Time-Dependent Dynamics of Fermionic Superfluids: from cold atomic gases, to nuclei and neutron stars

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Slides pptx with movies or pdf with no movies can be downloaded from
http://www.phys.washington.edu/users/bulgac/Media/Nordita_September_2014.pptx
http://www.phys.washington.edu/users/bulgac/Media/Nordita_September_2014.pdf

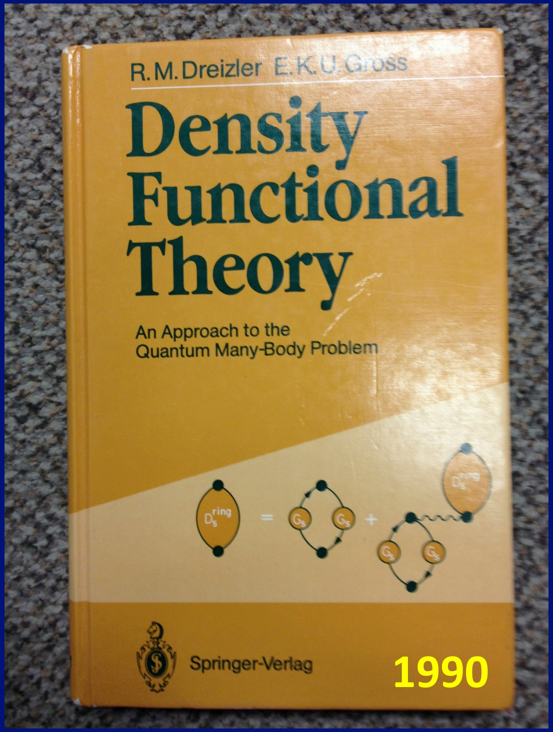
Why should one study fermionic superfluidity?

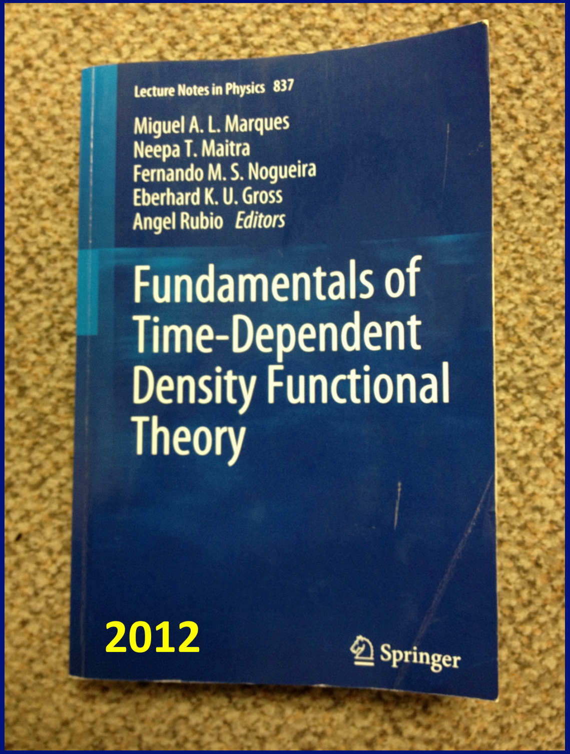
Superconductivity (discovered on April 8th, 1911) and superfluidity in Fermi systems are manifestations of quantum coherence at a macroscopic level

✓ Dilute atomic Fermi gases	$T_c \approx 10^{-9}$ eV
✓ Liquid ^{3}He	$T_c \approx 10^{-7}$ eV
✓ Metals, composite materials	$T_c \approx 10^{-3} - 10^{-2}$ eV
✓ Nuclei, neutron stars	$T_c \approx 10^{5} - 10^{6}$ eV
• QCD color superconductivity	$T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV ≈ 10^4 K)

Main Theoretical Tool





DFT has been developed and used mainly to describe normal (non-superfluid) electron systems – <u>50 years old theory</u>, Kohn and Hohenberg, 1964

A new local extension of DFT to superfluid systems and time-dependent phenomena was developed

Review: A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids*, Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)

Physical systems and processes we are interested in:

- ✓ Collective states in nuclei
- ✓ Nuclear large amplitude collective motion (LACM)
  (Induced) nuclear fission
- ✓ Excitation of nuclei with gamma rays and neutrons
- ✓ Coulomb excitation of nuclei with relativistic heavy-ions
- ✓ Nuclear reactions, fusion between colliding heavy-ions
- ✓ Neutron star crust and dynamics of vortices and their pinning mechanism

- ✓ Dynamics of vortices, Anderson-Higgs Mode
- ✓ Vortex crossing and reconnection and the onset of quantum turbulence
- ✓ Domain wall solitons and shock waves in collision of fermionic superfluid atomic clouds

One option is the two-fluid hydrodynamics (here at T=0, only one fluid)

N.B. There is no quantum statistics in two-fluid hydrodynamics

$$\frac{\partial n(\vec{r},t)}{\partial t} + \vec{\nabla} \bullet \left[\vec{\mathrm{v}}(\vec{r},t) n(\vec{r},t) \right] = 0$$

$$m \frac{\partial \vec{\mathrm{v}}(\vec{r},t)}{\partial t} + \vec{\nabla} \left\{ \frac{m \vec{\mathrm{v}}^2(\vec{r},t)}{2} + \mu\left[n(\vec{r},t) \right] + V_{ext}(\vec{r},t) \right\} = 0$$

Troubles:

- **These are classical equations, <u>no Planck's constant</u>, thus no quantized vortices (unless one imposes by hand quantization)**
- **No physically clear physical mechanism to describe superfluid to normal transition (no role for the critical velocity)**

Two-fluid hydrodynamics + vortex quantization is equivalent to a ``Bohr model" of a superfluid

Another option is the phenomenological Ginzburg-Landau model or the Gross-Pitaevskii equation:

$$i\hbar e^{i\gamma}\frac{\partial\Psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2\Delta\Psi(\vec{r},t)}{2M} + U\left(\left|\Psi(\vec{r},t)\right|^2\right)\Psi(\vec{r},t) + V_{ext}(\vec{r},t)\Psi(\vec{r},t) + fluct.$$

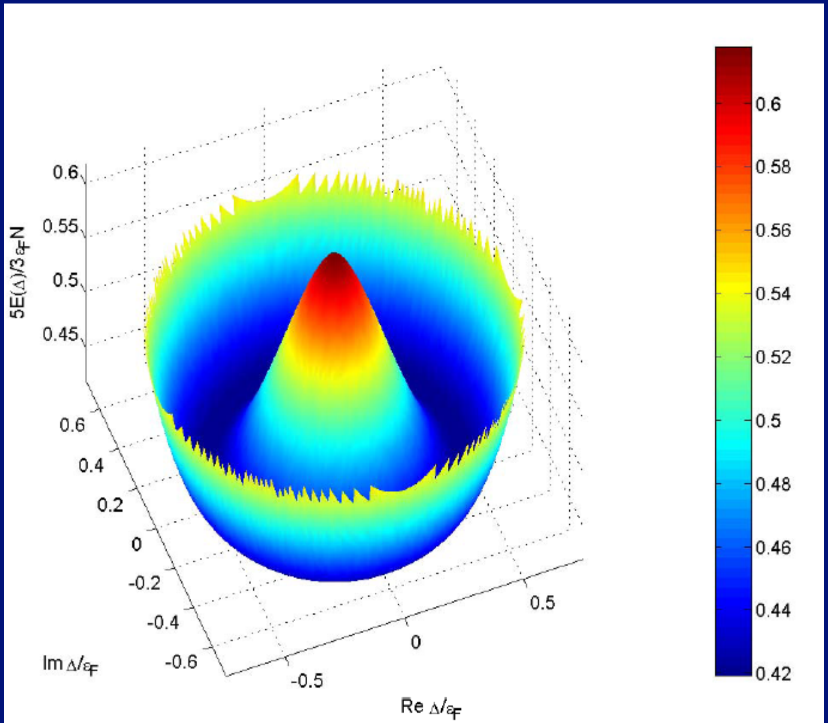
Troubles:

- **GLE valid only for temperatures near and below the critical temperature**
- **Even though is a quantum approach, it describes only the superfluid phase. There is no Cooper pair breaking mechanism**
- **GPE was the only microscopic equation available until recently, valid a superfluid of weakly interacting bosons at T=0**

Other issues:

There are a number of modes, such as the Anderson-Higgs mode, which cannot be describes in either of these phenomenological approaches.

Energy of a Fermi system as a function of the pairing gap: Anderson-Higgs mode



Both fail

$$\dot{n}+\vec{\nabla}\cdot\left[\vec{\mathrm{v}}n\right]=0$$

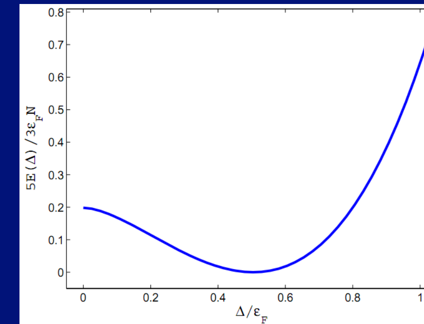
$$m\dot{\vec{\mathrm{v}}}+\vec{\nabla}\left\{\frac{m\vec{\mathrm{v}}^2}{2}+\mu\left[n\right]\right\}=0$$

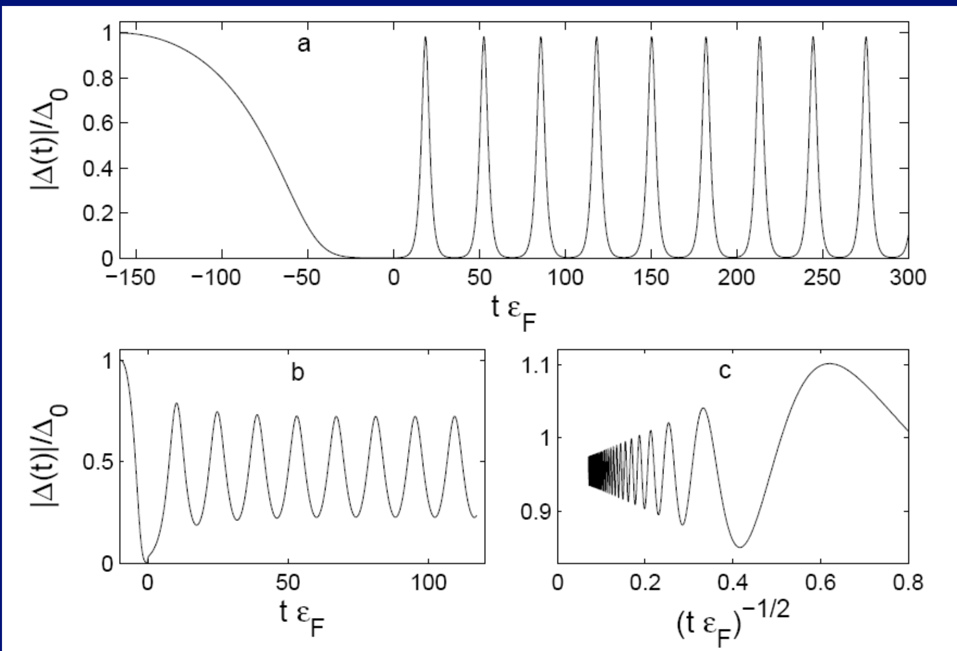
Landau's two-fluid hydrodynamics

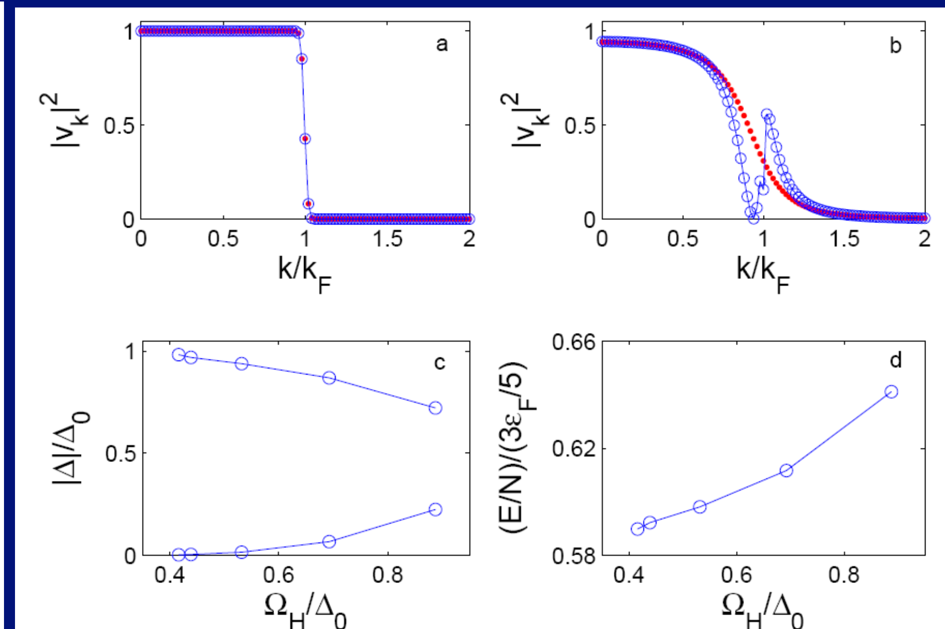
$$i\hbar\dot{\Psi}(\vec{r},t)=-\frac{\hbar^2}{4m}\Delta\Psi(\vec{r},t)+U\left(\left|\Psi(\vec{r},t)\right|^2\right)\Psi(\vec{r},t)$$

Ginzburg-Landau-like equation

Response of a unitary Fermi system to changing the scattering length with time







- All these modes have a very low frequency below the pairing gap, a very large amplitude and very large excitation energy

- None of these modes can be described either within two-fluid hydrodynamics or Ginzburg-Landau like approaches

Bulgac and Yoon, Phys. Rev. Lett. 102, 085302 (2009)

In order to treat this plethora of phenomena one needs to treat spatially inhomogeneous systems in real time!

Methods?

- **Quantum Monte Carlo is feasible for small particle numbers only and has been implemented so far only for static phenomena**

- **Density Functional Theory (large particle numbers)**

One needs:

1) **to find an Energy Density Functional (EDF)**
2) **to extend DFT to superfluid phenomena (SLDA)**
3) **to extend SLDA to time-dependent phenomena (TDSLDA)**
4) **to develop a stochastic extension (STDSLDA) if possible**

Kohn-Sham theorem (1965)

$$H = \sum_{i}^{N} T(i) + \sum_{i<j}^{N} U(ij) + \sum_{i<j<k}^{N} U(ijk) + \ldots + \sum_{i}^{N} V_{ext}(i)$$

$$H\Psi_0(1,2,\ldots N) = E_0\Psi_0(1,2,\ldots N)$$

$$n(\vec{r}) = \left\langle \Psi_0 \left| \sum_{i}^{N} \delta(\vec{r}-\vec{r}_i) \right| \Psi_0 \right\rangle$$

Injective map (one-to-one)

$$\Psi_0(1,2,\ldots N) \quad \Leftrightarrow \quad V_{ext}(\vec{r}) \quad \Leftrightarrow \quad n(\vec{r})$$

$$E_0 = \min_{n(\vec{r})} \int d^3r \left\{ \frac{\hbar^2}{2m^*(\vec{r})}\tau(\vec{r}) + \varepsilon\left[n(\vec{r})\right] + V_{ext}(\vec{r})n(\vec{r}) \right\}$$

$$n(\vec{r}) = \sum_{i}^{N} \left|\varphi_i(\vec{r})\right|^2, \qquad \tau(\vec{r}) = \sum_{i}^{N} \left|\vec{\nabla}\varphi_i(\vec{r})\right|^2$$

THEOREM: There exist an universal functional of particle density alone <u>independent of the external potential</u>

Normal Fermi systems only!

However, not everyone is normal!

I will illustrate the construction of the DFT functional for a superfluid unitary Fermi gas

What is a unitary Fermi gas and why would one want to study it?

One reason:

(for the nerds, I mean the hard-core theorists, not for the phenomenologists)

Bertsch's Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

What are the scattering length and the effective range?

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + \cdots$$

$$\sigma = \frac{4\pi}{k^2} \sin^2 \delta_0 + \cdots = 4\pi a^2 + \cdots$$

If the energy is small, only the s-wave scattering is relevant.

Let us consider a very old and simple example:

The hydrogen atom.

The ground state energy could only be a function of:

- ✓ Electron charge
- ✓ Electron mass
- ✓ Planck's constant

and then trivial dimensional arguments lead to

$$E_{gs} = \frac{e^4 m}{\hbar^2} \times \frac{1}{2}$$

Only the factor ½ requires some hard work.

Let us turn now to dilute fermion matter

The ground state energy is given by a function:

$$E_{gs} = f(N, V, \hbar, m, a, r_0)$$

Taking the scattering length to infinity and the range of the interaction to zero, we are left with:

$$E_{gs} = F(N, V, \hbar, m) = \frac{3}{5}\varepsilon_F N \times \xi$$

$$\frac{N}{V} = \frac{k_F^3}{3\pi^2}, \qquad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

Pure number (dimensionless)

The SLDA (DFT) energy density functional for unitary Fermi gas

Dimensional arguments, renormalizability, Galilean invariance, and symmetries ***determine the functional (energy density)***

$$\varepsilon(\vec{r}) = \frac{\hbar^2}{m}\left\{\left[\alpha\frac{\tau_c(\vec{r})}{2} + \gamma\frac{|\nu_c(\vec{r})|^2}{n^{1/3}(\vec{r})}\right] + \beta\frac{3(3\pi^2)^{2/3}n^{5/3}(\vec{r})}{5}\right\} - \frac{\hbar^2}{m}(\alpha-1)\frac{\vec{j}^{\,2}(\vec{r})}{2n(\vec{r})}$$

$$\Delta(\vec{r}) = \frac{\hbar^2}{m}\tilde{\Delta}(\vec{r})$$

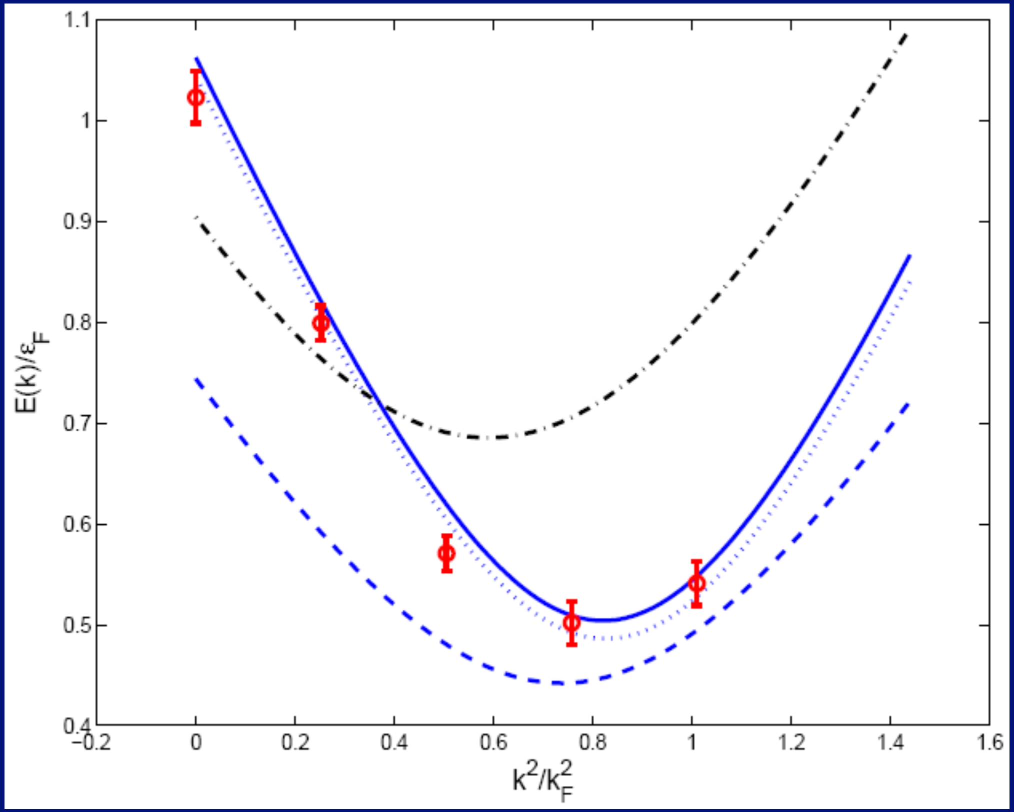
$$n(\vec{r}) = 2\sum_{0<E_k<E_c}|\mathrm{v}_\mathrm{k}(\vec{r})|^2, \quad \tau_c(\vec{r}) = 2\sum_{0<E_k<E_c}|\vec{\nabla}\mathrm{v}_\mathrm{k}(\vec{r})|^2,$$

$$\nu_c(\vec{r}) = \sum_{0<E<E_c}\mathrm{u}_\mathrm{k}(\vec{r})\mathrm{v}^*_\mathrm{k}(\vec{r}) \quad \Leftarrow \text{ divergent without a cutoff, need RG}$$

Three dimensionless constants α, β, and γ determining the functional are extracted from QMC for homogeneous systems by fixing the total energy, the pairing gap and the effective mass

The unitary Fermi gas and the dilute Bose gas are the only superfluids for which a microscopic framework exist to describe both statics and dynamics

Quasiparticle spectrum in homogeneous matter



solid/dotted blue line - SLDA based on homogeneous GFMC due to Carlson *et al*
red circles - GFMC due to Carlson and Reddy
dashed blue line - SLDA, homogeneous MC due to Juillet
black dashed-dotted line – meanfield at unitarity

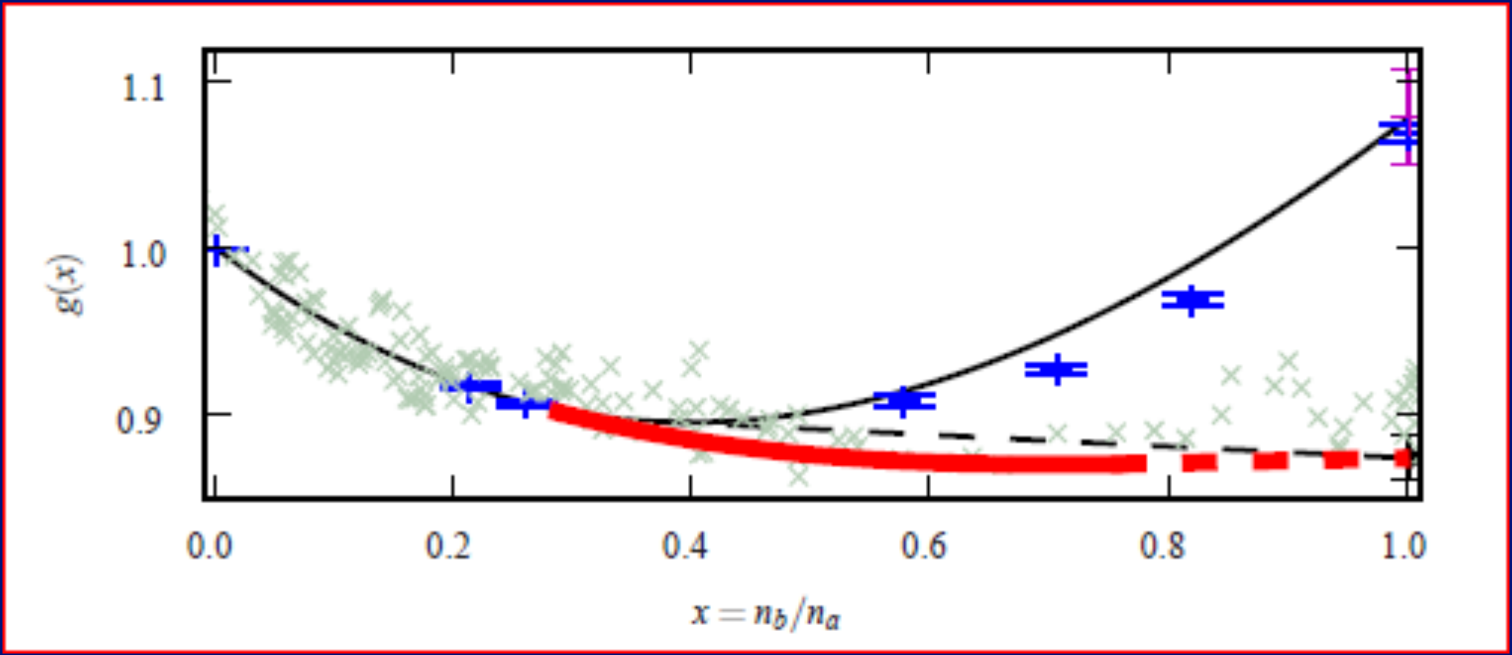
Bulgac, PRA 76, 040502(R) (2007)

Normal State			
(N_a,N_b)	E_{FNDMC}	E_{ASLDA}	(error)
(3,1)	6.6 ± 0.01	6.687	1.3%
(4,1)	8.93 ± 0.01	8.962	0.36%
(5,1)	12.1 ± 0.1	12.22	0.97%
(5,2)	13.3 ± 0.1	13.54	1.8%
(6,1)	15.8 ± 0.1	15.65	0.93%
(7,2)	19.9 ± 0.1	20.11	1.1%
(7,3)	20.8 ± 0.1	21.23	2.1%
(7,4)	21.9 ± 0.1	22.42	2.4%
(8,1)	22.5 ± 0.1	22.53	0.14%
(9,1)	25.9 ± 0.1	25.97	0.27%
(9,2)	26.6 ± 0.1	26.73	0.5%
(9,3)	27.2 ± 0.1	27.55	1.3%
(9,5)	30 ± 0.1	30.77	2.6%
(10,1)	29.4 ± 0.1	29.41	0.034%
(10,2)	29.9 ± 0.1	30.05	0.52%
(10,6)	35 ± 0.1	35.93	2.7%
(20,1)	73.78 ± 0.01	73.83	0.061%
(20,4)	73.79 ± 0.01	74.01	0.3%
(20,10)	81.7 ± 0.1	82.57	1.1%
(20,20)	109.7 ± 0.1	113.8	3.7%
(35,4)	154 ± 0.1	154.1	0.078%
(35,10)	158.2 ± 0.1	158.6	0.27%
(35,20)	178.6 ± 0.1	180.4	1%

Superfluid State			
(N_a,N_b)	E_{FNDMC}	E_{ASLDA}	(error)
(1,1)	2.002 ± 0	2.302	15%
(2,2)	5.051 ± 0.009	5.405	7%
(3,3)	8.639 ± 0.03	8.939	3.5%
(4,4)	12.573 ± 0.03	12.63	0.48%
(5,5)	16.806 ± 0.04	16.19	3.7%
(6,6)	21.278 ± 0.05	21.13	0.69%
(7,7)	25.923 ± 0.05	25.31	2.4%
(8,8)	30.876 ± 0.06	30.49	1.2%
(9,9)	35.971 ± 0.07	34.87	3.1%
(10,10)	41.302 ± 0.08	40.54	1.8%
(11,11)	46.889 ± 0.09	45	4%
(12,12)	52.624 ± 0.2	51.23	2.7%
(13,13)	58.545 ± 0.18	56.25	3.9%
(14,14)	64.388 ± 0.31	62.52	2.9%
(15,15)	70.927 ± 0.3	68.72	3.1%
(1,0)	1.5 ± 0.0	1.5	0%
(2,1)	4.281 ± 0.004	4.417	3.2%
(3,2)	7.61 ± 0.01	7.602	0.1%
(4,3)	11.362 ± 0.02	11.31	0.49%
(7,6)	24.787 ± 0.09	24.04	3%
(11,10)	45.474 ± 0.15	43.98	3.3%
(15,14)	69.126 ± 0.31	62.55	9.5%

Bulgac, Forbes, and Magierski, Lecture Notes in Physics (2012)

EOS for spin polarized systems



Red line: Larkin-Ovchinnikov phase (unitary Fermi supersolid)

Black line: normal part of the energy density
Blue points: DMC calculations for normal state, Lobo et al, PRL 97, 200403 (2006)
Gray crosses: experimental EOS due to Shin, Phys. Rev. A 77, 041603(R) (2008)

$$E(n_a, n_b) = \frac{3}{5}\frac{(6\pi^2)^{2/3}\hbar^2}{2m}\left[n_a g\left(\frac{n_b}{n_a}\right)\right]^{5/3}$$

Bulgac and Forbes,
Phys. Rev. Lett. 101, 215301 (2008)

Formalism for Time-Dependent Phenomena

"The time-dependent density functional theory is viewed in general as a reformulation of the exact quantum mechanical time evolution of a many-body system when only one-body properties are considered."

A.K. Rajagopal and J. Callaway, Phys. Rev. B 7, 1912 (1973)
V. Peuckert, J. Phys. C 11, 4945 (1978)
E. Runge and E.K.U. Gross, Phys. Rev. Lett. 52, 997 (1984)

http://www.tddft.org

$$E(t) = \int d^3r \left[\varepsilon(n(\vec{r},t), \tau(\vec{r},t), \nu(\vec{r},t), \vec{j}(\vec{r},t)) + V_{ext}(\vec{r},t) n(\vec{r},t) + \ldots \right]$$

$$\begin{cases} [h(\vec{r},t) + V_{ext}(\vec{r},t) - \mu]\mathrm{u}_{\mathrm{i}}(\vec{r},t) + [\Delta(\vec{r},t) + \Delta_{ext}(\vec{r},t)]\mathrm{v}_{\mathrm{i}}(\vec{r},t) = i\hbar \dfrac{\partial \mathrm{u}_{\mathrm{i}}(\vec{r},t)}{\partial t} \\ [\Delta^*(\vec{r},t) + \Delta^*_{ext}(\vec{r},t)]\mathrm{u}_{\mathrm{i}}(\vec{r},t) - [h(\vec{r},t) + V_{ext}(\vec{r},t) - \mu]\mathrm{v}_{\mathrm{i}}(\vec{r},t) = i\hbar \dfrac{\partial \mathrm{v}_{\mathrm{i}}(\vec{r},t)}{\partial t} \end{cases}$$

For time-dependent phenomena one has to add currents.
Galilean invariance determines the dependence on currents.

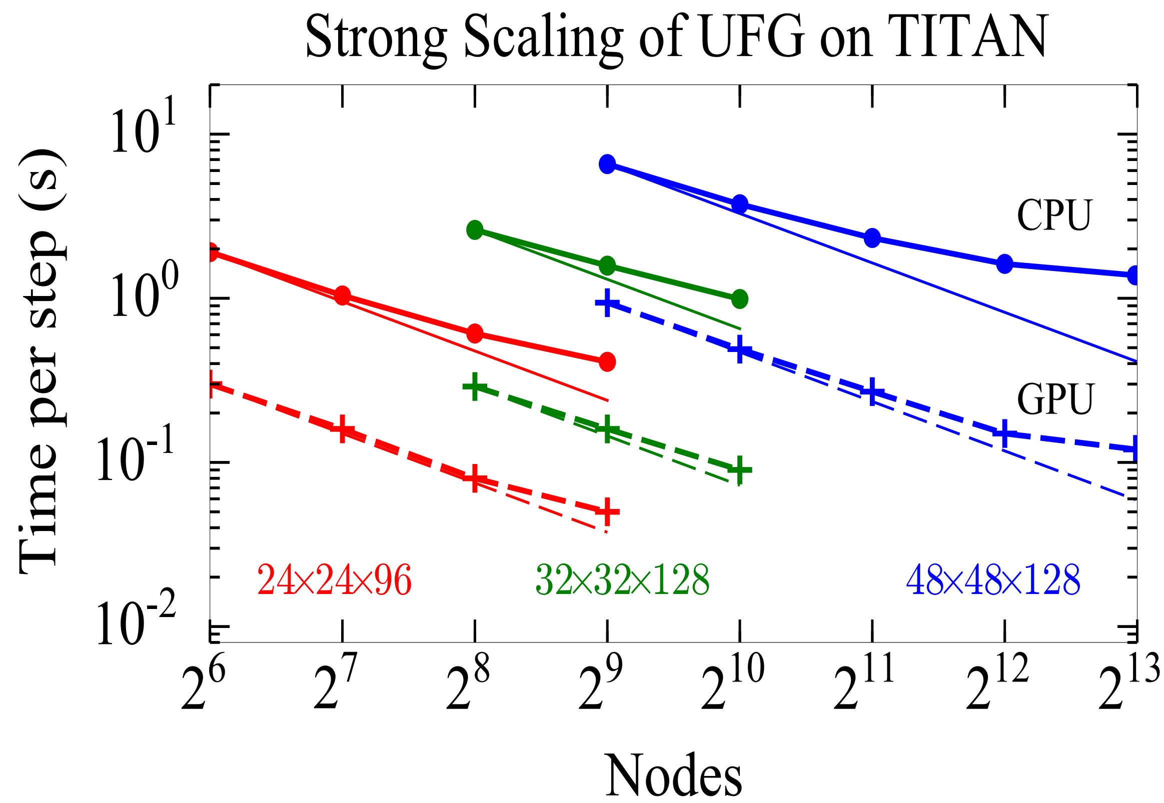
TDSLDA equations

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n\uparrow}(\vec{r},t) \\ u_{n\downarrow}(\vec{r},t) \\ v_{n\uparrow}(\vec{r},t) \\ v_{n\downarrow}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} \hat{h}_{\uparrow\uparrow}(\vec{r},t)-\mu & \hat{h}_{\uparrow\downarrow}(\vec{r},t) & 0 & \Delta(\vec{r},t) \\ \hat{h}_{\downarrow\uparrow}(\vec{r},t) & \hat{h}_{\downarrow\downarrow}(\vec{r},t)-\mu & -\Delta(\vec{r},t) & 0 \\ 0 & -\Delta^*(\vec{r},t) & -\hat{h}^*_{\uparrow\uparrow}(\vec{r},t)+\mu & -\hat{h}^*_{\uparrow\downarrow}(\vec{r},t) \\ \Delta^*(\vec{r},t) & 0 & -\hat{h}^*_{\downarrow\uparrow}(\vec{r},t) & -\hat{h}^*_{\downarrow\downarrow}(\vec{r},t)+\mu \end{pmatrix}\begin{pmatrix} u_{n\uparrow}(\vec{r},t) \\ u_{n\downarrow}(\vec{r},t) \\ v_{n\uparrow}(\vec{r},t) \\ v_{n\downarrow}(\vec{r},t) \end{pmatrix}$$

- **The system is placed on a large 3D spatial lattice (adequate representation of continuum)**
- **Derivatives are computed with FFTW (this insures machine accuracy) and is very fast**
- **Fully self-consistent treatment with fundamental symmetries respected (isospin, gauge, Galilean, rotation, translation)**
- **Adams-Bashforth-Milne fifth order predictor-corrector-modifier integrator**
- **No symmetry restrictions**
- ***Number of PDEs is of the order of the number of spatial lattice points – from 10,000s to 1-2,000,000***

$$\propto 4\left(\frac{2p_c L}{2\pi\hbar}\right)^3 = 4N_x N_y N_z$$

- **Initial state is the ground state of the SLDA (formally like HFB/BdG)**
- **The code was implemented on Jaguar, Titan, Franklin, Hopper, Edison, Hyak, Athena**
- **Initially Fortran 90, 95, 2003 …, presently C, CUDA, and obviously MPI, threads, etc.**



Sample Nuclear Code Comparisons (4-component qwfs)

$N_xN_yN_z$	N_{wf}	memory	CPU comp. + comm.	CPU comp.	GPU comp. + comm.	GPU comp.	# of GPUs	speedup
48^3	110592	10 TB	3.9s	2.4s	0.39s	0.023s	6912	10
64^3	262144	56 TB	20s	9.1s	0.80s	0.48s	16384	25

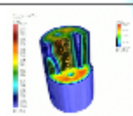
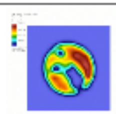
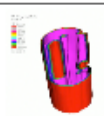
Over 1 million time-dependent 3D nonlinear complex coupled PDEs

Several hours of videos

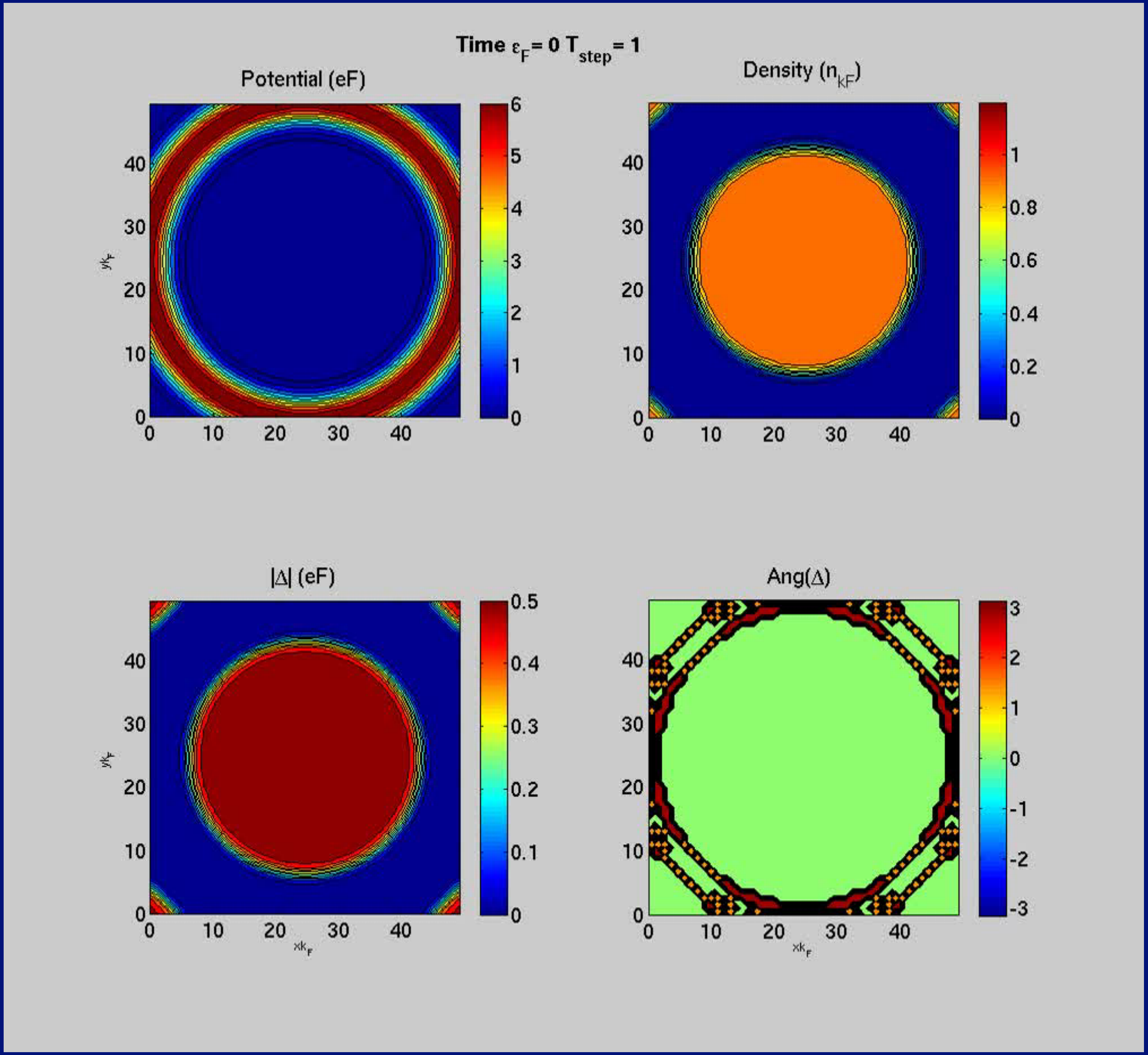
The Superfluid Local Density Approximation Applied to Unitary Fermi Gases -Supplementary Material

All simulations can be found here: http://www.phys.washington.edu/groups/qmbnt/UFG. The simulations can be categorized by the excitations: ball and rod, centered ball, centered small ball, centered big ball, centered supersonic ball, off-centered ball, and twisted stirrer. The following table matches simulations with numerical experiments. In several studies, we present multiple perspectives of the event as well as different plotting schemes to reveal different features of the dynamics.

3D Simulations

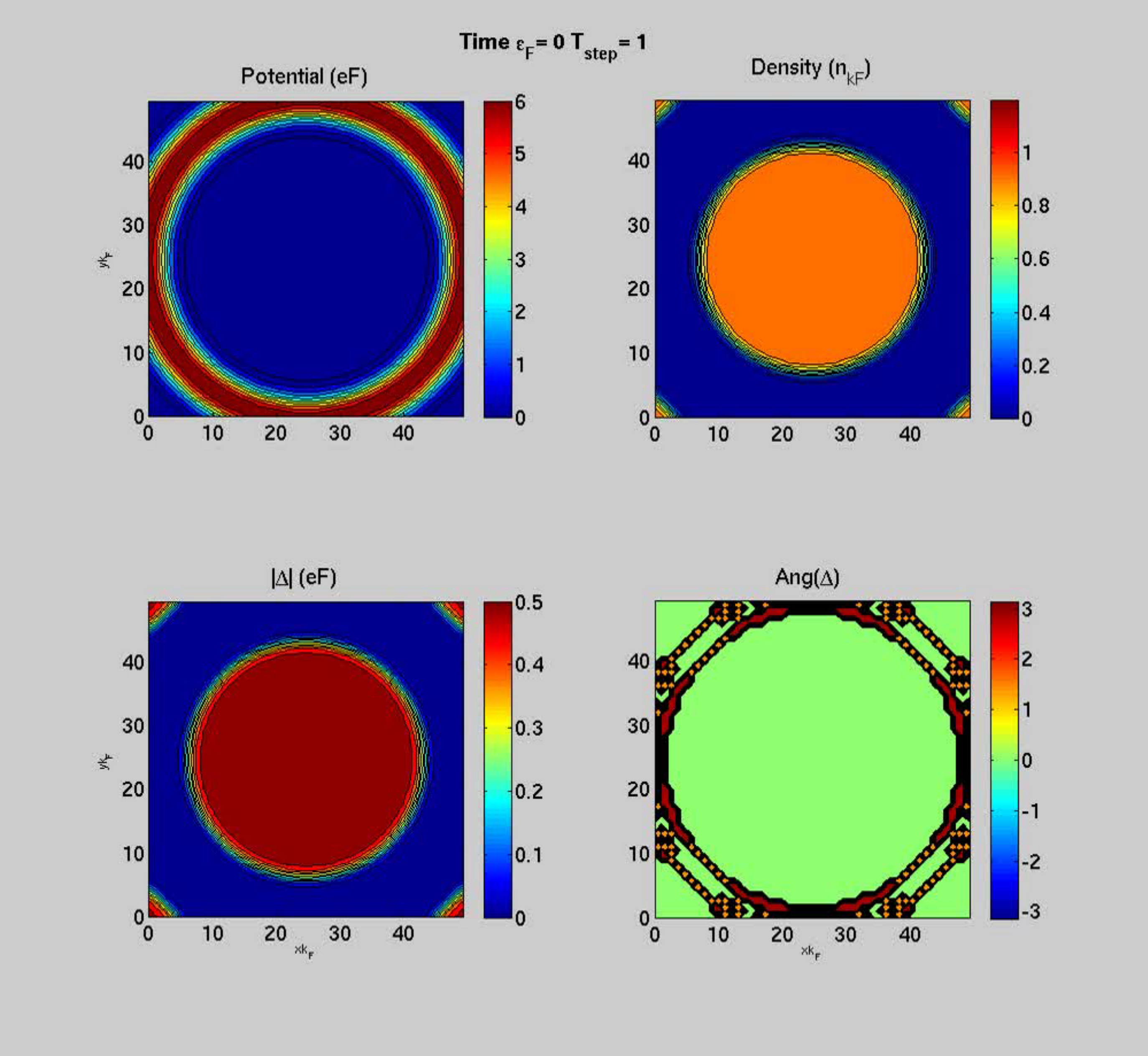
Excitation	Link	Description
Ball and Rod		
	nt-ball-rod-dns.m4v	density volume plot of magnitude of pairing field; front facing with quarter segment slice; 5m28s duration (20.9 MB)
	nt-ball-rod-dns-pln.m4v	density volume plot of magnitude of pairing field; 2D slice; 5m28s duration (9.8MB)
	nt-ball-rod-thin-angl.m4v	density contour plot of magnitude of pairing field focused on vortices ; angled front-facing with quarter segment slice; 5m28s duration (12.8MB)
Centered Ball		
	nt-ball-c.m4v	density contour plot of magnitude of pairing field focused on vortices; full geometry ; 3m29s

A. Bulgac, Y.-L. Luo, P. Magierski, K.J. Roche, Y. Yu
Science, 332, 1288 (2011)



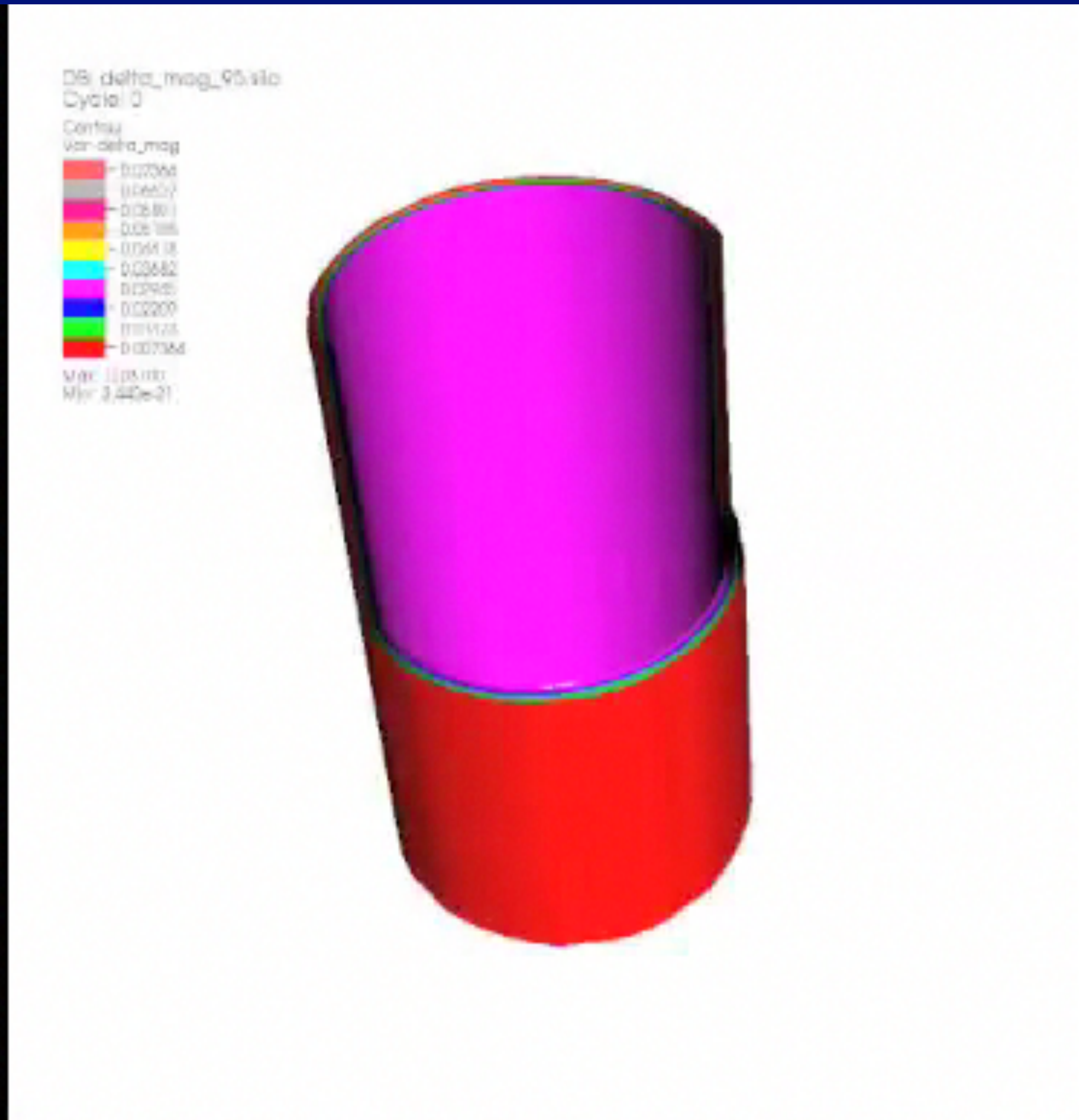
Time ε_F= 0 T_{step}= 1
Potential (eF)
Density (n_{kF})
|Δ| (eF)
Ang(Δ)
yk_F
xk_F

Movie

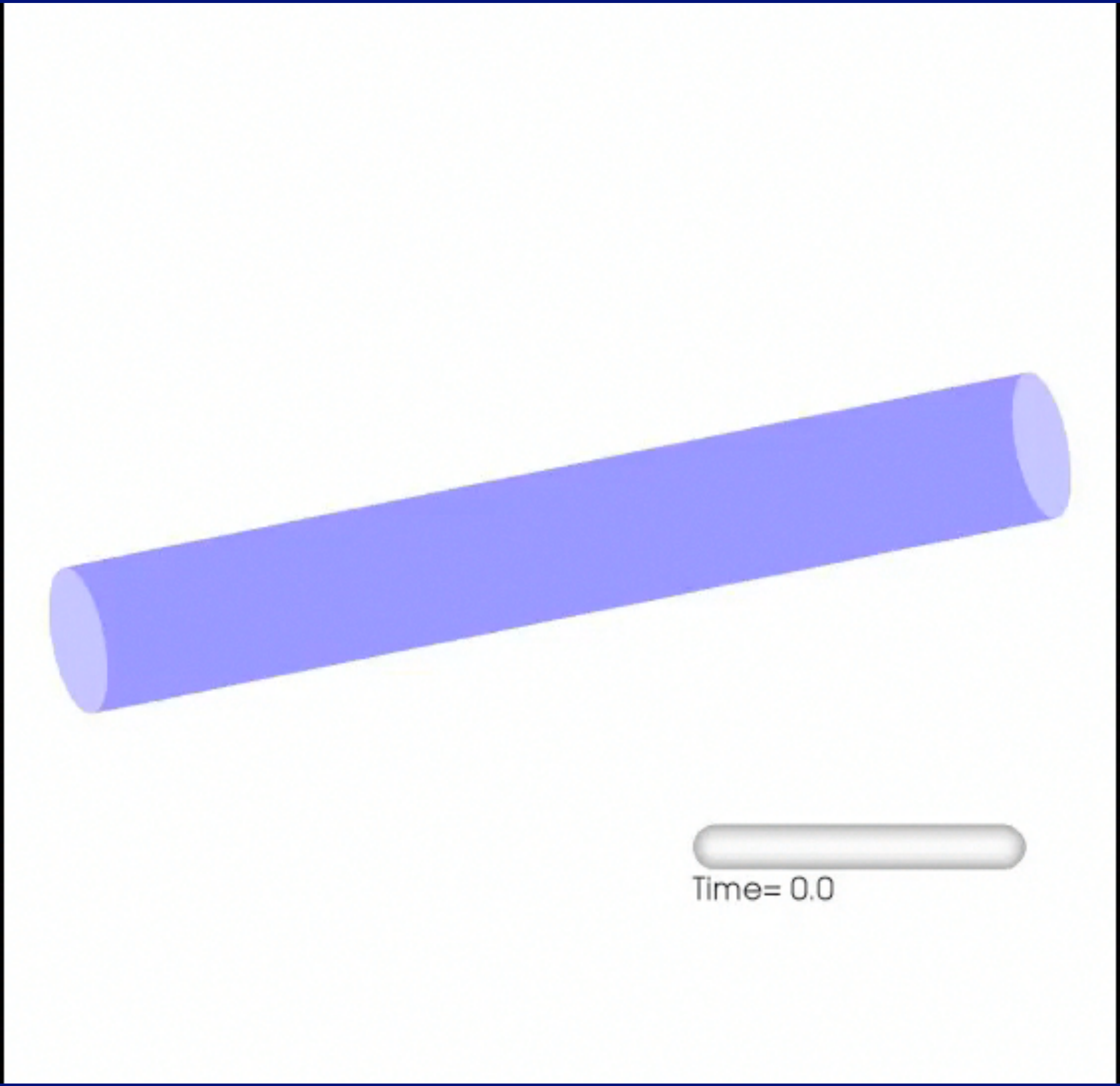


Time ε_F= 0 T_{step}= 1
Potential (eF)
Density (n_{kF})
|Δ| (eF)
Ang(Δ)
yk_F
xk_F

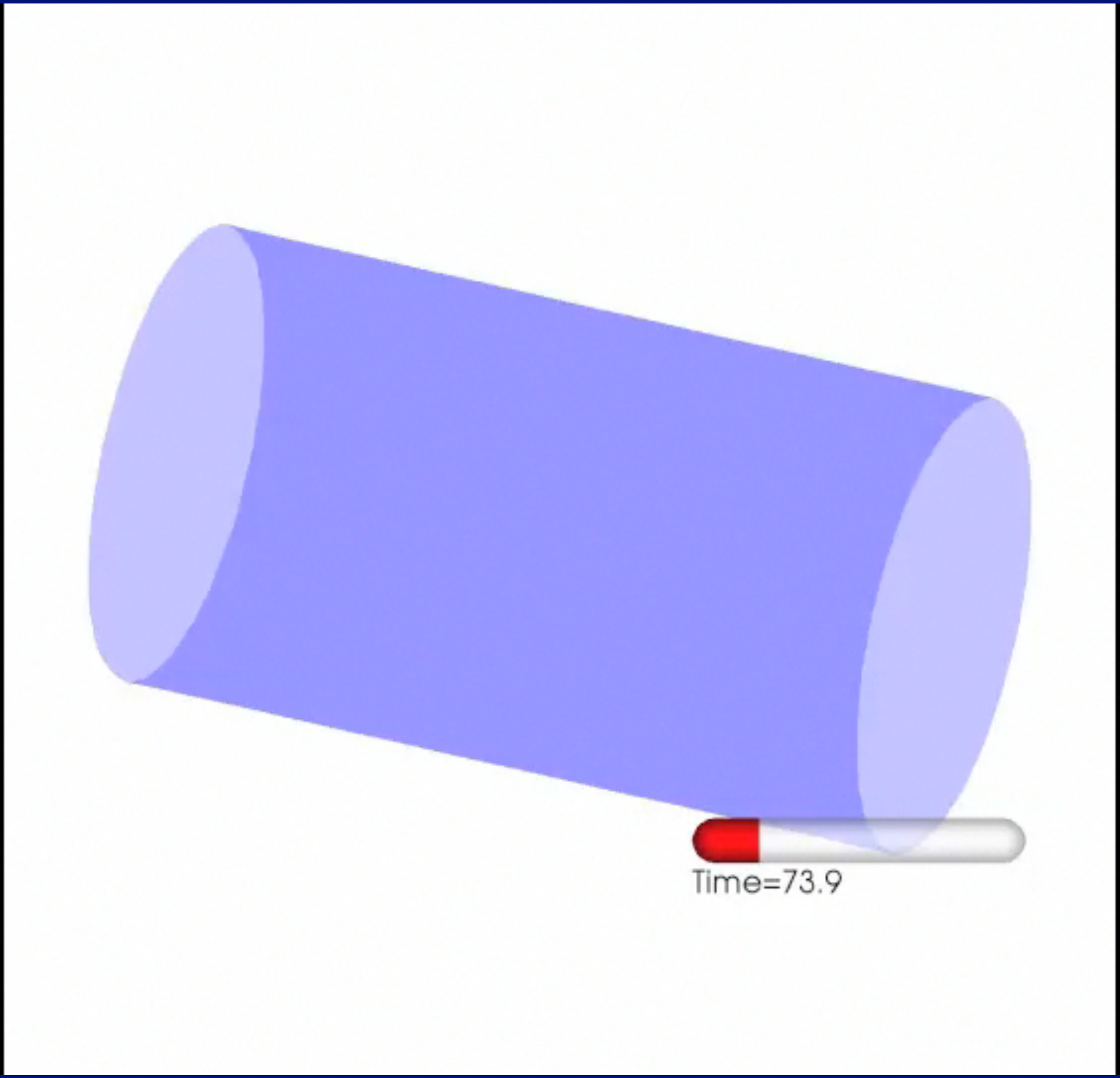
Movie



Movie



Movie



Movie

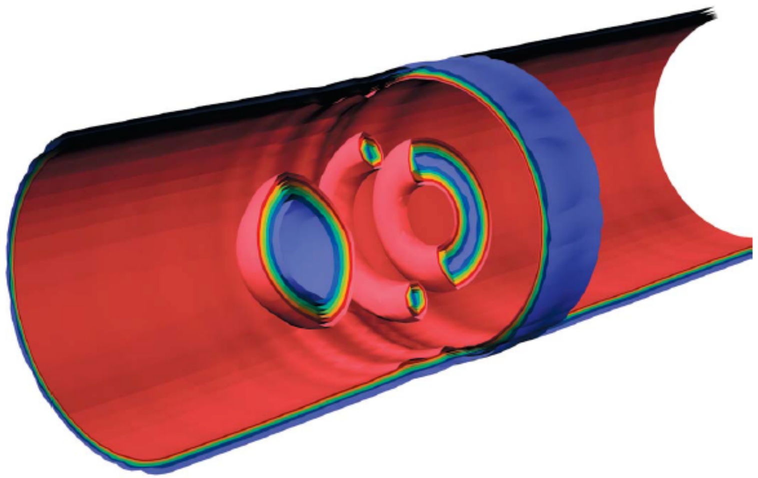


Fig. 2. A spherical projectile flying along the symmetry axis leaves in its wake two vortex rings.

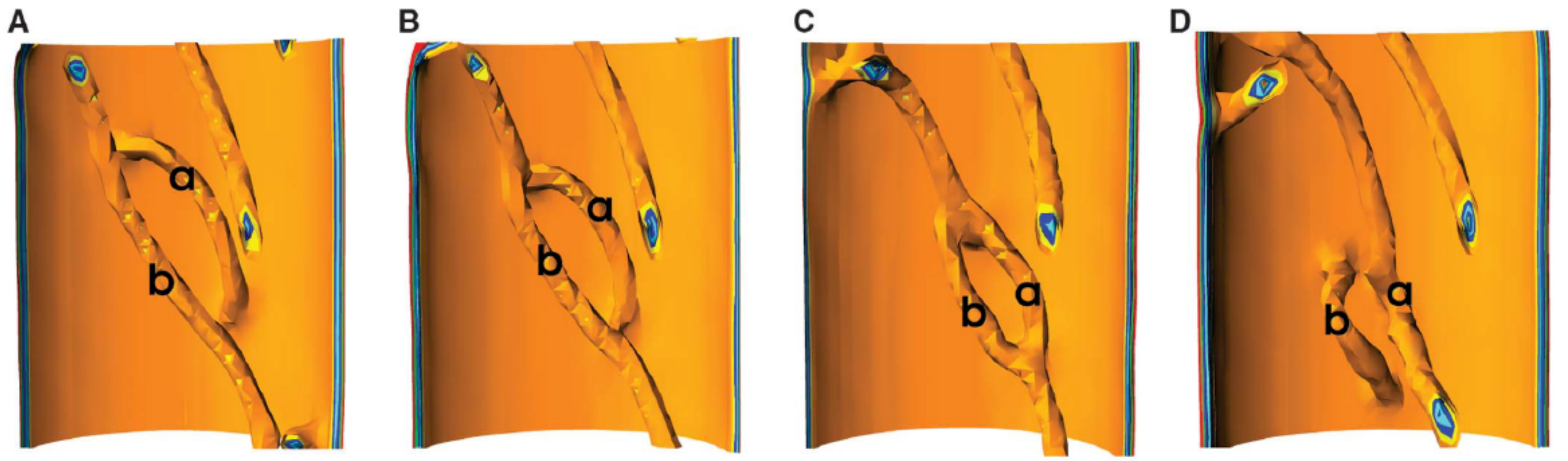
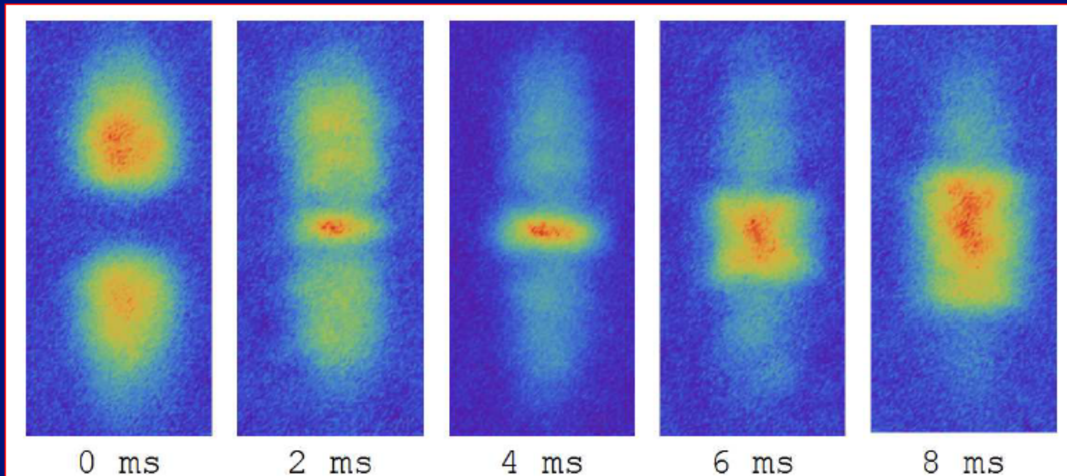
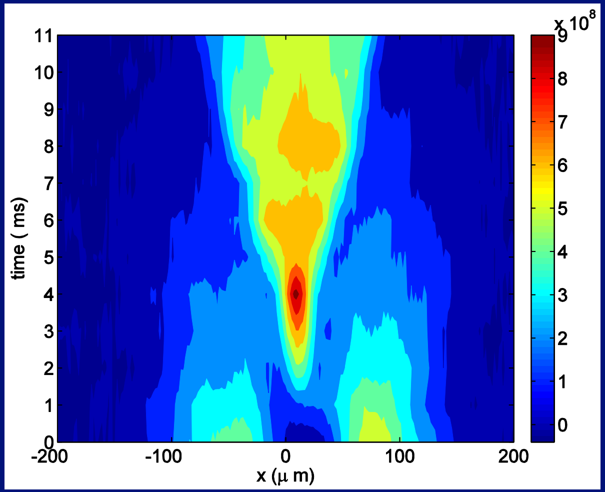


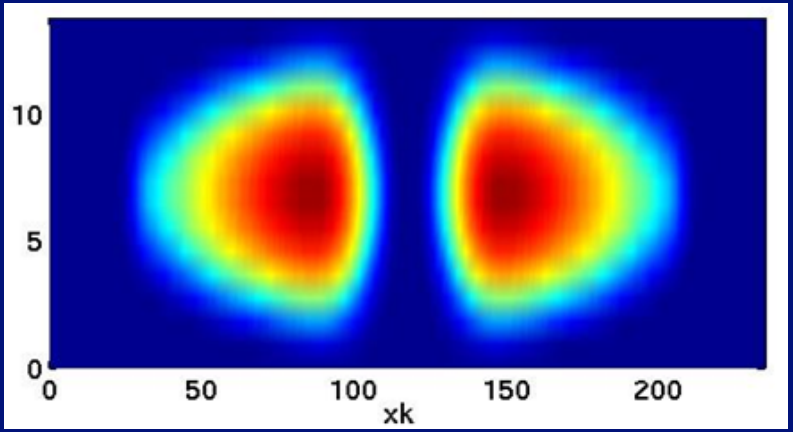
Fig. 3. (**A** to **D**) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.

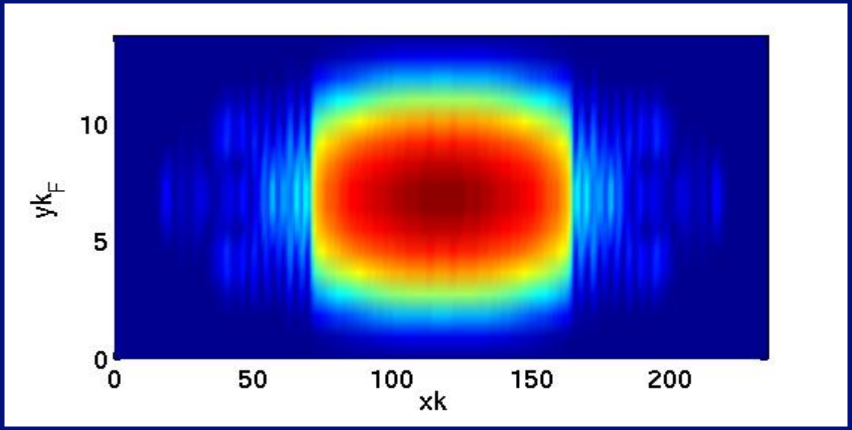
A. Bulgac, Y.-L. Luo, P. Magierski, K.J. Roche, Y. Yu
Science, 332, 1288 (2011)





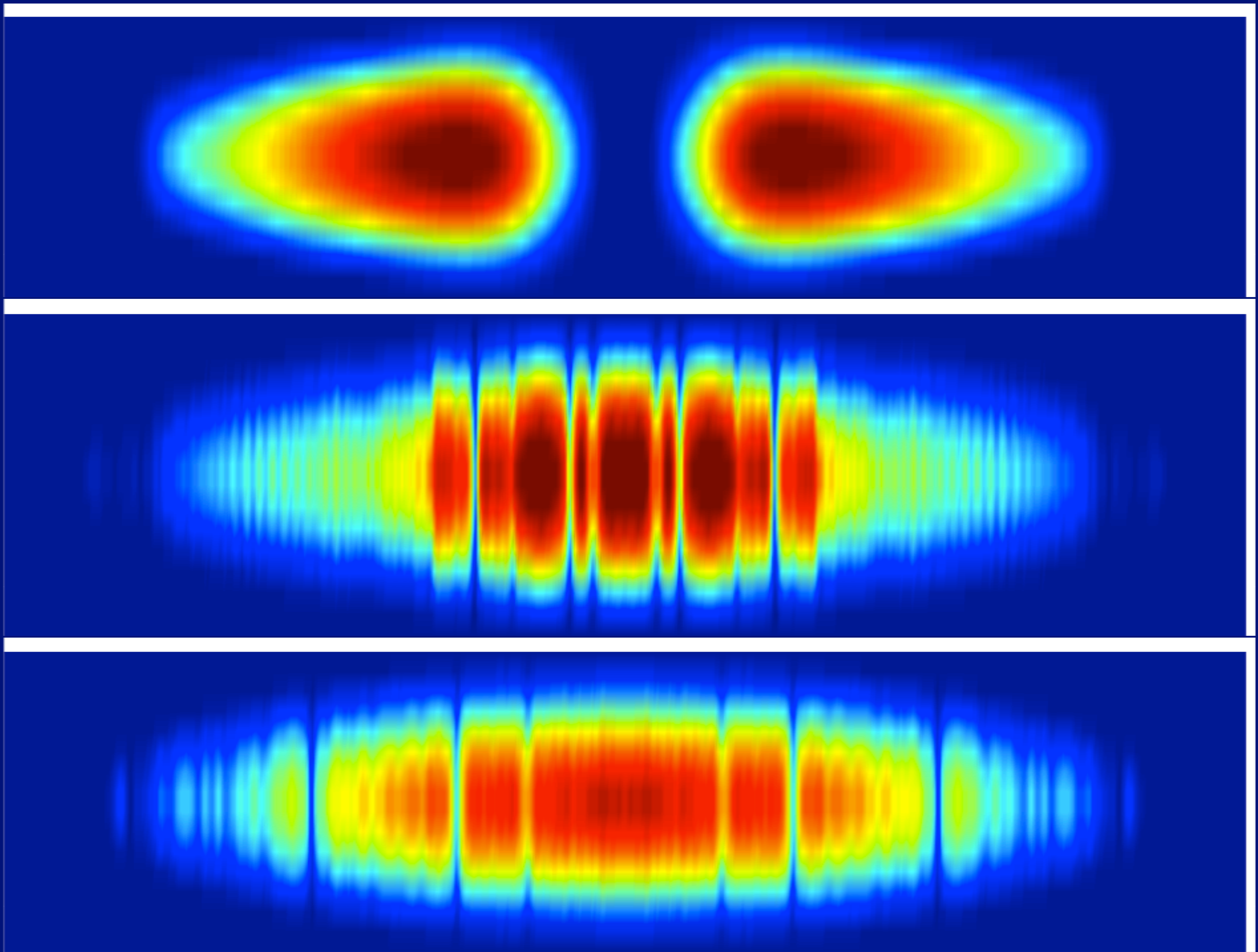
Observation of shock waves in a strongly interacting Fermi gas
J. Joseph, J.E. Thomas, M. Kulkarni, and A.G. Abanov PRL 106, 150401 (2011)



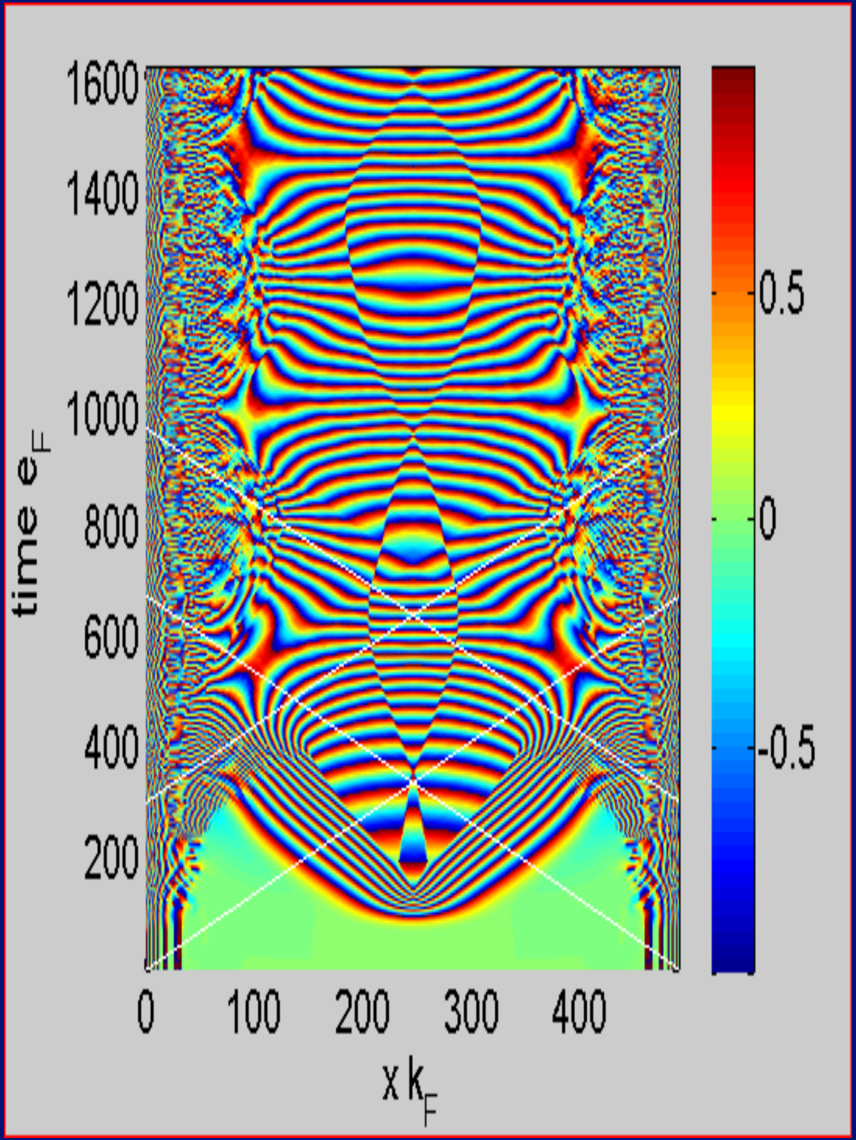


Number density of two colliding cold Fermi gases in TDSLDA
Bulgac, Luo, and Roche, Phys. Rev. Lett. 108, 150401 (2012)

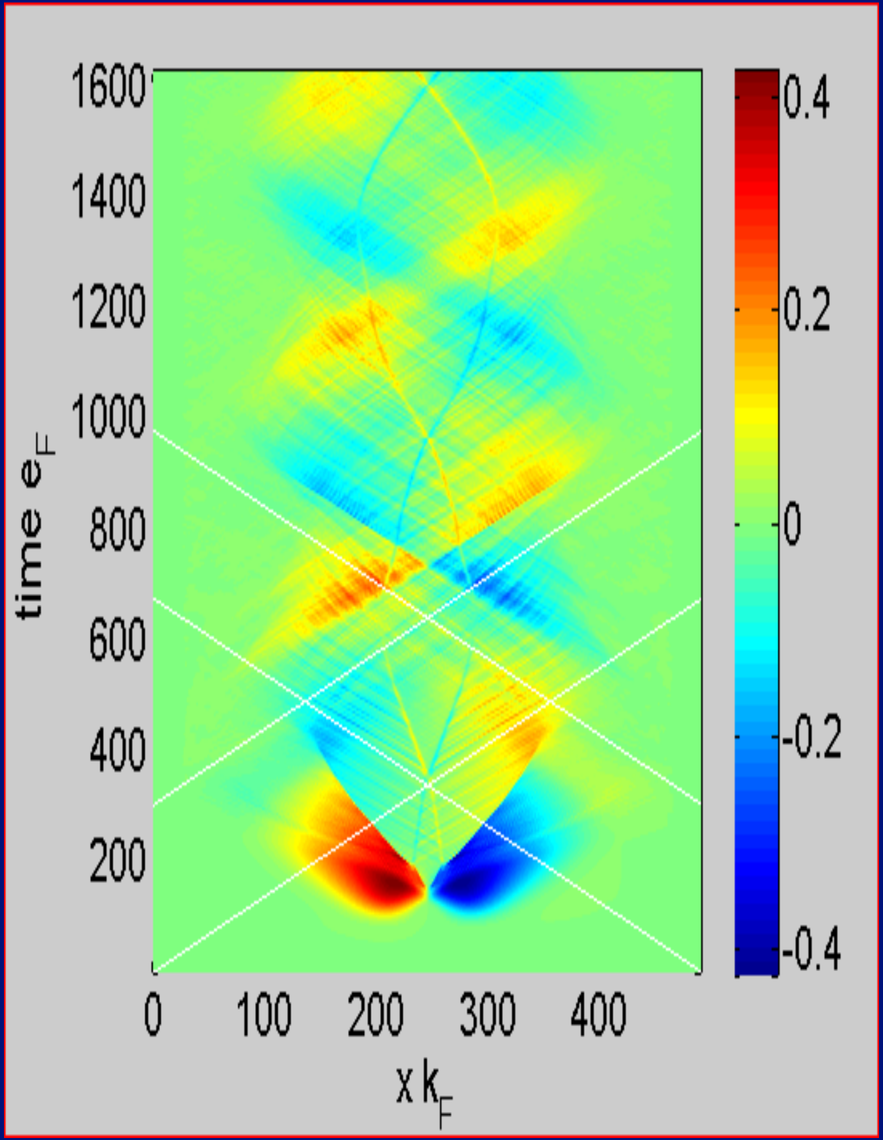
Collision of clouds with larger aspect ratio



Dark solitons/domain walls and shock waves in the collision of two UFG clouds



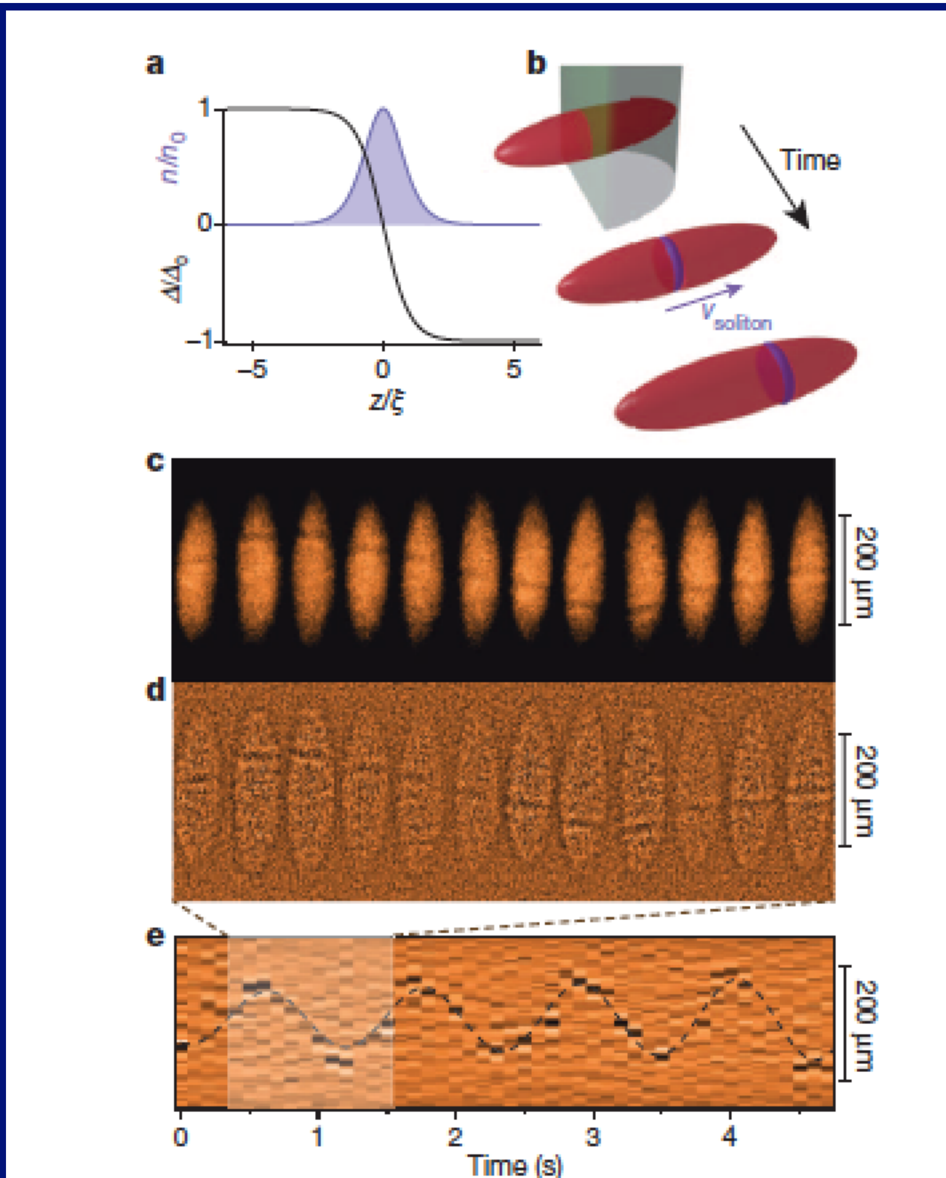
Phase of the pairing gap normalized to ε_F

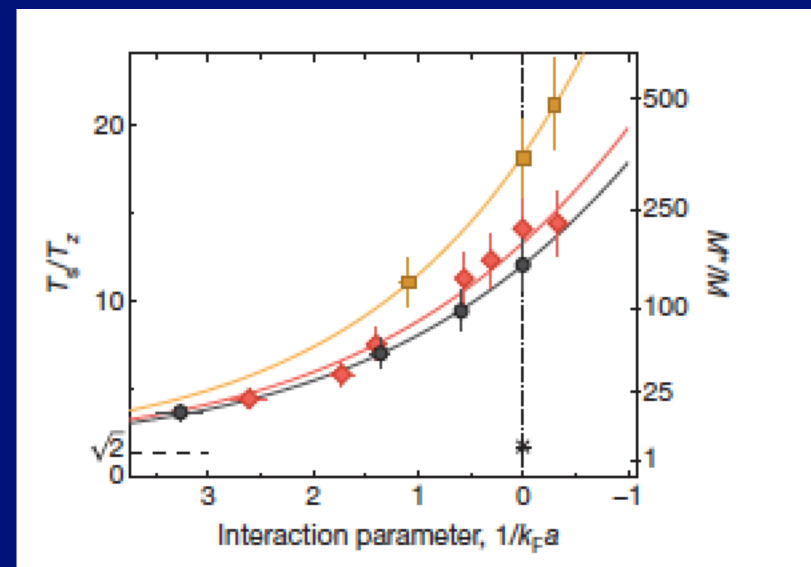


Local velocity normalized to Fermi velocity

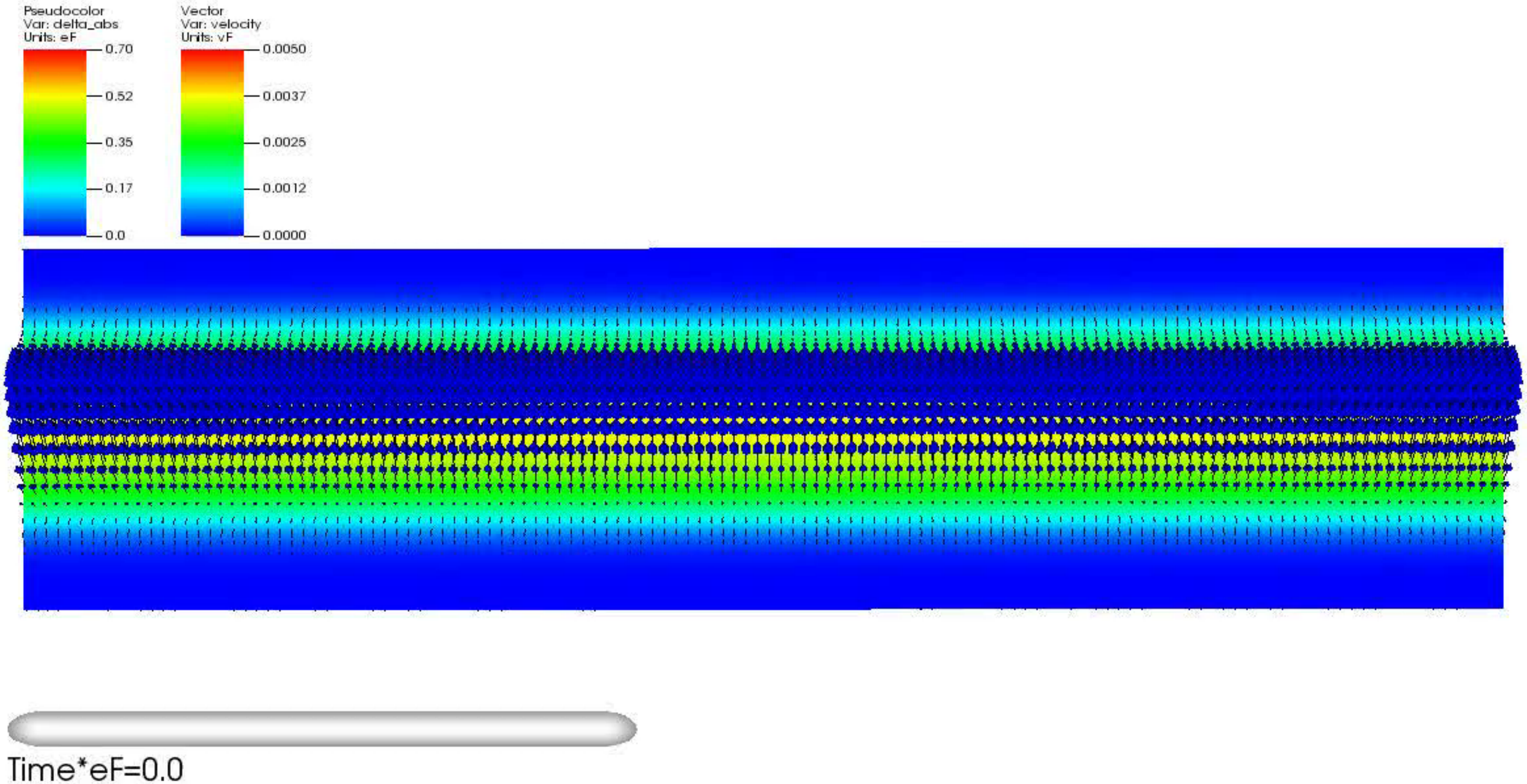
Heavy solitons in a fermionic superfluid

Tarik Yefsah[1], Ariel T. Sommer[1], Mark J. H. Ku[1], Lawrence W. Cheuk[1], Wenjie Ji[1], Waseem S. Bakr[1] & Martin W. Zwierlein[1]



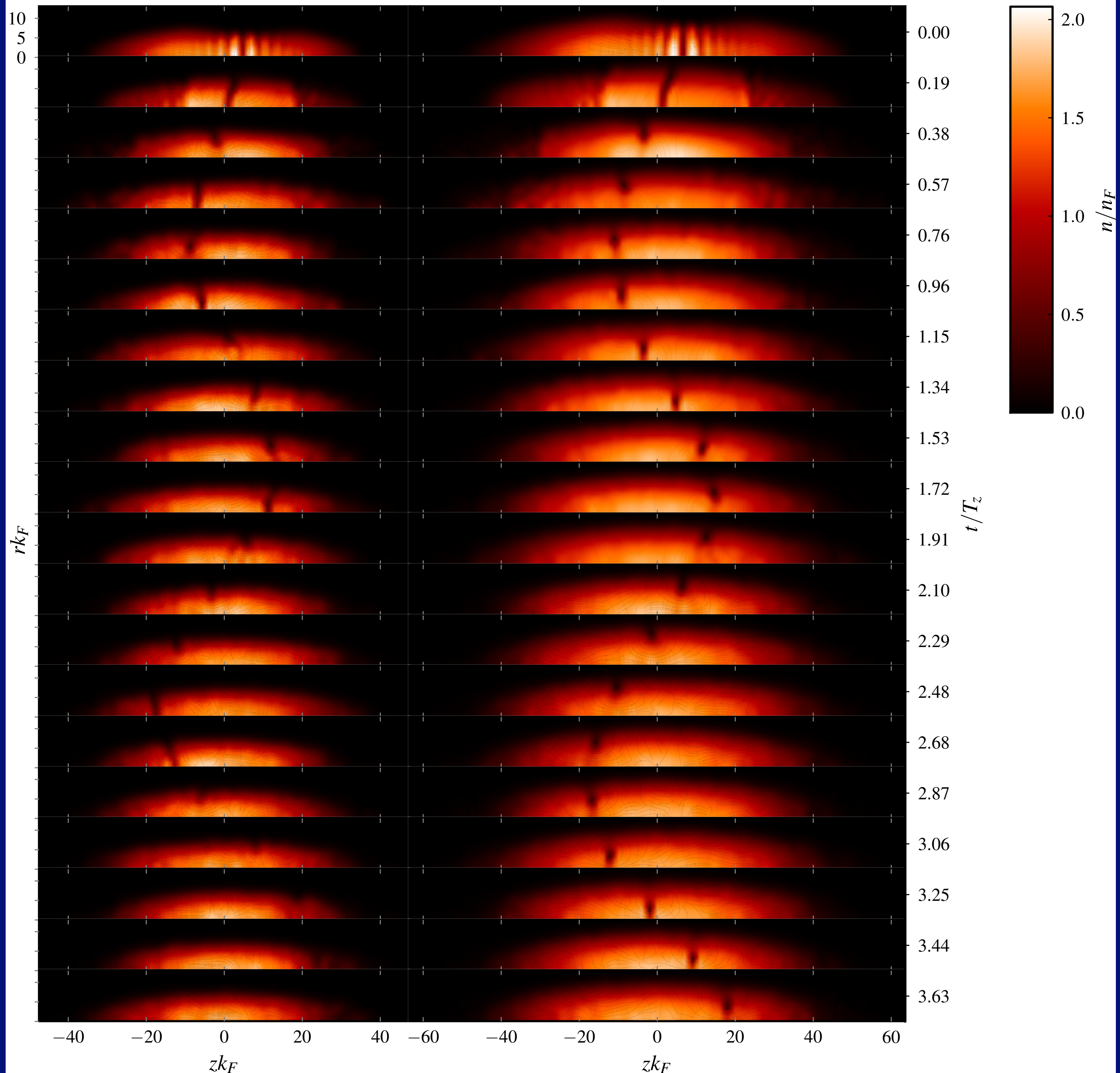


TDSLDA



Construction of ground state (adiabatic switching with quantum friction), generation of a domain wall using an optical knife, followed by the spontaneous formation of a vortex ring.
Aproximately 1270 fermions on a 48x48x128 spatial lattice, ≈ 260,000 complex PDEs, ≈ 309,000 time-steps, 2048 GPUs on Titan, 27.25 hours of wall time (initial code)
Wlazłowski et al, Phys. Rev. Lett. 112, 025301 (2014)

TDSLDA



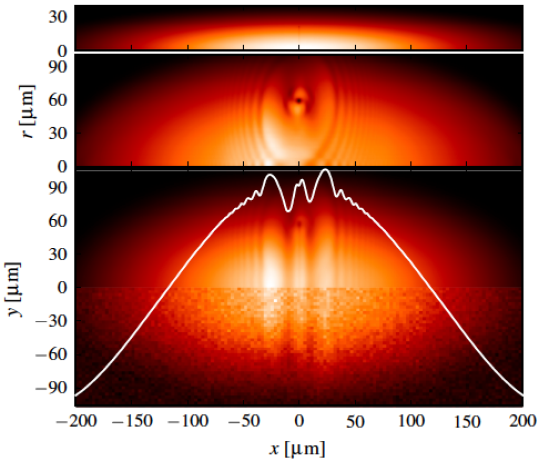
10
5
0
rk_F
−40
−20
0
20
40
zk_F
−60
−40
−20
0
20
40
60
zk_F
0.00
0.19
0.38
0.57
0.76
0.96
1.15
1.34
1.53
1.72
1.91
2.10
2.29
2.48
2.68
2.87
3.06
3.25
3.44
3.63
t/T_z
2.0
1.5
1.0
0.5
0.0
n/n_F

Imaging the vortex ring in experiment (movie)

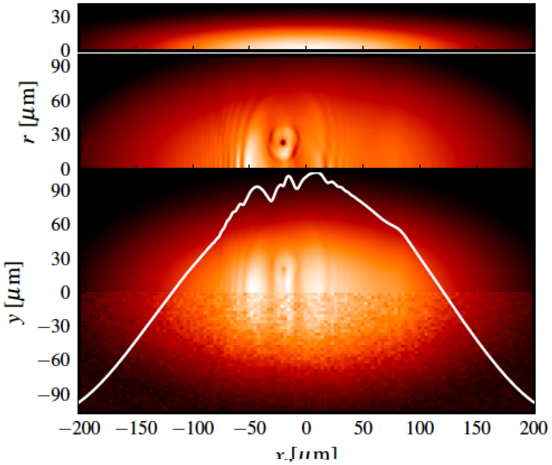
Large ring

Small ring

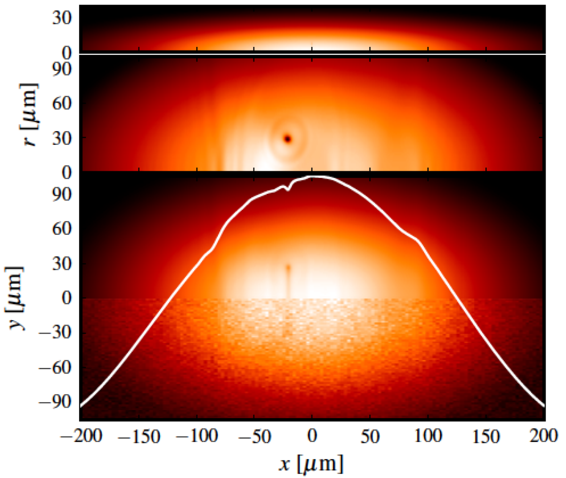
Too large B_{min}



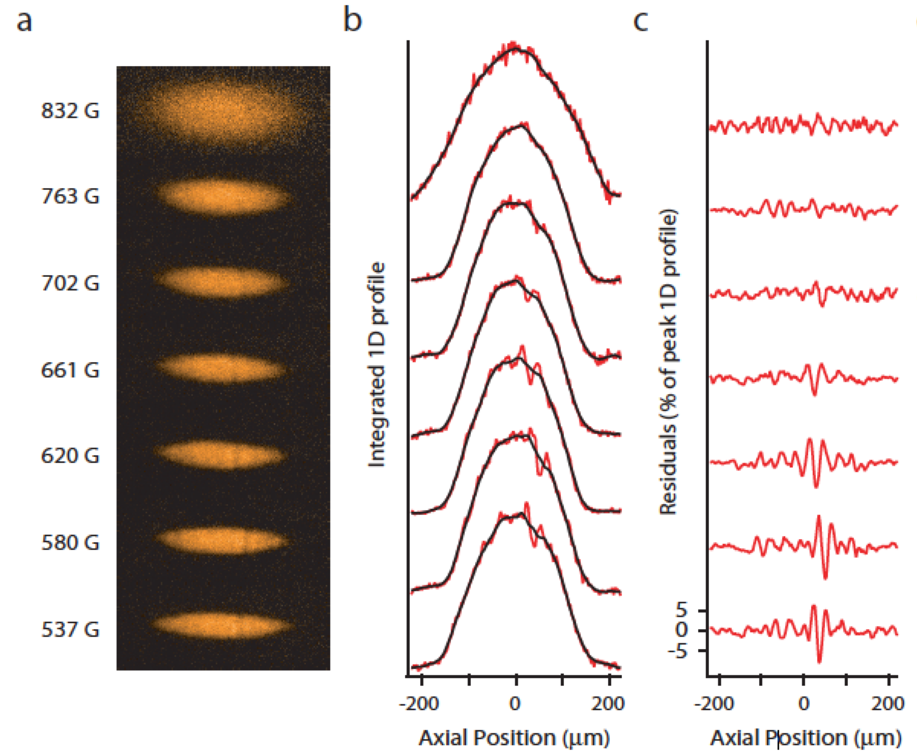
Large ring



Small ring



Insufficient ramping of magnetic field



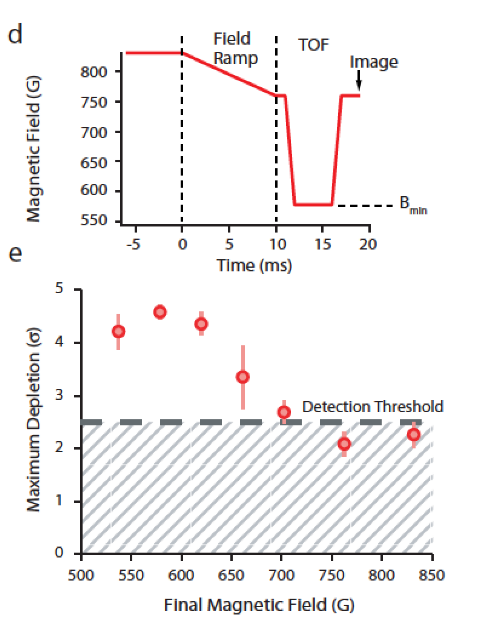
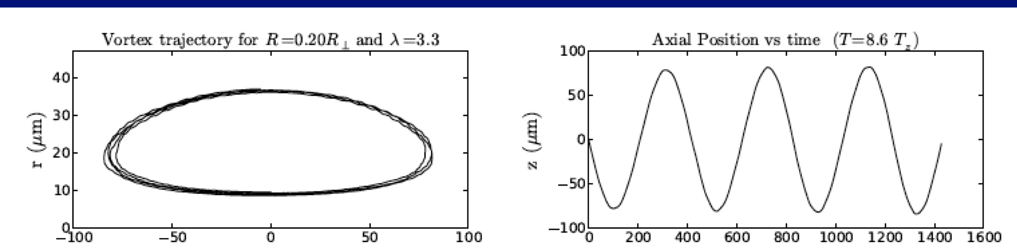


TABLE I. Dependence of the oscillation period on aspect ratio for a vortex ring imprinted with $R_0 = 0.30R_\perp$ at resonance. Note that the ETF consistently underestimates the period by about a factor of 0.56.

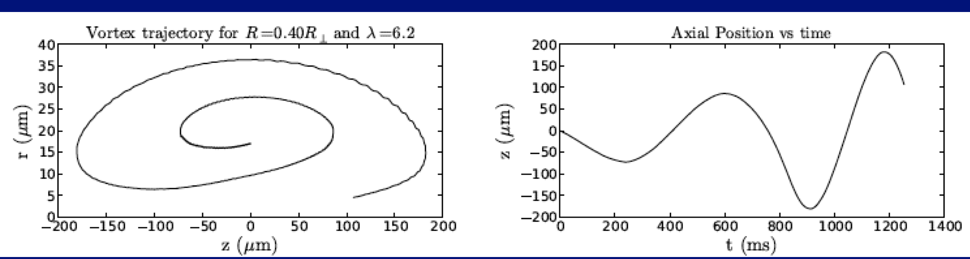
Aspect ratio	ETF period	Observed period [18]
$\lambda = 3.3$	$T = 9.9T_z$	$T = 18(2)T_z$
$\lambda = 6.2$	$T = 8.4T_z$	$T = 14(2)T_z$
$\lambda = 15$	$T = 6.7T_z$	$T = 12(2)T_z$

TABLE II. Benchmark of the ETF periods to the SLDA periods for sizes $24 \times 24 \times 96$, $32 \times 32 \times 128$, and $48 \times 48 \times 128$.

Size	T_{ETF}	T_{SLDA}	$T_{\mathrm{SLDA}}/T_{\mathrm{ETF}}$
$24 \times 24 \times 96$	$1.4T_z$	$1.7T_z$	1.2
$32 \times 32 \times 128$	$1.6T_z$	$1.9T_z$	1.2
$48 \times 48 \times 128$	$1.9T_z$	$2.6T_z$	1.4



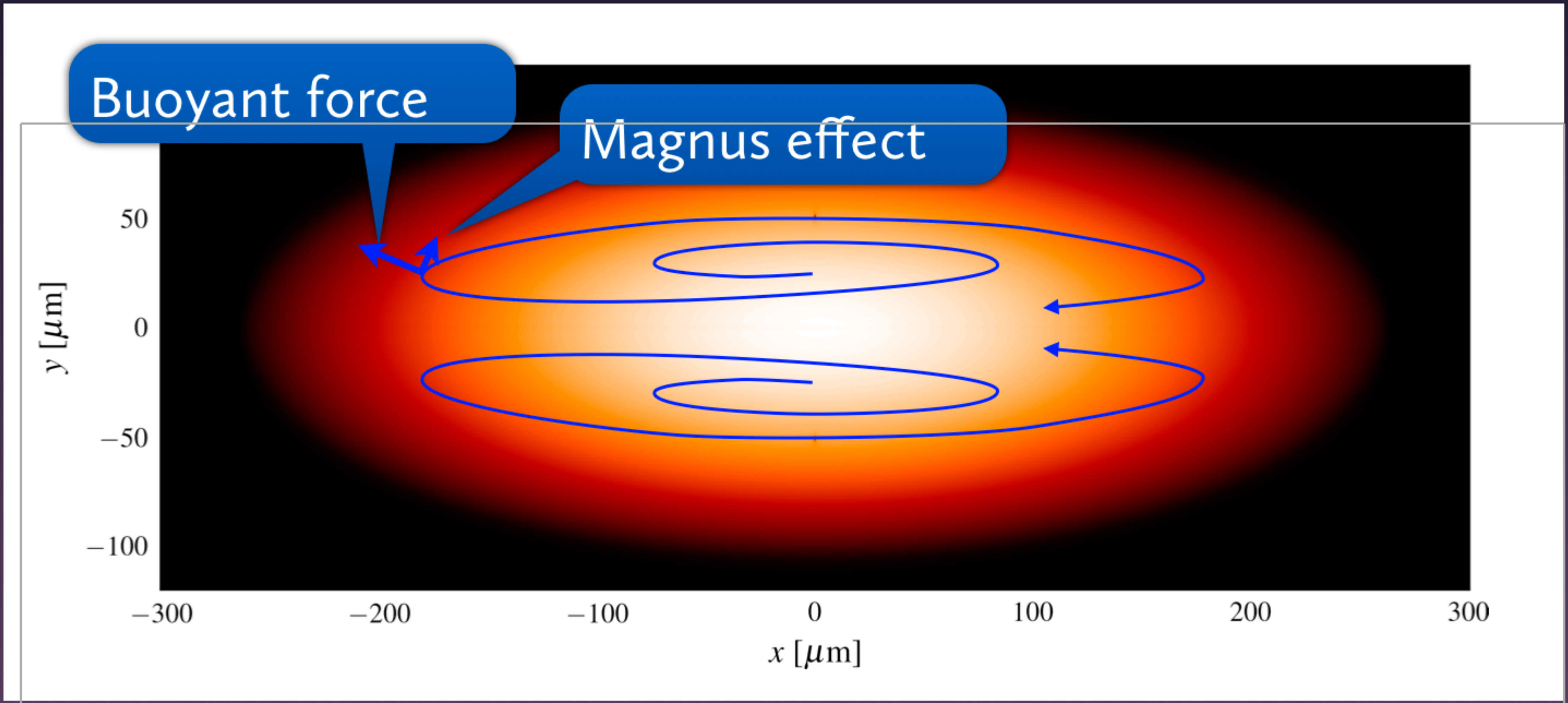
Near harmonic motion close to T=0 (very small number of phonons)



Anti-damping of the motion in the presence of a considerable number of phonons

TDSLDA (movie)

Vortex Ring Motion



Vortex ring motion (here in the presence of "thermal" noise, hence the inverse decay)

What TDSLDA tells us in the case of an axially non-symmetric trap, similar to the 2014 MIT experiment? (movie)

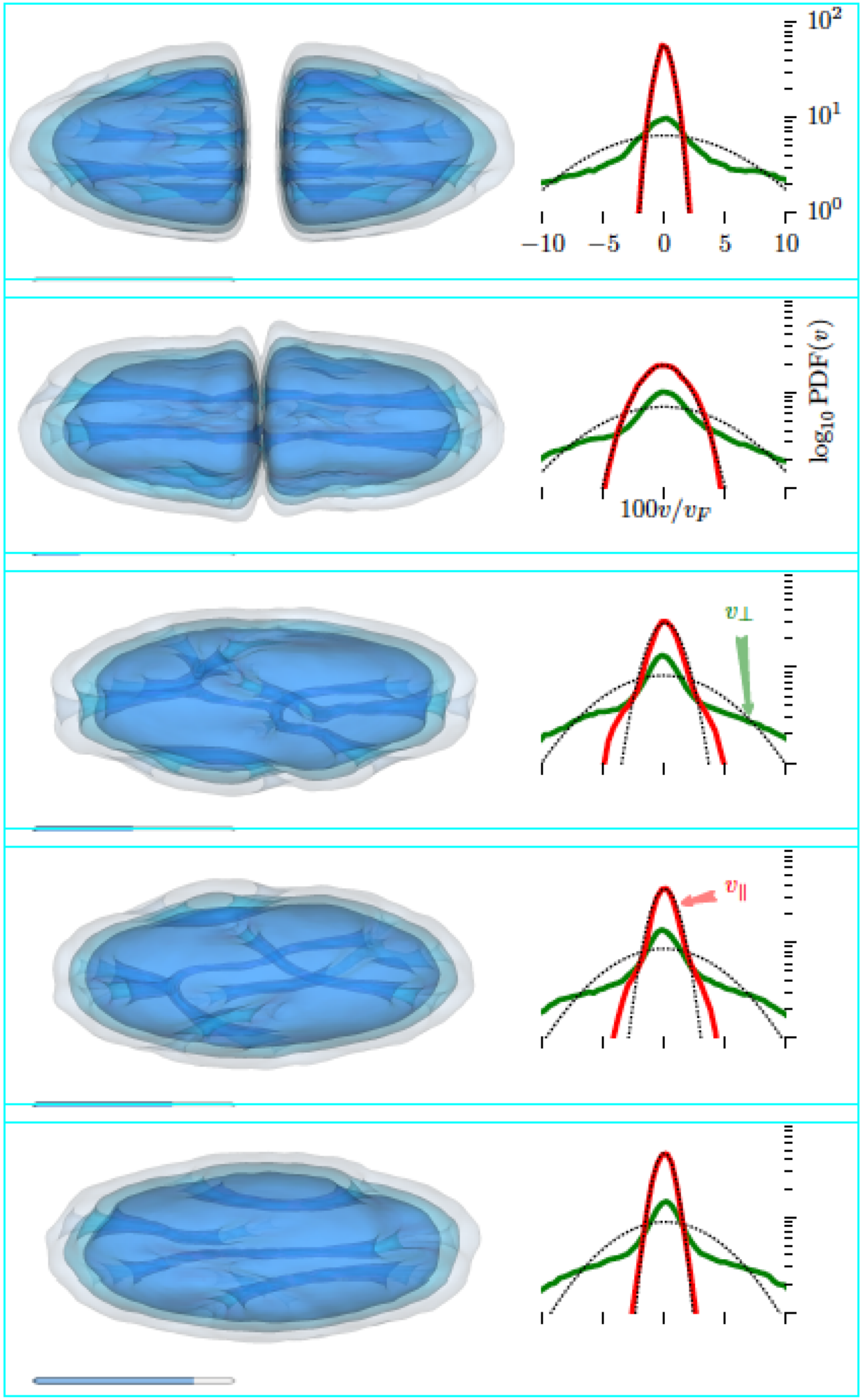
In agreement with the new experiment, when axial symmetry is broken a domain wall, converts to a vortex ring, which shortly becomes a vortex line.

View along the long axis
(y-axis vertical, movie)

In a slightly different geometry one can put directly in evidence in great detail the crossing and reconnection of vortex lines, the mechanism envisioned by Feynman in 1955 as the route to Quantum Turbulence (movie)

Quantum turbulence with no dissipation conjectured by Feynman (1955)
Exciting quantum turbulence in a unitary Fermi gas in a trap

Wlazłowski et al, arXiv:1404.1038



10^2
10^1
10^0
-10 -5 0 5 10
$\log_{10}$ PDF(v)
$100v/v_F$
$v_\perp$
$v_\parallel$

JETP LETTERS VOLUME 68, NUMBER 3 10 AUGUST 1998

Towards a universal nuclear density functional

S. A. Fayans
Kurchatov Institute Russian Science Center, 123182 Moscow, Russia

The total energy density of a nuclear system is represented as

$$\varepsilon=\varepsilon_{\rm kin}+\varepsilon_v+\varepsilon_s+\varepsilon_{\rm Coul}+\varepsilon_{sl}+\varepsilon_{\rm anom}, \qquad (1)$$

where $\varepsilon_{\rm kin}$ is the kinetic energy term which, since we are constructing a Kohn–Sham type functional, is taken with the free operator $t=p^2/2m$, i.e., with the effective mass $m^*=m$; all the other terms are discussed below.

The volume term in (1) is chosen to be in the form

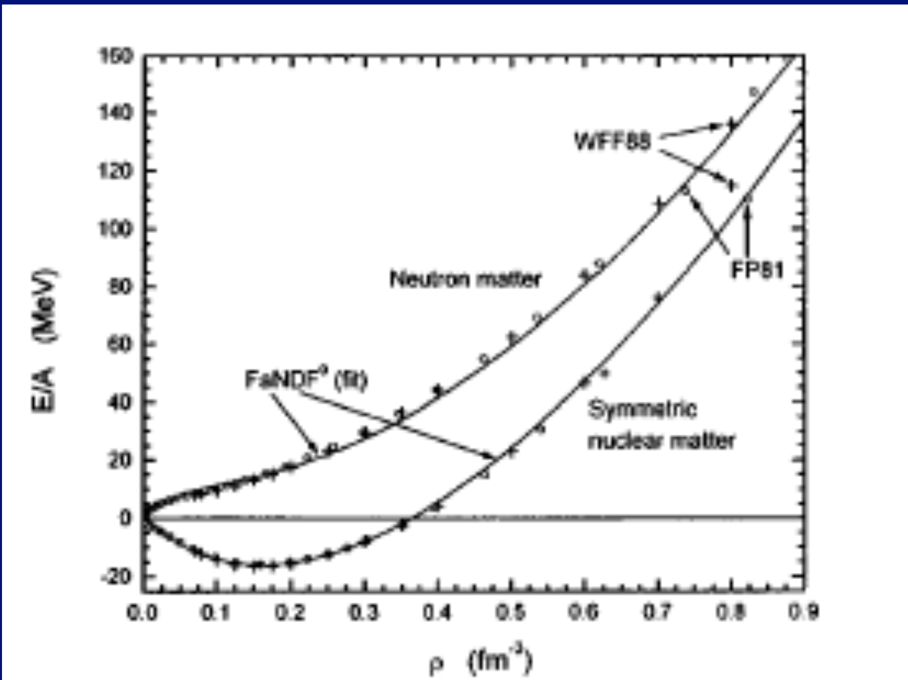
$$\varepsilon_v=\frac{2}{3}\epsilon_F^0\rho_0\left[a_+^v\frac{1-h_{1+}^v x_+^\sigma}{1+h_{2+}^v x_+^\sigma}x_+^2+a_-^v\frac{1-h_{1-}^v x_+}{1+h_{2-}^v x_+}x_-^2\right].$$

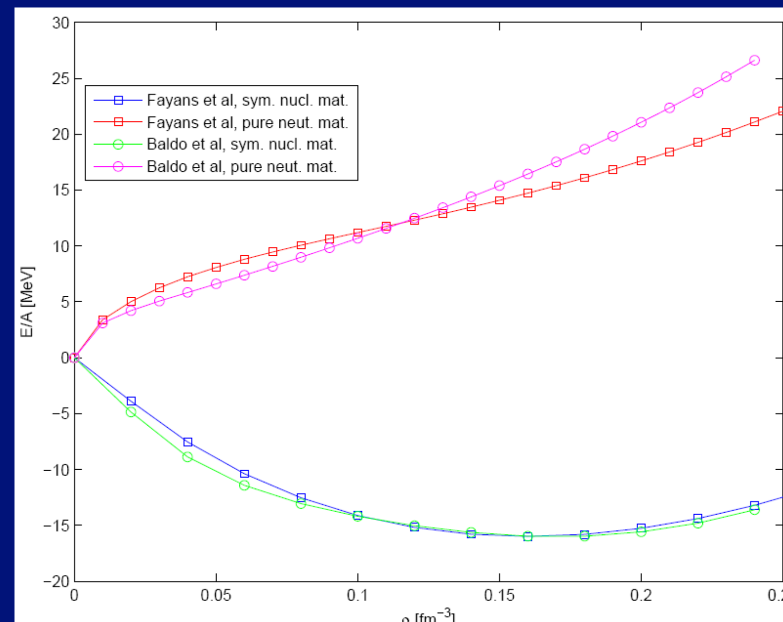
Here and in the following $x_\pm=(\rho_n\pm\rho_p)/2\rho_0$, $\rho_{n(p)}$ is the neutron (
$2\rho_0$ is the equilibrium density of symmetric nuclear matter wit

The surface part in Eq. (1) is meant to describe the finite-range and nonlocal in-medium effects which may presumably be incorporated phenomenologically within the EDF framework in a localized form by introducing a dependence on density gradients. It is taken as follows:

$$\varepsilon_s=\frac{2}{3}\epsilon_F^0\rho_0\frac{a_+^s r_0^2(\nabla x_+)^2}{1+h_+^s x_+^\sigma+h_\nabla^s r_0^2(\nabla x_+)^2}, \qquad (3)$$

with $h_+^s=h_{2+}^v$, a_+^s and h_∇^s the two free parameters. Such a form is obtained by adding





Baldo, Schuck, and Vinas, arXiv:0706.0658

Let us summarize some of the ingredients of the SLDA in nuclei

Energy Density (ED) describing the normal system

ED contribution due to superfluid correlations

$$E_{gs} = \int d^3r \left\{ \varepsilon_N[\rho_n(\vec{r}),\rho_p(\vec{r})] + \varepsilon_S[\rho_n(\vec{r}),\rho_p(\vec{r}),\nu_n(\vec{r}),\nu_p(\vec{r})] \right\}$$

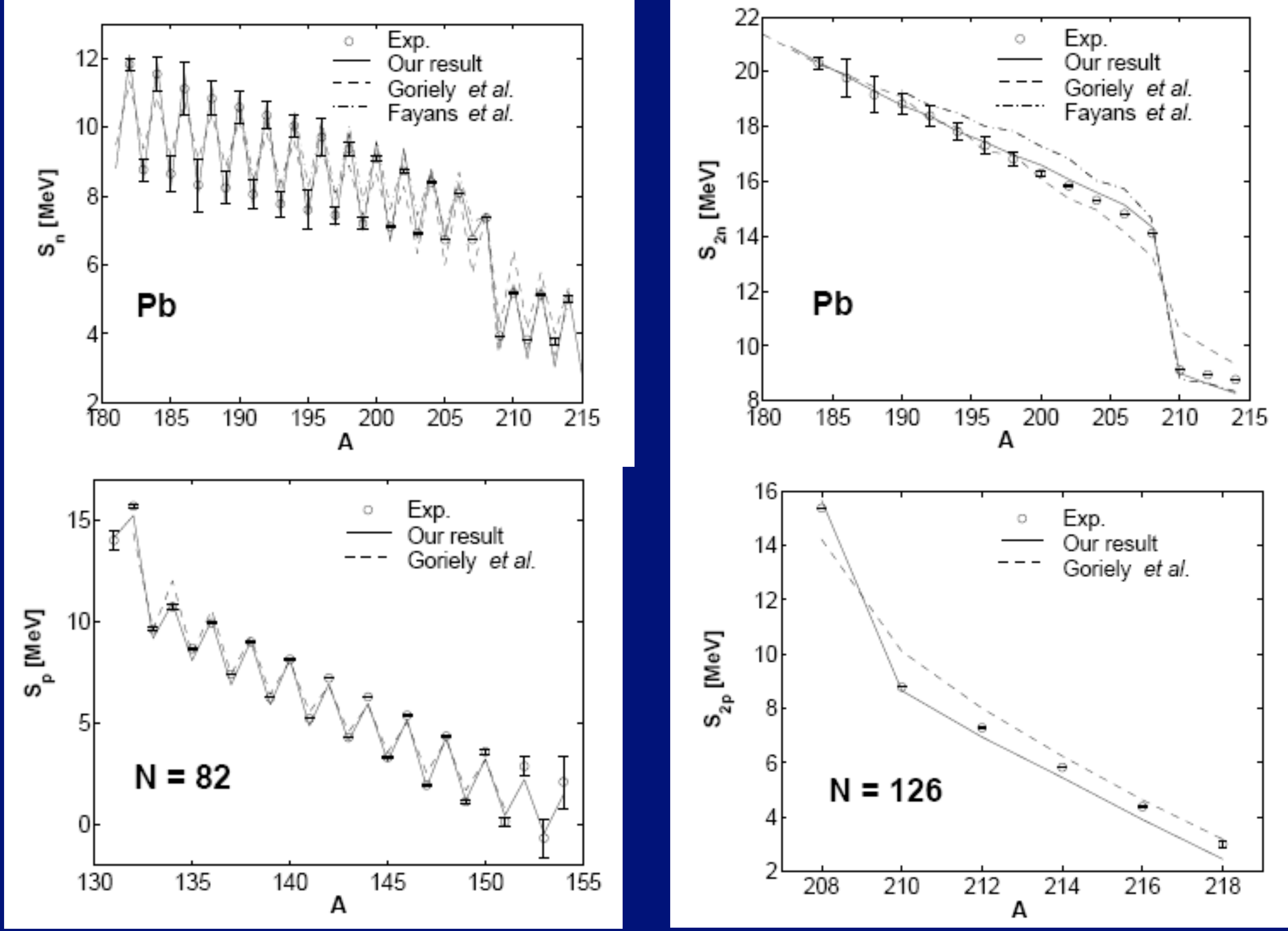
$$\begin{cases} \varepsilon_N[\rho_n(\vec{r}),\rho_p(\vec{r})] = \varepsilon_N[\rho_p(\vec{r}),\rho_n(\vec{r})] \\ \varepsilon_S[\rho_n(\vec{r}),\rho_p(\vec{r}),\nu_n(\vec{r}),\nu_p(\vec{r})] = \varepsilon_S[\rho_p(\vec{r}),\rho_n(\vec{r}),\nu_p(\vec{r}),\nu_n(\vec{r})] \end{cases}$$

Isospin symmetry constraints
(Coulomb energy and other relatively small terms not shown here.)

$$\varepsilon_S\left[\rho_n,\rho_p,\nu_p,\nu_n\right] = g(\rho_p,\rho_n)[|\nu_p|^2 + |\nu_n|^2] + f(\rho_p,\rho_n)[|\nu_p|^2 - |\nu_n|^2]\ \frac{\rho_p-\rho_n}{\rho_p+\rho_n}$$

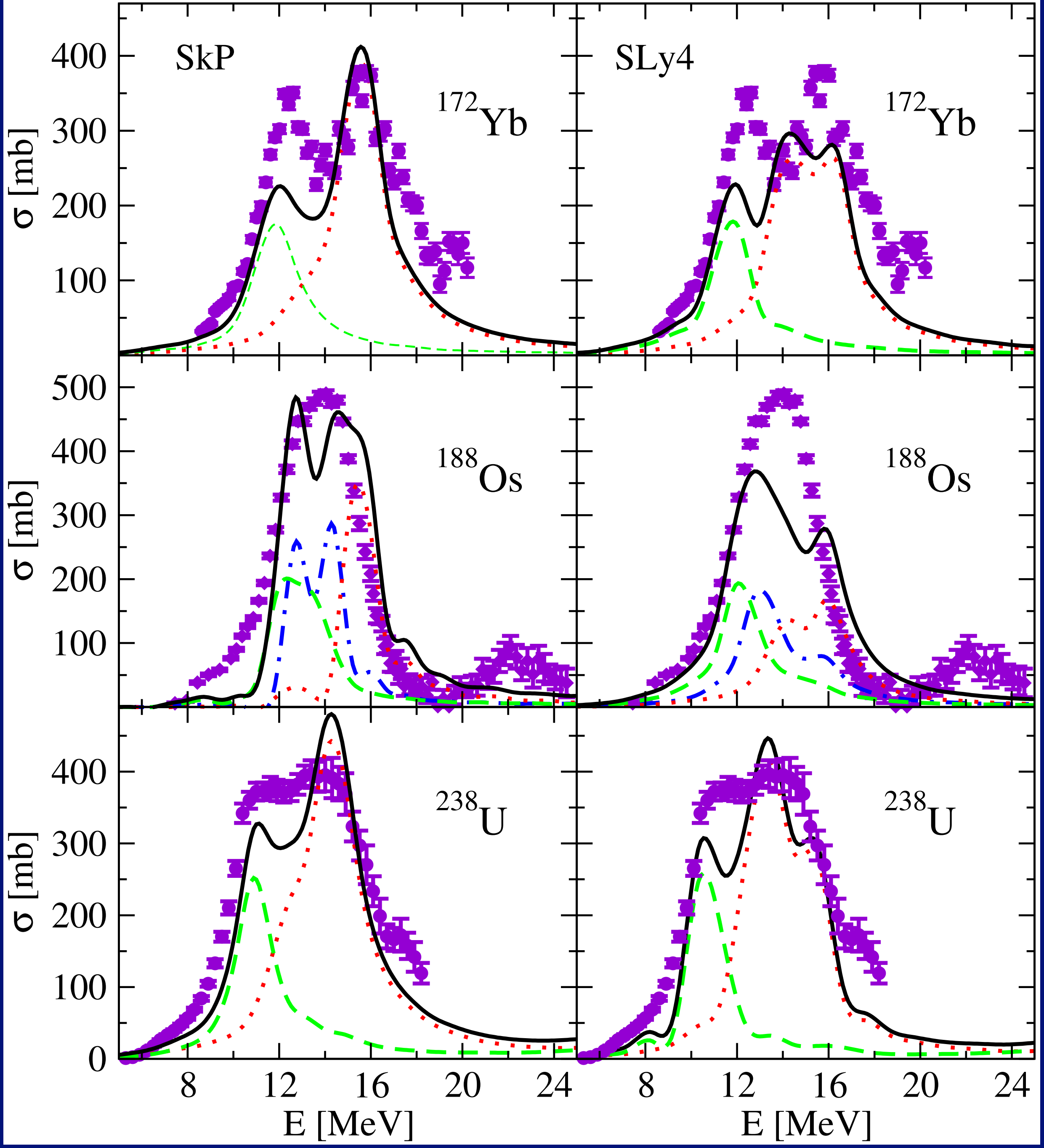
where $g(\rho_p,\rho_n) = g(\rho_n,\rho_p)$

and $f(\rho_p,\rho_n) = f(\rho_n,\rho_p)$



<u>A single universal parameter for pairing!</u>

Yu and Bulgac, PRL, 90, 161101 (2003)

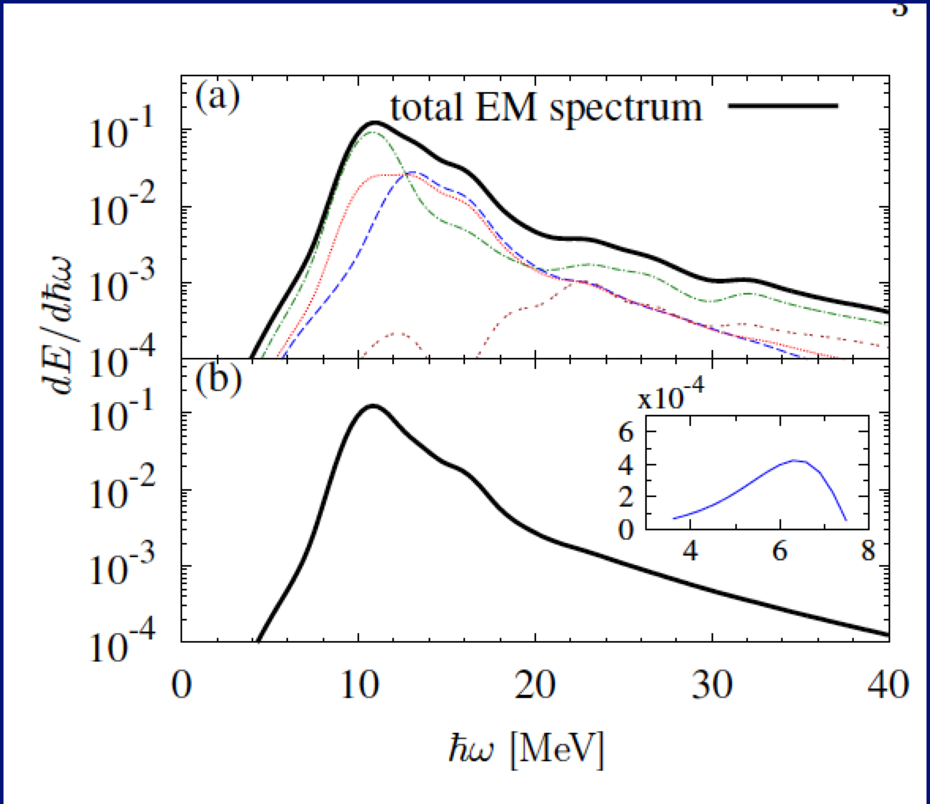


Giant Dipole Resonance deformed and superfluid nuclei

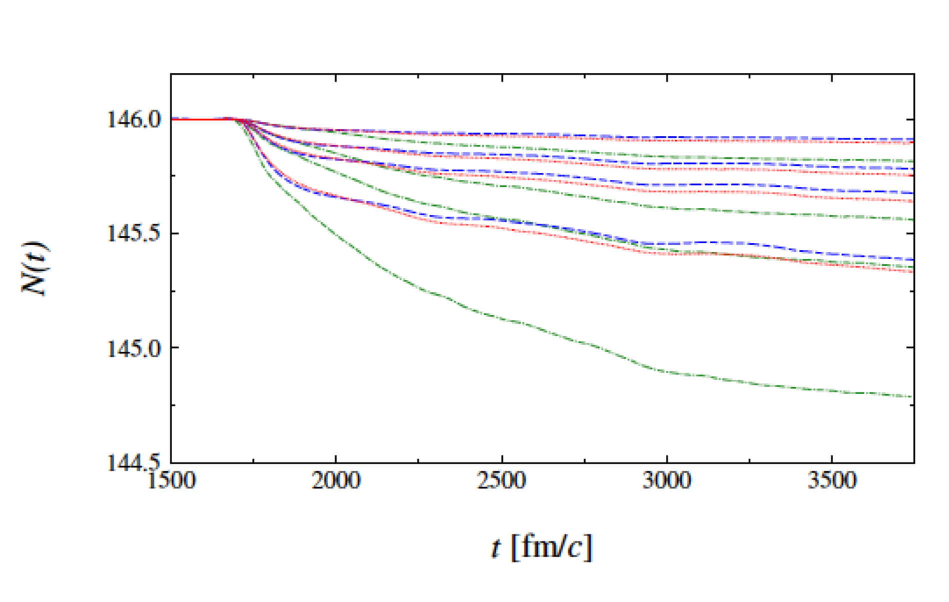
Osmium is triaxial, and both protons and neutrons are superfluid.

Stetcu, *et al.*, Phys. Rev. C 84, 051309(R) (2011)

Coulomb excitation of target ^{238}U with relativistic projectile ^{238}U

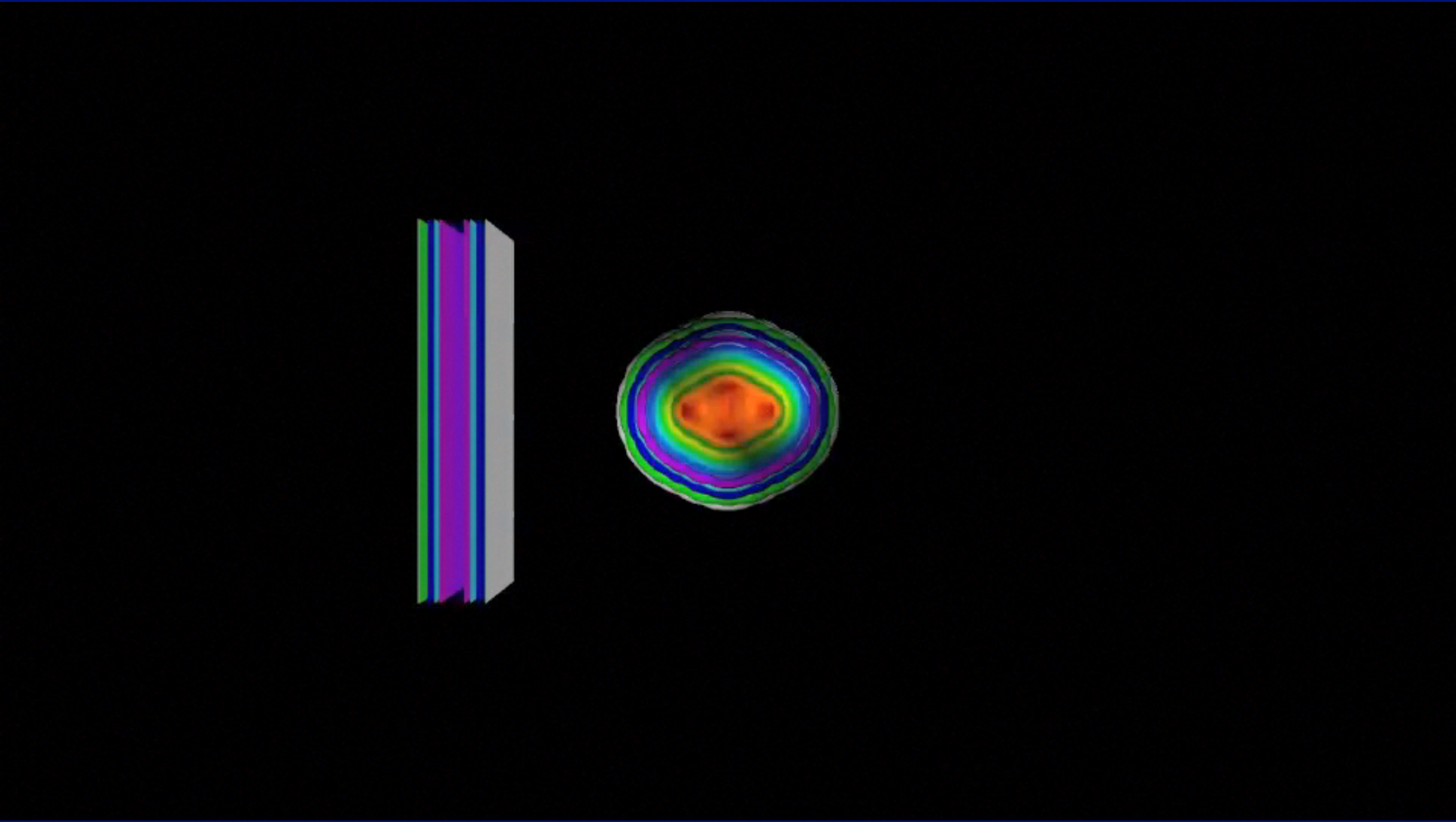


Spectrum of emitted radiation



Number of neutrons in a sphere of radius 15 fm

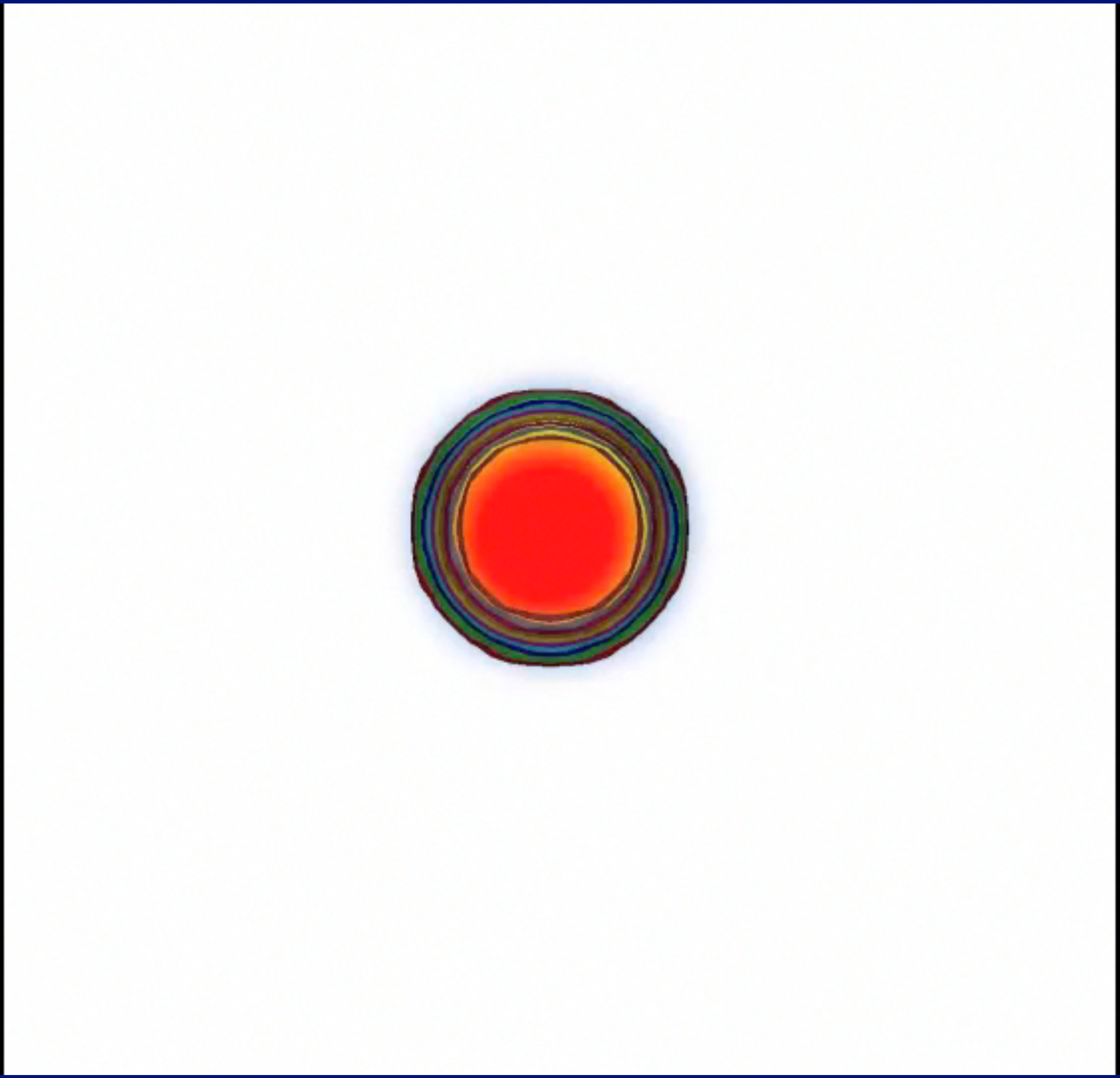
Stetcu *et al.*, arXiv:1403.2671



Neutron scattering of ^{238}U computed in TDSLDA

I. Stetcu *et al.*

Movie

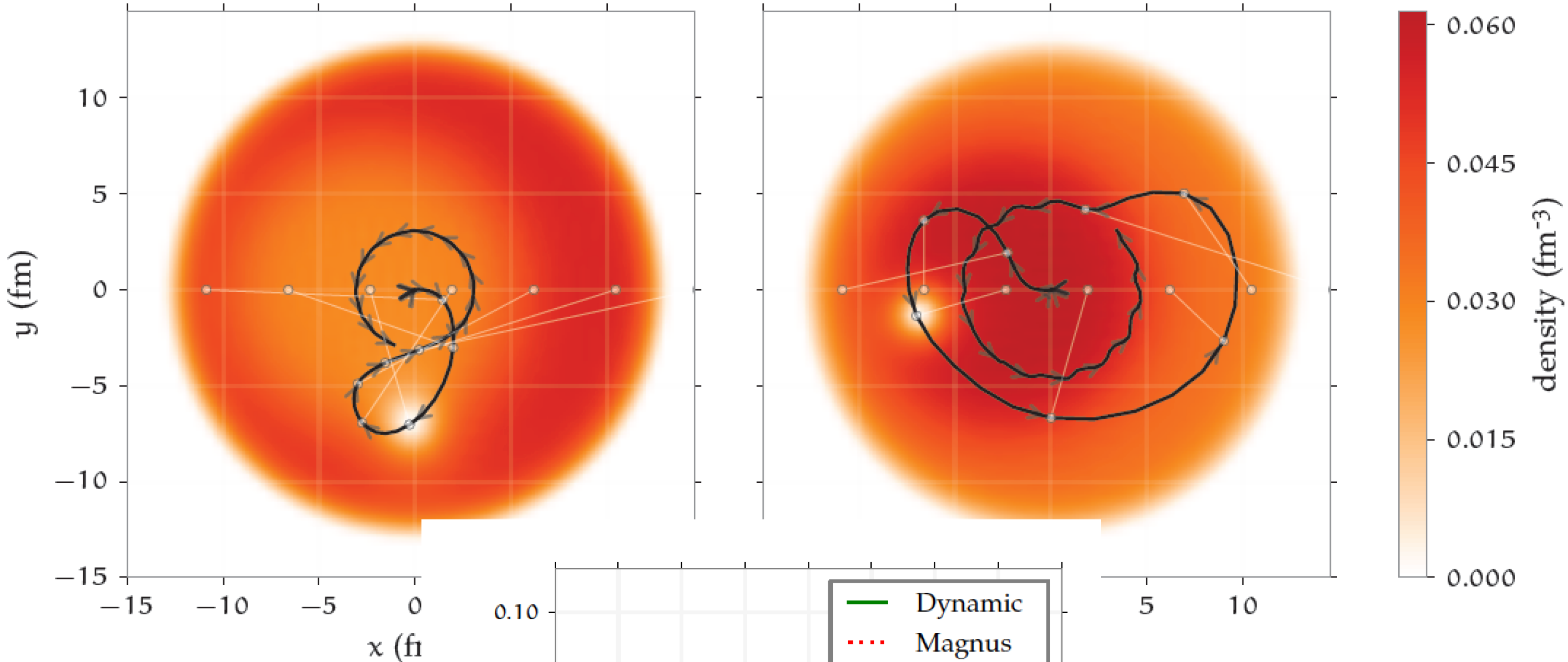


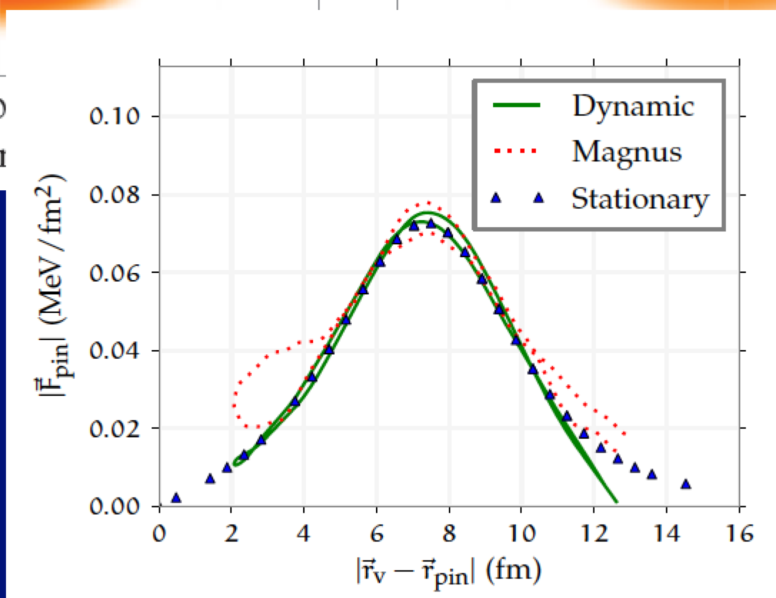
Real-time induced fission of ^{280}Cf computed in TDSLDA

I. Stetcu *et al.*

Movie

How to compute the pinning energy of a vortex on nucleus in the neutron star crust





Attraction

Repulsion

Bulgac, Forbes, and Sharma, Phys. Rev. Lett. 110, 241102 (2013)