Why strongly interacting fermion gases are interesting to a many-body theorist?

Aurel Bulgac
University of Washington, Seattle

People I have been lucky to work with on these problems:




Clockwise (starting top left corner) : G.F. Bertsch (Seattle), J.E. Drut (Seattle) , P. Magierski (Warsaw, Seattle), Y. Yu (Seattle, Lund, Wuhan), M.M. Forbes (Seattle), A. Schwenk (Seattle, Vancouver), A. Fonseca (Lisbon), P. Bedaque (Seattle, Berkeley, College Park),

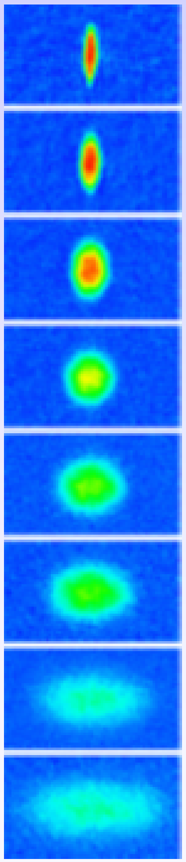



What will be covered in this talk:

- Lots of others people results (experiment and theory) throughout the entire presentation
- What is the unitary regime?
- The two-body problem, how one can manipulate the two-body interaction?
- What many/some theorists know and suspect that is going on?
- What experimentalists have managed to put in evidence so far and how that agrees with theory?

Why Study Fermi Gases ?

Duke Physics
Atom Cooling and Trapping



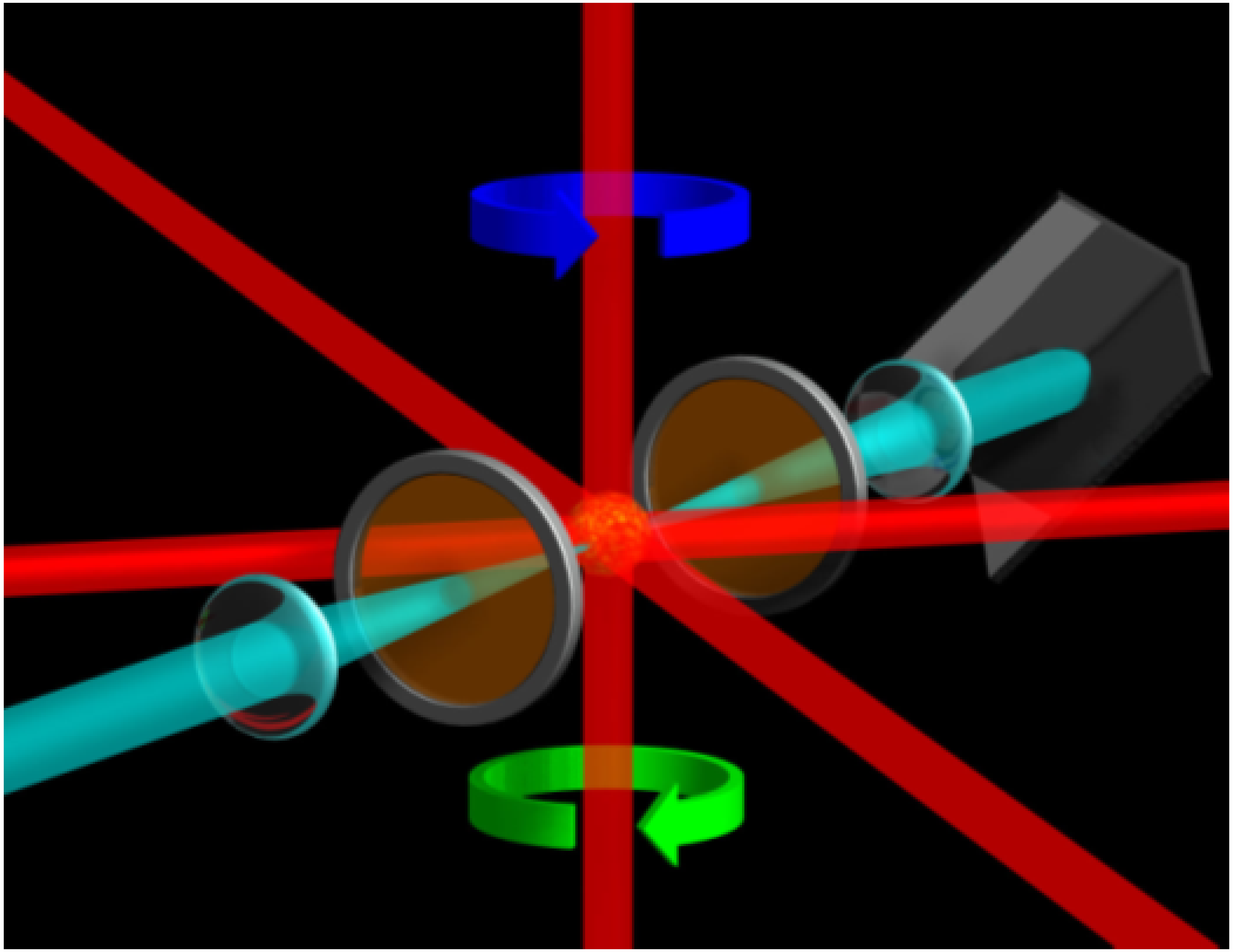
O'Hara et al., Science 2002

- Fermions are the building blocks of matter
- Strongly-interacting Fermi gases are stable
- Link to other interacting Fermi systems:
  - High-T_C superconductors – Neutron stars
  - Lattice field theory
  - Quark-gluon plasma of Big Bang
  - String theory!

From a talk of J.E. Thomas (Duke)

Optical Trap Loading

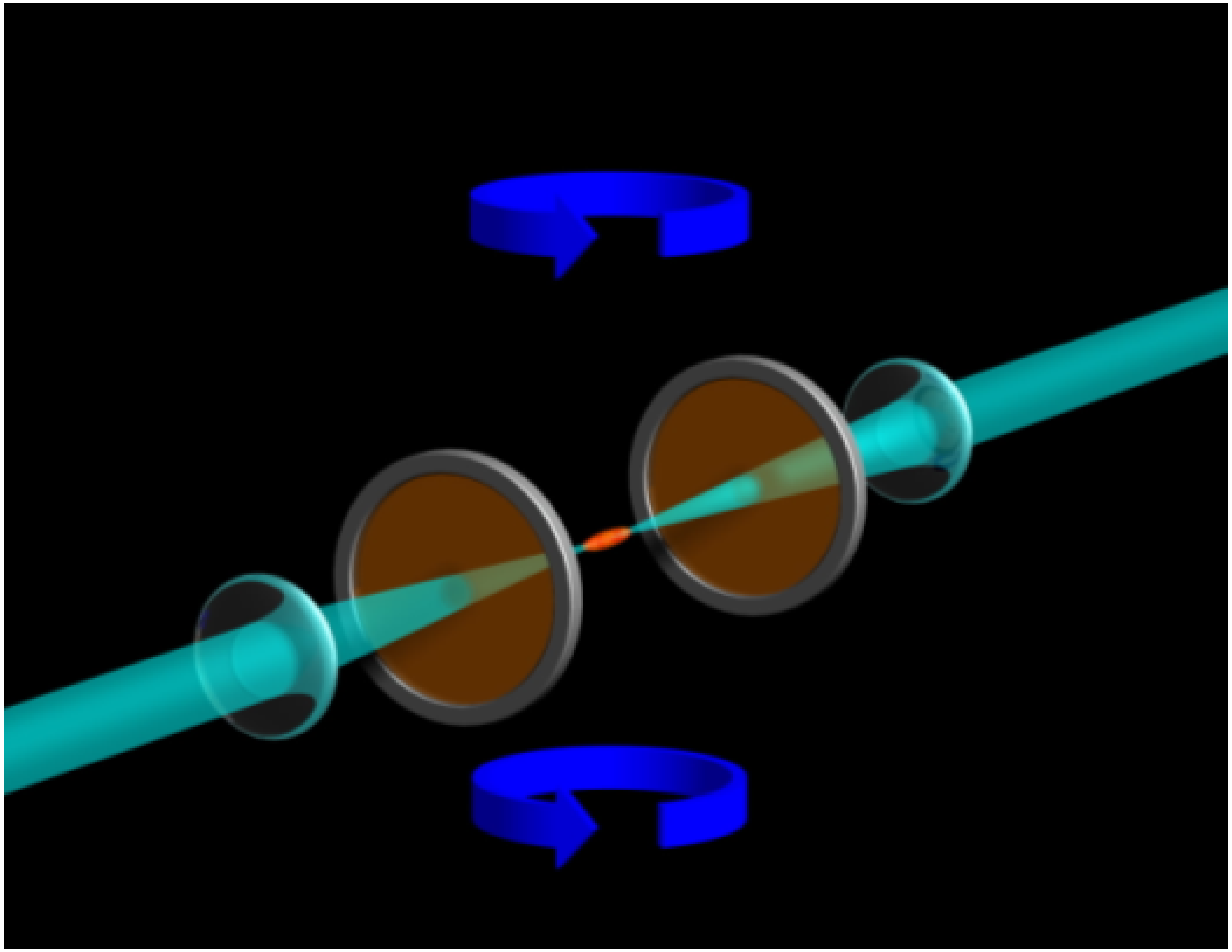
Duke Physics
Atom Cooling and Trapping



From a talk of J.E. Thomas (Duke)

Forced Evaporation

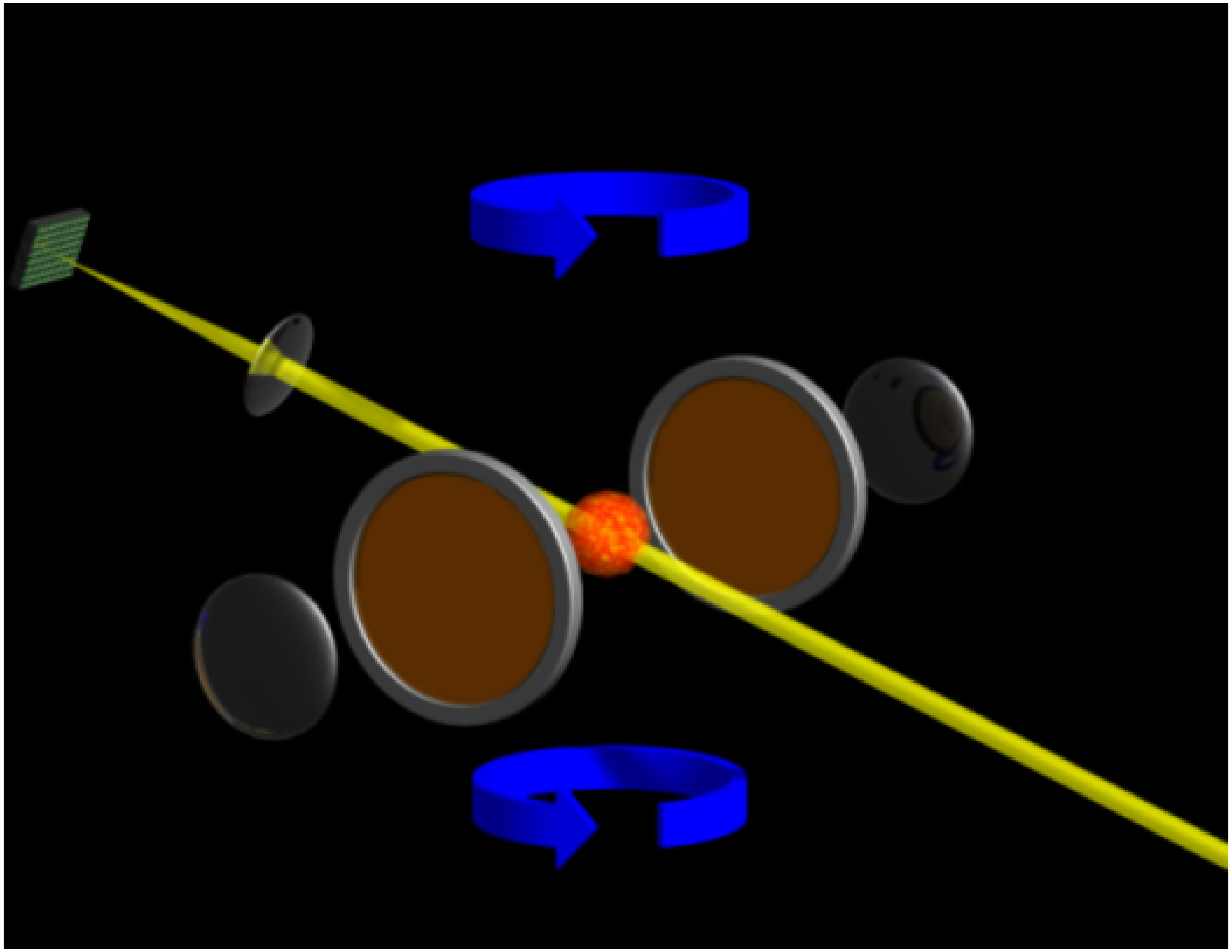
Duke Physics
Atom Cooling and Trapping



From a talk of J.E. Thomas (Duke)

High-Field Imaging

Duke Physics
Atom Cooling and Trapping



From a talk of J.E. Thomas (Duke)

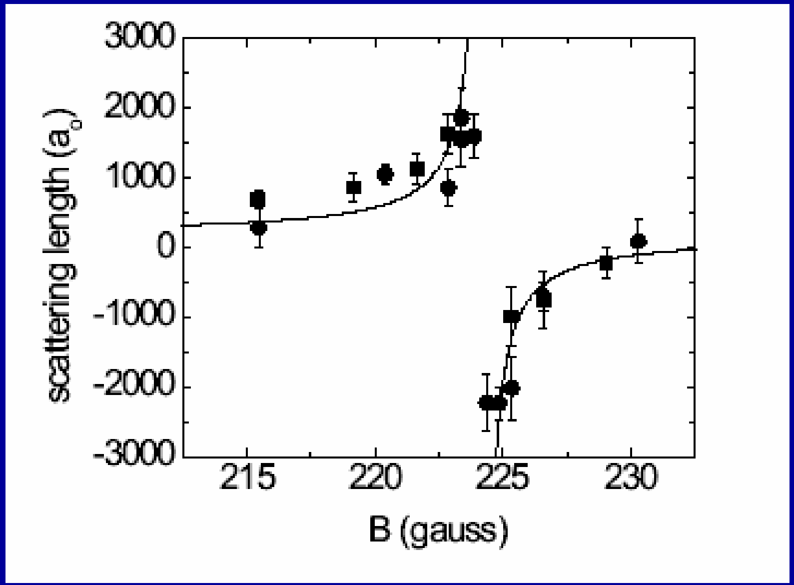
Feshbach resonance

$$H = \frac{\vec{p}^{\,2}}{2\mu_r} + \sum_{i=1}^{2}(V_i^{hf} + V_i^{Z}) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \cancel{V^d}$$

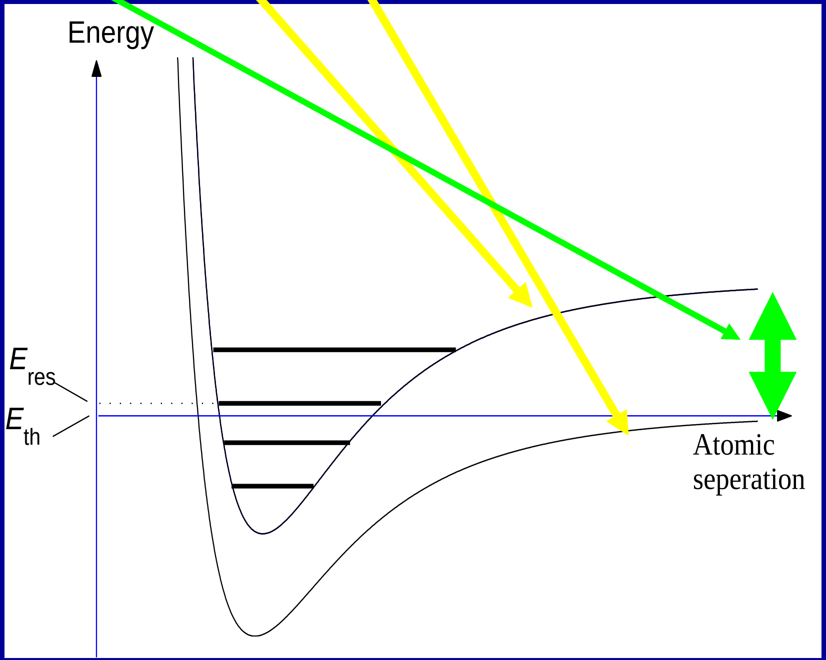
$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{S}^{e}\cdot\vec{S}^{n},\; V^{Z} = (\gamma_e S_z^e - \gamma_n S_z^n)B$$

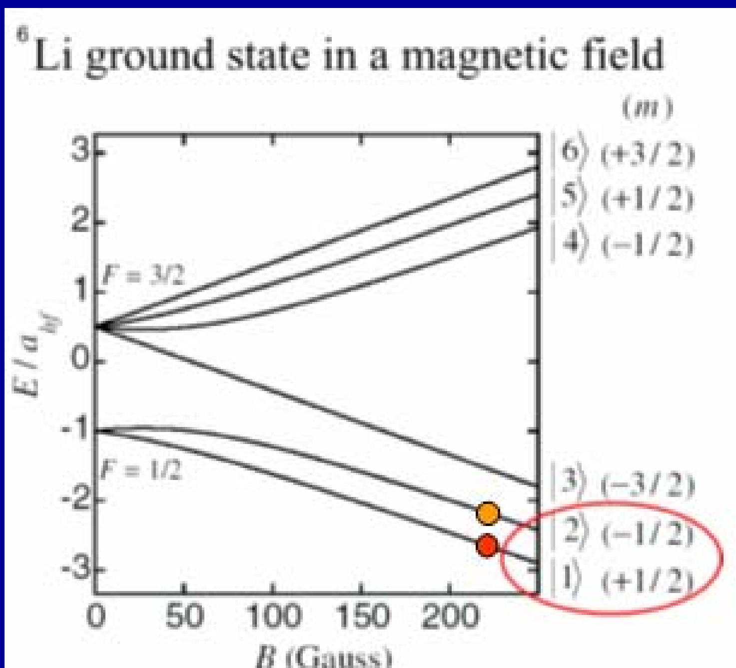
Channel coupling

Tiesinga, Verhaar, Stoof
Phys. Rev. A47, 4114 (1993)



Regal and Jin
Phys. Rev. Lett. 90, 230404 (2003)





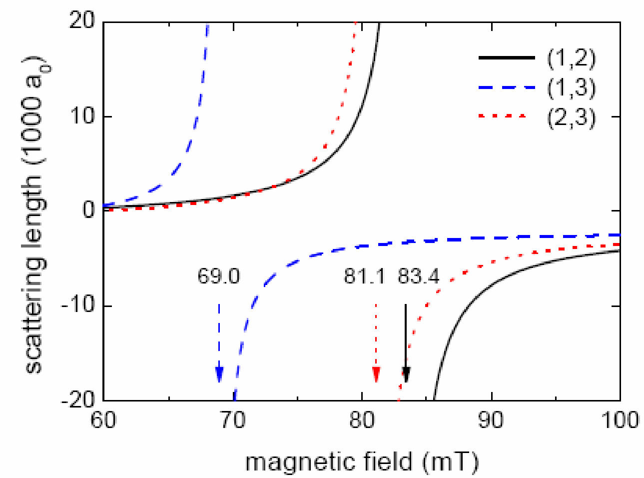
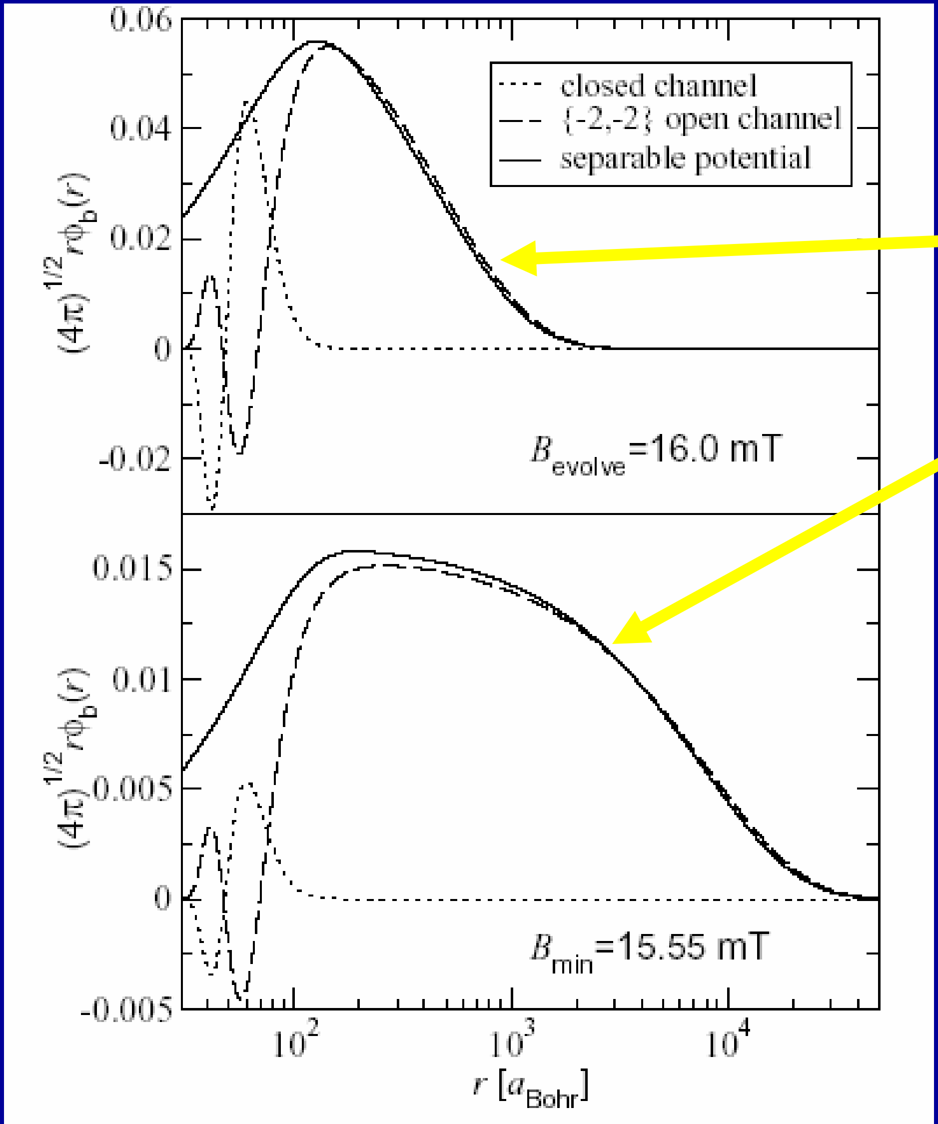


FIG. 4: Scattering lengths versus magnetic field from multi-channel quantum scattering calculations for the (1, 2), (1, 3), and (2, 3) scattering channels. The arrows indicate the resonance positions.

Bartenstein *et al*. Phys. Rev. Lett. **94**, 103201 (2005)

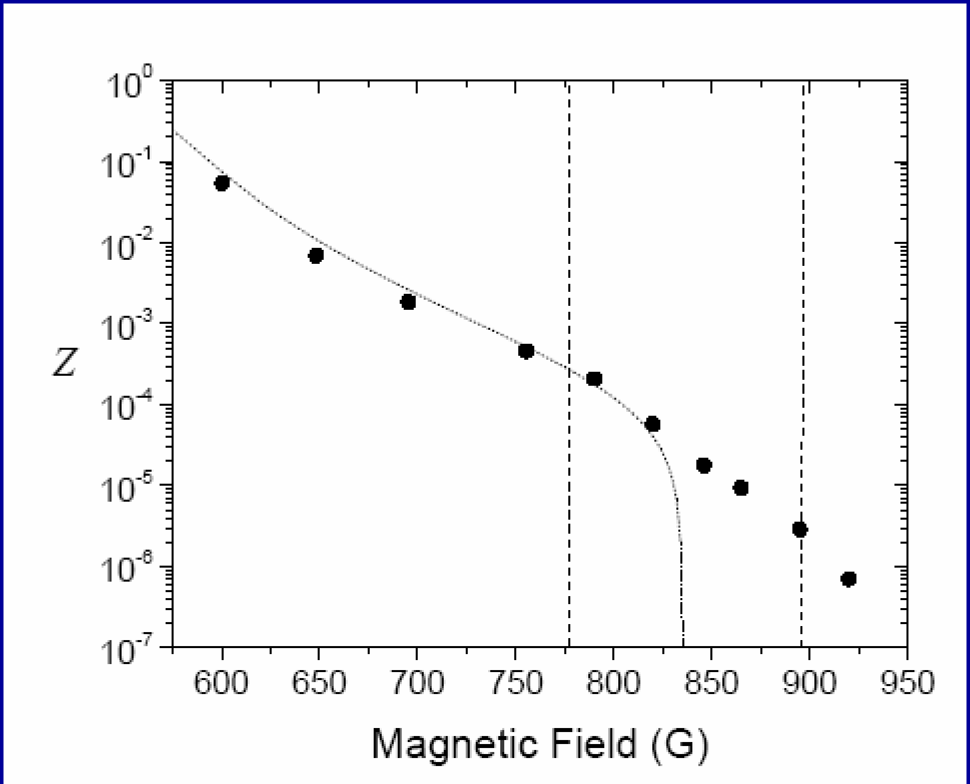


Halo dimer
(open channel)

$$\frac{P(r > r_0)}{P(r < r_0)} \propto \frac{a}{r_0} \quad >> \ 1$$

Most of the time two atoms are at distances greatly exceeding the range of the interaction!

Köhler *et al.* Phys. Rev. Lett. 91, 230401 (2003), inspired by Braaten *et al.* cond-mat/0301489



Z – measured probability to find the two atoms in a singlet state (closed channel)

Dots - experiment of Partridge *et al.* cond-mat/0505353

➢ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

The system is very dilute, but strongly interacting!

$$n\, r_0^3 \ll 1$$

$$n\, |a|^3 \gg 1$$

n - number density

$$r_0 \ll n^{-1/3} \approx \lambda_F/2 \ll |a|$$

r_0 - range of interaction

a - scattering length

Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not.

- ***systems of bosons are unstable (Efimov effect)***
- ***systems of three or more fermion species are unstable (Efimov effect)***
- ***a number of theorists believed that the two species fermions systems are unstable as well***

- **Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)**
- **Chang et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided best the theoretical estimates for the ground state energy of such systems.**
- **Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.**

BCS $\rightarrow$BEC crossover

Leggett (1980), Nozieres and Schmitt-Rink (1985), Randeria *et al.* (1993),…

If a<0 at T=0 a Fermi system is a BCS superfluid

$$\Delta \approx \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right) << \varepsilon_F, \quad \text{iff} \quad k_F\,|a|<<1 \text{ and } \xi = \frac{1}{k_F}\frac{\varepsilon_F}{\Delta} >> \frac{1}{k_F}$$

If $|a|=\infty$ and $nr_0^3 \ll 1$ a Fermi system is strongly coupled and its properties are universal. Carlson *et al*. PRL 91, 050401 (2003)

$$\frac{E_{\text{normal}}}{N} \approx 0.54\frac{3}{5}\varepsilon_F, \qquad \frac{E_{\text{superfluid}}}{N} \approx 0.44\frac{3}{5}\varepsilon_F \quad \text{and } \xi = O(\lambda_F),\ \Delta = O(\varepsilon_F)$$

If a>0 ($a \gg r_0$) and $na^3 \ll 1$ the system is a dilute BEC of tightly bound dimers

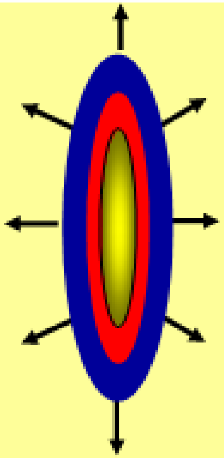
$$\varepsilon_2 = -\frac{\hbar^2}{ma^2} \quad \text{and} \quad n_b a^3 << 1, \quad \text{where} \quad n_b = \frac{n_f}{2} \text{ and } a_{bb} = 0.6a > 0$$

Finite Temperature Hydrodynamics

Duke Physics

Atom Cooling and Trapping

2) Finite Temperature Hydrodynamics: Breathing Mode or Expansion



Scale factor

$\sigma_x(t) = \sigma_x b_x(t)$

$\mathbf{u}(\mathbf{x},t) =$ velocity field

$$\nabla P_0(\tilde{\mathbf{x}}) + n_0(\tilde{\mathbf{x}})\nabla U(\tilde{\mathbf{x}}) = 0$$

If isentropic: $\mathbf{u}_n = \mathbf{u}_s = \mathbf{u}$

$$m\frac{\partial \mathbf{u}}{\partial t} = -\nabla\left(\tfrac{m}{2}\mathbf{u}^2 + U(\mathbf{x})\right) - \frac{\nabla P(\mathbf{x})}{n(\mathbf{x},t)}$$

$$x = \tilde{x}\, b_x(t) \qquad u_x = \tilde{x}\, \dot{b}_x(t) \qquad n(\mathbf{x},t) = \frac{n_0(\tilde{\mathbf{x}})}{\Gamma}$$

$$P(n,T) = \tfrac{2}{3} n \varepsilon_F(n) f_E\left[\frac{T}{T_F(n)}\right]$$

If isentropic expansion: $\Rightarrow$ $P(\mathbf{x}) = \dfrac{P_0(\tilde{\mathbf{x}})}{\Gamma^{5/3}}$

$$m\frac{\partial \mathbf{u}}{\partial t} = -\nabla\left(\frac{m}{2}\mathbf{u}^2 + U(\mathbf{x}) - \frac{U(\tilde{\mathbf{x}})}{\Gamma^{2/3}}\right)$$

Temperature and Density Independent!

Experiments $\rightarrow$ isentropic behavior

From a talk of J.E. Thomas (Duke)

Ground state properties of unitary gases

Consider Bertsch's MBX challenge (1999): "Find the ground state of infinite homogeneous neutron matter interacting with an infinite scattering length."

$$r_0 \to 0 \quad << \quad \lambda_F \quad << \quad |a| \to \infty$$

- Carlson, Morales, Pandharipande and Ravenhall, PRC 68, 025802 (2003), with Green Function Monte Carlo (GFMC)

$$\frac{E_N}{N} = \alpha_N \frac{3}{5}\varepsilon_F, \qquad \alpha_N = 0.54$$

normal state

- Carlson, Chang, Pandharipande and Schmidt, PRL 91, 050401 (2003), with GFMC

$$\frac{E_S}{N} = \alpha_S \frac{3}{5}\varepsilon_F, \qquad \alpha_S = 0.44$$

superfluid state

This state is half the way from BCS→BEC crossover, the pairing correlations are in the strong coupling limit and HFB invalid again.

Theory (QMC)

$$\frac{E}{N} = \frac{3}{5}\varepsilon_F \xi, \qquad \xi = 0.42(2)$$

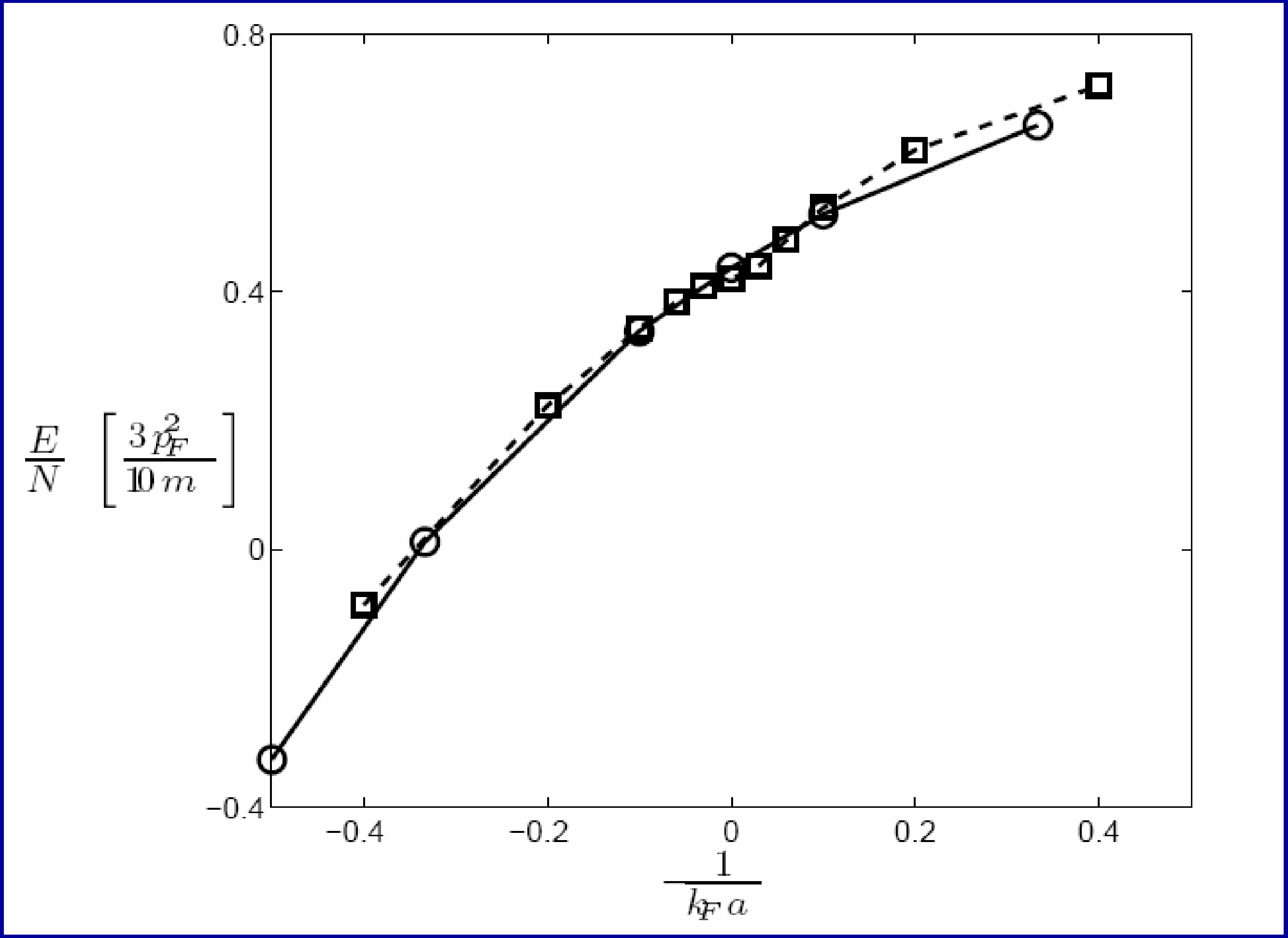
$$\Delta = \varepsilon_F \eta, \qquad \eta = 0.504(24)$$

Experiment

$$\xi = \begin{cases} 0.74(4), & \text{Duke (2002)} \\ 0.51(4), & \text{Duke (2005)} \\ 0.32(12), & \text{Innsbruck (2004)} \\ 0.36(15), & \text{Paris (2004)} \\ 0.46(5), & \text{Rice (2005)} \\ 0.45(5), & \text{JILA (2006)} \\ 0.41(15), & \text{Paris (2007)} \end{cases}$$

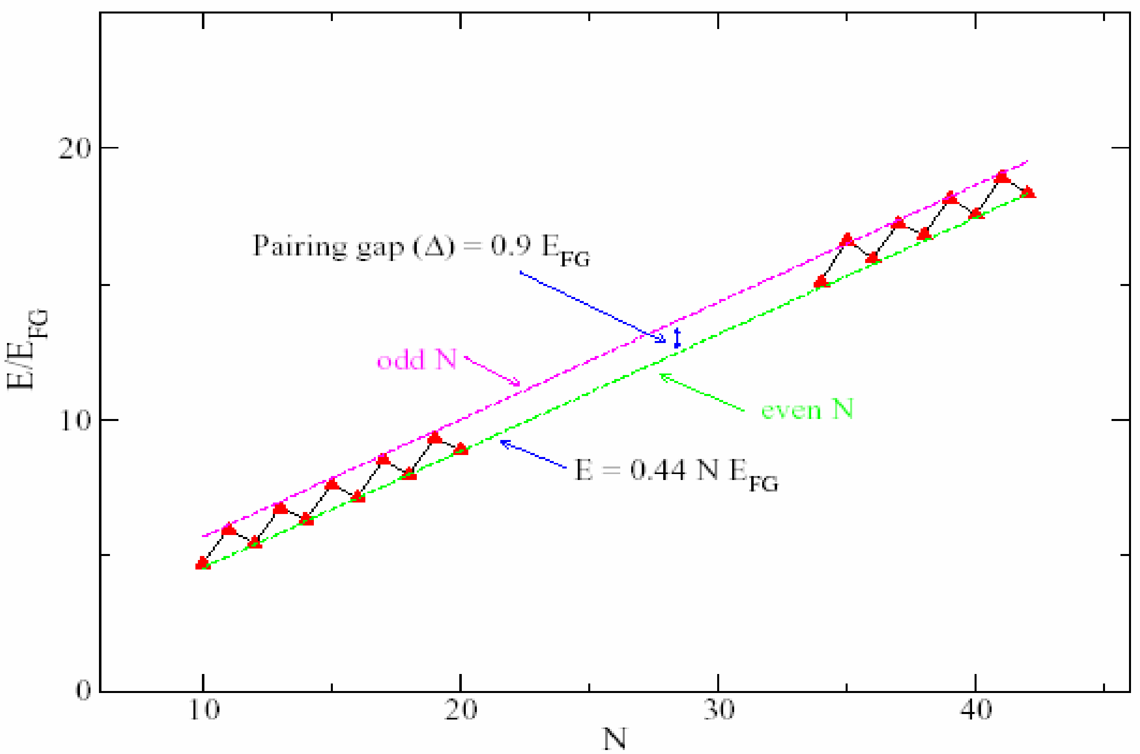
Note, no reliable experimental determination of the pairing gap yet!

BEC side BCS side



Solid line with open circles – Chang *et al.* physics/0404115
Dashed line with squares - Astrakharchik *et al.* cond-mat/0406113

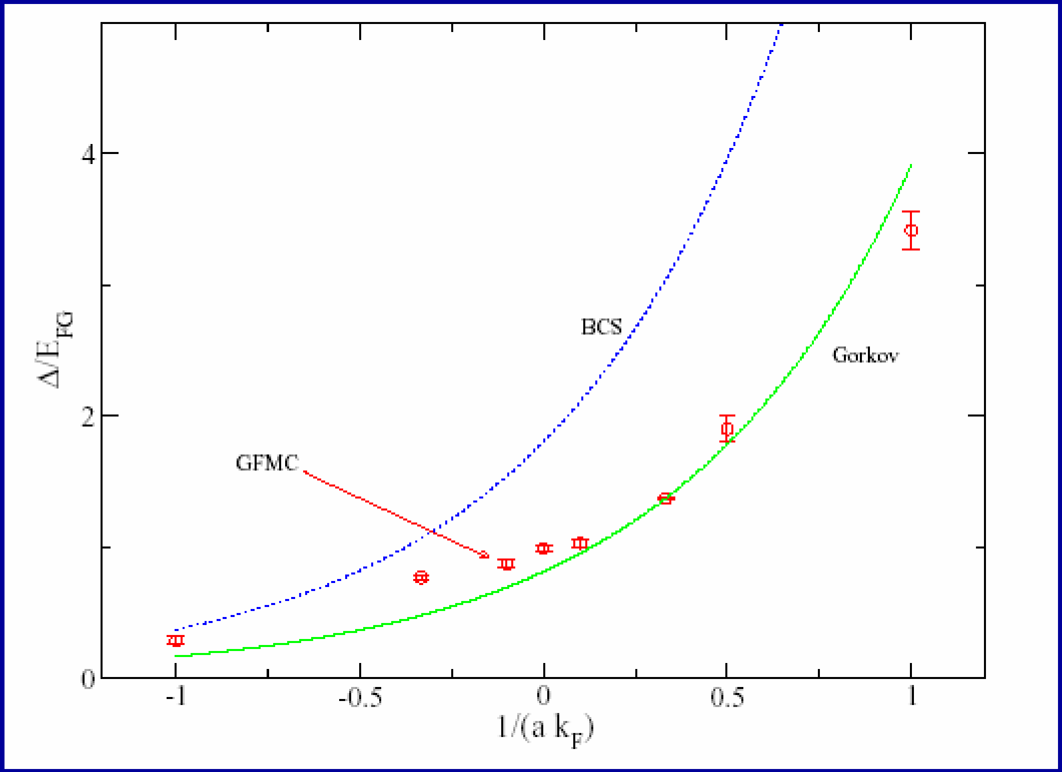
$$\Delta(2n+1) = E(2n+1) - \frac{1}{2}(E(2n) + E(2n+2))$$



Result for $ak_F = -\infty$

$$E_{FG} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}$$

Green Function Monte Carlo with Fixed Nodes
S.-Y. Chang, J. Carlson, V. Pandharipande and K. Schmidt
physics/0403041



$$\Delta_{Gorkov} = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)$$

$$\Delta_{BCS} = \frac{8}{e^2} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2k_F a}\right)$$

Fixed node GFMC results, S.-Y. Chang *et al.* (2003)

Determining the critical temperature for the superfluid to normal phase transition in theory and experiment

Grand Canonical Path-Integral Monte Carlo calculations on 4D-lattice

$$H = T + V = \int d^3x \left[\psi_\uparrow^\dagger(\vec{x}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \psi_\uparrow(\vec{x}) + \psi_\downarrow^\dagger(\vec{x}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \psi_\downarrow(\vec{x}) \right] - g \int d^3x \, n_\uparrow(\vec{x}) n_\downarrow(\vec{x})$$

$$N = \int d^3x \left[n_\uparrow(\vec{x}) + n_\downarrow(\vec{x}) \right]$$

Trotter expansion (*trotterization* of the propagator)

$$Z(\beta) = \mathrm{Tr}\, \exp\left[-\beta \left(H - \mu N \right) \right] = \mathrm{Tr} \left\{ \exp\left[-\tau \left(H - \mu N \right) \right] \right\}^{N_\tau}, \qquad \beta = \frac{1}{T} = N_\tau \tau$$

Recast the propagator at each time slice and use FFT

$$\exp\left[-\tau \left(H - \mu N \right) \right] \approx \exp\left[-\tau \left(T - \mu N \right)/2 \right] \exp(-\tau V) \exp\left[-\tau \left(T - \mu N \right)/2 \right] + O(\tau^3)$$

Discrete Hubbard-Stratonovich transformation

$$\exp(-\tau V) = \prod_{\vec{x}} \sum_{\sigma_\pm(\vec{x}) = \pm 1} \left\{ 1 + \sigma_\pm(\vec{x}) A \left[n_\uparrow(\vec{x}) + n_\downarrow(\vec{x}) \right] \right\}, \qquad A = \sqrt{\exp(\tau g) - 1}$$

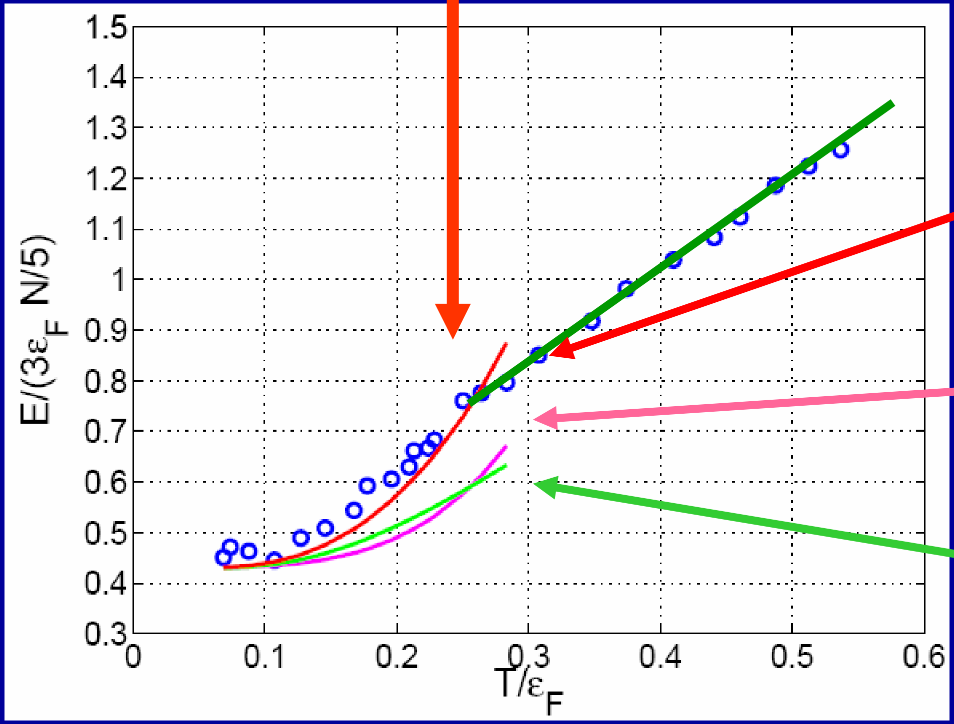
σ-fields fluctuate both in space and imaginary time

$$\frac{m}{4\pi\hbar^2 a} = -\frac{1}{g} + \frac{m k_{cut-off}}{2\pi^2 \hbar^2}$$

Running coupling constant g defined by lattice

A. Bulgac, J.E. Drut and P.Magierski

Superfluid to Normal Fermi Liquid Transition



Bogoliubov-Anderson phonons and quasiparticle contribution (red line)

Bogoliubov-Anderson phonons contribution only (magenta line)

People never consider this ???

Quasi-particles contribution only (green line)

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.44$$

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}}\exp\left(-\frac{\Delta}{T}\right)$$

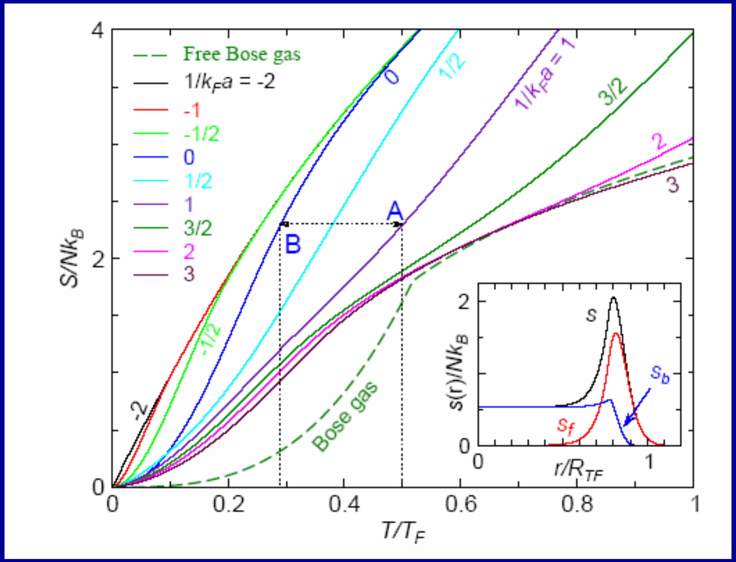
$$\Delta = \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

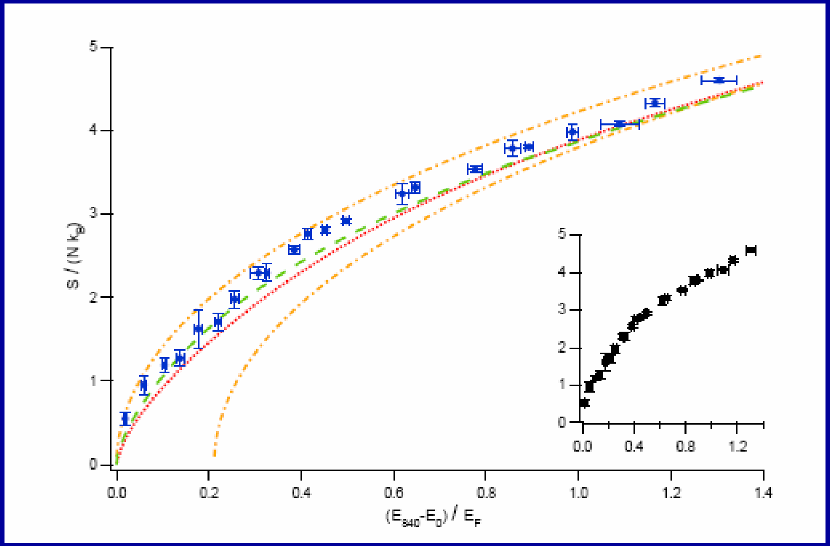
- **Lattice size:**
  from 6^3 x 112 at low T
  to 6^3 x 30 at high T

- **Number of samples:**
  Several 10^5's for T
- **Also calculations for 4^3 lattices**
- **Limited results for 8^3 lattices**

Experimentally one needs a thermometer!

Solution: Measure the entropy S as a function of the of the energy E and use T = dE/dS





Chen et al, PRL 95, 260405 (2005)

Exp: Luo et al, PRL (2007)

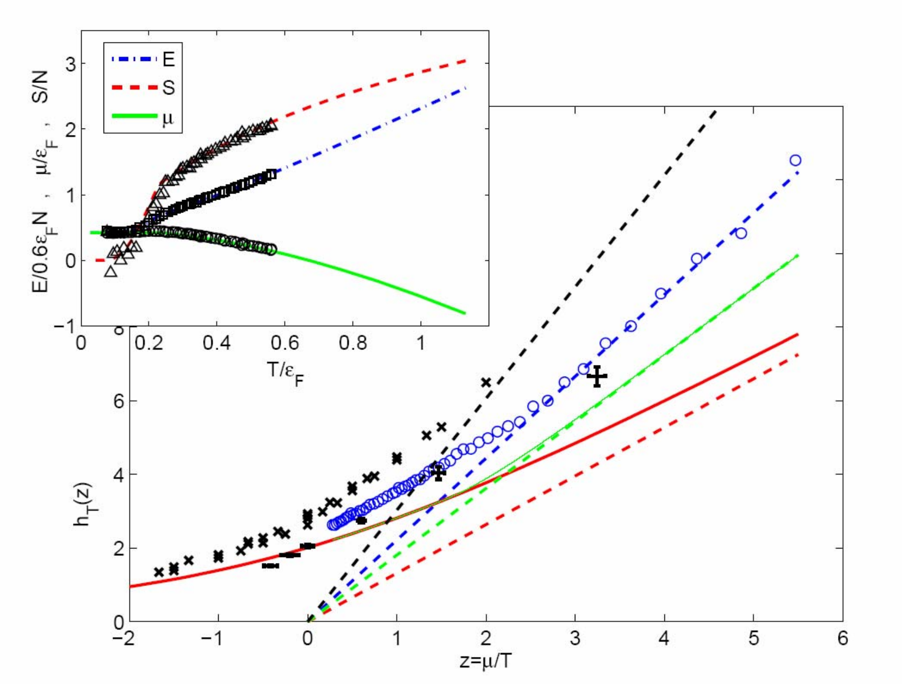
How to use Quantum Monte Carlo results to describe what is going on in an atomic trap at finite temperature and confront theory and experiment?

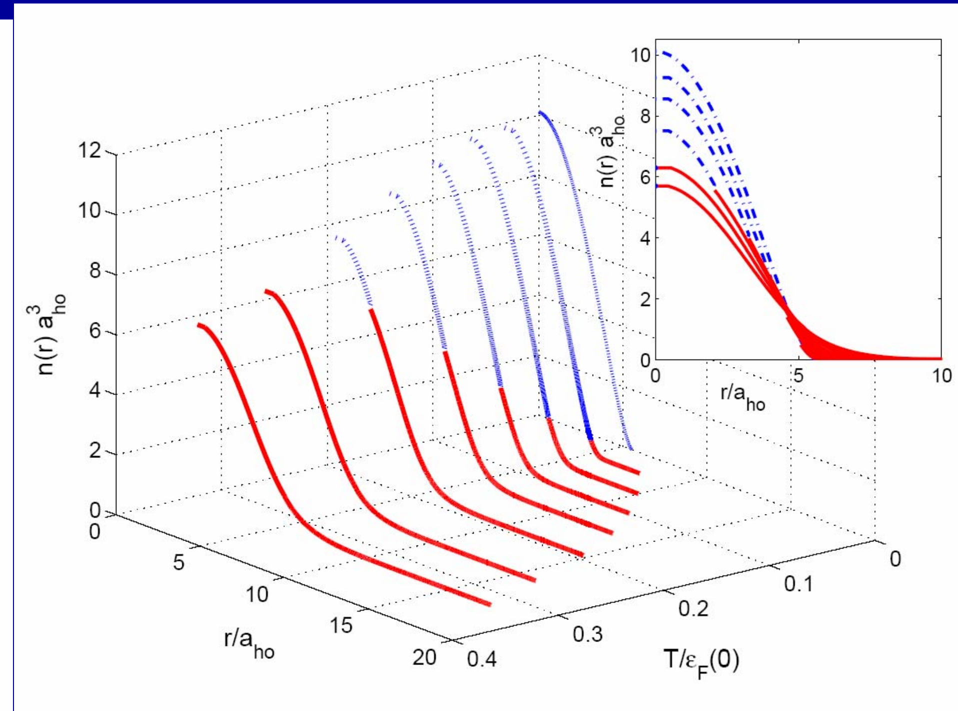
Use Local Density Approximation (LDA) in conjunction with the Universality of Fermi Gases

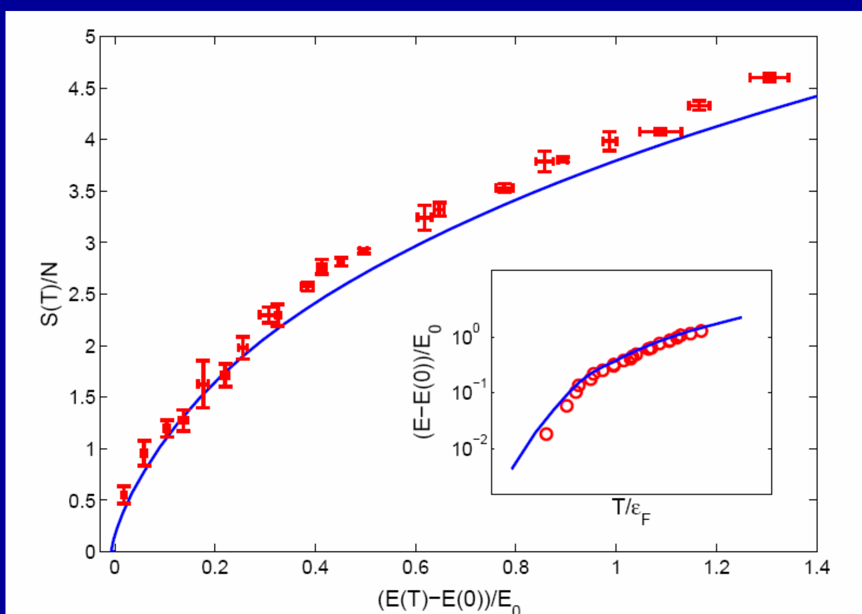
$$F(T)=\int d^3r\left[\frac{3}{5}\frac{\hbar^2\left(3\pi^2\right)^{2/3}n^{5/3}(\vec{r})}{2m}\xi\left(\frac{T}{n^{2/3}(\vec{r})}\right)-T\sigma\left(\frac{T}{n^{2/3}(\vec{r})}\right)n(\vec{r})+(V_{trap}(\vec{r})-\lambda)n(\vec{r})\right]$$

energy density

entropy density







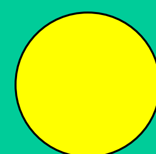
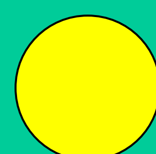
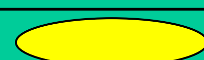
Experiment – red symbols
Luo et al, PRL (2007)

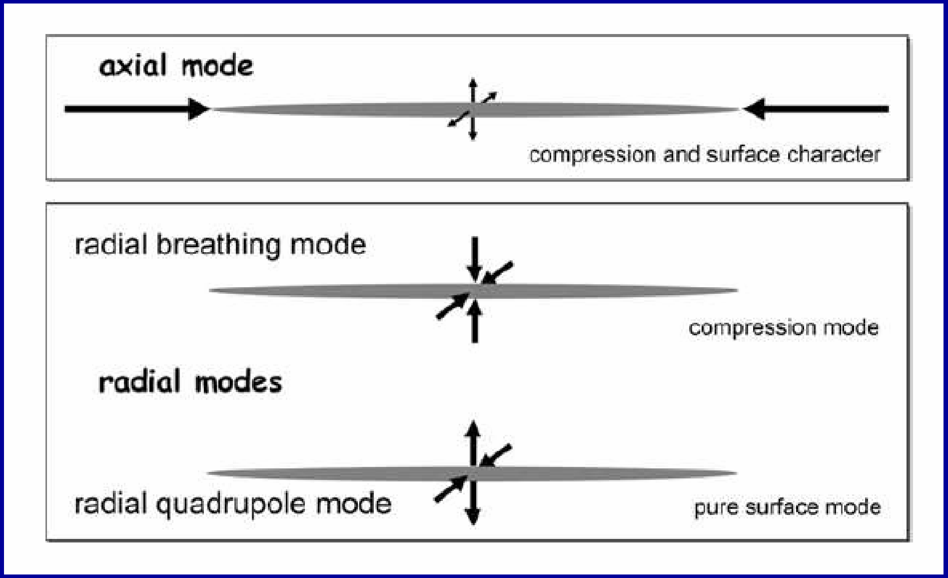
Blue curves – pure theory

Collective modes

Sound in infinite fermionic matter

$$\omega = v_s k$$

	Local shape of Fermi surface	Sound velocity	
Collisional Regime - <u>high T!</u> **Compressional mode**	Spherical 	$v_s \approx \frac{v_F}{\sqrt{3}}$	First sound
Superfluid collisionless- <u>low T!</u> **Compressional mode**	Spherical 	$v_s \approx \frac{v_F}{\sqrt{3}}$	Bogoliubov-Anderson sound
Normal Fermi fluid collisionless - <u>low T!</u> **(In)compressional mode**	 Elongated along propagation direction	$v_s = s v_F$ $s > 1$	Landau's zero sound Need repulsion !!!



Grimm, cond-mat/0703091

$$\varepsilon(n)=\frac{3}{5}\frac{\hbar^2k_F^2}{2m}\left[\xi-\frac{\varsigma}{k_Fa}-\frac{5\iota}{(k_Fa)^2}+O\left(\frac{1}{(k_Fa)^3}\right)\right]$$

$$\xi\approx 0.44,\qquad \varsigma\approx 1,\qquad \iota\approx 1$$

$$U=\frac{m\omega_0^2\left(x^2+y^2+\lambda^2z^2\right)}{2}$$

$$\frac{\delta\omega^2}{\omega^2}=\frac{\varsigma}{\xi}\frac{1}{k_F(0)a}K$$

Adiabatic regime
Spherical Fermi surface

Bogoliubov-Anderson modes in a trap

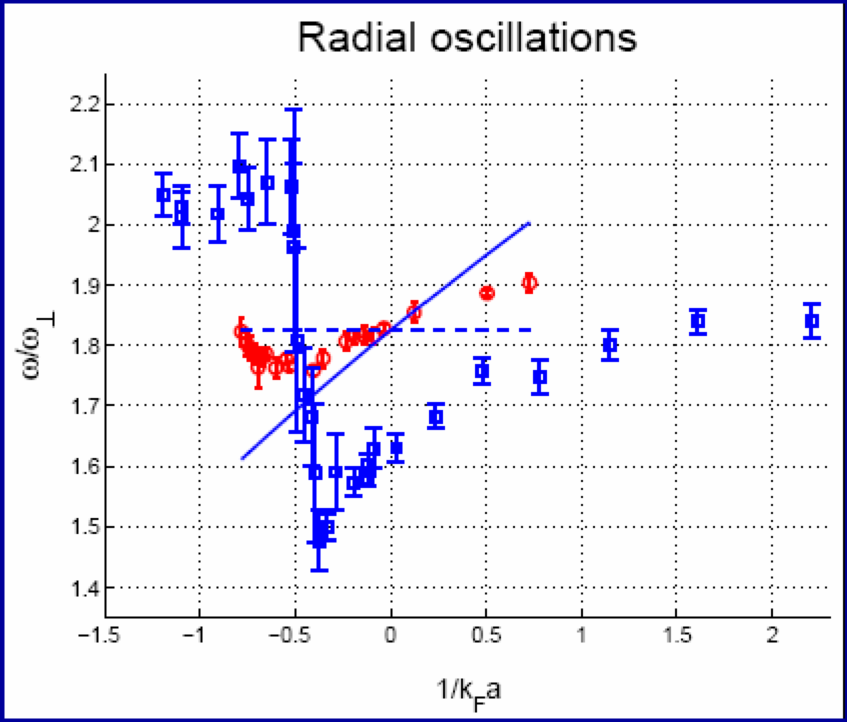
Perturbation theory result using GFMC equation of state in a trap

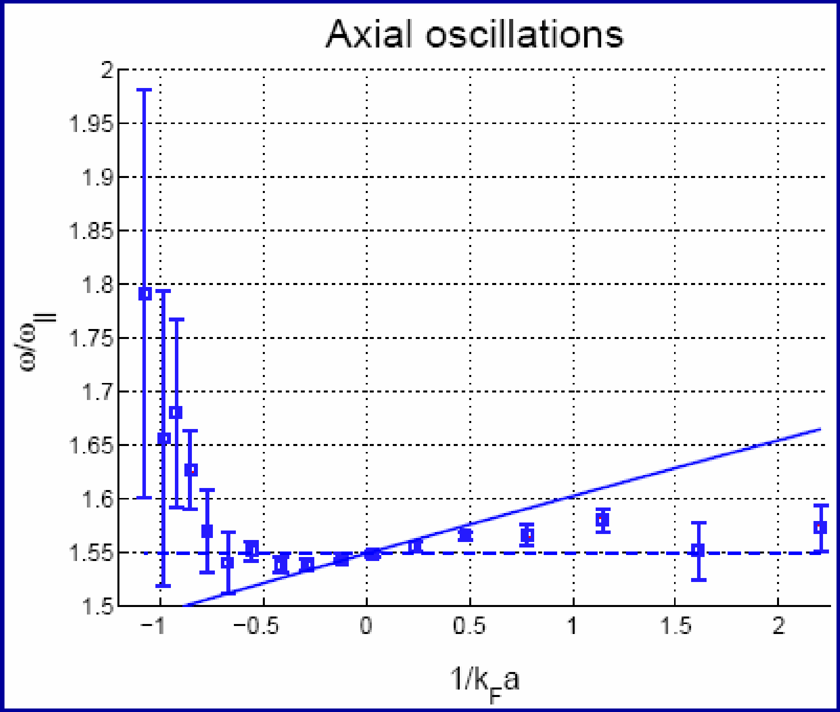
TABLE II: Results for K.

trap type	mode	f_1	ω	K
spherical	dipole	z	ω_0	0
$\lambda=1$	monopole	$1-2r^2$	$2\omega_0$	$\frac{256}{525\pi}$
	quadrupole	xy	$\sqrt{2}\omega_0$	0
axial	$M=\pm 2$	$xy,\ x^2-y^2$	$\sqrt{2}\omega_0$	0
$\lambda\ll 1$	$M=\pm 1$	$xz,\ yz$	ω_0	0
	radial	$x^2+y^2+\frac{2}{5}\lambda^2z^2-\frac{2}{5}$	$\sqrt{\frac{10}{3}}\omega_0$	$\frac{1024}{2625\pi}$
	axial	$1-6\lambda^2z^2$	$\sqrt{\frac{12}{5}}\lambda\omega_0$	$\frac{256}{2625\pi}$

Only compressional modes are sensitive to the equation of state and experience a shift!

Innsbruck's results - blue symbols
Duke's results - red symbols



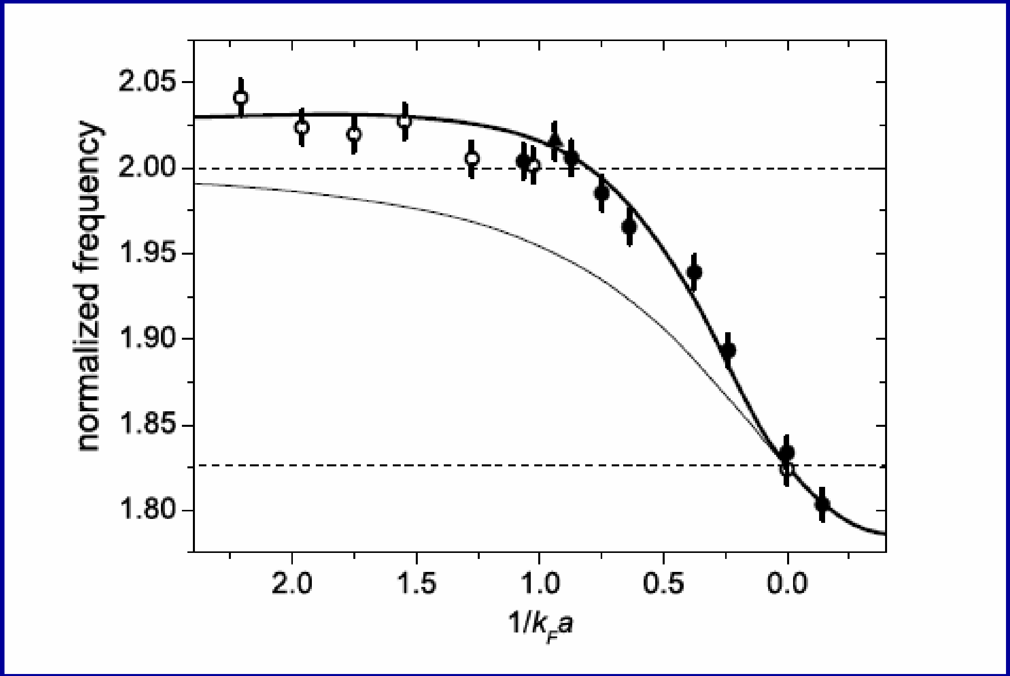


First order perturbation theory prediction (blue solid line)

Unperturbed frequency in unitary limit (blue dashed line)
Identical to the case of non-interacting fermions

If the matter at the Feshbach resonance would have a bosonic character then the collective modes will have significantly higher frequencies!

High precision results



Radial breathing mode: theory vs experiment
meanfield EoS – light curve, MC EoS – thick curve

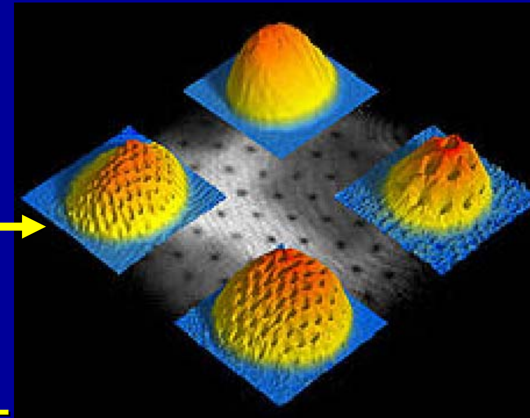
Grimm, cond-mat/0703091

If we set our goal to prove that these systems become superfluid, there is no other way but to show it!

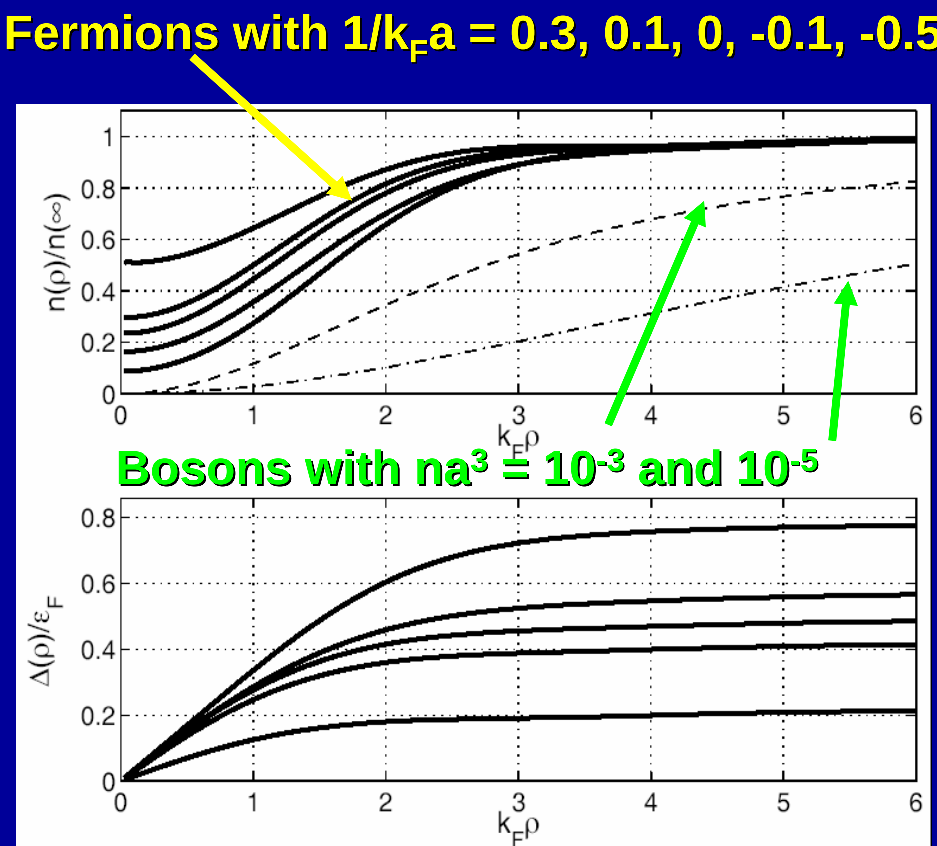
Is there a way to put directly in evidence the superflow?

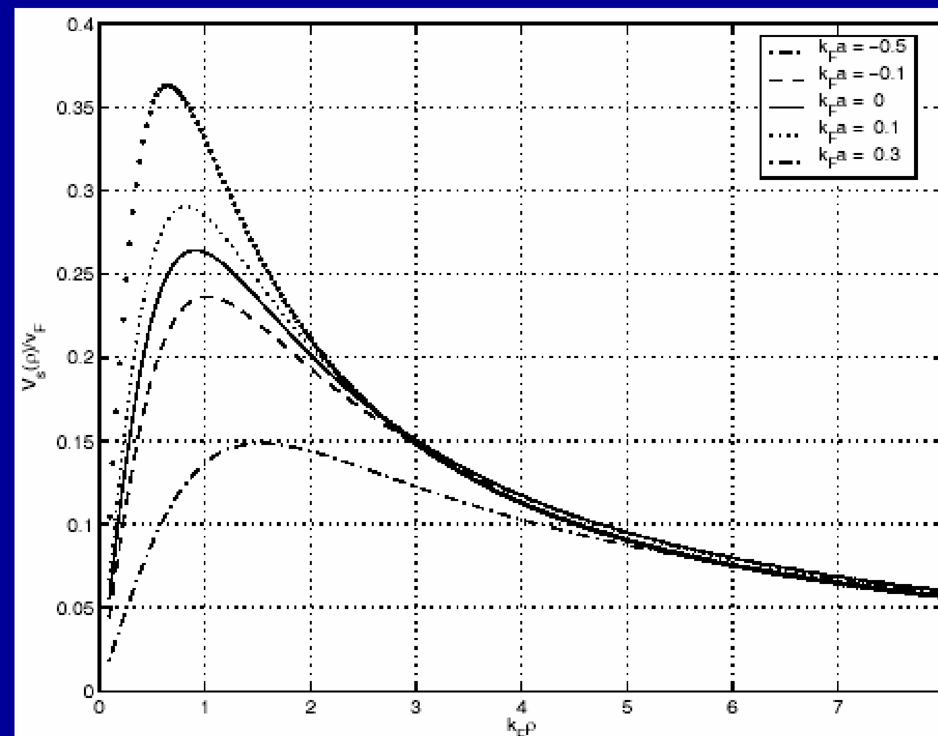
Vortices!

The depletion along the vortex core is reminiscent of the corresponding density depletion in the case of a vortex in a Bose superfluid, when the density vanishes exactly along the axis for 100% BEC.



From Ketterle's group





Extremely fast quantum vortical motion!

Number density and pairing field profiles

Local vortical speed as fraction of Fermi speed

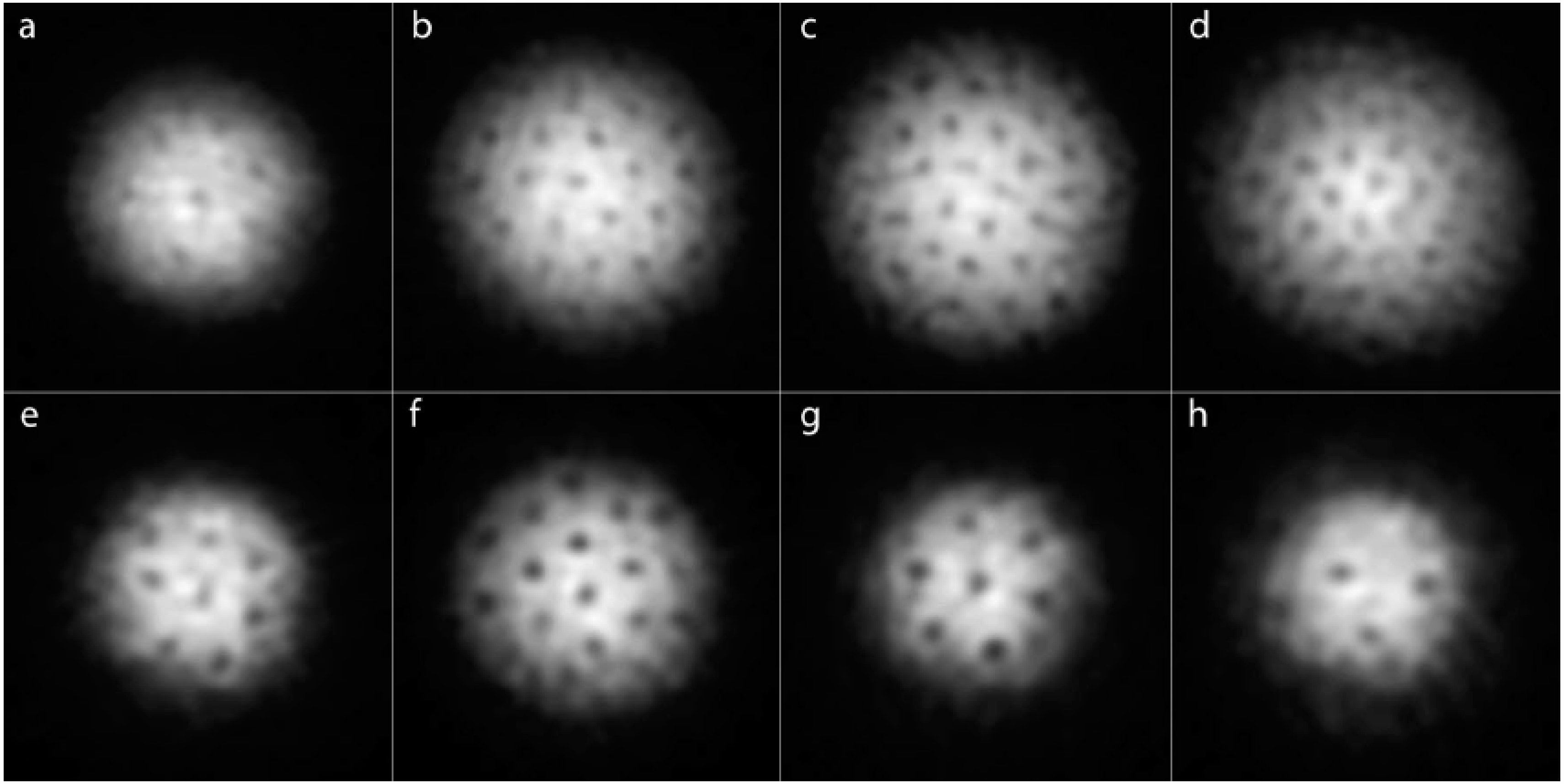


Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is $880\ \mu\mathrm{m} \times 880\ \mu\mathrm{m}$.

Zweirlein et al. cond-mat/0505653

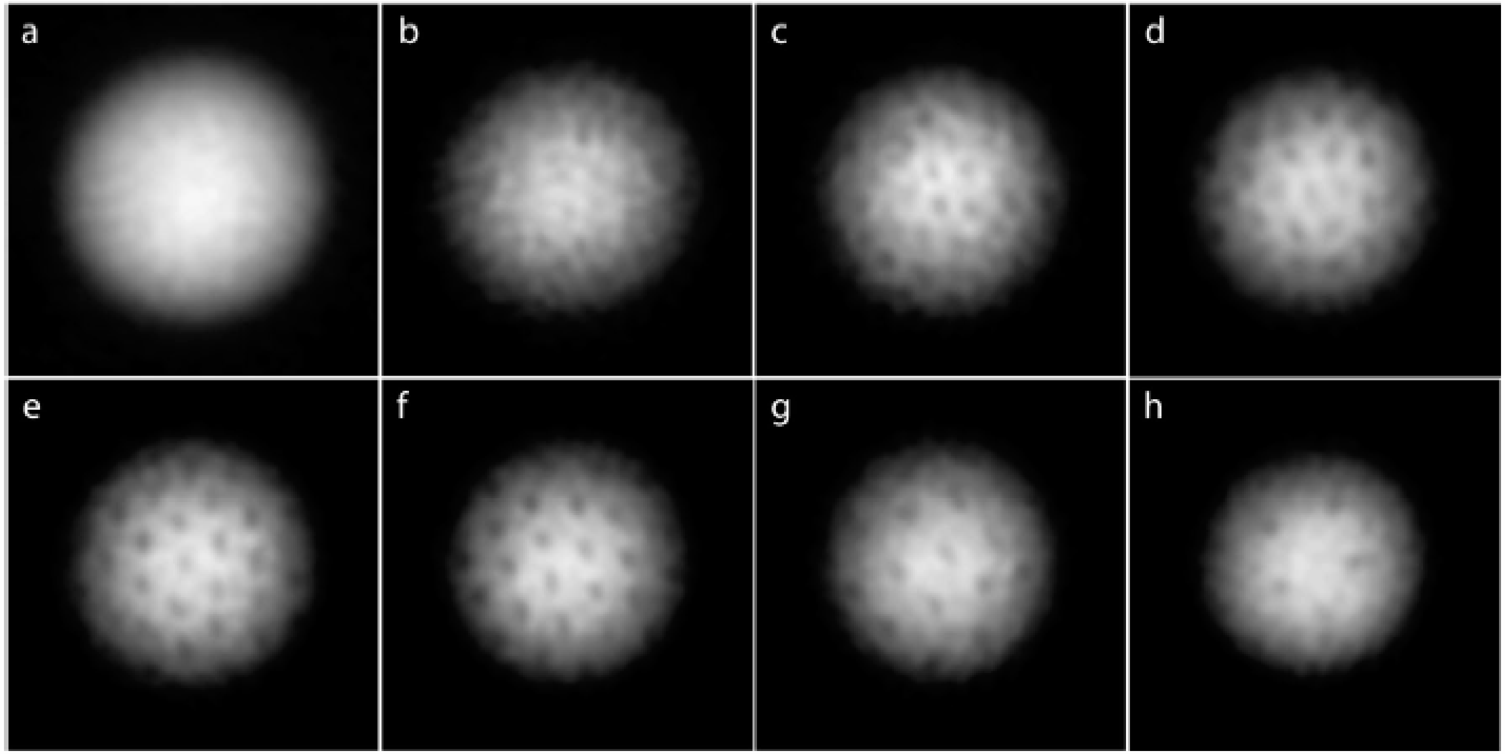
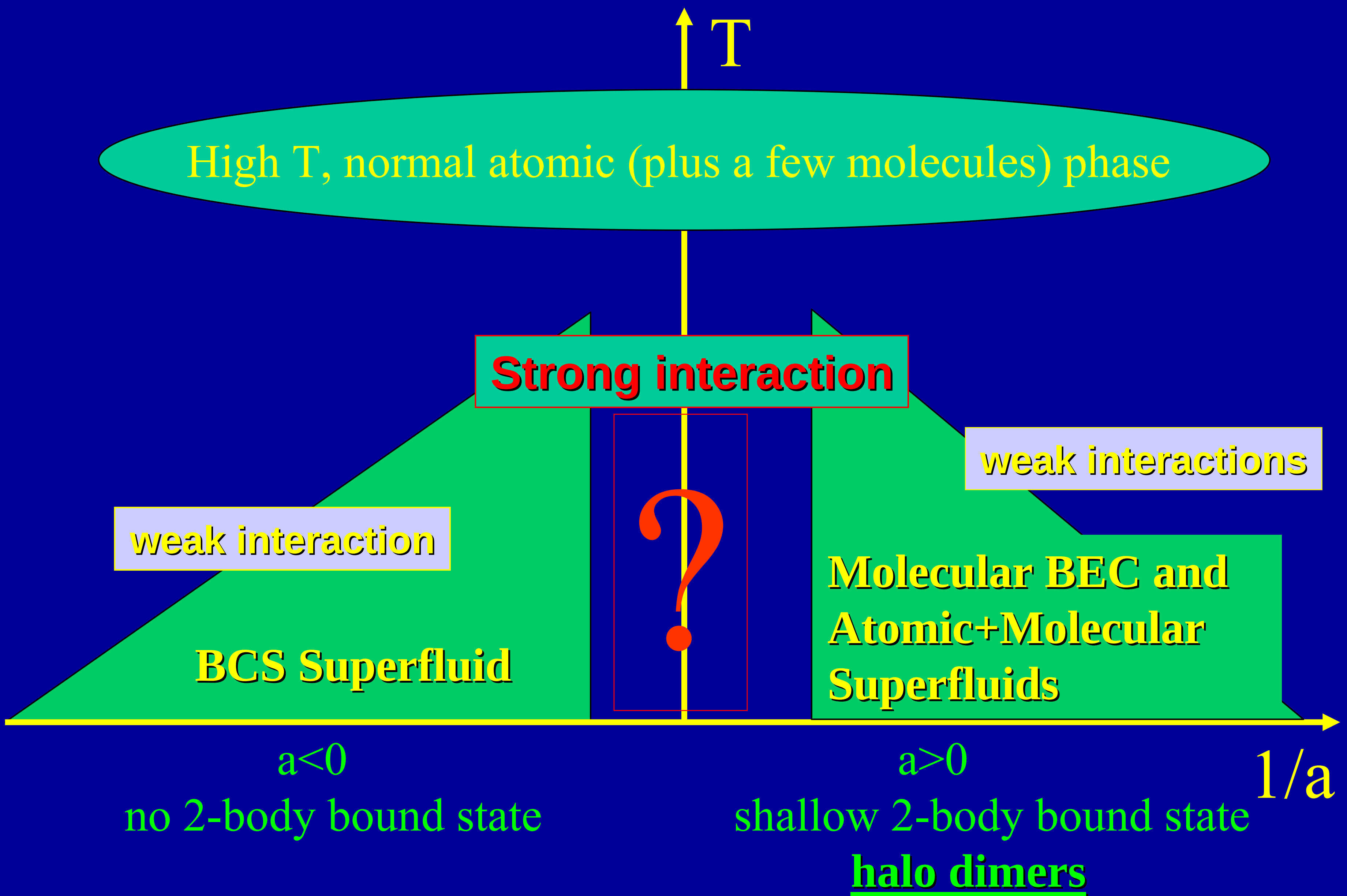


Fig. 6: Formation and decay of a vortex lattice in a fermion pair condensate on the BEC-side close to the Feshbach resonance. A molecular condensate, prepared at 766 G as shown in (a), was stirred for 800 ms. The field was then ramped to 812 G in 20 ms for equilibration. At this field, $1/k_F a = 0.35$, and the condensate was deep in the strongly interacting regime. To observe the vortex lattice, the field was ramped in 25 ms to 735 G ($1/k_F a = 2.3$), where the condensate was released from the trap and imaged after 12 ms time-of-flight. The equilibration times after the end of the stirring were (b) 40 ms, (c) 240 ms, (d) 390 ms, (e) 790 ms, (f) 1140 ms, (g) 1240 ms and (h) 2940 ms. Due to stirring, evaporation and vibrational relaxation, the number of fermion pairs decayed from 3×10^6 (a) to 1×10^6 (b-h). The field of view of each image is $830\ \mu\text{m}\times830\ \mu\text{m}$.

Zweirlein et al. cond-mat/0505653

Phases of a two species dilute Fermi system BCS-BEC crossover

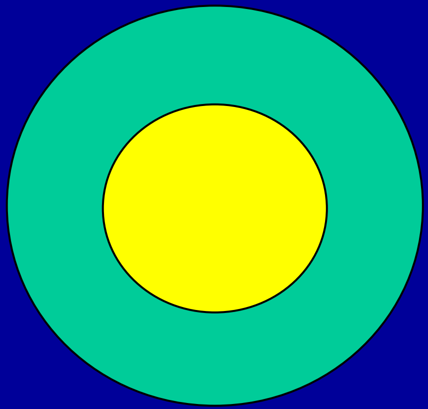


Until now we kept the numbers of spin-up and spin-down equal.

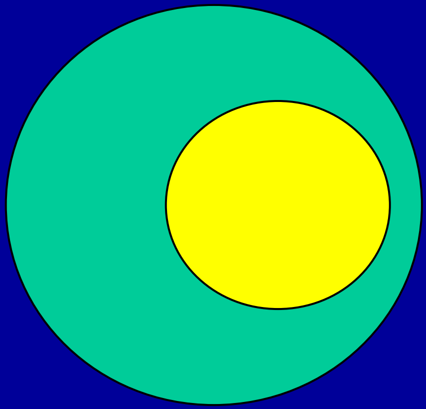
What happens when there not enough partners for everyone to pair with?

What theory tells us?

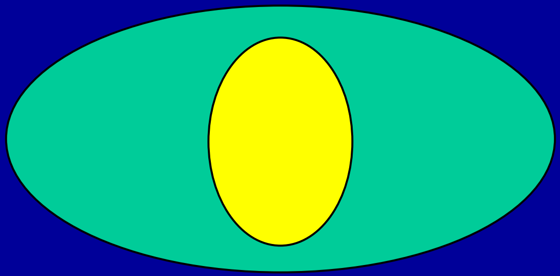
Green – spin up
Yellow – spin down



If $\left|\mu_\uparrow - \mu_\downarrow\right| < \frac{\Delta}{\sqrt{2}}$ the same solution as for $\mu_\uparrow = \mu_\downarrow$



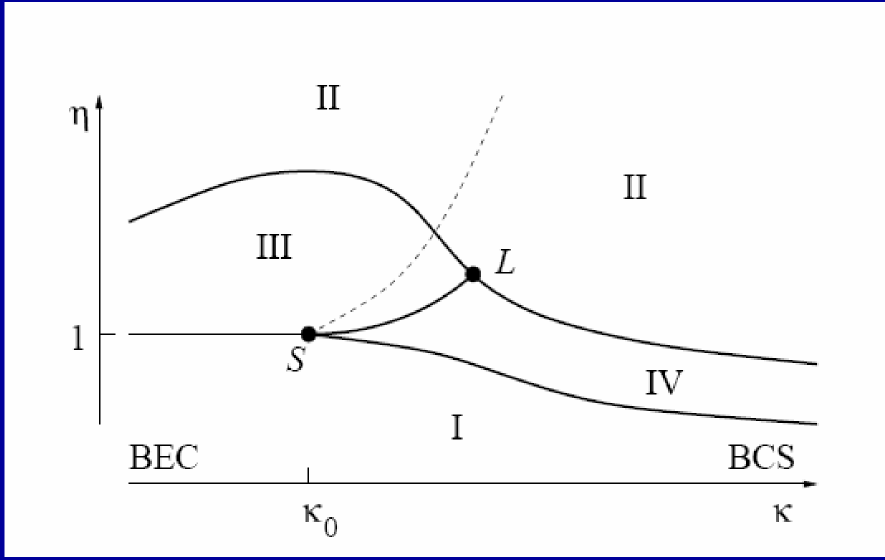
LOFF solution (1964)
Pairing gap becomes a spatially varying function
Translational invariance broken



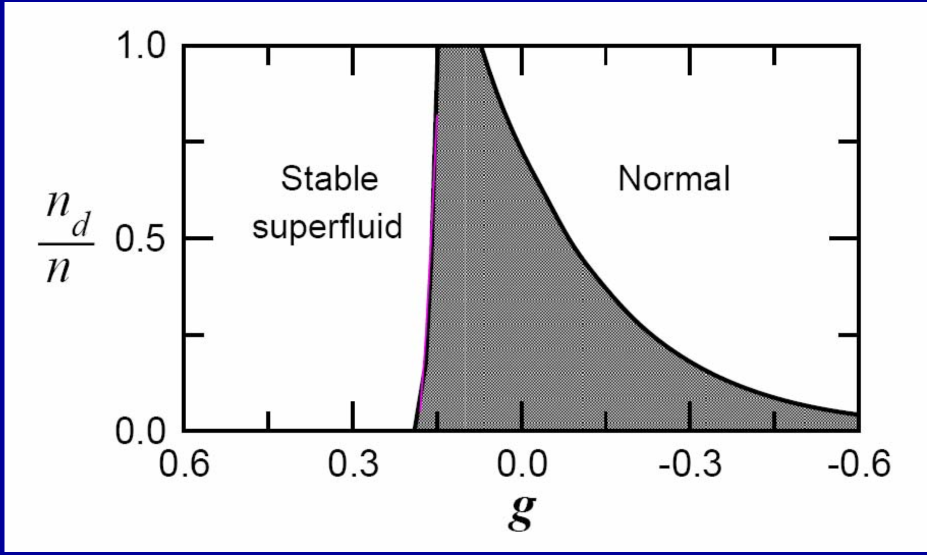
Muether and Sedrakian (2002)
Translational invariant solution
Rotational invariance broken

LOFF and Deformed Fermi Surfaces pairing can occur only for relatively small imbalances.

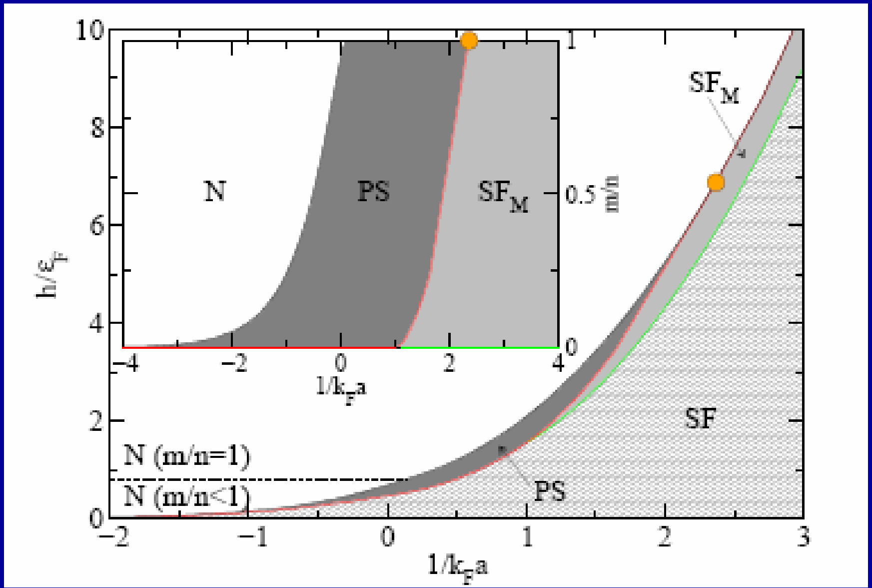
What happens if the imbalances become large?



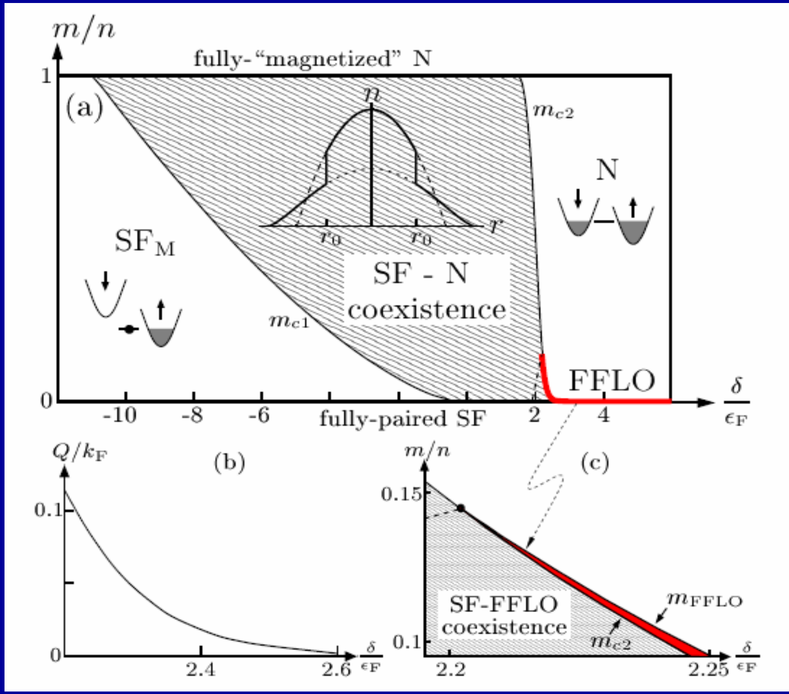
Son and Stephanov, cond-mat/0507586



Pao, Wu, and Yip, PR B 73, 132506 (2006)



Parish, Marchetti, Lamacraft, Simons cond-mat/0605744

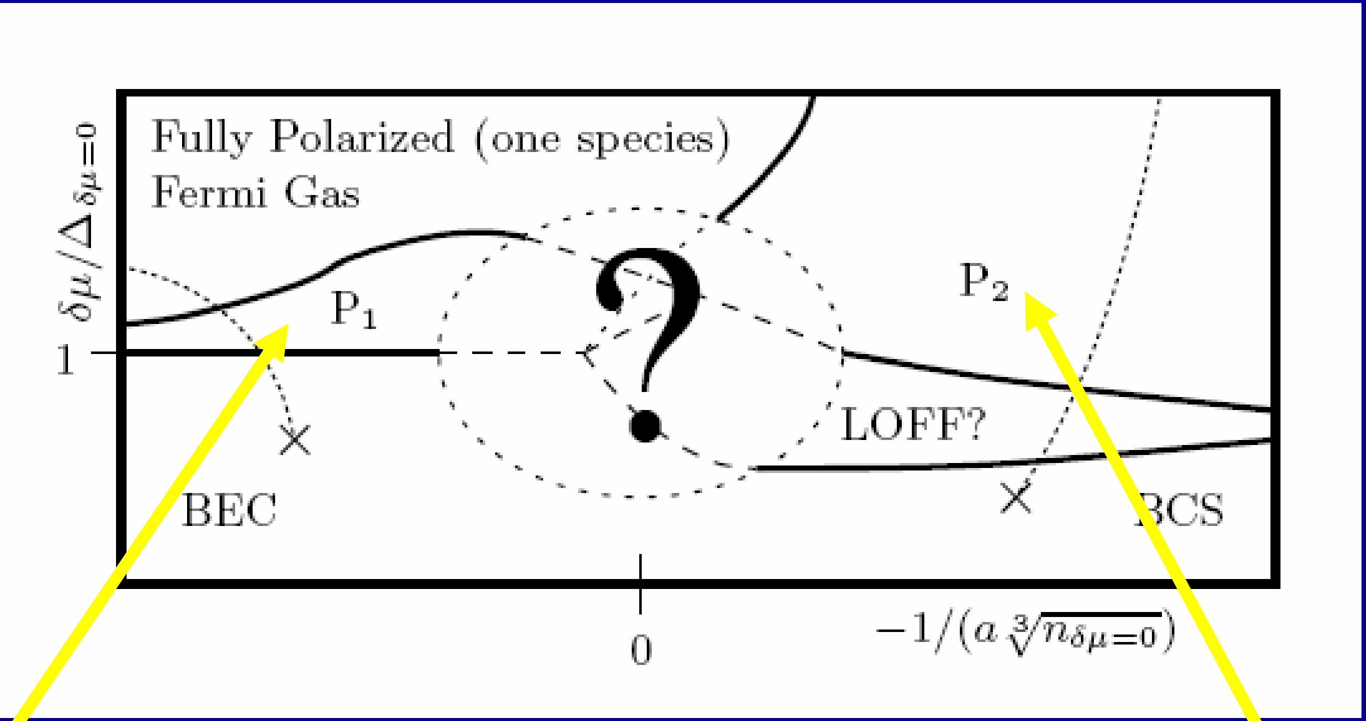


Sheeny and Radzihovsky, PRL 96, 060401(2006)

What really is happening!

Induced *p*-wave superfluidity in asymmetric Fermi gases

Two new superfluid phases where before they were not expected



Bulgac, Forbes, Schwenk

One Bose superfluid coexisting with one P-wave Fermi superfluid

Two coexisting P-wave Fermi superfluids

BEC regime

- **all minority (spin-down) fermions form dimers and the dimers organize themselves in a Bose superfluid**

- **the leftover/un-paired majority (spin-up) fermions will form a Fermi sea**

- **the leftover spin-up fermions and the dimers coexist and, similarly to the electrons in a solid, the leftover spin-up fermions will experience an attraction due to exchange of Bogoliubov phonons of the Bose superfluid**

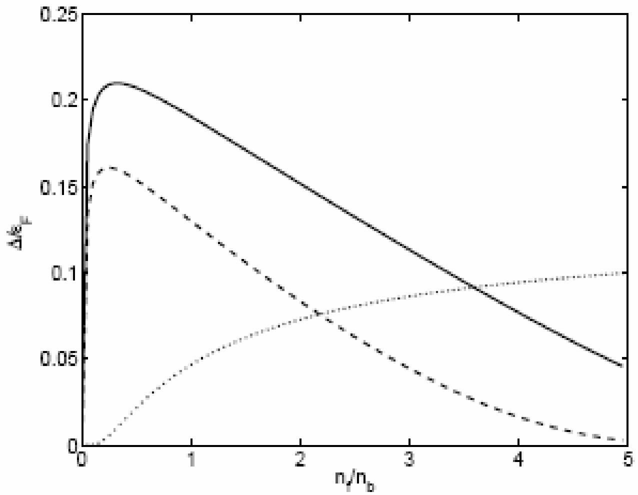


FIG. 1: The ratio Δ/ε_F ($\varepsilon_F = \hbar^2 k_F^2/2m$) as a function of n_f/n_b, for a fixed boson number density $n_b = 10^{13}$ cm^{-3} and $n_b a^3 = 0.064$ (solid line) and $n_b a^3 = 0.037$ (dashed line) respectively. The dots show the value of the gap in the case of p-wave paring for $n_b a^3 = 0.064$.

p-wave gap

Bulgac, Bedaque, Fonseca, cond-mat/030602

$$\Delta_p \sim \varepsilon_F \exp\left(-0.44\frac{n_b}{n_f}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \ll k_F a \ll 1$$

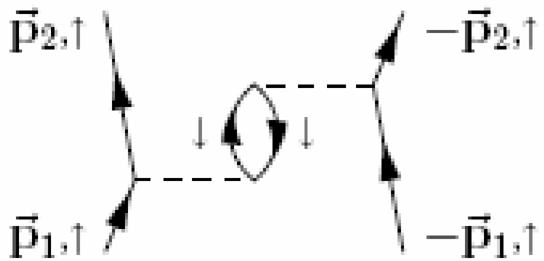
$$\Delta_p \sim \varepsilon_F \exp\left(-\frac{6\pi^2}{\alpha_{fb}^2 (k_F a)^2 \ln\left(x^2\right)}\frac{n_b}{n_f}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \gg k_F a, \qquad x^2 = \left(\frac{\hbar k_F}{m_b c}\right)^2$$

$$\Delta_p\Big|_{\max} \sim \varepsilon_F \exp\left(-\frac{5.6}{k_F a}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \approx 0.44 k_F a \ll 1$$

!!!

BCS regime:

$$U^{\uparrow\uparrow}_{\mathrm{ind}}(0, \vec{p}_1 - \vec{p}_2) =$$



$$= -N_{\mathrm{F}}^{\downarrow} \left(\frac{4\pi a \hbar^2}{m} \right)^2 L\left(|\vec{p}_1 - \vec{p}_2| / (2\hbar k_{\mathrm{F}}^{\downarrow}) \right) .$$

The same mechanism works for the minority/spin-down component

$$\Delta_p^{\uparrow} \sim \varepsilon_F^{\uparrow} \exp\left(\frac{1}{N_F^{\uparrow} U_p^{\uparrow\uparrow}}\right) = \varepsilon_F^{\uparrow} \exp\left(-\frac{\pi^2}{4 k_F^{\uparrow} k_F^{\downarrow} a^2 L_p\left(\frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right)}\right)$$

$$L_p(z) = \frac{5z^2 - 2}{15z^4} \ln\left|1 - z^2\right| - \frac{z^2 + 5}{30z} \ln\left|\frac{1 - z}{1 + z}\right| - \frac{z^2 + 2}{15z^2}$$

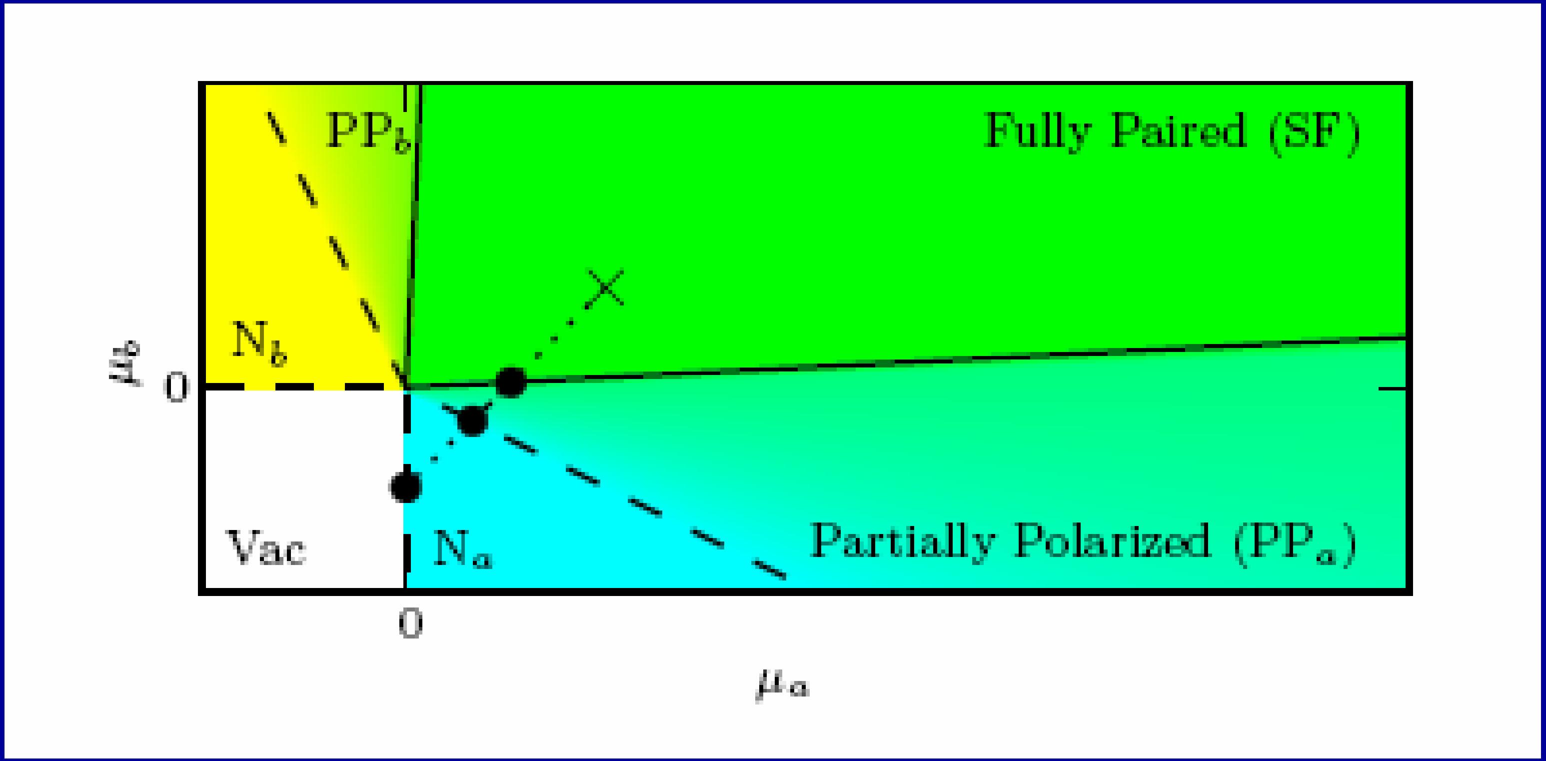
$$\Delta_p^{\uparrow}\Big|_{\max} \sim \varepsilon_F^{\uparrow} \exp\left(-\frac{\pi^2}{0.11\left(2k_F^{\uparrow}a\right)^2}\right), \qquad \text{for } k_F^{\downarrow} \approx 0.77 k_F^{\uparrow} \text{ and fixed } k_F^{\downarrow}$$

$$\left.\begin{aligned} \Delta_p^{\uparrow} &\sim \varepsilon_F^{\uparrow} \exp\left(-\frac{3\pi^2}{2\left(2k_F^{\uparrow}a\right)^2 \ln\left(\frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right)} \frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right) \\ \Delta_p^{\downarrow} &\sim \varepsilon_F^{\downarrow} \exp\left(-\frac{18\pi^2}{\left(2k_F^{\downarrow}a\right)^2} \frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right) \end{aligned}\right\} \quad \text{for } k_F^{\uparrow} \gg k_F^{\downarrow}$$

➢T=0 thermodynamics in asymmetric Fermi gases at unitarity in a trap

Use Local Density Approximation (LDA)

$$\lambda_{\{a,b\}}(\vec{r}) = \mu_{\{a,b\}} - V_{trap}(\vec{r})$$



Now let us concentrate on a spin unbalanced Fermi gas at unitarity

At unitarity almost everything is a function of the densities alone at T=0!

We use both micro-canonical and grand canonical ensembles

$$x = \frac{n_b}{n_a} \le 1, \qquad y = \frac{\mu_b}{\mu_a} \le 1$$

$$E(n_a, n_b) = \frac{3}{5}\alpha \left[n_a g(x)\right]^{5/3}$$

$$P(\mu_a, \mu_b) = \frac{2}{5}\beta\left[\mu_a h(y)\right]^{5/2} = \mu_a n_a + \mu_b n_b - E(n_a, n_b) = \frac{2}{3}E(n_a, n_b)$$

$$y = \frac{g'(x)}{g(x) - xg'(x)}, \qquad h(y) = \frac{1}{g(x) - xg'(x)}$$

$$x = \frac{h'(y)}{h(y) - yh'(y)}, \qquad g(x) = \frac{1}{h(y) - yh'(y)}$$

The functions g(x) and h(y) determine fully the thermodynamic properties and only a few details are relevant

Both g(x) and h(y) are convex functions of their argument.

Non-trivial regions exist!

$$h(y) = \begin{cases} 1 & \text{if} \quad y \le y_0 \\ \dfrac{1+y}{(2\xi)^{3/5}} & \text{if} \quad y \in [y_1, 1] \end{cases}$$

$$h''(y) \ge 0$$

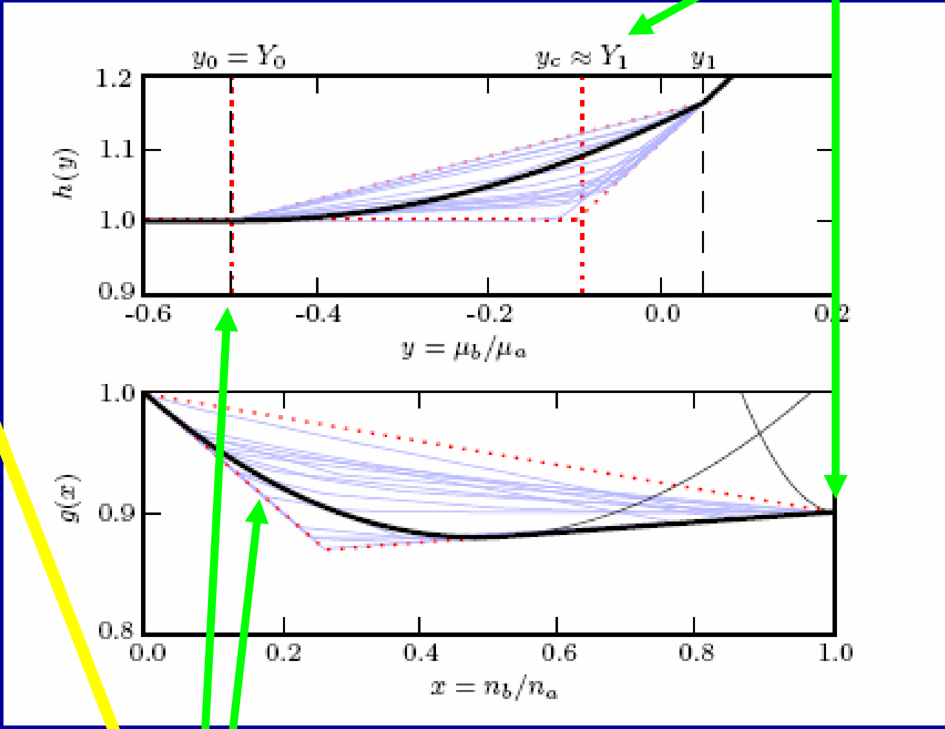
$$y_0 \le Y_0 < y_c < Y_1 \le 1, \qquad y_c = (2\xi)^{3/5} - 1$$

$$g(0) = 1, \qquad g(x) = (2\xi)^{3/5}$$

$$g''(x) \ge 0$$

$$g'(0) \le Y_0 \quad \text{and} \quad g'(1) \in \left[\frac{g(1)}{1+Y_1^{-1}}, \frac{g(1)}{2}\right]$$

Bounds given by GFMC



Bounds from the energy required to add a single spin-down particle to a fully polarized Fermi sea of spin-up particles

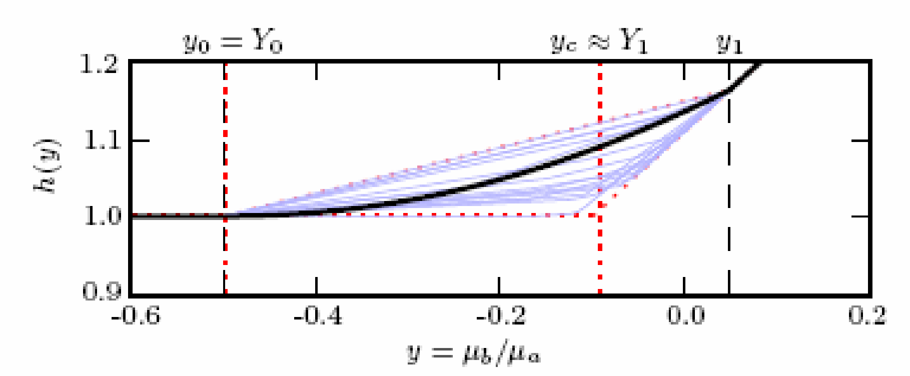
Now put the system in a trap

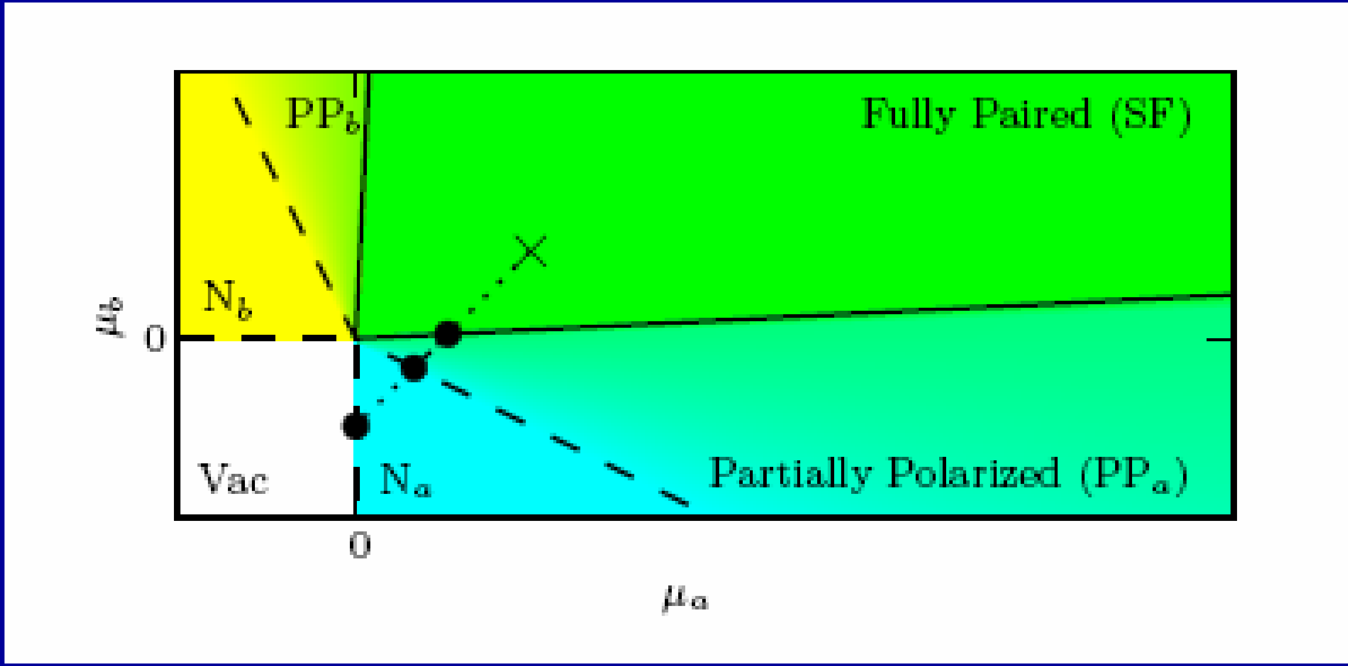
$$\mu_{a,b}(\vec{r}) = \lambda_{a,b} - V(\vec{r}), \qquad y(\vec{r}) = \frac{\mu_b(\vec{r})}{\mu_a(\vec{r})}$$

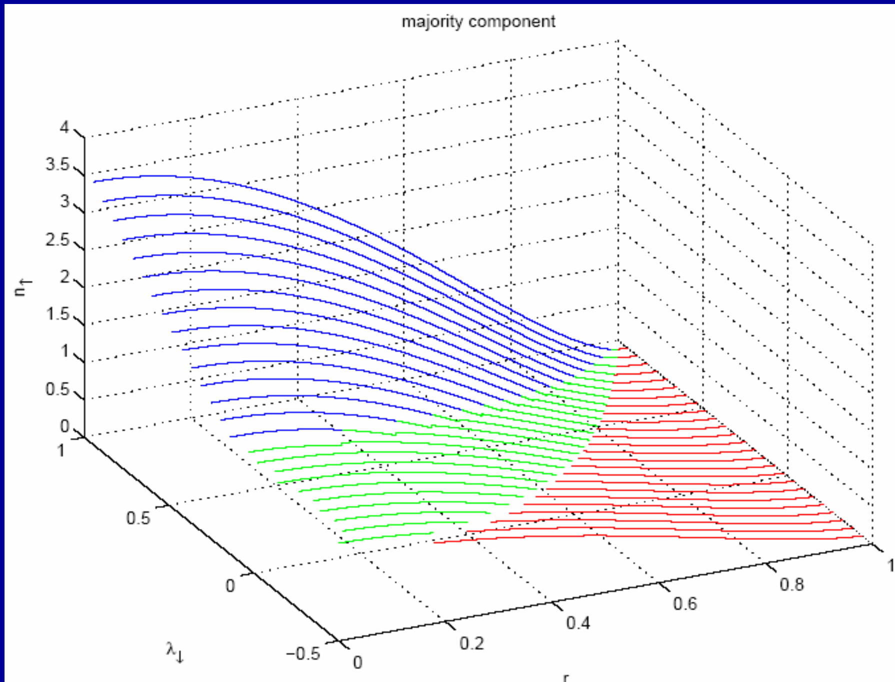
$$2\mu_- = \lambda_a - \lambda_b$$

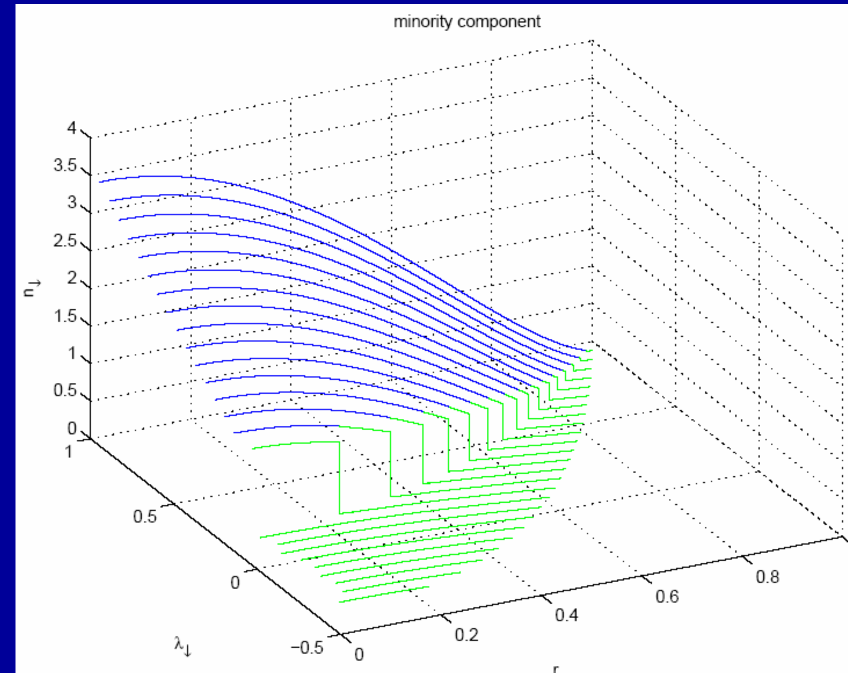
$$n_a(\vec{r}) = \beta\left[\mu_a(\vec{r})h(y(\vec{r}))\right]^{3/2}\left[h(y(\vec{r})) - y(\vec{r})h'(y(\vec{r}))\right]$$

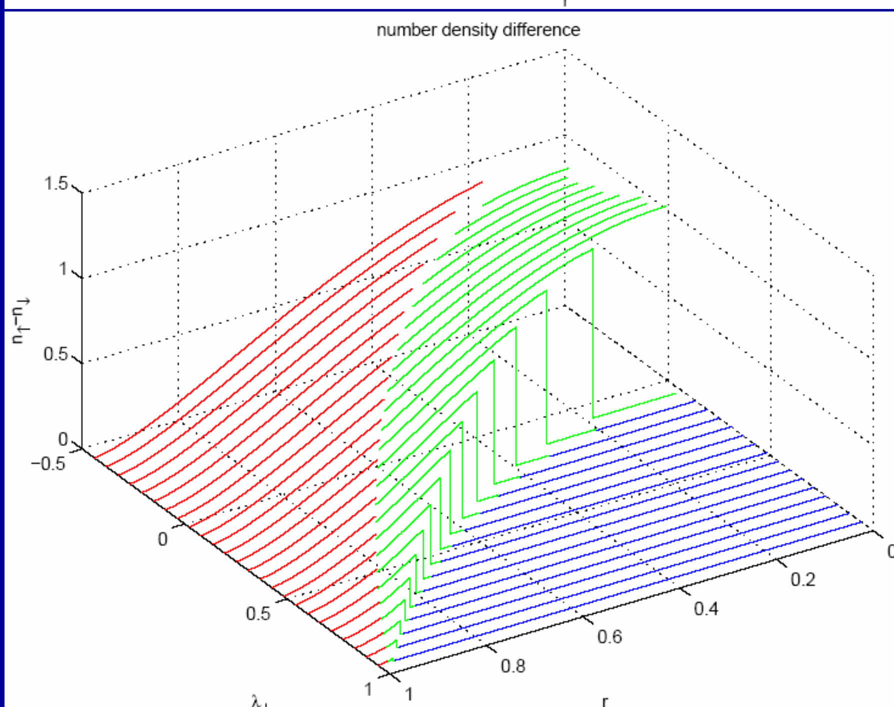
$$n_b(\vec{r}) = \beta\left[\mu_a(\vec{r})h(y(\vec{r}))\right]^{3/2} h'(y(\vec{r}))$$









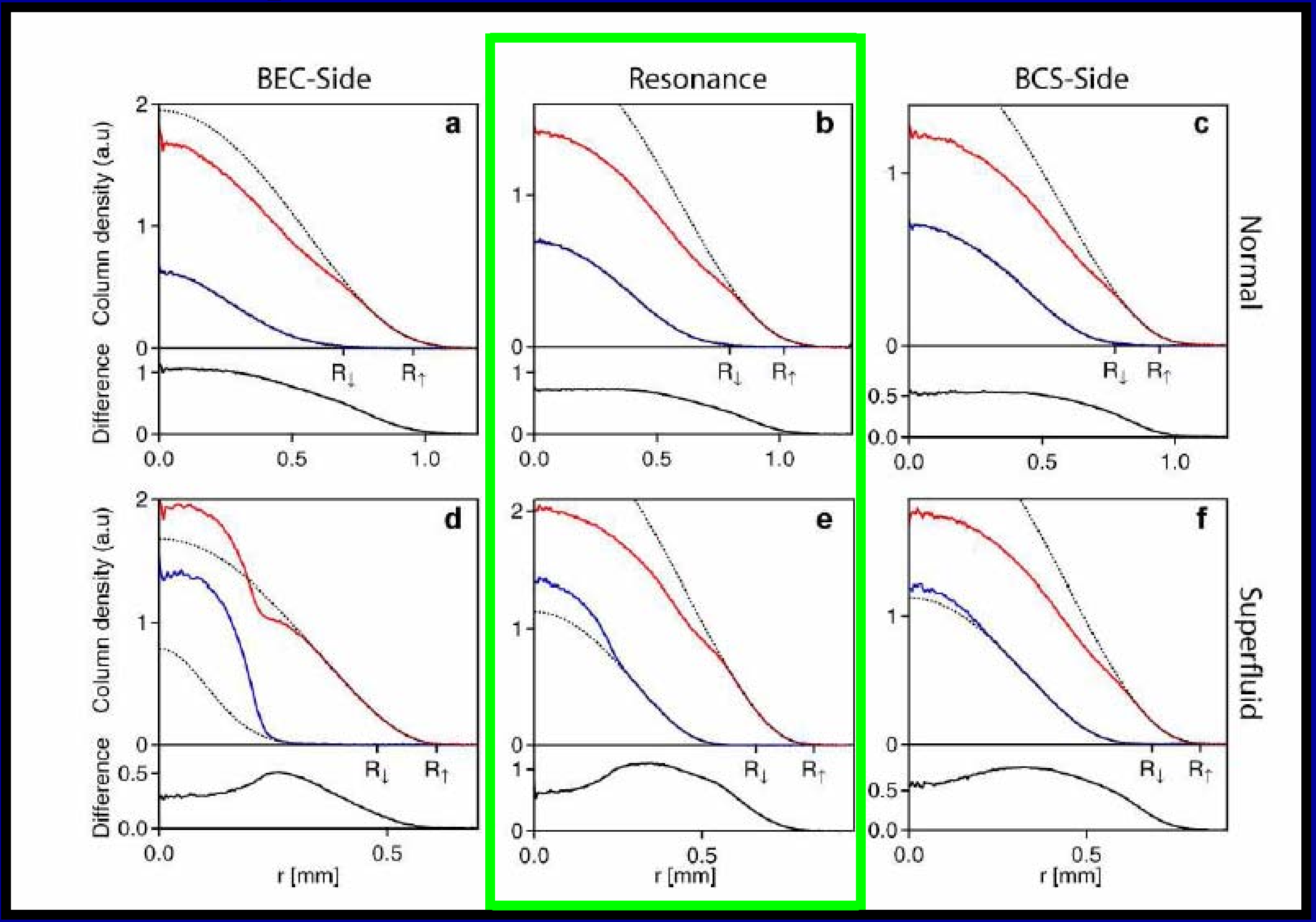


- **blue - P = 0 region**
- **green - 0 < P < 1 region**
- **red - P= 1 region**

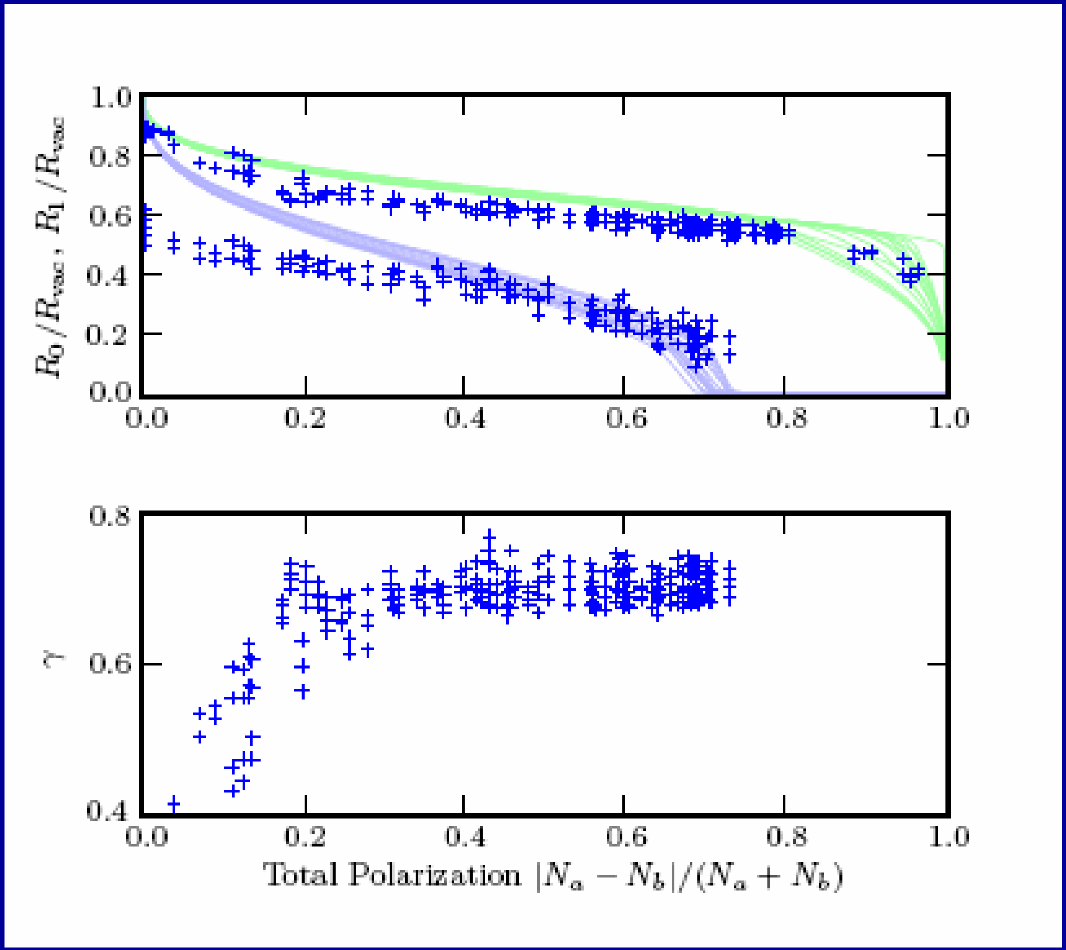
Column densities (experiment)

Normal

Superfluid



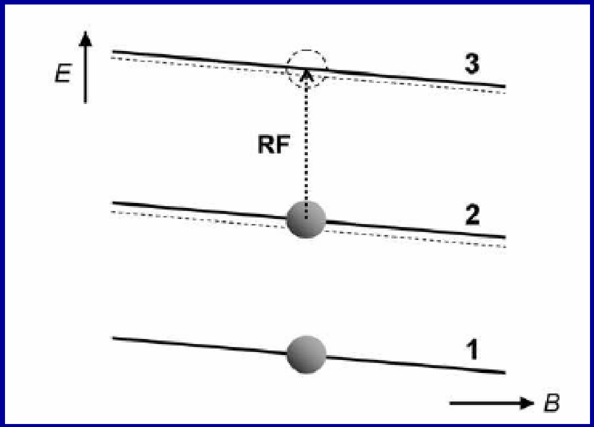
Zweirlein *et al.* cond-mat/0605258



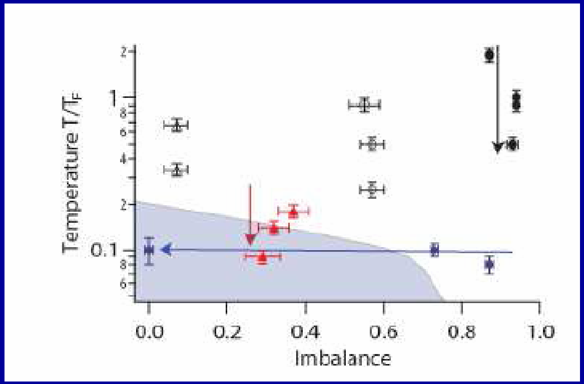
$$\gamma = \frac{y_1 - 1}{y_0 - 1} = \frac{R_0^2 - R_{vac}^2}{R_1^2 - R_{vac}^2} \approx 0.70(5)$$

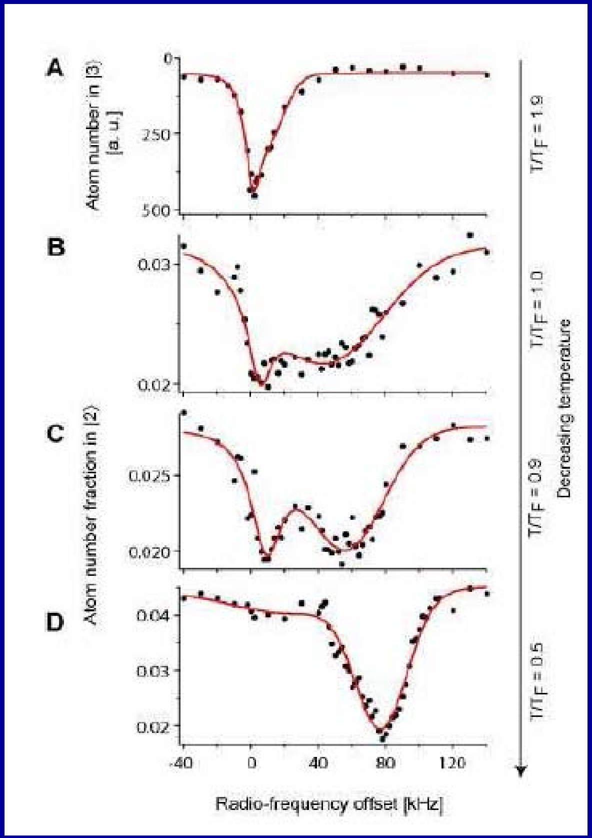
Experimental data from Zwierlein *et al.* cond-mat/0605258

Rf -spectroscopy



Grimm, cond-mat/0703091





Schunck et al, cond-mat/0702066

The unitary Fermi gases (at resonance and off resonance) have an extremely rich structure and a very rich phase diagram

The tunability of the interaction is a non-paralleled feature of these systems

Theoretically these systems can be described essentially exactly and they present an extraordinary opportunity to test lots of many-body techniques and develop new ones

It can have a major impact on other fields

One can simulate both theoretically and experimentally lots of systems encountered in condensed matter physics, nuclear physics and astrophysics