www.lsw.uni-heidelberg.de/~mcamenzi/NS_Mass.html

Landau criterion for superflow stability
(flow without dissipation)

Consider a superfluid flowing in a pipe with velocity v_s:

$$E_0 + \frac{Nmv_s^2}{2} < E_0 + \varepsilon_{\vec{p}} + \vec{v}_s \cdot \vec{p} + \frac{Nmv_s^2}{2} \Rightarrow v_s < \frac{\varepsilon_{\vec{p}}}{p}$$

no internal excitations

One single quasi-particle excitation with momentum p

In the case of a Fermi superfluid this condition becomes

$$v_s < \frac{\Delta}{\hbar k_F}$$

Vortex in neutron matter

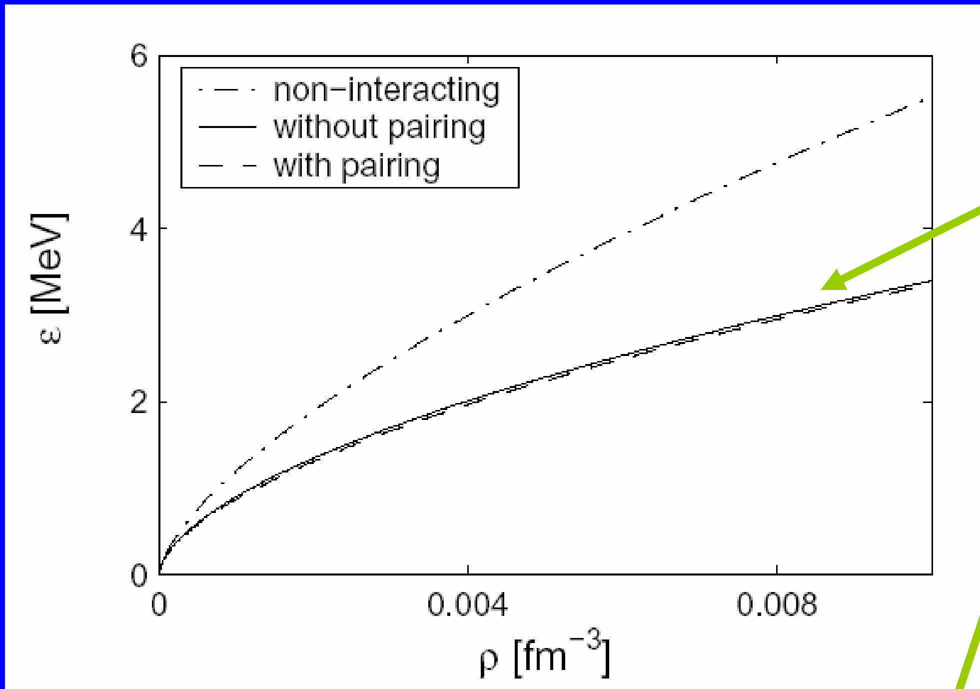
$$\begin{pmatrix} \mathrm{u}_{\alpha\,\mathrm{kn}}(\vec{r}) \\ \mathrm{v}_{\alpha\,\mathrm{kn}}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \mathrm{u}_{\alpha}(r)\exp[i(n+1/2)\phi - ikz] \\ \mathrm{v}_{\alpha}(r)\exp[i(n-1/2)\phi - ikz] \end{pmatrix}, \quad \mathrm{n\ -\ half\text{-}integer}$$

$$\Delta(\vec{r}) = \Delta(r)\exp(i\phi), \qquad \vec{r} = (r,\phi,z)\ \text{[cyllindrical coordinates]}$$

Oz - vortex symmetry axis

Ideal vortex, Onsager's quantization (one $\hbar$ per Cooper pair)

$$\vec{\mathrm{V}}_{\mathrm{v}}(\vec{r}) = \frac{\hbar}{2mr^2}(y,-x,0) \quad \Leftarrow \quad \frac{1}{2\pi}\oint_C \vec{\mathrm{V}}_{\mathrm{v}}(\vec{r})\cdot d\vec{r} = \frac{\hbar}{2m}$$

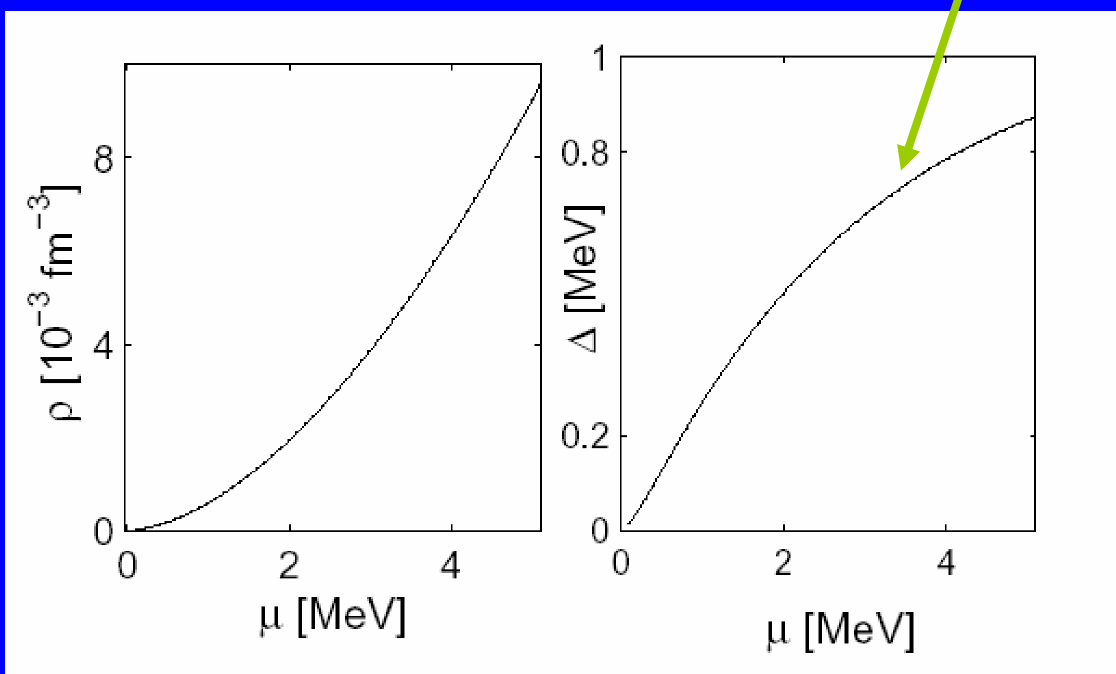


Fayans's FaNDF^0

$$\Delta = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(-\frac{\pi}{2\tan\delta(k_F)}\right)$$

An additional factor of **1/(4e)$^{1/3}$** is due to induced interactions

Again, HFB not valid.



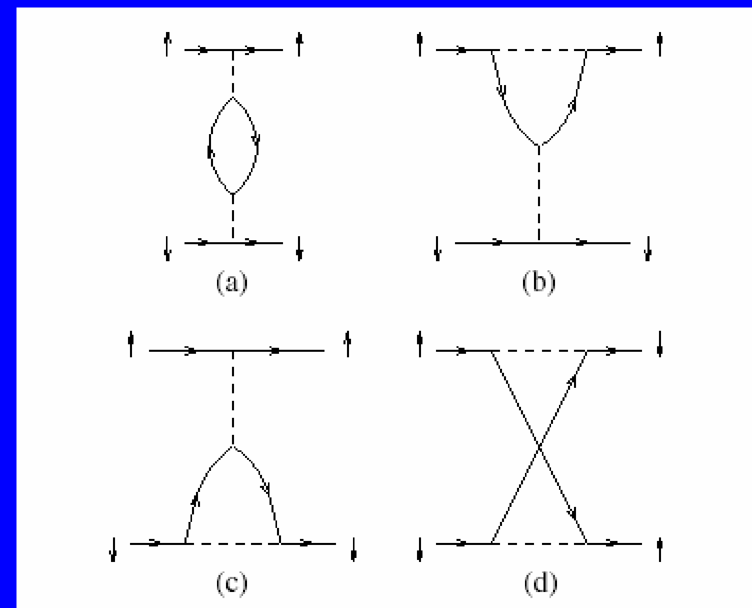
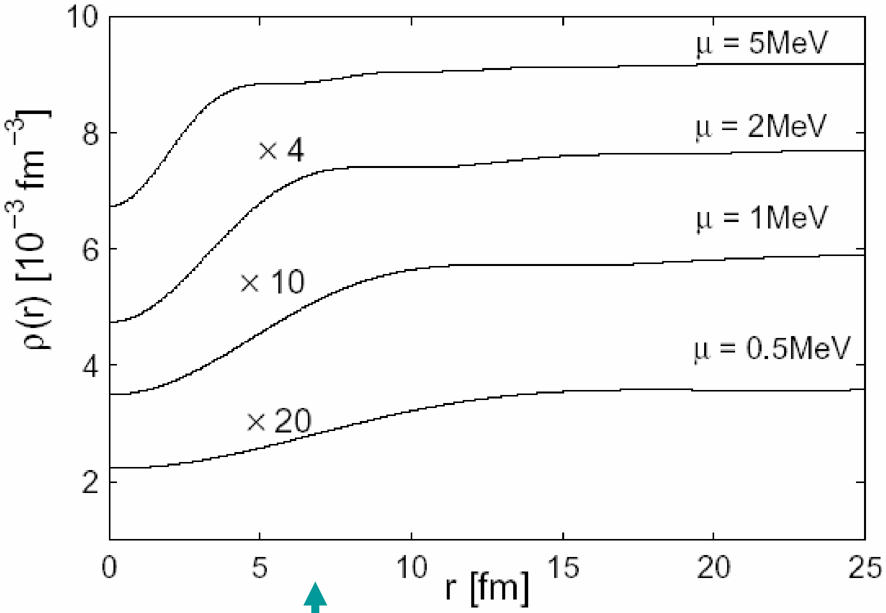
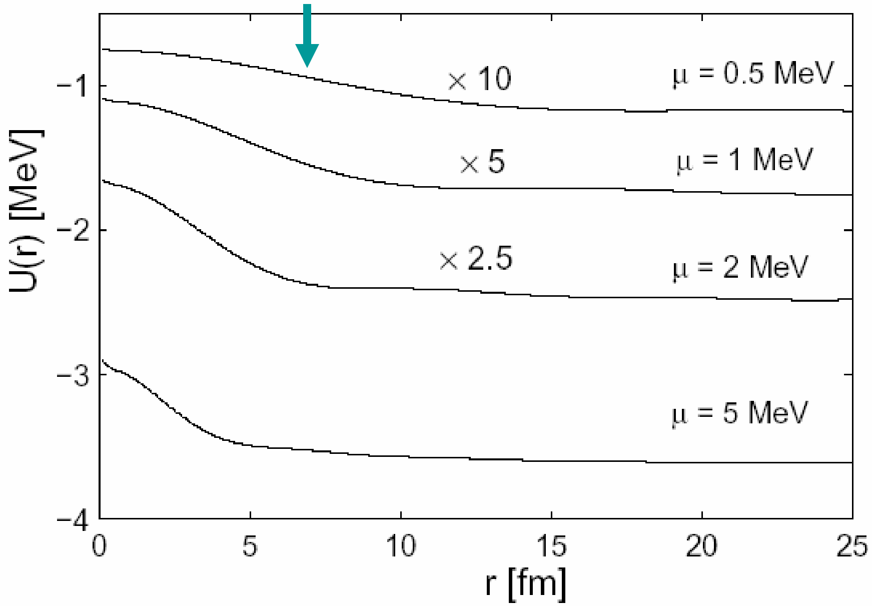


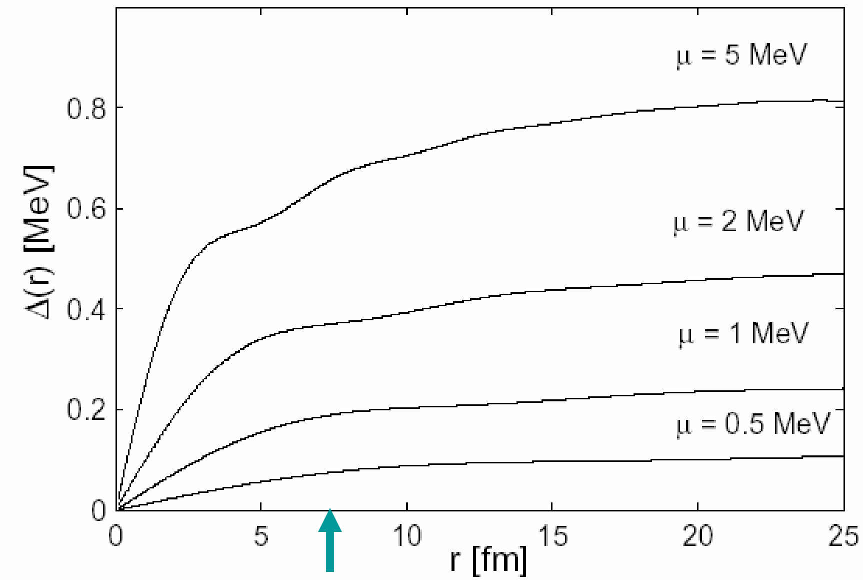
FIG. 1. Diagrams for the induced interactions between two fermions in different internal states to second order in the effective interaction.

from Heiselberg et al
Phys. Rev. Lett. 85, 2418, (2000)

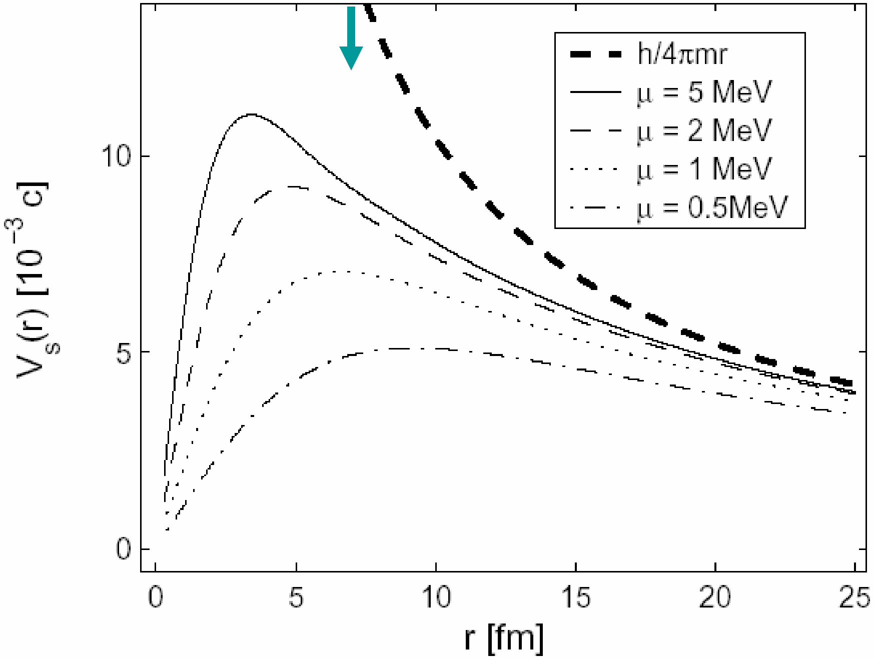


Distances scale with l_F





Distances scale with $x \gg l_F$



Dramatic structural changes of the vortex state naturally lead to significant changes in the energy balance of a neutron star

- In low density region $\varepsilon(\rho_{out})\rho_{out} > \varepsilon(\rho_{in})\rho_{in}$

which thus leads to a large anti-pinning energy $E^V_{pin} > 0$:

$$E^V_{pin} = \left[\varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in}\right]V$$

- The energy per unit length is going to be changed dramatically when compared to previous estimates, by

$$\frac{\Delta \mathrm{E}_{\mathrm{vortex}}}{\mathrm{L}} \approx \left[\varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in}\right]\pi R^2$$

- Specific heat, transport properties are expected to significantly affected as well.

Vortices in dilute atomic Fermi systems in traps

- ✓ 1995 BEC was observed.
- ✓ 2000 vortices in BEC were created, thus BEC confirmed un-ambiguously.
- ✓ In 1999 DeMarco and Jin created a degenerate atomic Fermi gas.
- ✓ 2002 O'Hara, Hammer, Gehm, Granada and Thomas observed expansion of a Fermi cloud compatible with the existence of a superfluid fermionic phase.

Observation of stable/quantized vortices in Fermi systems would provide the ultimate and most spectacular proof for the existence of a Fermionic superfluid phase.

How can one put in evidence a vortex in a Fermi superfluid?

Hard to see, since density changes are not expected, unlike the case of a Bose superfluid.

What we learned from the structure of a vortex in low density neutron matter can help however.

If the gap is not small one can expect a noticeable density depletion along the vortex core, and the bigger the gap the bigger the depletion.

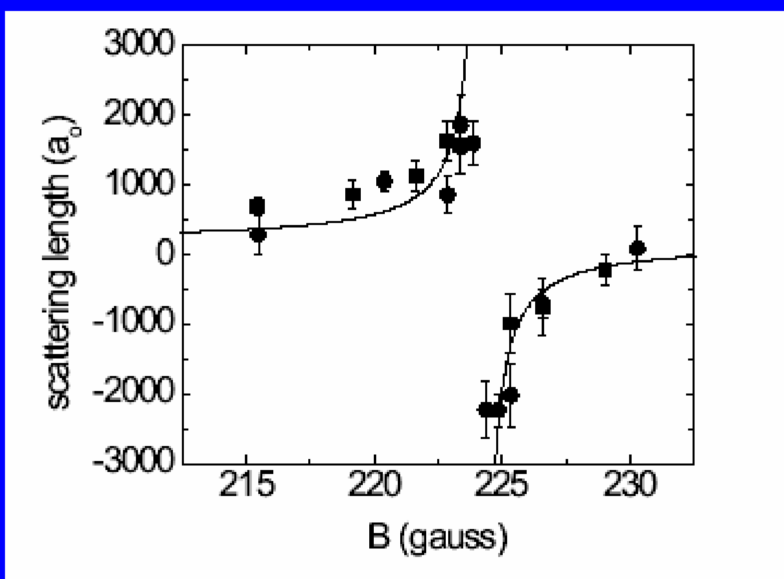
One can change the magnitude of the gap by altering the scattering length between two atoms with magnetic fields by means of a Feshbach resonance.

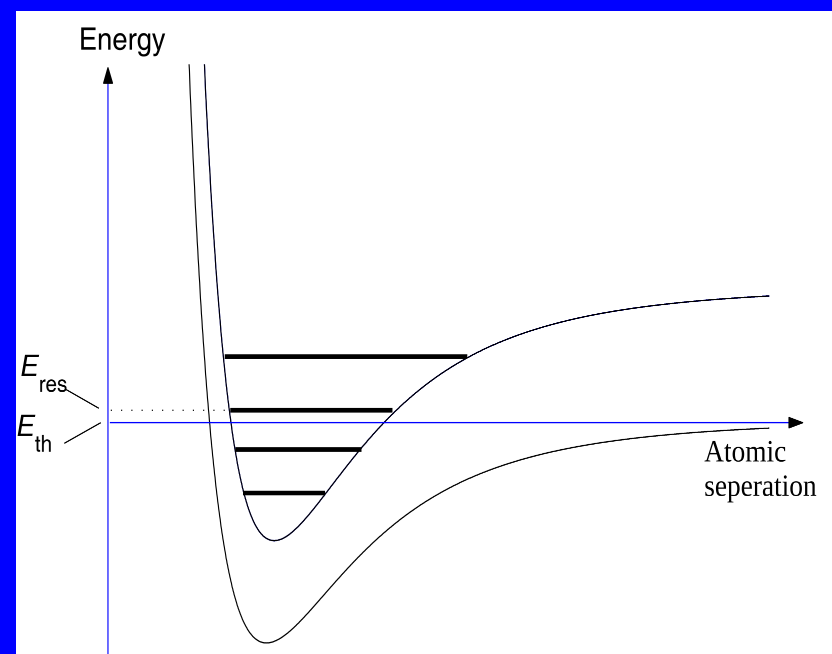
Feshbach resonance

$$H = \frac{\vec{p}^{\,2}}{2\mu_r} + \sum_{i=1}^{2}(V_i^{hf} + V_i^{Z}) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + V^d$$

$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{S}^e \cdot \vec{S}^n, \quad V^Z = (\gamma_e S_z^e - \gamma_n S_z^n)B$$

Tiesinga, Verhaar, Stoof
Phys. Rev. A47, 4114 (1993)





Regal and Jin
Phys. Rev. Lett. 90, 230404 (2003)

Consider Bertsch's MBX challenge (1999): "Find the ground state of infinite homogeneous neutron matter interacting with an infinite scattering length."

- Carlson, Morales, Pandharipande and Ravenhall, nucl-th/0302041, with Green Function Monte Carlo (GFMC)

$$\frac{E_N}{N} = \alpha_N \frac{3}{5}\varepsilon_F, \qquad \alpha_N = 0.54$$

normal state

- Carlson, Chang, Pandharipande and Schmidt, physics/0303094, with GFMC

$$\frac{E_S}{N} = \alpha_S \frac{3}{5}\varepsilon_F, \qquad \alpha_S = 0.44$$

superfluid state

This state is half the way from BCS→BEC crossover, the pairing correlations are in the strong coupling limit and HFB invalid again.

Now one can construct an LDA functional to describe this new state of Fermionic matter

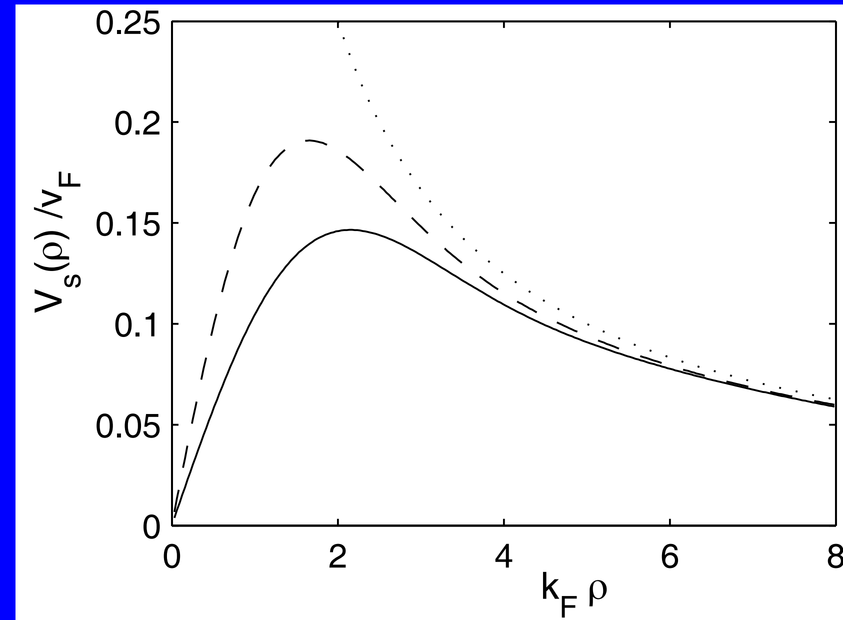
$$\mathcal{E}(\boldsymbol{r})n(\boldsymbol{r}) = \frac{\hbar^2}{m}\left[\frac{m}{2m^*}\tau(\boldsymbol{r}) + \beta n(\boldsymbol{r})^{5/3} + \gamma\frac{|\nu(\boldsymbol{r})|^2}{n(\boldsymbol{r})^{1/3}}\right],$$

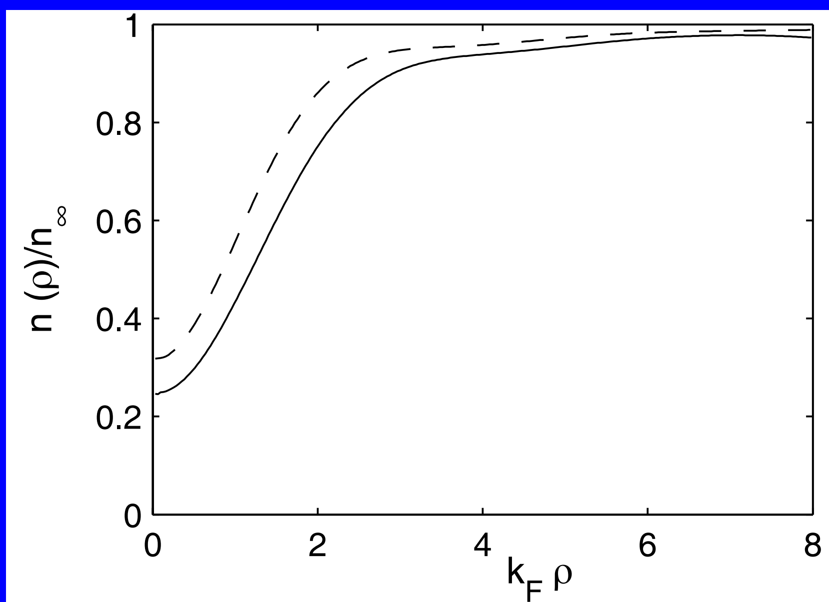
$$n(\boldsymbol{r}) = \sum_\alpha |v_\alpha(\boldsymbol{r})|^2, \quad \tau(\boldsymbol{r}) = \sum_\alpha |\boldsymbol{\nabla} v_\alpha(\boldsymbol{r})|^2,$$

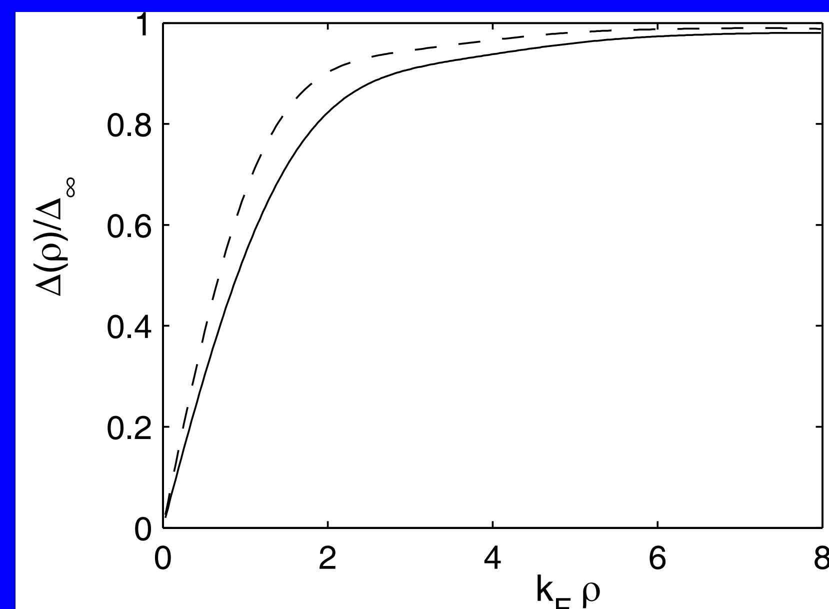
$$\nu(\boldsymbol{r}) = \sum_\alpha v_\alpha^*(\boldsymbol{r})u_\alpha(\boldsymbol{r}).$$

- This form is not unique, as one can have either: b=0 (set I) or b≠0 and m*=m (set II).
- Gradient terms not determined yet (expected minor role).

The depletion along the vortex core is reminiscent of the corresponding density depletion in the case of a vortex in a Bose superfluid, when the density vanishes exactly along the axis for 100% BEC.



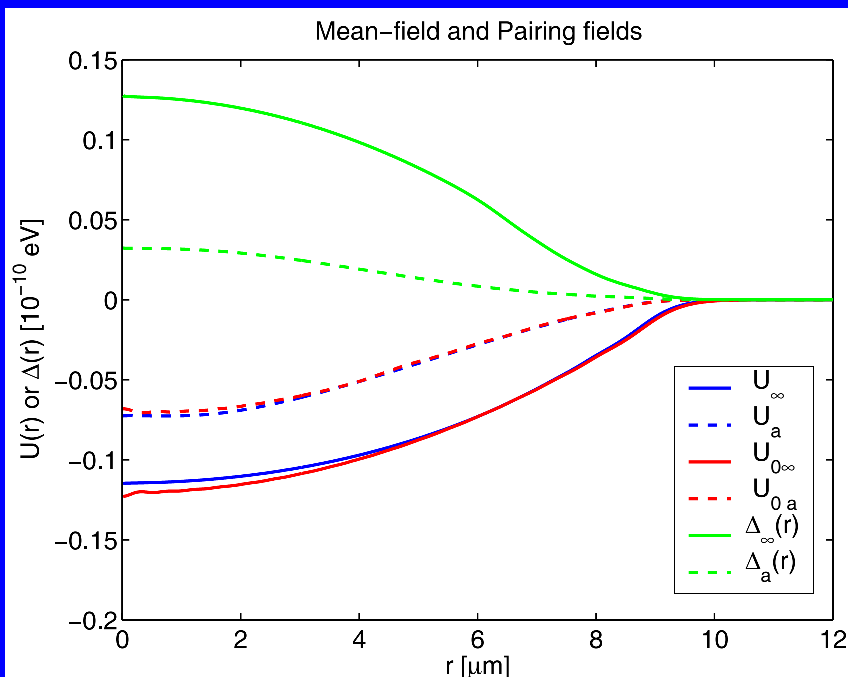


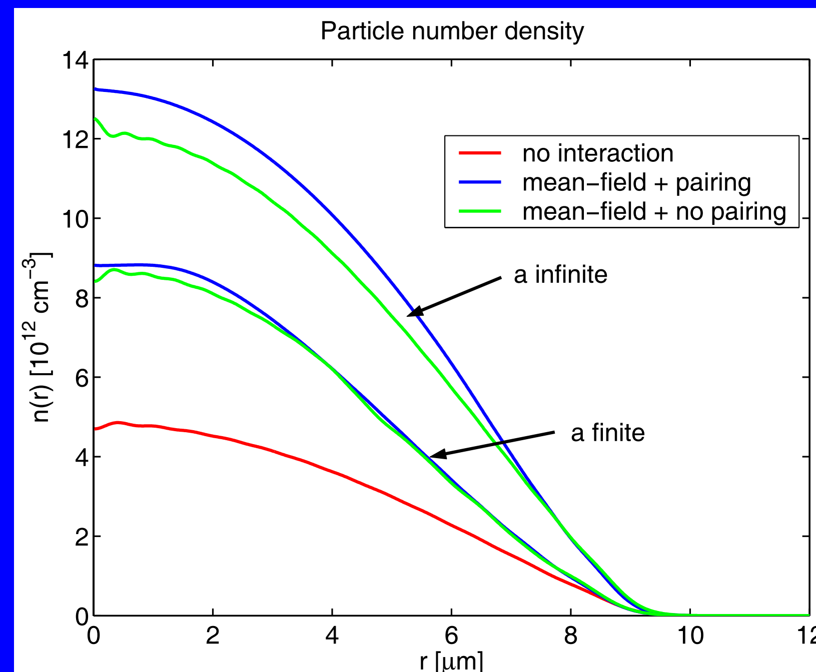


Solid lines are results for parameter set I, dashed lines for parameter set II (dots – velocity profile for ideal vortex)

40K (Fermi) atoms in a spherical harmonic trap

Effect of interaction, with and without weak and strong pairing correlations with fixed chemical potential.

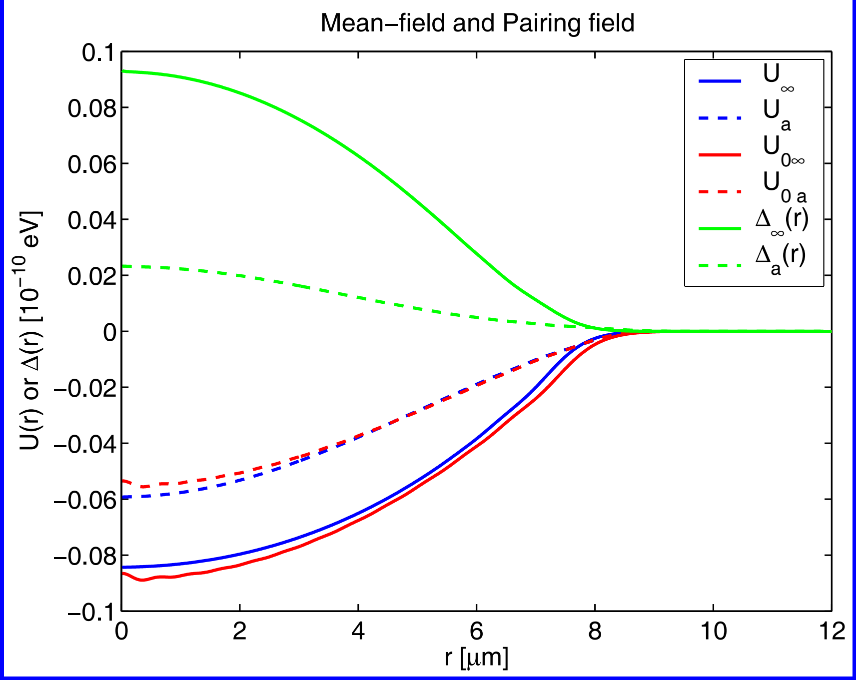


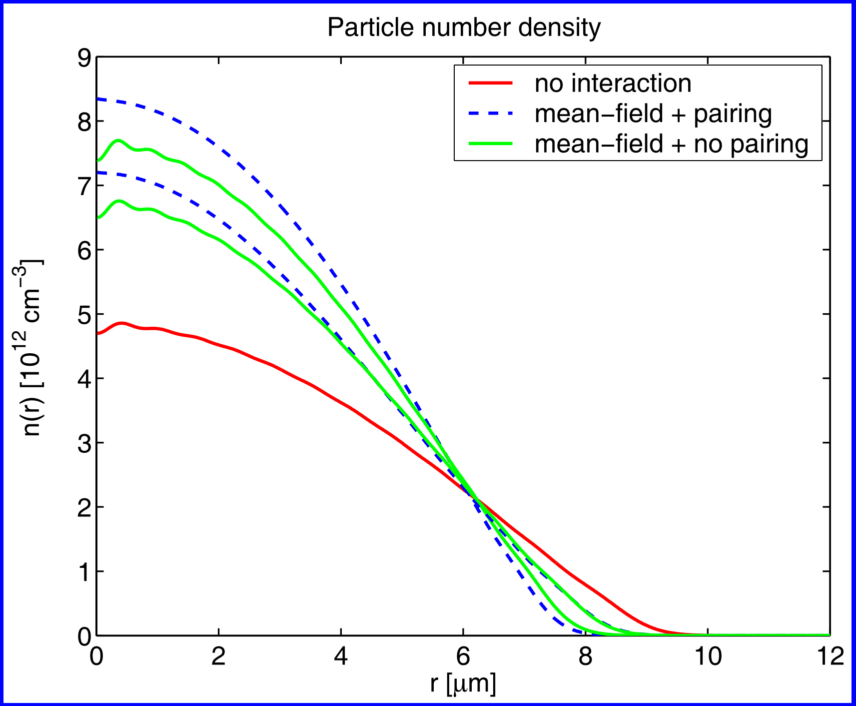


m= 0.14µ10^{-10}eV, ħw=0.568 µ10^{-12}eV,
a = -12.63nm (when finite)

40K (Fermi) atoms in a spherical harmonic trap

Effect of interaction, with and without weak and strong pairing correlations with fixed particle number, N = 5200.





$\hbar w=0.568\ \mu 10^{-12}$eV, a = -12.63nm (when finite)

Lessons:

- ✓ There are essentially no systems where the naïve BCS/HFB approximation works with reasonable accuracy. In nuclear/neutron matter one would need densities lower than 10^{-6} fm^{-3} for the simple BCS formula and one would still be off by a factor of 2 in the gap.

- ✓ Even in dilute Fermi gases corrections to the BCS/HFB formalism are essential, even though often (but not always) $n|a|^3$ á 1

- ✓ Nuclei and neutron stars are essentially in the regime of strong coupling, even though in particular nuclei the pairing gaps are relatively small

- ✓ No need for a finite range of the interaction so far in pairing and the big question is whether there are any nuclear phenomena in particular where the role of the finite range corrections in medium could be ascertained <u>unambiguously!?</u>

Summary

- ✓ LDA for Fermi systems with superfluid correlation is simple and easy to implement with <u>all nuclear symmetries satisfied</u>.
- ✓ The agreement between experiment and theory for one- and two-nucleon separation energies is spectacular, and likely there is lot of room for improvement.
- ✓ The simple form of the LDA functional suggest new facets of pairing worthy investigation ($g_0 \neq g_1$).
- ✓ Application to other physical systems: neutron stars, dilute (and not) atomic systems straightforward and offering new qualitative results.
- ✓ A number of theoretical developments desirable: isoscalar and isovector density dependence of the pairing coupling, "effective range" corrections(?), linear response, EDF dependences on anomalous densities other than quadratic, pairing in other partial waves, coupling between neutron and proton pairing vibrations.