On Superfluid Properties of Asymmetric Dilute Fermi Systems

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Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

- ✓ Dilute atomic Fermi gases — $T_c \approx 10^{-12} – 10^{-9}$ eV
- ✓ Liquid ^{3}He — $T_c \approx 10^{-7}$ eV
- ✓ Metals, composite materials — $T_c \approx 10^{-3} – 10^{-2}$ eV
- ✓ Nuclei, neutron stars — $T_c \approx 10^{5} – 10^{6}$ eV
- QCD color superconductivity — $T_c \approx 10^{7} – 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

Outline:

- **Induced *p*-wave superfluidity in asymmetric Fermi gases**

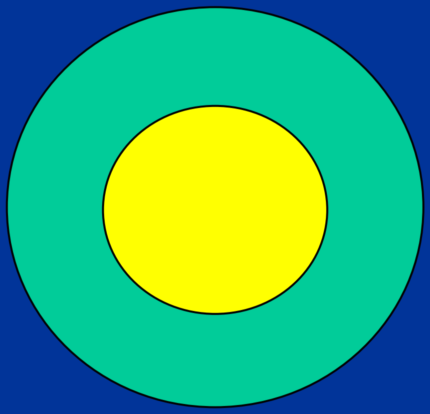
  Bulgac, Forbes, and Schwenk, cond-mat/0602274, PRL in press

- **T=0 thermodynamics in asymmetric Fermi gases at unitarity**

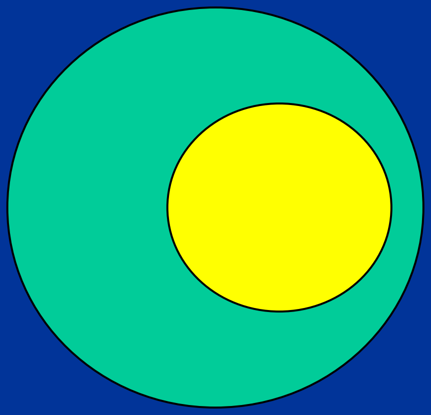
  Bulgac and Forbes, cond-mat/0606043

Induced *p*-wave superfluidity in asymmetric Fermi gases

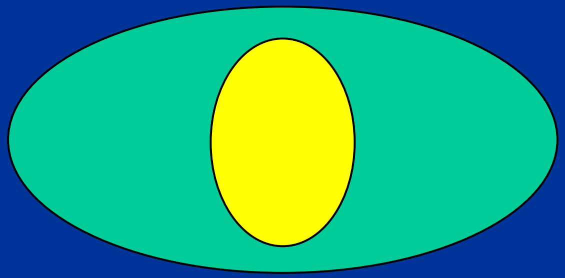
Green – spin up
Yellow – spin down



If $\left|\mu_{\uparrow} - \mu_{\downarrow}\right| < \frac{\Delta}{\sqrt{2}}$ the same solution as for $\mu_{\uparrow} = \mu_{\downarrow}$



LOFF (1964) solution
Pairing gap becomes a spatially varying function
Translational invariance broken



Muether and Sedrakian (2002)
Translational invariant solution
Rotational invariance broken

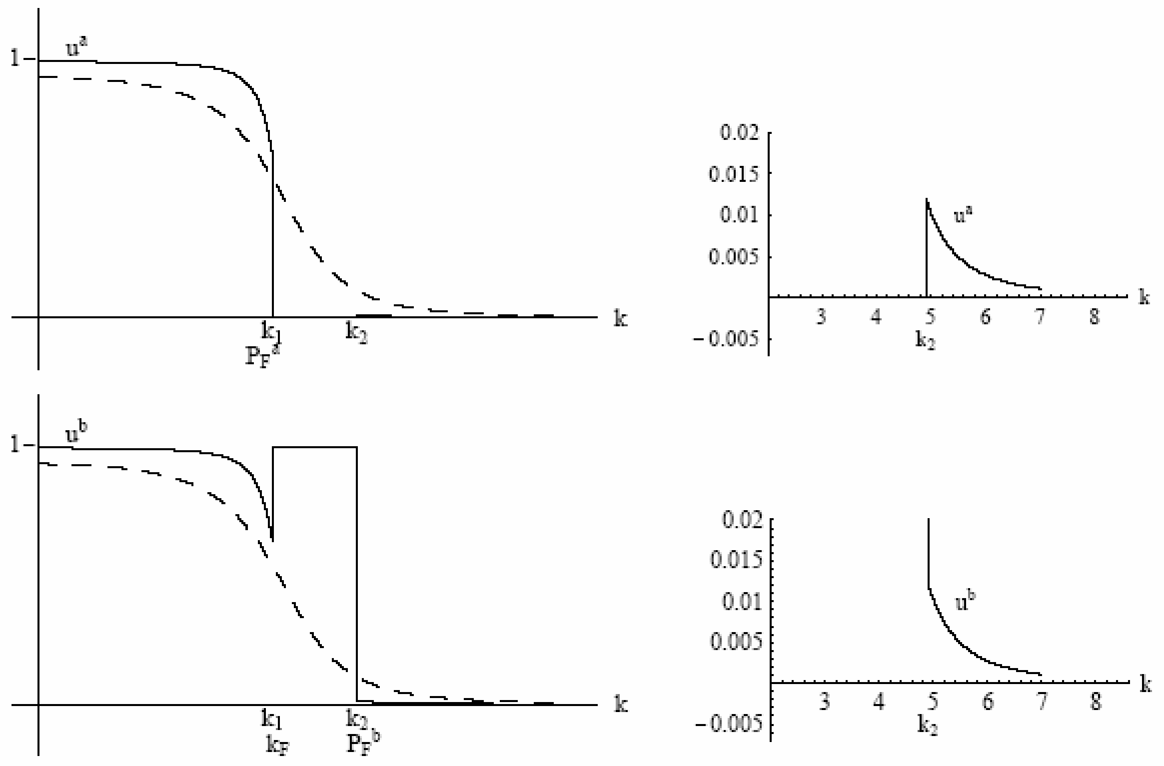
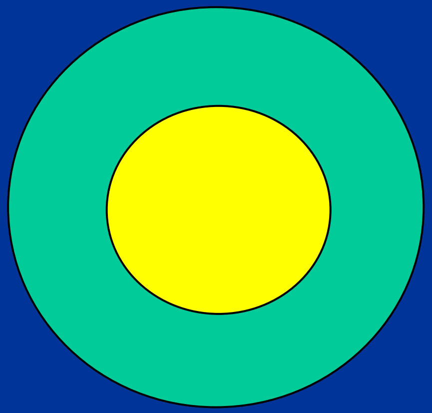
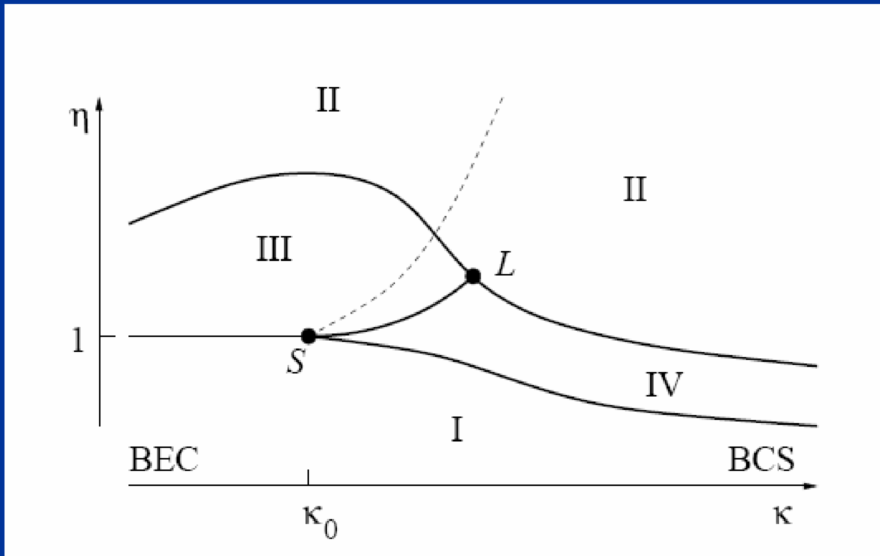


FIG. 2: *Figures on top: On the left it is shown the particle occupation number* u^a *for the specie a in the Sarma phase as compared with the BCS state (dashed curve). On right there is a amplified plot of the specie a occupation number for* $k \geq k_2$*. Figures on bottom: On left it is shown the particle occupation number* u^b *for the specie b in the Sarma Phase also compared with the BCS state (dashed curve). Again we show on right a amplified plot of the specie b occupation number for* $k \geq k_2$*. We also show the hierarchy between the momenta involved,* $P_F^a < k_1 < k_F < k_2 < P_F^b$.

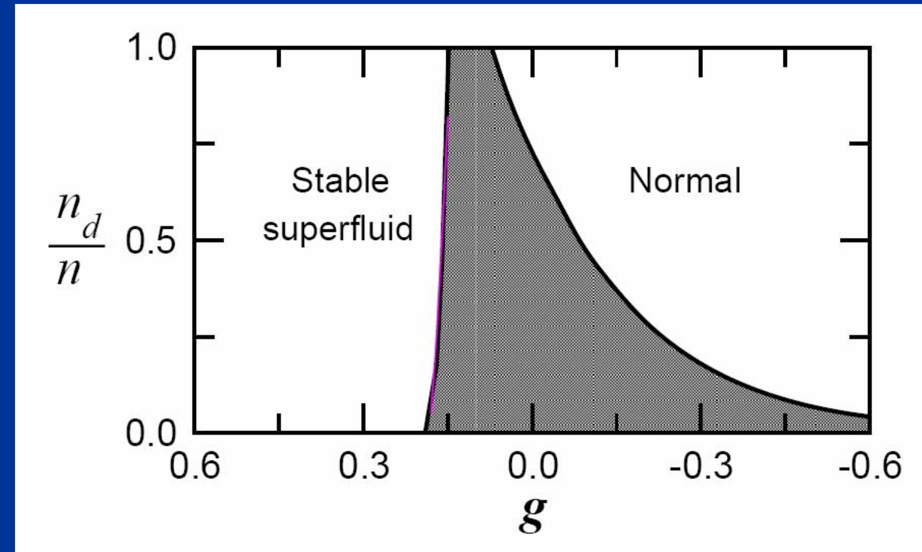


Sarma solution (1962)

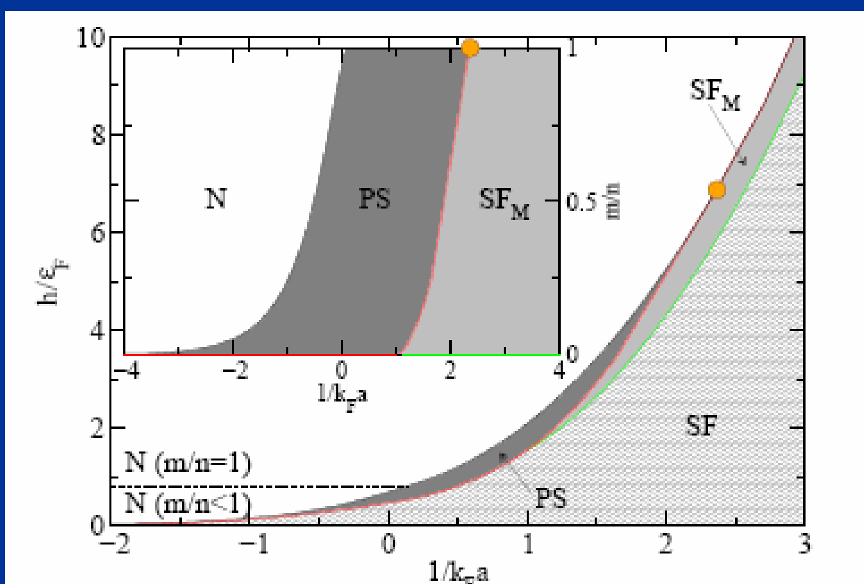
(similar to nonvanishing seniority in nuclear physics introduced by Racah in late 1940's, some people call it now breached pairing)



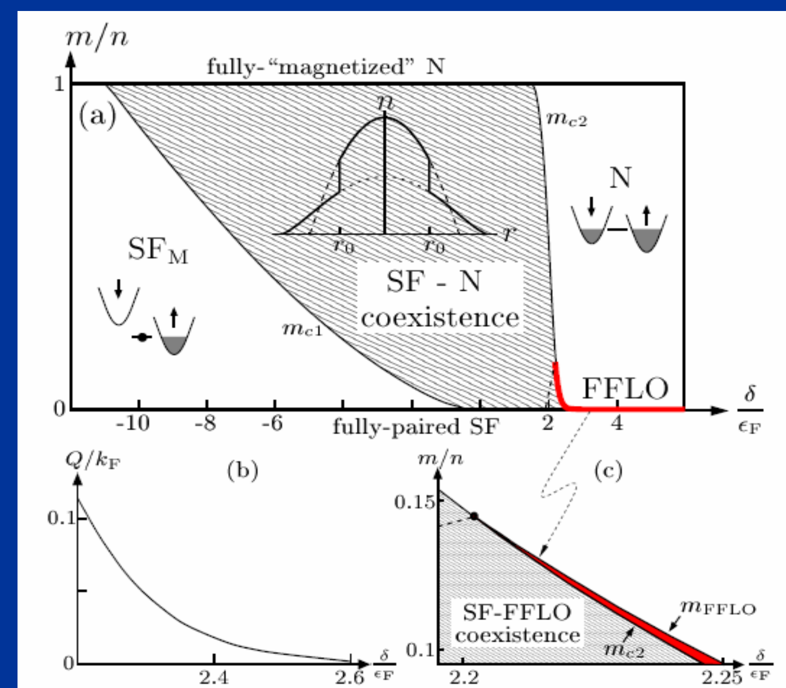
Son and Stephanov, cond-mat/0507586



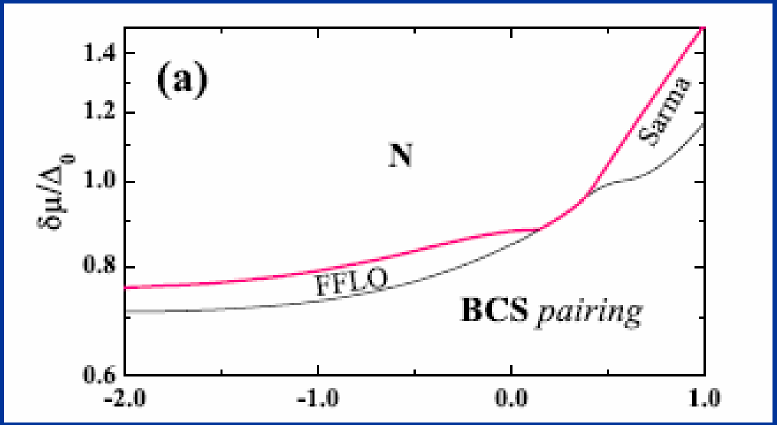
Pao, Wu, and Yip, PR B 73, 132506 (2006)



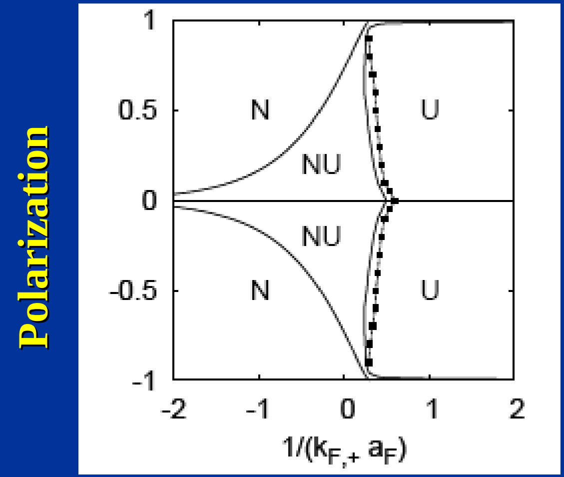
Parish, Marchetti, Lamacraft, Simons cond-mat/0605744



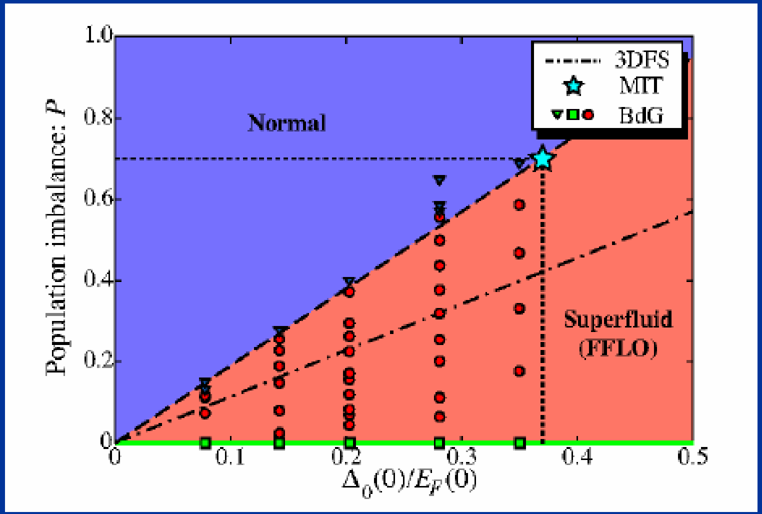
Sheeny and Radzihovsky, PRL 96, 060401(2006)



Liu, Hu, cond-mat/0606322



Iskin, Sa de Mello, cond-mat/0604184



Machida, Mizhushima, Ichioka
cond-mat/0604339

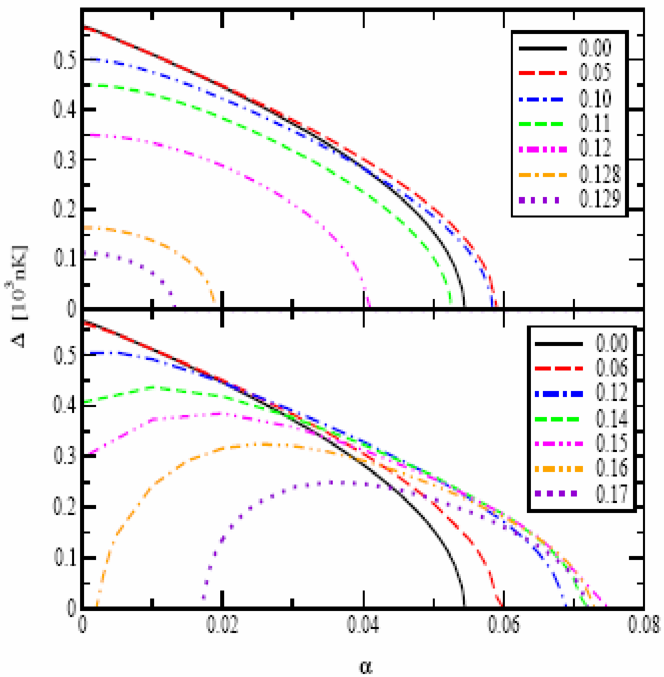


FIG. 1: (Color online) The dependence of the pairing gaps in the LOFF phase (upper panel) and the DFS phase (lower panel) on the asymmetry parameter for several values of the the total momentum P/k_F and deformation parameter $\delta\epsilon$ which are indicated in the panels.

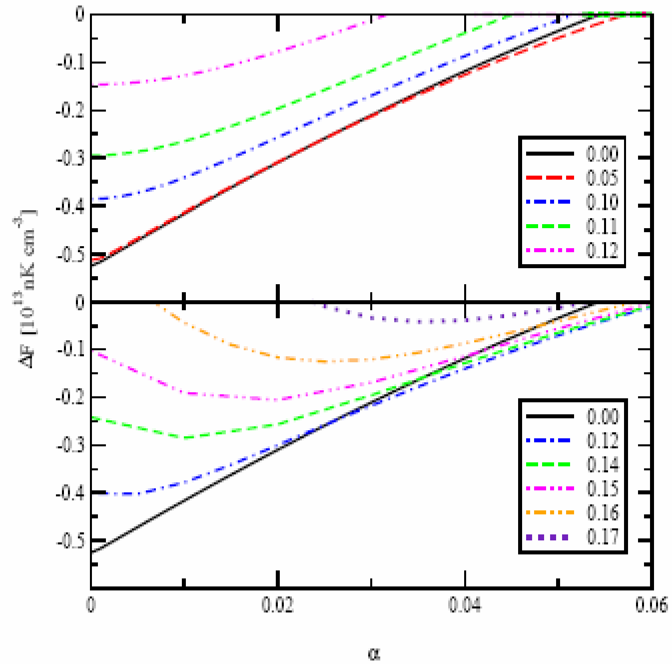
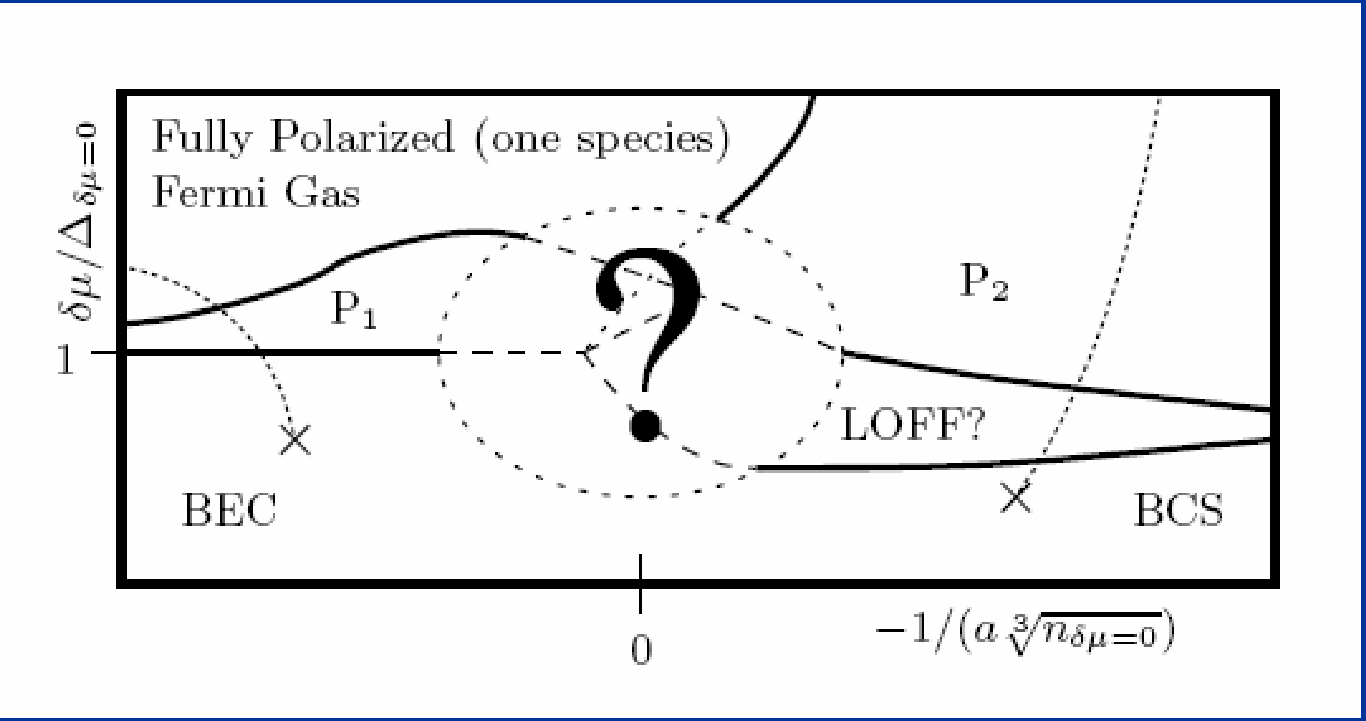


FIG. 4: (Color online) The dependence of the free energy of the LOFF (upper panel) and the plane-wave DFS phase (lower panel) on the asymmetry parameter for several values of the deformation parameter $\delta\epsilon$ and the total momentum P/k_F which are indicated in the panels.

Sedrakian, Mur-Petit, Polls, Muether
Phys. Rev. A 72, 013613 (2005)

What we suggest!



Bulgac, Forbes, Schwenk

BEC regime

- all minority (spin-down) fermions form dimers and the dimers organize themselves in a Bose superfluid

- the leftover/un-paired majority (spin-up) fermions will form a Fermi sea

- the leftover spin-up fermions and the dimers coexist and, similarly to the electrons in a solid, the leftover spin-up fermions will experience an attraction due to exchange of Bogoliubov phonons of the Bose superfluid

Mean-field energy density

$$\frac{E}{V} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} n_f + U_{fb} n_f n_b + \frac{1}{2} U_{bb} n_b^2 + \varepsilon_2 n_b$$

$$U_{fb} = \frac{\pi\hbar^2 a}{m}\alpha_{fb} \approx \frac{\pi\hbar^2 a}{m} 3.6, \qquad U_{bb} = \frac{\pi\hbar^2 a}{m}\alpha_{bb} \approx \frac{\pi\hbar^2 a}{m} 1.2$$

$$n_b = \frac{n_\downarrow}{2}, \qquad n_f = n_\uparrow - n_\downarrow = \frac{k_F^3}{6\pi^2}, \qquad \varepsilon_2 = -\frac{\hbar^2}{ma^2}$$

Induced interaction between the un-paired spin-up fermions (Bardeen, Baym, Pines, 1967)

$$U_{ind}(q_0, \vec{q}) = U_{fb}^2 \frac{2n_b \varepsilon_{\vec{q}}}{q_0^2 - \varepsilon_{\vec{q}}(\varepsilon_{\vec{q}} + 2n_b U_{bb})}, \qquad \varepsilon_{\vec{q}} = \frac{q^2}{2m_b}, \qquad m_b = 2m$$

$$U_{ind}(0, \vec{q}) = -\frac{\pi\hbar^2 a}{m}\frac{\alpha_{fb}^2}{\alpha_{bb}}\frac{1}{1 + \dfrac{q^2}{2m_b^2 c^2}}, \qquad m_b c^2 = U_{bb} n_b = \frac{2\pi\hbar^2 a \alpha_{bb} n_b}{m_b}$$

p-wave induced interaction

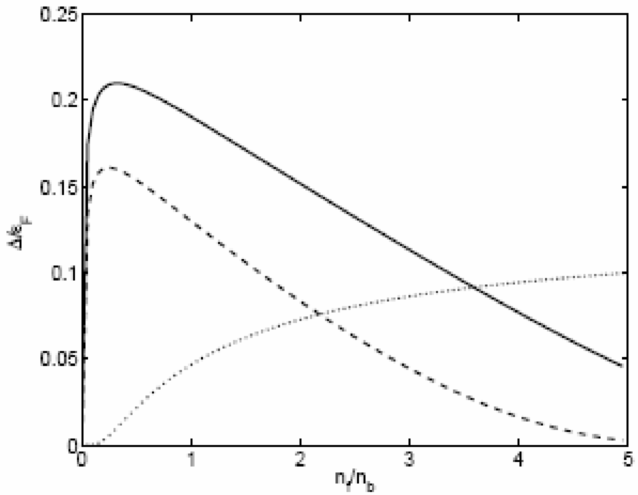
$$U_P = \int_{-1}^{1} \frac{d\cos\theta}{2} \cos\theta U_{ind}(0, \vec{p}_1 - \vec{p}_2) = -\frac{\pi\hbar^2 a}{m} \frac{\alpha_{fb}^2}{\alpha_{bb}} R_1\left(\frac{\hbar k_F}{m_b c}\right)$$

$$R_1(x) = \frac{2}{x^2}\left[\left(\frac{1}{x^2} + \frac{1}{2}\right)\ln\left(1 + x^2\right) - 1\right], \qquad x^2 = \left(\frac{\hbar k_F}{m_b c}\right)^2 = \frac{3\pi}{\alpha_{bb} k_F a} \frac{n_f}{n_b}$$

$$R_1(x)\begin{cases} \dfrac{x^2}{6}, & \text{if} \quad x \ll 1 \\ \dfrac{\ln x^2}{x^2}, & \text{if} \quad x \gg 1 \end{cases}$$

Pairing gap

$$\Delta_p \sim \varepsilon_F \exp\left(\frac{1}{N_F U_p}\right), \qquad N_F = \frac{m k_F}{2\pi^2\hbar^2}$$



p-wave gap

FIG. 1: The ratio Δ/ε_F ($\varepsilon_F = \hbar^2 k_F^2/2m$) as a function of n_f/n_b, for a fixed boson number density $n_b = 10^{13}\ cm^{-3}$ and $n_b a^3 = 0.064$ (solid line) and $n_b a^3 = 0.037$ (dashed line) respectively. The dots show the value of the gap in the case of p-wave paring for $n_b a^3 = 0.064$.

Bulgac, Bedaque, Fonseca, cond-mat/030602

$$\Delta_p \sim \varepsilon_F \exp\left(-0.44\frac{n_b}{n_f}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \ll k_F a \ll 1$$

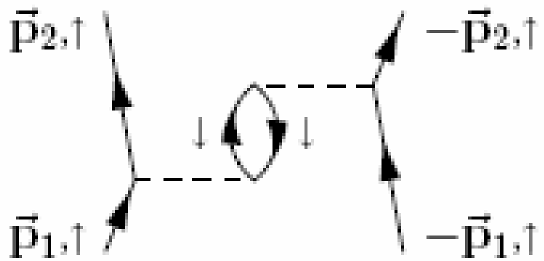
$$\Delta_p \sim \varepsilon_F \exp\left(-\frac{6\pi^2}{\alpha_{fb}^2 (k_F a)^2 \ln\left(x^2\right)}\frac{n_b}{n_f}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \gg k_F a, \qquad x^2 = \left(\frac{\hbar k_F}{m_b c}\right)^2$$

$$\Delta_p\Big|_{\max} \sim \varepsilon_F \exp\left(-\frac{5.6}{k_F a}\right), \qquad \text{if} \qquad \frac{n_f}{n_b} \approx 0.44 k_F a \ll 1$$

!!!

BCS regime:

$$U_{\mathrm{ind}}^{\uparrow\uparrow}(0, \vec{p}_1 - \vec{p}_2) =$$



$$= -N_{\mathrm{F}}^{\downarrow} \left(\frac{4\pi a \hbar^2}{m} \right)^2 L\left(|\vec{p}_1 - \vec{p}_2| / (2\hbar k_{\mathrm{F}}^{\downarrow}) \right) .$$

The same mechanism works for the minority/spin-down component

$$\Delta_p^{\uparrow} \sim \varepsilon_F^{\uparrow} \exp\left(\frac{1}{N_F^{\uparrow} U_p^{\uparrow\uparrow}}\right) = \varepsilon_F^{\uparrow} \exp\left(-\frac{\pi^2}{4 k_F^{\uparrow} k_F^{\downarrow} L_p\left(\frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right)}\right)$$

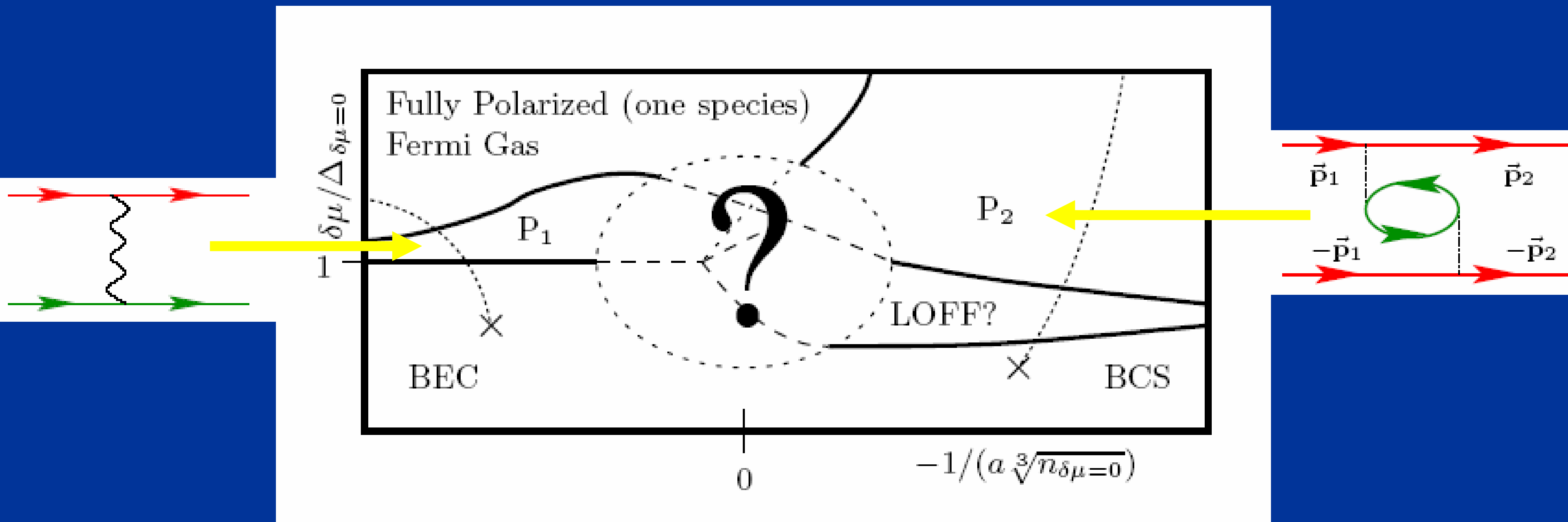
$$L_p(z) = \frac{5z^2 - 2}{15z^4} \ln\left|1 - z^2\right| - \frac{z^2 + 5}{30z} \ln\left|\frac{1 - z}{1 + z}\right| - \frac{z^2 + 2}{15z^2}$$

$$\Delta_p^{\uparrow}\Big|_{\max} \sim \varepsilon_F^{\uparrow} \exp\left(-\frac{\pi^2}{0.11\left(2k_F^{\uparrow} a\right)^2}\right), \qquad \text{for } k_F^{\downarrow} \approx 0.77 k_F^{\uparrow} \text{ and fixed } k_F^{\downarrow}$$

$$\left.\begin{aligned} \Delta_p^{\uparrow} &\sim \varepsilon_F^{\uparrow} \exp\left(-\frac{3\pi^2}{2\left(2k_F^{\uparrow} a\right)^2 \ln\left(\frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right)} \frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right) \\ \Delta_p^{\downarrow} &\sim \varepsilon_F^{\downarrow} \exp\left(-\frac{18\pi^2}{\left(2k_F^{\downarrow} a\right)^2} \frac{k_F^{\uparrow}}{k_F^{\downarrow}}\right) \end{aligned}\right\} \quad \text{for } k_F^{\uparrow} \gg k_F^{\downarrow}$$

To summarize:

- **at weak coupling (a<0) the gaps are smaller than s-wave gap, and this mechanism does not destabilize LOFF**
- **may became large at unitarity**



In a trap:

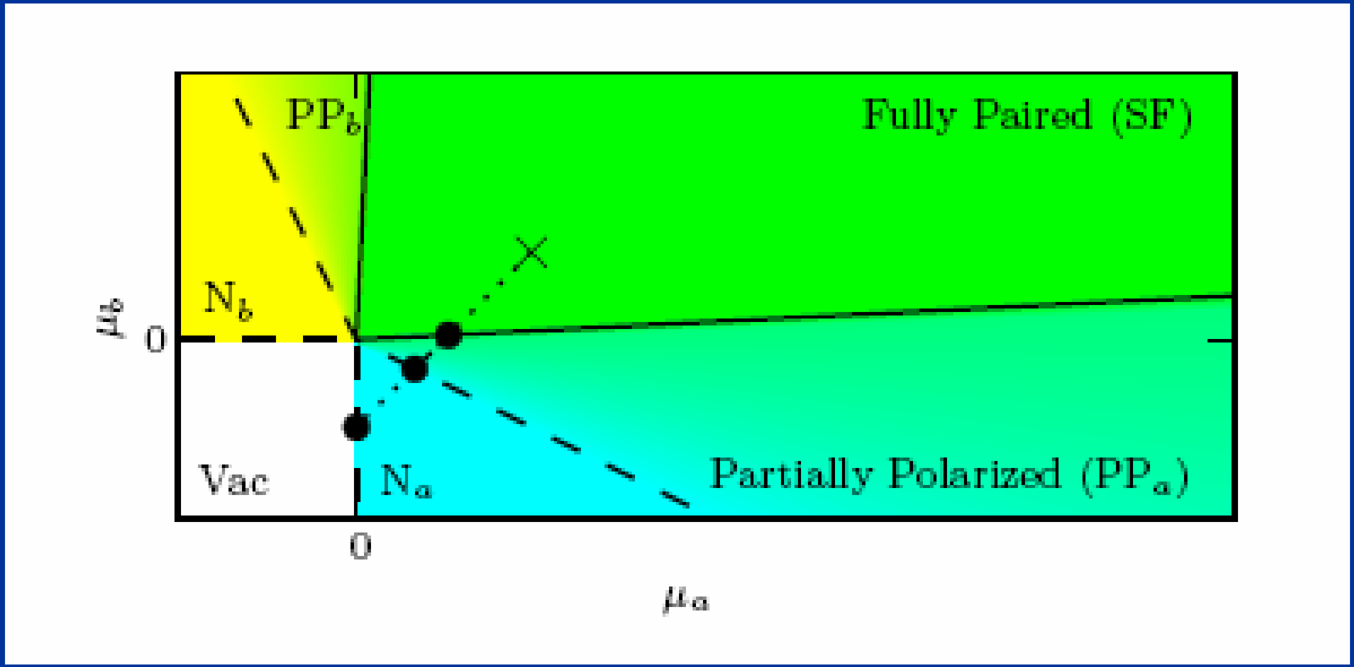
$$\mu_{\uparrow,\downarrow}(\vec{r}) = \lambda_{\uparrow,\downarrow} - V(\vec{r})$$

$$\delta\mu = \frac{\lambda_\uparrow - \lambda_\downarrow}{2}$$

T=0 thermodynamics in asymmetric Fermi gases at unitarity

What we think is going on:

At unitarity the equation of state of a two-component fermion system is subject to rather tight theoretical constraints, which lead to well defined predictions for the spatial density profiles in traps and the grand canonical phase diagram is:



In the grand canonical ensemble there are only two dimensionfull quantities

We use both micro-canonical and grand canonical ensembles

$$x = \frac{n_b}{n_a} \le 1, \qquad y = \frac{\mu_b}{\mu_a} \le 1$$

$$E(n_a, n_b) = \frac{3}{5}\alpha\left[n_a g(x)\right]^{5/3}$$

$$P(\mu_a, \mu_b) = \frac{2}{5}\beta\left[\mu_a h(y)\right]^{5/2} = \mu_a n_a + \mu_b n_b - E(n_a, n_b) = \frac{2}{3}E(n_a, n_b)$$

$$y = \frac{g'(x)}{g(x) - xg'(x)}, \qquad h(y) = \frac{1}{g(x) - xg'(x)}$$

$$x = \frac{h'(y)}{h(y) - yh'(y)}, \qquad g(x) = \frac{1}{h(y) - yh'(y)}$$

The functions g(x) and h(y) determine fully the thermodynamic properties and only a few details are relevant

$$n_a = \frac{\partial P}{\partial \mu_a} = \beta \left[\mu_a h(y) \right]^{3/2} \left[h(y) - y h'(y) \right]$$

$$n_b = \frac{\partial P}{\partial \mu_b} = \beta \left[\mu_a h(y) \right]^{3/2} h'(y)$$

$$\mu_a = \frac{\partial E}{\partial n_a} = \alpha \left[n_a g(x) \right]^{2/3} \left[g(x) - x g'(x) \right]$$

$$\mu_b = \frac{\partial E}{\partial n_b} = \alpha \left[n_a g(x) \right]^{2/3} g'(x)$$

Both g(x) and h(y) are convex functions of their argument.

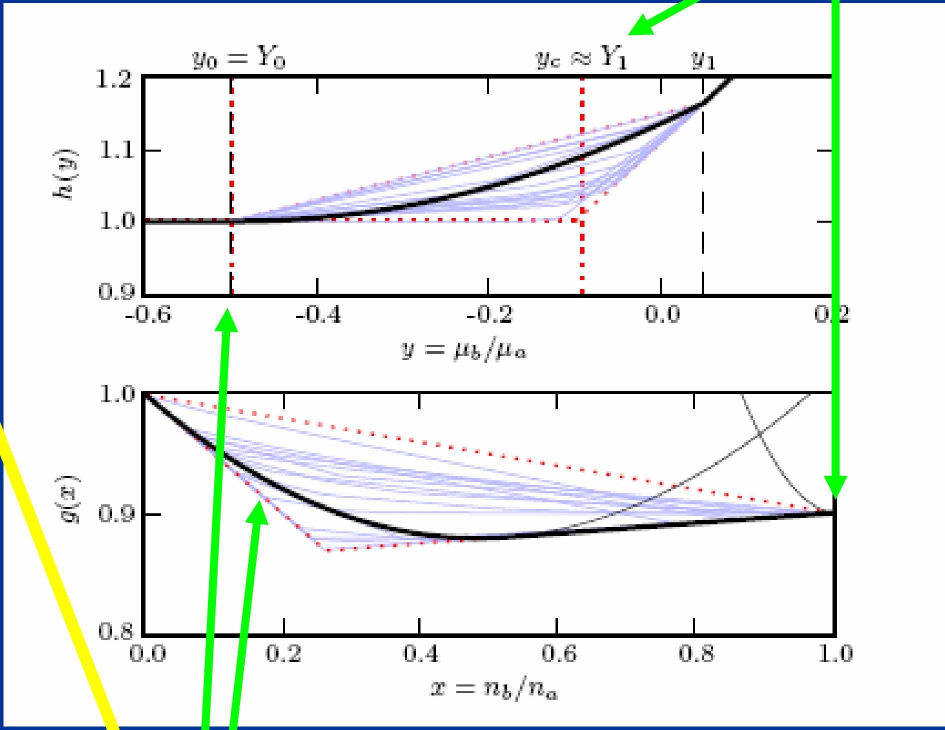
Non-trivial regions exist!

Bounds given by GFMC

$$h(y) = \begin{cases} 1 & \text{if} \quad y \le y_0 \\ \dfrac{1+y}{(2\xi)^{3/5}} & \text{if} \quad y \in [y_1, 1] \end{cases}$$

$$h''(y) \ge 0$$

$$y_0 \le Y_0 < y_c < Y_1 \le 1, \qquad y_c = (2\xi)^{3/5} - 1$$



$$g(0) = 1, \qquad g(x) = (2\xi)^{3/5}$$

$$g''(x) \ge 0$$

$$g'(0) \le Y_0 \quad \text{and} \quad g'(1) \in \left[\frac{g(1)}{1+Y_1^{-1}}, \frac{g(1)}{2}\right]$$

Bounds from the energy required to add a single spin-down particle to a fully polarized Fermi sea of spin-up particles

Estimate of the energy required to add a spin-down particle to a fully polarized Fermi Sea of spin-up particles

$$\phi(\vec{r}_0,\{\vec{r}_n\}) = \sum_n A_n \frac{\exp(-\kappa r_{0n})}{r_{0n}}$$

$$\left(-\kappa + \frac{1}{a}\right) A_n + \sum_{m\neq n} A_m \frac{\exp(-\kappa r_{mn})}{r_{mn}} = 0$$

$$\Psi(\vec{r}_0,\{\vec{r}_n\}) = \Phi_{SD}\left(\{\vec{r}_n\}\right)\phi(\vec{r}_0,\{\vec{r}_n\})$$

$$e_0 \approx -\frac{\hbar^2\kappa^2}{m} \Rightarrow \begin{cases} \dfrac{4\pi\hbar^2 n_a a}{m} & \text{if} \quad a\to 0^- \\ -\dfrac{0.7\hbar^2\left(4\pi n_a\right)^{2/3}}{m} & \text{if} \quad a\to\infty \\ -\dfrac{\hbar^2}{ma^2} & \text{if} \quad a\to 0^+ \end{cases}$$

$$\mu_b = \alpha n_a^{2/3} g'(0) = \alpha n_a^{2/3} Y_0 = e_0$$

$$Y_0 \approx -0.5 < y_c = -0.99(15)$$

Non-trivial regions exist!

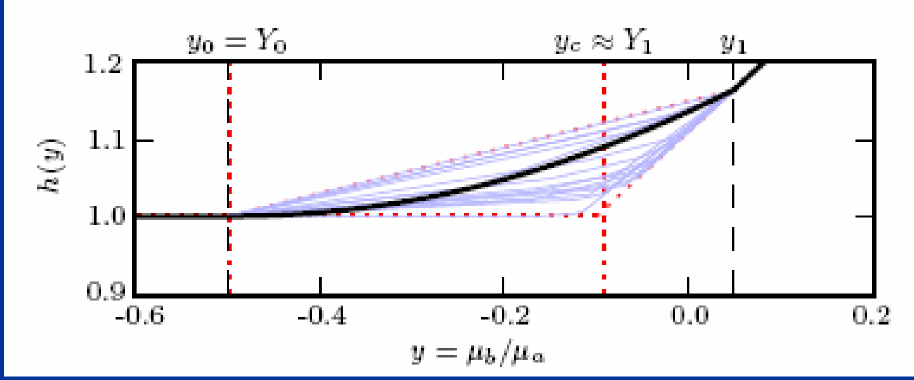
Now put the system in a trap

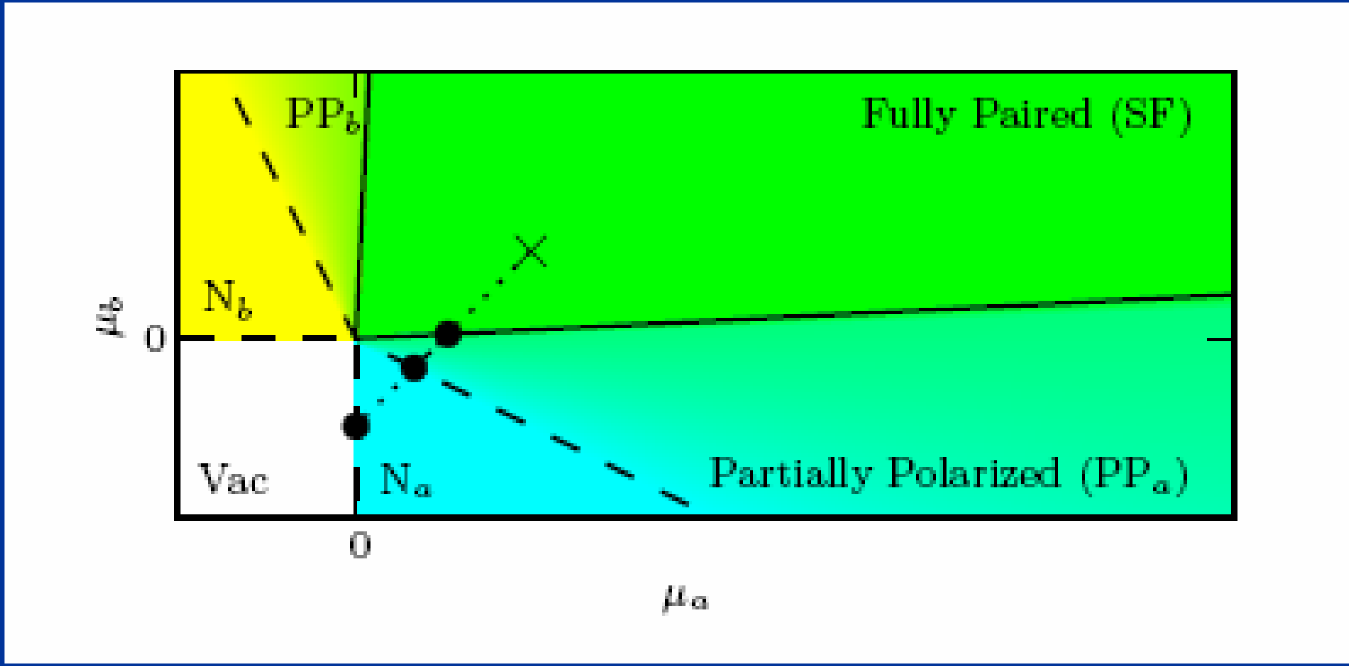
$$\mu_{a,b}(\vec{r}) = \lambda_{a,b} - V(\vec{r}), \qquad y(\vec{r}) = \frac{\mu_b(\vec{r})}{\mu_a(\vec{r})}$$

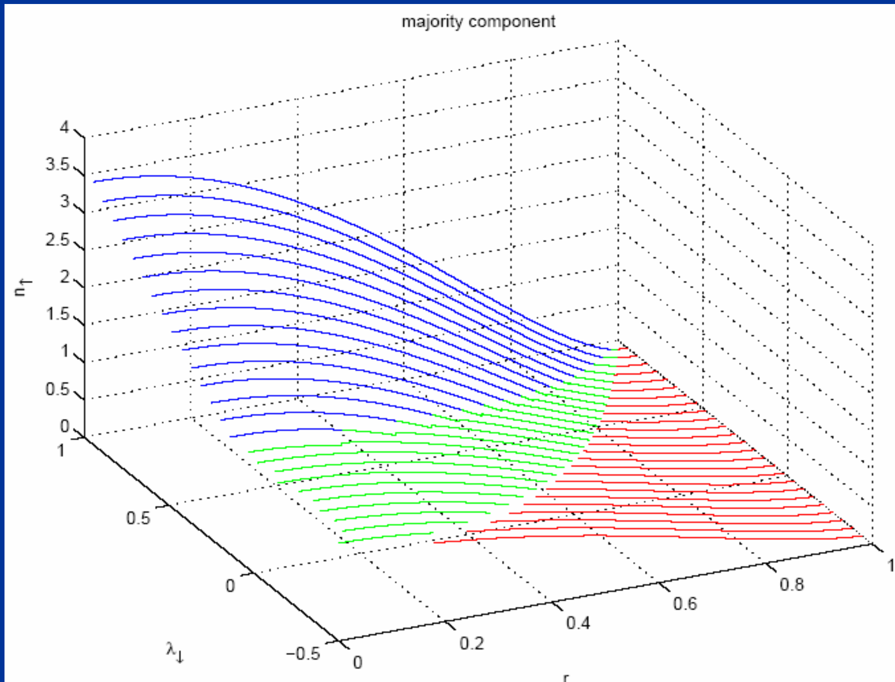
$$2\mu_- = \lambda_a - \lambda_b$$

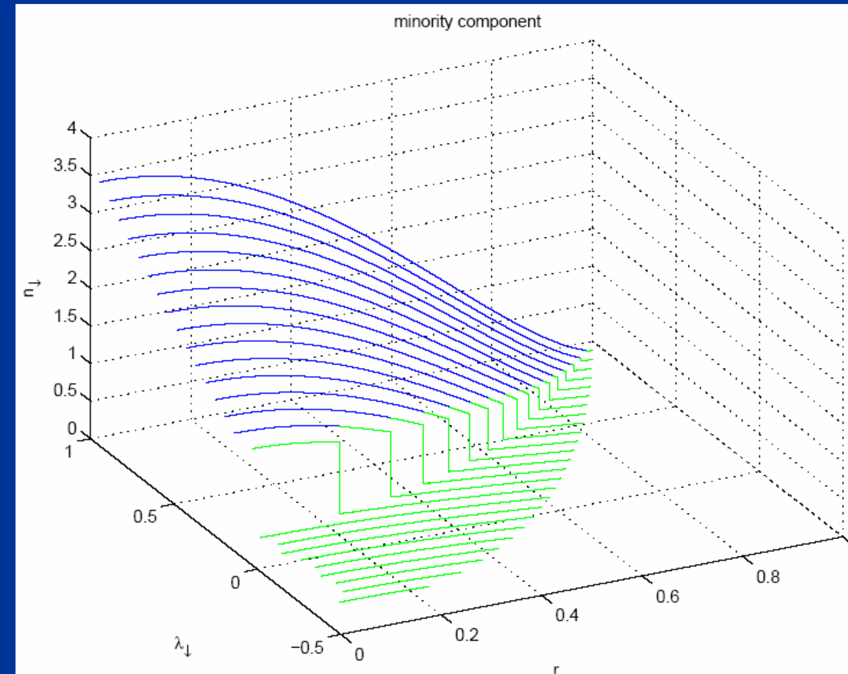
$$n_a(\vec{r}) = \beta\left[\mu_a(\vec{r})h(y(\vec{r}))\right]^{3/2}\left[h(y(\vec{r})) - y(\vec{r})h'(y(\vec{r}))\right]$$

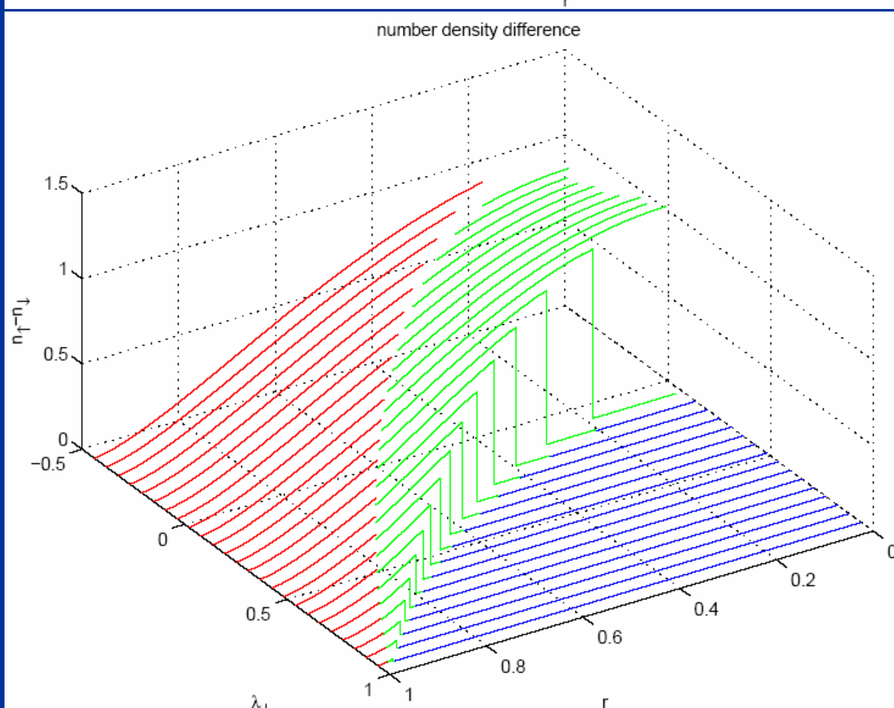
$$n_b(\vec{r}) = \beta\left[\mu_a(\vec{r})h(y(\vec{r}))\right]^{3/2} h'(y(\vec{r}))$$









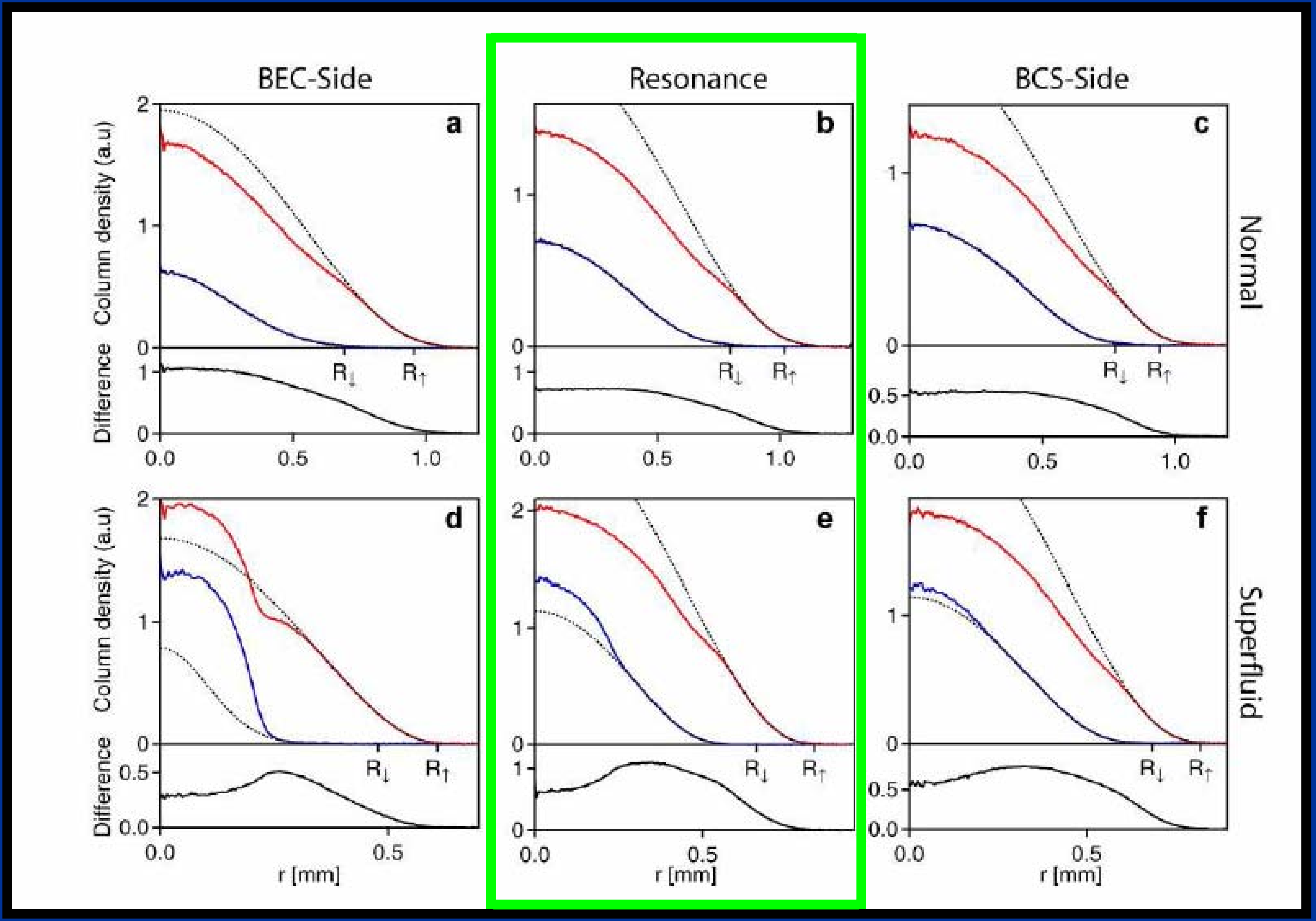


- **blue - P = 0 region**
- **green - 0 < P < 1 region**
- **red - P= 1 region**

Column densities

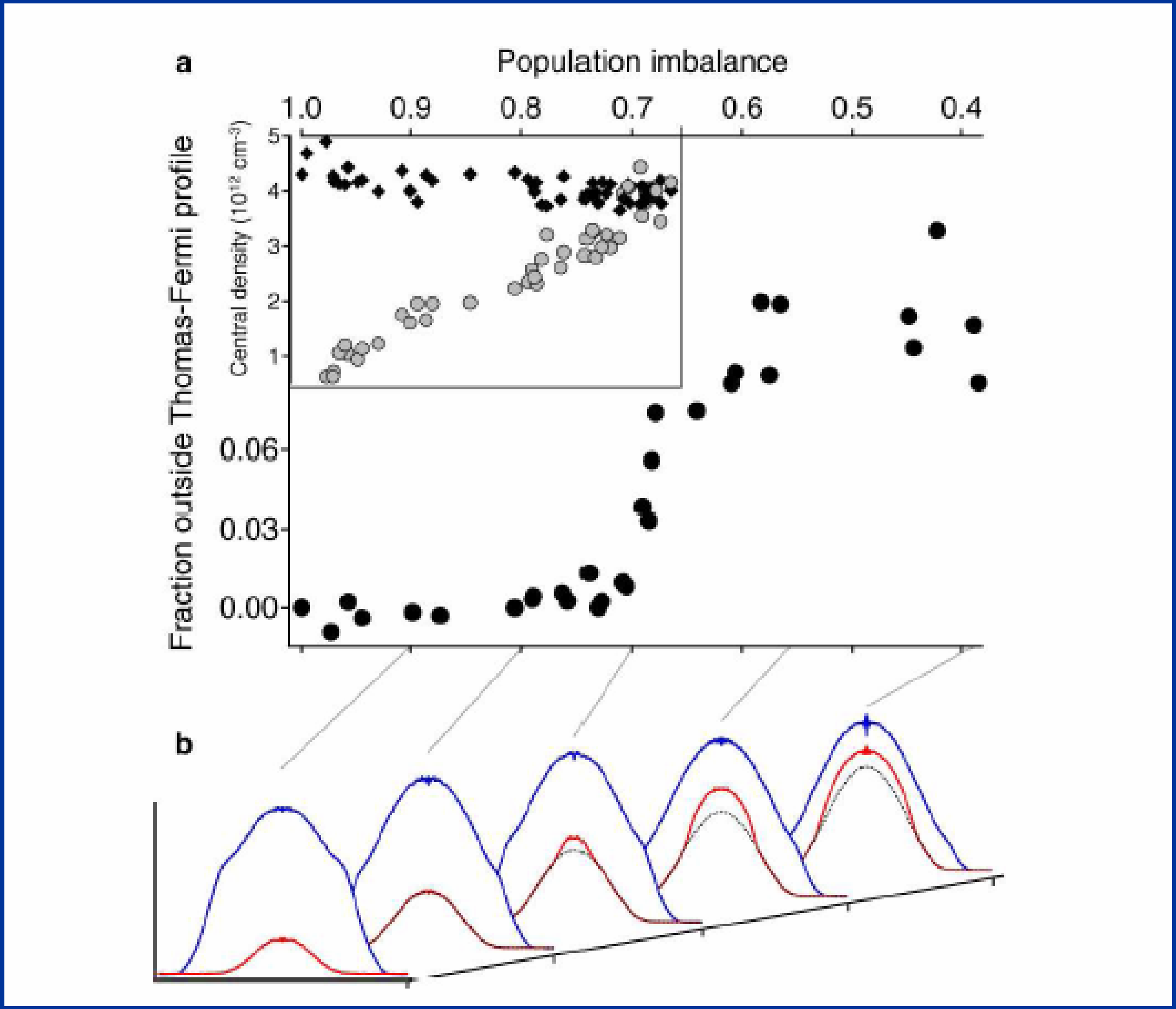
Normal

Superfluid



Zweirlein *et al.* cond-mat/0605258

Column densities



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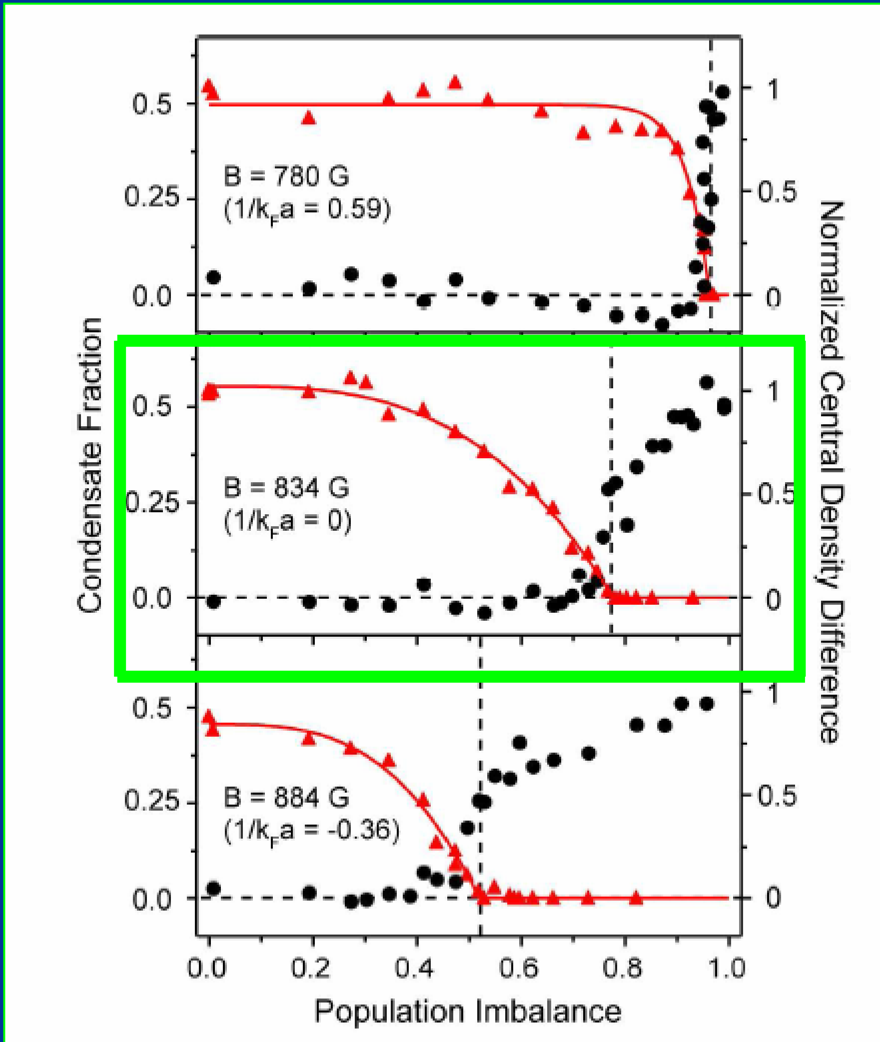
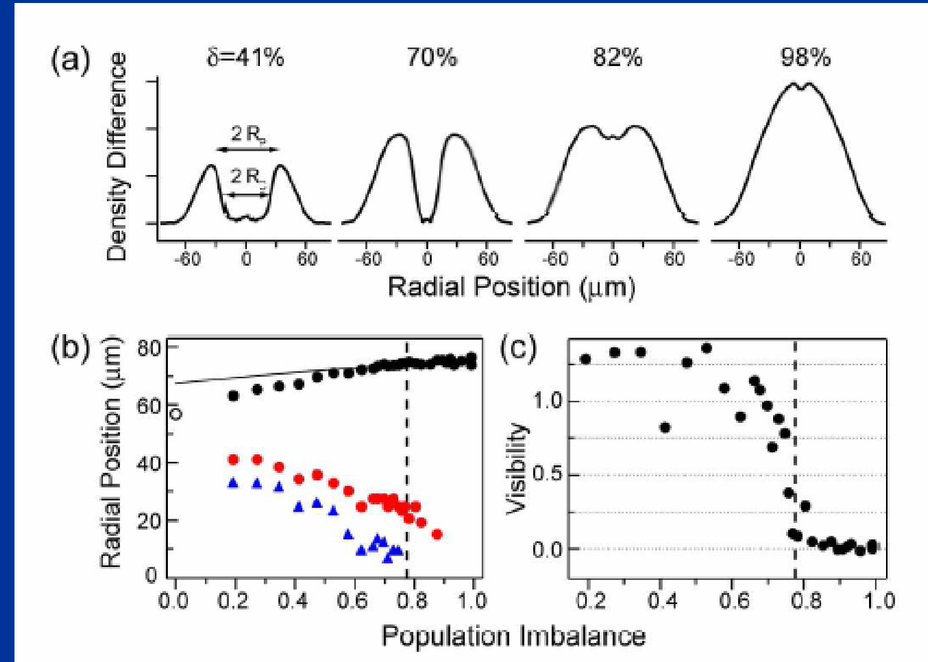
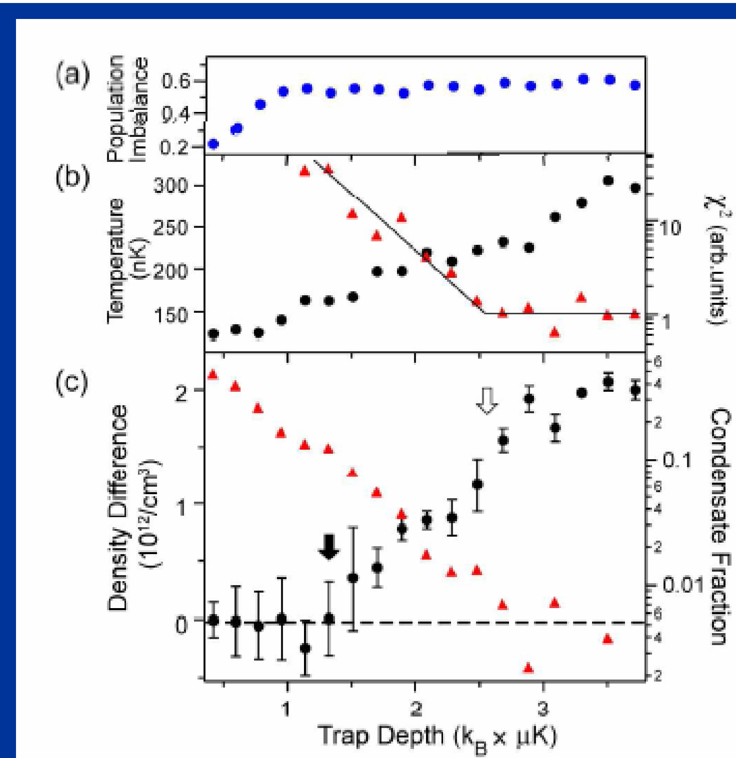
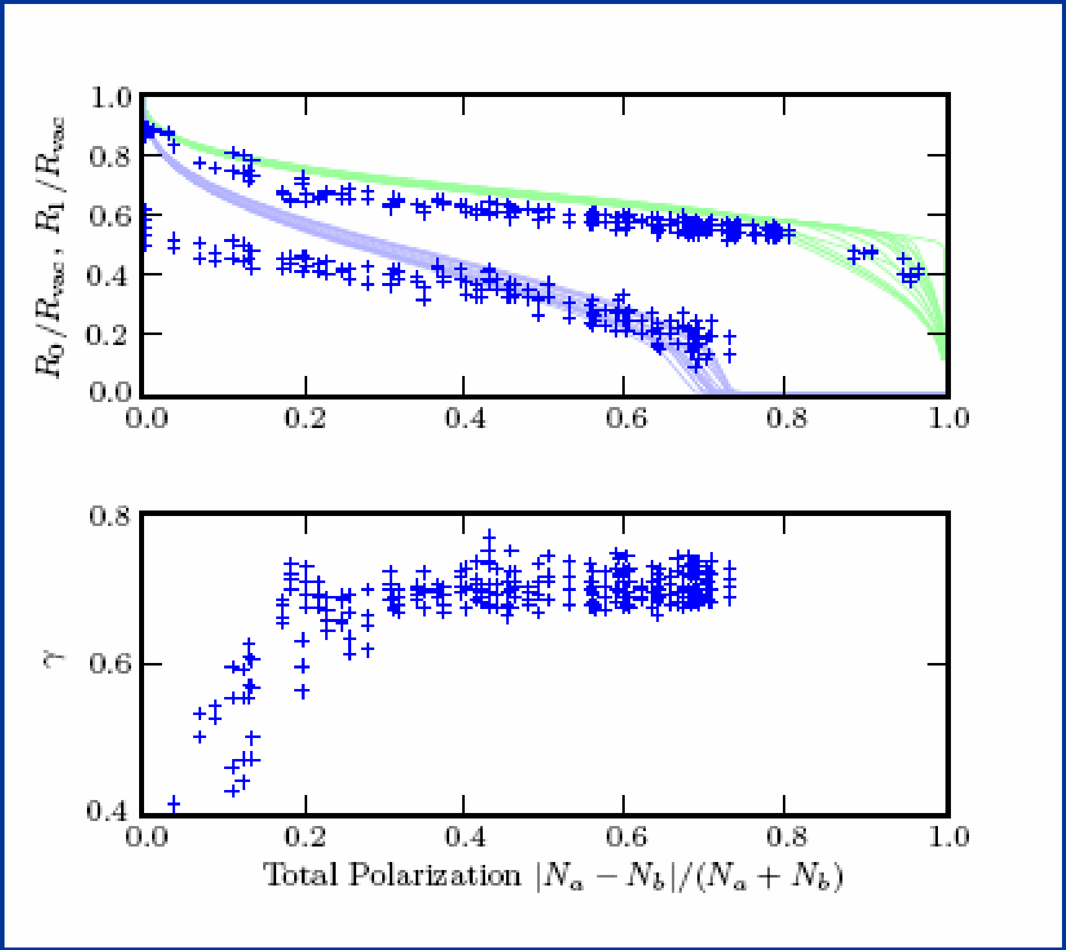






Shin *et al.* cond-mat/0606432

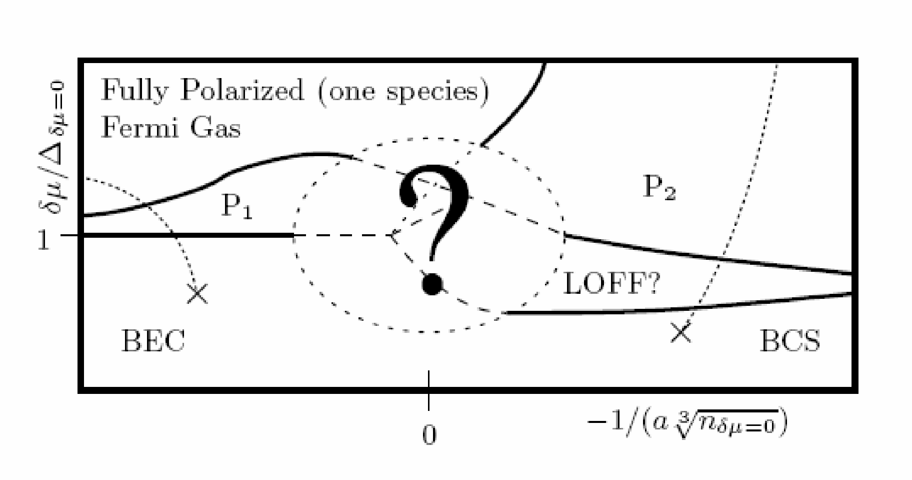


$$\gamma = \frac{y_1 - 1}{y_0 - 1} = \frac{R_0^2 - R_{vac}^2}{R_1^2 - R_{vac}^2} \approx 0.70(5)$$

Experimental data from Zwierlein *et al.* cond-mat/0605258

Main conclusions:

- At T=0 a two component fermion system is always superfluid, irrespective of the imbalance and a number of unusual phases should exists.



- At T=0 and unitarity an asymmetric Fermi gas has non-trivial partially polarized phases